LARGE LINEAR COLLIDERS*

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1. Introduction

This lecture is a status report on work we have doing at SLAC on studies of large linear colliders (LLC) with energies far beyond those attainable with either the SLC or LEP. I had intended to begin with an extensive discussion of the physics capabilities of electron colliders, but after the preceding talks on theory, and the discussion on the experimental difficulties in searches for relatively rare processes with proton colliders, I will considerably shorten my remarks on these topics. Suffice it to say that electron-positron annihilation studies have played a leading role in the last decade in establishing our "standard" view of the elementary constituents of matter and their interactions, through such things as studies of charmonium, "b"-onium; the discoveries of the tau, jets, B, D, Λ_c, F, gluons; determination of α_s and the beginnings of measurements on its evolution with energy; etc.

The power of electron-positron annihilation studies comes from the fact that in the annihilation of these point particles one obtains a well defined center-of-mass energy and all of the elementary constituents produced in the final state are produced on a more or less equal footing. In contrast, in proton-proton collisions, the collisions are between composite particles, and light elementary constituents are produced much more copiously than heavy ones. Thus, though the rate of production of D mesons, for example, is very much larger at proton machines than at electron-positron machines, the large signal to noise ratio at the electron machines has meant that the discovery and elucidation of most of the properties of these particles has been done by experiments at electron machines. While it is certain that proton machines will have an advantage in total center-of-mass energy for the forseeable future, the electron machines will, I believe, continue to have an advantage in delineating the properties of new entities.

In the next sections I will discuss energy and luminosity requirements for a LLC, the design equations for these machines, the work that we have been doing on optimization

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of parameters, and finally tell you where I think we are in this process. The work I will describe has been contributed to by M. Allen, K. Bane, K. Brown, A. Chao, D. Farkas, W. Herrmannsfeldt, G. Loew, R. Miller, R. B. Neal, R. Noble, W. K. H. Panofsky, B. Richter, L. Rifkin, D. Ritson, R. Ruth, C. Sinclair, R. Stiening, and P. Wilson. All these people are of course only working part time on this problem.

2. Energy and Luminosity Requirements

In considering the design of the large LLC's of the future, the first question one has to address is the energy that will be required. There is no precise answer to this question for electron-positron machines, just as there is no precise answer for proton-proton machines. Were there some expected sharp new threshold at energies much beyond the Z^o, we would have a number to specify. However, there is no such threshold and we can only use the same qualitative arguments that have been used in the setting of the energy of the next generation large proton machine at something between 10 and 40 TeV in the center-of-mass. We might require that any LLC have the same "effective" energy as one of these large proton machines. The question, then, becomes what is the same "effective" energy.

Ellis[1)] and Kane[2)] have discussed this question. In a proton machine the upper limit of detectability for a given process occurs when the cross section for that process falls to a value such that there are too few events to give a meaningful result. This cross section is a function of the invariant mass of the state and this effective mass limit is the energy of an equivalent electron-positron machine. Table I is from Ellis and shows the fraction of the proton-proton center-of-mass energy required in an electron-positron machine for several processes. The result of this analysis indicates very roughly that a 1.5 TeV e^+e^- machine is "equivalent" to a 10 TeV proton machine, while a 3 TeV electron-positron machine is "equivalent" to a 40 TeV proton machine. Of course this kind of analysis is very rough but it suffices to set the scale of energy required in a LLC that would be roughly equivalent to the proton machines that have been under discussion — we need an e^+e^- energy of about 2 TeV.

We also have to set the luminosity for these LLC's and Eq. (1) shows the yield (Y) in events per effective year (6 months running) as a function of S. The square of the center-of-mass energy, and R, the ratio of the cross-section to the electromagnetic muon pair cross section.

$$Y = \frac{1200}{S\,(\mathrm{TeV}^2)} \cdot \frac{\mathcal{L}\,(\mathrm{cm}^{-2}\mathrm{s}^{-1})}{10^{33}} \cdot R \qquad (1)$$

R is process dependent and ranges from about 7 for the sum over all of the "old" quarks to about 20 for W^+W^- pairs. It is evident from Eq. (1) that we need a luminosity of around 10^{33} $cm^{-2}s^{-1}$ for a practical machine.

Table I. Fraction of $p-p$ cm energy required in an e^+e^- collider to study various processes at the upper limit of detectability in the proton machine.

	e^+e^- Energy Fraction	
Process	10 TeV $p-p$	40 TeV $p-p$
Jet Pairs	0.36	0.20
L^+L^-	0.03	0.01
Z'	0.12	0.04
W'	0.29	0.08
η_T	0.20	0.08
$\tilde{g}$	0.24	0.12
$Q\bar{Q}$	0.13	0.07
Geometric Mean	0.16	0.07

3. Design Equations

I will begin this discussion by assuming that the cross section of the beam at the collision point is round. I will discuss later what happens if one drops this assumption. We first examine the effect of the beam-beam interaction. Figure 1 shows the trajectory of a particle in one beam as it passes through the other beam at the collision point. The fields in these beams are extremely strong — on the order of several megagauss — and thus strongly perturb the cross section of the beams as they pass through each other. In a single particle case, illustrated in Fig. 1, we can define a disruption parameter (D) such that

$$D = \frac{\sigma_z}{F} = \frac{r_e \sigma_z N}{\gamma \sigma_{r_0}^2} \tag{2}$$

where σ_z and F are defined in the figure, r_e is the classical electron radius, N is the number of particles in the bunch, γ is the energy in rest mass units and σ_{r_0} is the radius of the bunch. This beam-beam interaction tends to increase the luminosity in the collision region for large values of D. As D approaches 1, a strong mutual pinch of the two colliding bunches occurs and the average value of the beam radius during the collision time is reduced. As the value of D approaches 2, a kind of saturation sets in wherein the bunches oscillate in cross section during the collision period and the time average value of the bunch radii no longer decreases. Since the luminosity in this collision depends on $\langle\sigma_r^{-2}\rangle$, the luminosity increases over that which would obtain in the absence of this pinch effect in the fashion shown in Fig. 2. The luminosity is given by

$$\mathcal{L} = \frac{N^2 f\langle\sigma_r^{-2}\rangle}{4\pi} \equiv \frac{N^2 fH(D)}{4\pi\sigma_{r_0}^2} \qquad (3)$$

where f is the collision frequency, H is the enhancement factor shown in Fig. 2, and σ_{r_0} is the unperturbed radius of the colliding bunches.

As I mentioned earlier, the macroscopic fields in the collision region are very large and this gives rise to a new constraint in the design of electron colliders — synchrotron radiation from the collision region that can markedly increase the energy spread in the beams. If one treats this synchrotron radiation (Beamstrahlung) in the classical fashion, one can easily show that the mean energy loss from synchrotron radiation in the collision region is proportional to the square of the beam energy times the square of the macroscopic fields. In the case of a round beam the field is proportional to σ_r^{-1} and the fractional energy spread coming from synchrotron radiation in this classical approximation is proportional to the beam energy times the luminosity.

I have used the word "classical" several times in the description above. I have done so for this approximation is incorrect for the beam parameters coming out of the optimization described below. The problem is that the critical energy in the synchrotron radiation is on the order of the beam energy itself and so the standard synchrotron radiation equations do not apply. We are still investigating this problem and do not really understand yet how to properly handle synchrotron radiation in the optimization procedure. In what follows synchrotron radiation effects are treated in classical approximation and the optimization is carried out as if this were the correct way to proceed. Be warned! We do not yet know whether how proper treatment will markedly change the optimization conditions.

A summary of the design equations is given below.

$$\epsilon_n \equiv \gamma\sigma_x\sigma_x' \qquad (4)$$

$$\mathcal{L} = \frac{\gamma N^2 fH(D)}{4\pi\,\epsilon_n\,\beta^*} = \frac{3.5\times 10^{31}\,P(\mathrm{MW})D\,H(D)}{\sigma_z(\mathrm{mm})} \qquad (5)$$

$$D = \frac{r_e \sigma_z N}{\epsilon_n \beta^*} \tag{6}$$

$$\frac{\sigma_{E^*}}{E^*} = \frac{414\, E(\text{TeV})\, \mathcal{L}(10^{33}\ \text{cm}^{-2}\text{s}^{-1})}{\sigma_z(\text{mm})\ f(\text{Hz})} \tag{7}$$

$$\eta_\ell = K\ \frac{Nb}{\lambda^2 G} \tag{8}$$

Equation (4) defines the invariant emittance of the beam. Equation (5) gives the luminosity in $\text{cm}^{-2}\text{s}^{-1}$. Note that P is the power in one beam in megawatts, and β^* is the lattice function at the collision point. Equation (6) defines the disruption parameter again. Equation (7) gives the center-of-mass energy spread coming from classical synchrotron radiation. Equation (8) gives the efficiency for converting RF energy in the linac to energy in the beam. In this equation K is a structure dependent constant, b is the number of bunches per linac pulse, λ is the RF accelerating wavelength, and G is the accelerating gradient. This equation is an approximation and is good for efficiencies up to about 20%.

4. Machine Parameters and Optimization

The optimization of linear colliders is much more complex than the optimization of electron storage rings. Storage rings are a mature technology and unit costs are well known. There are collection of simple equations relating RF power requirements to beam energy and machine radius which can easily be differentiated to yield a minimum cost. The result of this optimization is that the radius of a machine is proportional to the square of its energy. Such a minimization was done, for example, for LEP and the costs arrived at by that procedure were within about 10% of the costs arrived at after a detailed design effort. After 25 years of design and construction of storage rings, cost optimization is a relatively simple procedure.

This is not the case for linear colliders. The cost of a given machine comes from a complex interplay of factors effecting energy efficiency and factors effecting performance. Energy efficiency factors such as RF wavelength, accelerating gradient, particles per bunch, and bunches per pulse are intertwined with other factors such as bunch length, disruption parameter, final focus system design, and synchrotron radiation at the collision point. Let me begin then with some example solutions for big linear colliders that have not been optimized. All machines discussed in what follows will have 2 TeV in the center-of-mass, a luminosity of $10^{33}\ \text{cm}^{-2}\text{s}^{-1}$, and a Beamstrahlung driven energy spread of 10% (standard deviation).

Table II shows a partly optimized solution where we have minimized the AC power for the assumed beam parameters. A final focus system has been completely designed for this example and raytraced with a program that does beam optics through second order. The beam size at the collision point is slightly elliptical due to aberrations in its final focus system and gives a beam of 0.15 microns by 0.2 microns for an energy spread coming out of the linac of $\pm 0.5\%$. Transverse wake field effects in the linac have been evaluated (the interaction of the beam back on itself through the resonant accelerating structure) and in both the 10 cm and 5 cm RF wavelength examples the steering tolerances with respect to the axis of the accelerator are 50 microns arising from the short range wake field. In addition, we have assumed 6 experimental areas sharing the luminosity and have designed the beam splitters and combiners necessary for this kind of operation.

Table II. An incompletely optimized example of a 2 TeV LLC at 10^{33} luminosity and 10% σ_{E^*}/E^* for two RF wavelength.

β_y^* (cm)	1	
D	2	
σ_z (mm)	2	
ϵ_n (mrad)	4×10^{-6}	
N	1.4×10^{10}	
f (Hz)	2000	
b	12	
P_b (MW per beam)	4.7	
λ (cm)	10	5
G (MV/m)	20	40
L (km - each linac)	50	25
Number of Klystrons	3500	3500
Total Input AC Power (MW)	390	290

The machines are not too long, 100 km and 50 km respectively, when compared to the 100 km circumference of the large proton machines that give equivalent physics performance. The klystrons required to drive the linac at either wavelength have peak RF powers about a factor or 5 higher than those now available but have average powers considerably lower. The input AC power assumptions are based on an assumed klystron efficiency of 50%. These machines seem to be in the multi-billion dollar class and use a great deal of input AC power.

We have just begun to work on optimization procedures. Ritson has written a computer program that maps a set of input parameters (energy, luminosity, Beamstrahlung energy spread, final spot size, RF wavelength, klystron efficiency, etc.) to a set of output parameters describing the machine (length, wall plug power, repetition rate, accelerating gradient, etc.). Adding in estimates of cost per unit length of accelerator structure, klystron cost vs frequency and peak and average power, AC power cost, etc., the program can search for a cost minimum. For example, Fig. 3 shows the relative cost of a 2 TeV, $\mathcal{L} = 10^{33}$ collider as a function of RF wavelength for two different assumed final spot sizes. The curve shows that for a spot radius of 0.25 microns, the minimum cost occurs for an RF wavelength of 6 cm and gives 50 km for the total length of the two linacs at the minimum. Reducing the spot size by a factor of two, shifts the optimum wavelength to 5 cm, shortens the linac by 20%, and seems to reduce cost by 25%.

The optimization procedures are still primitive. We don't yet include constraints from such things as transverse and longitudinal wake fields, limits on accelerating gradients in the linac, technical limits on our power sources, etc. We still have much to do in understanding the basic limits of technology and beam dynamics for LLC's before we can take these optimization procedures seriously. For now, they are useful in pointing out regions of the parameter space where LLC's might be less costly than in other regions.

The proceeding analysis has assumed a round beam at the collision point and that assumption makes the Beamstrahlung induced energy spread proportional to the energy times the luminosity. If the beam could be made elliptical at the collision point, this relation can be broken and it is possible in principle to get an arbitrary late large reduction in Beamstrahlung at a given luminosity. Physically, the luminosity is inversely proportional to the beam area, while the Beamstrahlung is proportional to the square of the magnetic field in the beam. This magnetic field is, in turn, proportional to the inverse of the beam perimeter. If aberrations in the final focus system do not prevent it, it is possible to make the area divided by the square of the perimeter arbitrarily small and so remove all constraints on system design coming from Beamstrahlung.

Bassetti and Gygi-Hanney[3)] have calculated the effect on the beam induced energy spread of using elliptical beams. Figure 4 shows their results for the relative Beamstrahlung induced energy spread at constant beam area as a function of the ratio of the major to minor axies of the beam. Large reduction in Beamstrahlung require large aspect ratios. However aberrations in the final focus system will limit the size of the small dimension of the spot and the use of flat beams will be more difficult in practice than the use of round beams for they will require more exotic higher order correction schemes.

As an example, we have compared what can be done with second order corrected schemes for round and flat beams in a specific case. Both beams would have an area at the collision point of 0.06 square microns in linear transport theory. Our tracking program indicates the area of the "round" beam increases to 0.09 square microns while that of the flat beam, with a 20 to 1 aspect ratio, increases to 0.16 square microns.

We have run this flat beam example through the optimization program and find a very different optimization for our 2 TeV machine. Some of the relevant parameters are:

Accelerating Gradient	125 MV per m
Total Length	16 km
RF Wavelength	3.5 cm
Total Input AC Power	145 MW
Fractional Energy Spread	0.7%

These parameters look interesting — particularly the reduction in the total AC power consumption. However, before one gets too excited about this kind of beam, remember that the constraints have not yet been put in the optimization program. Thus, for example, we don't know it's possible to get an accelerating gradient of 125 MV per meter at 3.5 cm. Also the effects of the wake fields in the linac in this example are extremely severe. Alignment tolerances due to wake fields are proportional to the accelerating gradient times the cube of the RF wavelength and while the high gradient in this example helps the alignment's problem, the small wavelength hurts it badly. Alignment tolerances for this example are 9 microns in the linac.

5. Conclusion

This lecture has been more in the nature of a status report on our program to design LLC's than a description of a design of a practical machine. There remains a great deal of work to do in the areas of theoretical studies of the behavior of intense beams in accelerating structures, the development of the technology required to build these machines at reasonable cost, and in the understanding of the tradeoffs between parameters that will lead to minimum cost solutions. At SLAC we are working in the following areas.

- Theoretical studies of beam dynamics in linear colliders.
- Optimized accelerator structures giving minimum wake field effects for a given accelerating gradient.
- Experimental studies of the breakdown strength of copper accelerating structures to determine the maximum accelerating gradient that can be maintained as a function of RF wavelength and pulse length.
- The development of high efficiency RF power sources including a proof of principle laser driven source with 100 MW of peak power and efficiency of greater than 70%.
- Theoretical studies of low aberration final focus systems.
- Improved system optimization programs to minimize the costs of a given facility.

There is an enormous amount of work to do and I would feel much more comfortable if we had more resources to put on these important problems.

Probably the single most important milestone on the road to LLC's is the completion and successful operation of the SLC. This machine, to be completed at SLAC in late 1986, is really the proof of principle for the entire concept. When it runs, we will all be more confident that linear colliders can supplant storage rings and give the very high energies in the electron-positron system that will be required for future experiments..

I have often been asked how soon I believe a serious proposal for an LLC could be made. I always try to avoid answering this question, but sometimes I can't get out of it. The difficulty in answering it comes because the stakes in the game are now so high. What is wanted by the physics community is a TeV machine — a factor of 20 larger than the energy of the first (not yet completed) linear collider, the SLC. That large a step is unprecedented in the development of a new accelerator technology.

Electron storage rings have been around for about 25 years and the typical step in energy from one machine generation to the next has been about a factor of three. Proton synchrotron have been around even longer and a typical step in energy between generations has been around a factor of five. It is true that the SSC now under development in the United States involves a step of 20 in energy over the largest machine now operating, the Tevatron at FNAL, but a great deal of experience has been accumulated about proton synchrotrons and about the behavior of stored beams through studies in many electron and a few proton storage rings.

With all of these disclaimers out of the way, I will be optimistic. I think that there is a reasonable chance of being ready to make a serious proposal in 4 to 5 years. By that time the SLC will have been running for a few years, much new technology will have been developed, and we will understand our optimization procedures better. Perhaps at the next ICFA seminar we can describe a serious design for an LLC driven by a conventional linac as is under study at SLAC and Novosibirsk, or by some new technology such as laser acceleration or wake field acceleration.

We all know that the future in high energy physics is in higher energy machines and given a reasonable fraction of the effort devoted to machine development of that now devoted to experiments on existing machines and those under construction. I believe that we can be ready for the next step in a timely fashion.

References

1) J. Ellis, Proceedings of the XIVth International Symposium on Multiparticle Dynamics (1983).

2) G. Kane, Presentation at this conference.

3) M. Bassetti and M. Gygi-Hanney, LEP note 221, CERN (1980).

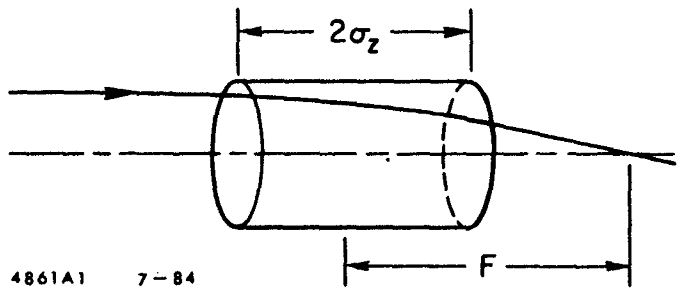

Fig. 1. The effect of the intense fields in a bunch on a particle of the opposite sign passing through the bunch.

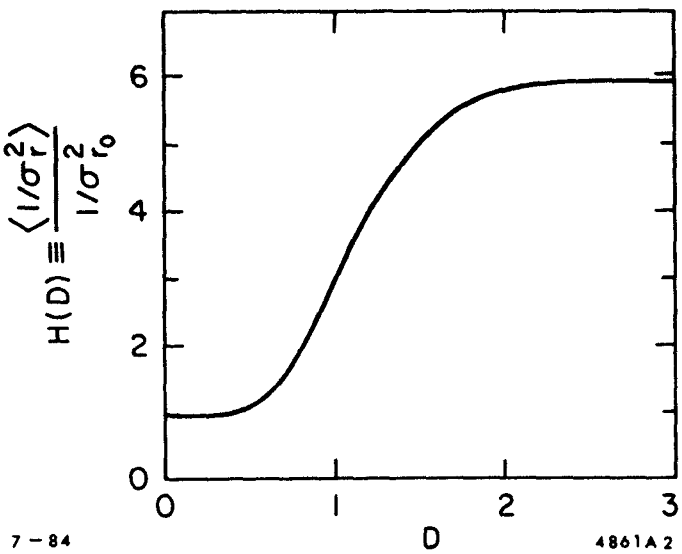

Fig. 2. Luminosity enhancement factor (H) as a function of the disruption parameter (D).

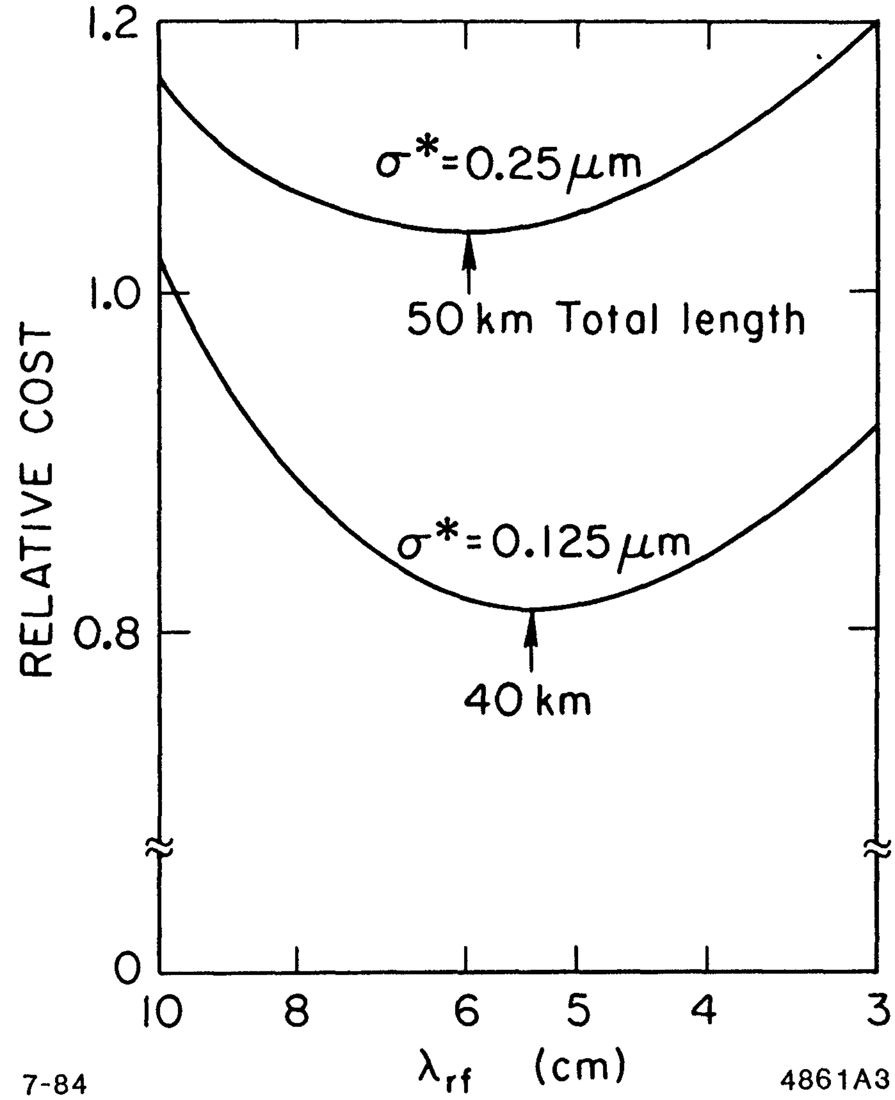

Fig. 3. Relative cost of a 2 TeV, 10^{33} luminosity collider as a function of RF wavelength for two different final spot radii.

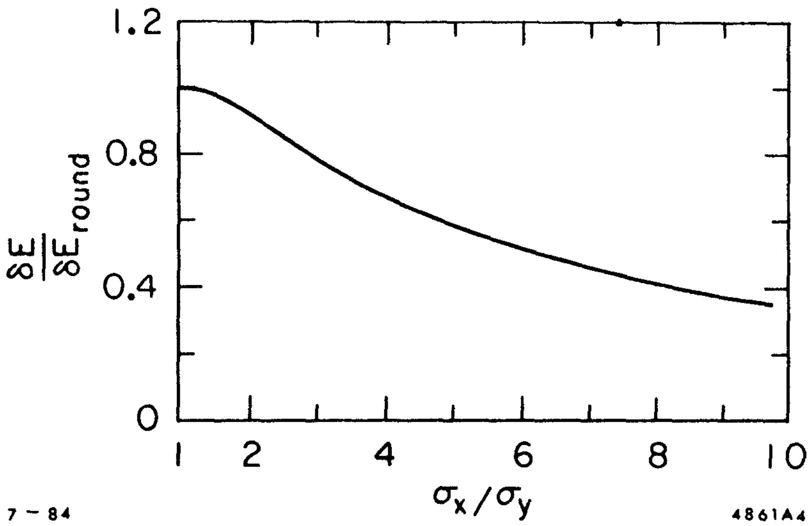

Fig. 4. Ratio of synchrotron radiation energy loss for an elliptical beam and a round beam as a function of the aspect ratio of the elliptical beam.