THE WAKE FIELD ACCELERATION MECHANISM

by

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ABSTRACT

The wake fields of dense bunches of relativistic electrons are used to accelerate secondary beams of positrons and electrons. The basic principle is the transformation of wake forces by means of geometric structures with different impedances at different locations. In such *wake field transformers* beams of a few GeV energy can accelerate secondary beams to ten times the energy of the driving particles. Two 50 GeV colliding beam linear accelerators based on this mechanism occupy less than 1300 meters total length.

CONTENTS

1. INTRODUCTION

Wake fields are electromagnetic fields excited by bunches of charged particles passing structures of varying shape. The best known wake fields are those excited in accelerating cavities in linear accelerators and storage rings. Fig. 1 shows the electric fields generated by a Gaussian bunch passing on axis a single cell of the PETRA/1/ accelerating cavity, computed by a direct solution of Maxwell's equations in time domain/2/. Particles inside or behind the bunch will feel a longitudinal force due to these fields. The force integrated over the total passage time is usually called the *wake potential* w(s):

$$w(s) = e \int E_z(z=ct-s,t)\, dz$$

where s denotes the relative position of test particles in the bunch frame with respect to the bunch center. Fig 2 shows a typical example of such a wake potential. Particles sitting in front of the bunch do not loose much energy since they can only see fields generated by particles in front of them. Particles at the tail may even be accelerated. Unfortunately it follows from causality and simple symmetry in integrals for the wake potential that the absolute value of the wake potential behind a symmetric bunch of finite length is always less or equal to the maximum absolute value of the wake inside the bunch. In addition it follows that the wake in the middle of the symmetric bunch is exactly half the maximum of the wake for all positions s /3/. Therefore it does not seem economical to use this wake effect for particle acceleration since no significant gain in particle energy can be achieved by such a mechanism.

The goal of a wake field accelerator would obviously be a wake potential that is much higher behind the bunch than inside the bunch. Such a wake would allow excitation of very high gradients by means of low energy high current beams. From the above it follows that such a system *must* have the wake exciting beam and the accelerated beam at different locations: The high current low energy beam has to pass the structure at a location of low impedance; and the accelerated bunch has to pass the same structure at a location of high impedance, i.e. one needs a *wake potential transformer*.

There are many possible geometries yielding such a transformation effect/4/; and some will be described in this paper. Since available computational means nowadays are restricted to structures of cylindrical symmetry, we will basically study a cylindrical wake field transformer and show how such a system can generate gradients up to a few hundred MeV/m. It will also be shown that two 50 GeV colliding linear accelerators can be built within a total length of about 1300 m.

2. THE WAKE FIELD ACCELERATION PRINCIPLE

The simplest wake field transformer is shown in figure 3. It is a pill box with a beam pipe at the center through which the accelerated bunch penetrates. Near the outer boundary a slot around the circumference on both sides of the box allows a hollow ring beam to pass. In figure 3 one sees schematically the electric field lines of the hollow beam coming into the structure. It is interesting to note that these field lines exist only at the outside of the hollow beam since relativistic particles are assumed. During the passage, fields are radiated into the outer part of the pill box. These fields are reverted in sign at the outer boundary and subsequently they travel towards the center of the structure. During this time the electromagnetic energy density increases and so does the electric field strength. Matched with a proper time distance, a second (normal) bunch follows on the axis seeing an accelerating field much higher than the decelerating field for the driving beam.

For a realistic transformer one would use many of such pill boxes stapled together as shown in figure 4. Figure 5 shows the electric field (for eight time points) generated by an annular beam passing a seven unit accelerating structure having an outer radius of 6 cm. At t=287 ps the fields have reached the center and a concentrated electric field is observed. Subsequently the pulse is reflected at the axis and travels to the outside where it will be reflected again. At each reflection the pulse is inverted in sign thus yielding an acceleration potential for positrons and electrons generated by just electrons. In order to improve the first reflection of the fields at the outer boundary small triangles may be used as shown in figure 5. It can be seen from these field patterns that the pulse is actually a triple pulse with a negative head and tail embedding the main pulse for acceleration.

3. A 50 GeV COLLIDING BEAM WAKE FIELD LINEAR ACCELERATOR

A possible layout of a 50 GeV colliding linear accelerator is shown in figure 7. The electron rings may be produced by a ring cathode or an electron ring accelerator of the ERA type/5/. The rings and the particles to be accelerated pass a conventional linear accelerator which brings both beams to a medium energy. Roughly of the same length is the subsequent wake field transformer which boosts up the energy by a factor of ten for the central beam. During this process the outer driving beam loses all its energy. The particles are focussed by a solenoid in the wake transformer. A parameter list of such a linear accelerator in colliding beam mode is given in Table I. Table II summarizes the beam dynamics parameters.

4. BEAM DYNAMICS

There are several aspects of beam dynamics in such a wake field accelerator. So far we considered the collective effects only in a rather preliminary way but it is clear in which way more detailed calculations have to go.

The longitudinal collective effects are described by the self induced wake potentials for both beams. Figures 10 and 11 show the longitudinal wake potentials for both beams. Particles in the central beam are decelerated by about 15 MeV/m. The boosting in amplitude for the wake at the center can easily be seen in figure 8. Figure 9 gives a detailed graph of the accelerating pulse at the center having a full length of about 25 picoseconds. Figure 11 shows the longitudinal self induced wake potential in the accelerated beam. This wake has to be added to the accelerating wake coming from the hollow outer beam. A proper phasing between these two wakes yields a minimized energy spread in the high energy beam/6/. Apart from this fact the wake also reduces the effective gradient. Figure 10 shows the longitudinal wake potential in the outer beam showing a peak deceleration of roughly 17 MeV/m.

The transverse wake field effects for the outer beam can be calculated with the computer code TBCI/7/. Let us consider a hollow bunch with total charge Q and major radius R. The dipole moment of a ring bunch displaced by the amount δ in radial direction is δ. A filamentary beam with the corresponding charge $Q'=Q\cdot\delta/R$ has the same dipole moment and thus creates the same dipole wake potential. In a normal cavity with beam pipes on both ends the transverse dipole wake potential affects all the particles at a certain position s in the same way independant of their individual radial position and the amplitude of the wake is given by the bunch charge and the average bunch displacement. Thus the wake forces may w be written in cartesian and cylindrical coordinates, see fig.12:

$$
\begin{aligned}
w_x &= w_0 \quad , \quad & w_r(r,\varphi) &= w_0 \cdot \cos\varphi \ , \\
w_y &= 0 \quad , \quad & w_\varphi(r,\varphi) &= -w_0 \cdot \sin\varphi \ ,
\end{aligned}
$$

In the case of the outer beam this is no longer true and the force varies along the ring coordinate φ. It is somewhat obvious that particles at $\varphi=0$, see figure 12, are kicked more than the particles at $\varphi= 90^\circ$. The different azimuthal and radial wake potentials generated by a hollow beam holding $6\cdot10^{12}$ particles beeing 100 μm displaced are shown in figure 14. As suspected the radial kick is dominant and increases almost continuosly towards the tail of the driving bunch. Figure 15 shows the transverse wakes induced by the central high energy bunch holding 10^{11} particles for a similiar displacment of 100 μm. As mentionned above the azimuthal wake is exactly the inverse to the radial wake thus generating a pure parametric dipole force.

The full beam dynamics can only be adequately treated by tracking procedures. Here we want to give a rough estimate to what happens transversely to a particle sitting at five standart deviations behind the center of the driving bunch. Given the transverse kick per unit length $\Delta p'$ caused by a constantly displaced beam by 100 μm -which is otherwise rigid- the particle trajectories in a longitudinal solenoid field of strength B=7T are given by, see Fig13:

$$\omega = (e/m_0) \cdot B$$

$$y(t) = (\Delta p'/eB) \cdot \{ ct - \sin \omega t \; / \; (\omega/c) \}$$

$$y_{max} = (\Delta p' / e B) \cdot L_{tot}$$

$$= (7\mathrm{keV/m \cdot c}) / (e \; 7 \; \mathrm{T}) \cdot 300 \; \mathrm{m} = .001 \; \mathrm{m}$$

$$x_{max} = (E \cdot c \cdot \Delta p') / (ecB)^2$$

$$= 5.5\mathrm{GeV} \cdot c \cdot 7 \; \mathrm{keV/m} / (e \; c \; 7\mathrm{T})^2 = 8.7 \; \mu\mathrm{m}$$

L_{tot} = total length of the wake accelerator

E = nominal particle energy

B = solenoid field strength

It is rather obvious that this estimate is a worst case approximation. First of all one is not bothered too much about the particles sitting in the fifth standart deviation. Furthermore the driving beam will not have a constant offset but randomly distributed displacements. Thus there is a justified hope that one can focus sufficiently with less solenoid field strength and/or less tight tolerances. Detailed calculations taking all the wakes and changes in transverse particle distribution into account are presently being worked on.

5. SCALING LAWS

The wake transformer discussed above represents not at all the maximum gradient that can be achieved. Some of the following obvious scaling laws show that there is alomost no theoretical limit to the gradient. The gradient scales as follows:

$G_{wake} \propto$ Number of particles in the driving bunch

$\propto 1/(\text{ spatial dimensions })^2$

Taking the parameters that could be achieved *easily* in an electron ring accelerator/5/ one gets over 600 MeV/m:

$N = 5 \cdot 10^{12}$

$R = 2.7$ cm

$G = 170\ \text{MeV/m} \cdot (5 \cdot 10^{12}/6 \cdot 10^{12}) \cdot (5.5\text{cm}/2.7\text{cm})^2 = \underline{600\ \text{MeV/m}}$

6. OTHER WAKE FIELD TRANSFORMER GEOMETRIES

As already mentioned there are many other geometries possible based on the same physical principle/4/. The choice of the cylindrical tranformer discussed above was mainly influenced by the availability of computational tools for the wake field computation. Just to show two more possible geometries we have the elliptical transformer and a multi beam star transformer, see figure 16.

The elliptical wake transformer is based on the fact the the impedance of such a structure depends on the ratio hole size / bunch length, i.e. for a given bunch length of the two seperate beams the impedance is different at the two focal points. In first order approximation all the fields radiated from one focus reach the other at the same time.

Another possibility can be deduced from the cylindrical wake transformer by splitting the annular beam into several single beams. The connecting channels between the beams may then be seperated, see figure 16b. At the expense of an increased parasitic wake for the driving beams keeping the same charge as in a ring one gains in simplicity of handling the single beams.

In principle one could use any metallic box with two sets of tubes for the beams and different coupled impedances. The numbers of merit for such transformers are the *transformer ratio* and the *impedance per unit length* seen by the driving beam.

7. SUMMARY

The basic idea of *wake field transformation* by means of structures with different impedances at different locations offers a promising way for reaching gradients of a few hundred MeV/m. The fact that the wake transformers are simply shaped metallic boxes without any frequency tuning or phasing devices will make this kind of accelerator unexpensive and reliable. Breakdown limits are considered to be less severe than in any other accelerating near field device since the high field strength exists only at a limited area and olny for a very short time of typically a few tens of picoseconds. Compared to conventional storage rings the over all size of a colliding linac using wake transformers is down by a very large factor.

8. LITERATURE

/1/ The PETRA study group,PETRA proposal, DESY 1974 and 1976
/2/ T.Weiland, Proceedings of XI International Conference on High Energy Accelerators,Geneva 1980, pp.570-575
/3/ P.B.Wilson, CERN/ISR-TH/78-23, 1978 and T.Weiland and B.Zotter, Particle Accellerator, Vol 11(1981), pp.143-151
/4/ G.A.Voss and T.Weiland, DESY M-82-10, April 1982
/5/ U.Schumacher, this conference
/6/ SLC design report, SLAC-Report 229, June 1980
/7/ T.Weiland, DESY 82-015, March 1982

TABLE I: General parameters of a 50 Gev e^+ e^- colliding beam linear wake field accelerator

E_0	nominal particle energy	50 GeV
L_{tot,e^-}	total length of the electron linac	550 m
L_{tot,e^+}	total length of the positron linac	650 m
G_{conv}	gradient of the conventional linac	25 MeV/m
G_{wake}	gradient in the wake field transformer	170 MeV/m
$<P>$	average power consumption	8+8 MW
P	peak power	3900 MW
N_1	number of high energy particles per bunch	10^{11}
N_2	number of particles in the driving bunch	$6 \cdot 10^{12}$
η	efficiency of the wake transformer	16 %
f	repetition frequency	100 Hz
σ	r.m.s. bunch length of both beams	0.2 cm
t	wake field transformation gain	10.2

TABLE II: Beam dynamics

Driving Beam:

N_2	number of particles	$6 \cdot 10^{12}$
E_{20}	energy at the entrance of the wake transf.	5.5 GeV
E_{21}	energy at the end of the wake transf.	0.5 GeV
$\Delta\varphi$	maximum phase slip between driving beam and and accelerated beam	0.5 ps
U_p	maximum particle energy loss (self fields)	16.8 MeV/m
$\Delta p'$	peak transverse momentum kick per unit length due to delf fields	6.9 keV/mc
B	solenoid field strength	7 T
y_{max}	maximum particle deviation for a constant beam misalignement of δ = 100 μm	1 mm

High Energy Beam:

N_1	number of particles	10^{11}
U_p	maximum particle energy loss (self fields)	15.2 MeV/m
$\Delta p'$	peak transverse momentum kick per unit length due to self fields	18.9 keV/m

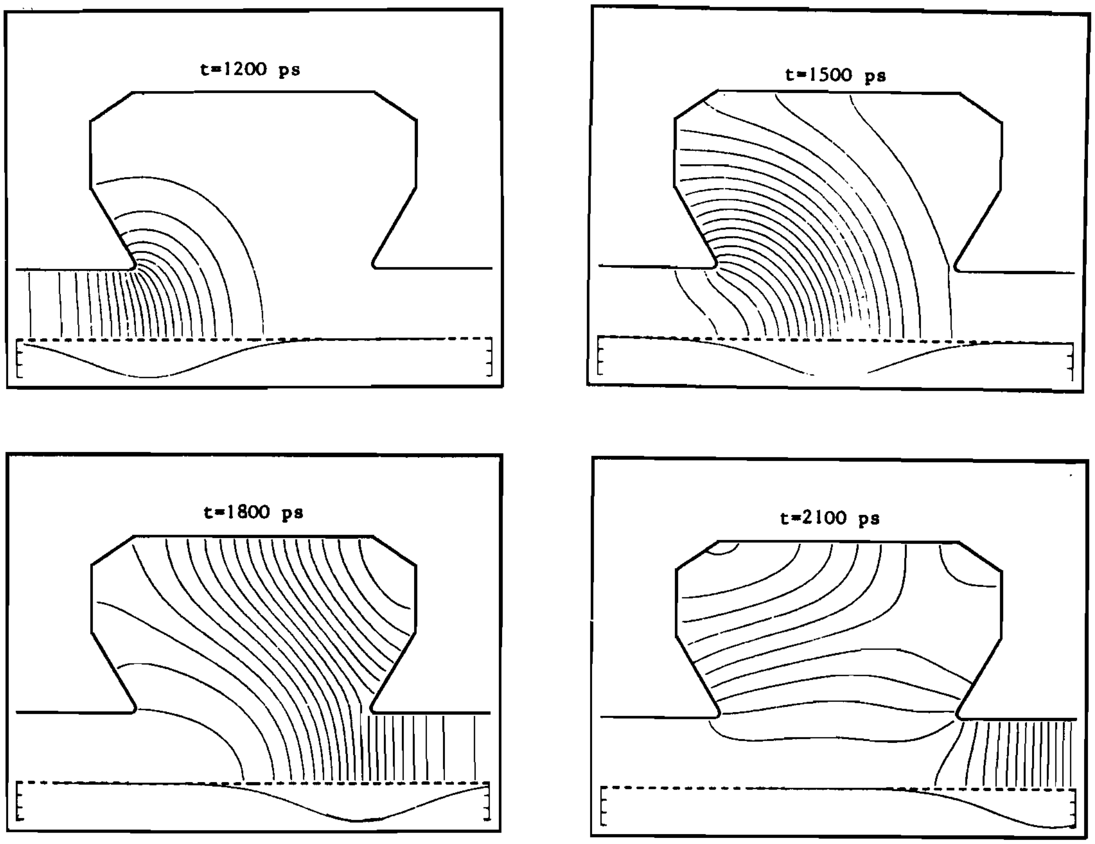


Figure 1: Electromagnetic fields excited by a Gaussian bunch of r.m.s. length 5cm passing a single cell of the PETRA accelerating cavity on axis. The charge density of the Gaussian bunch is shown in relative units at the bottom of each picture, i.e. below the symmetry axis.

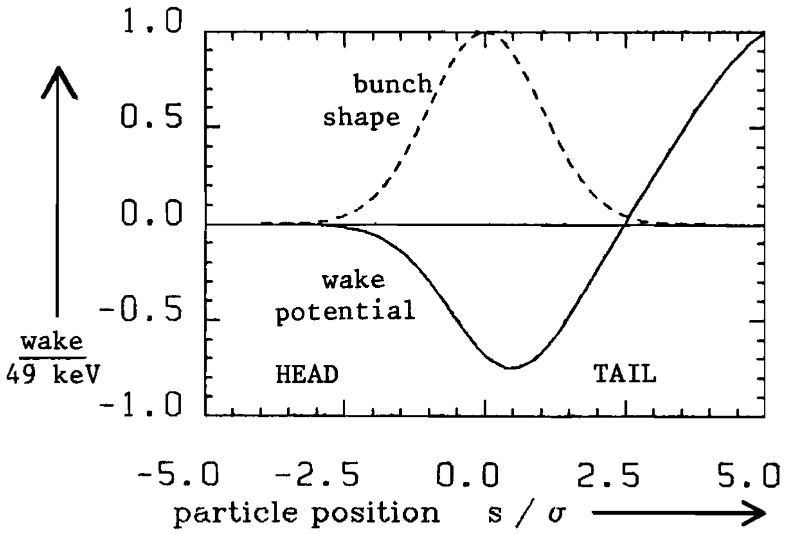


Figure 2: Wake potential inside a Gaussian bunch (σ=5cm,N=10^{12}) due to the fields shown in figure 1. The particle position s is given in units of bunch length and the head particles are located at the left hand side.

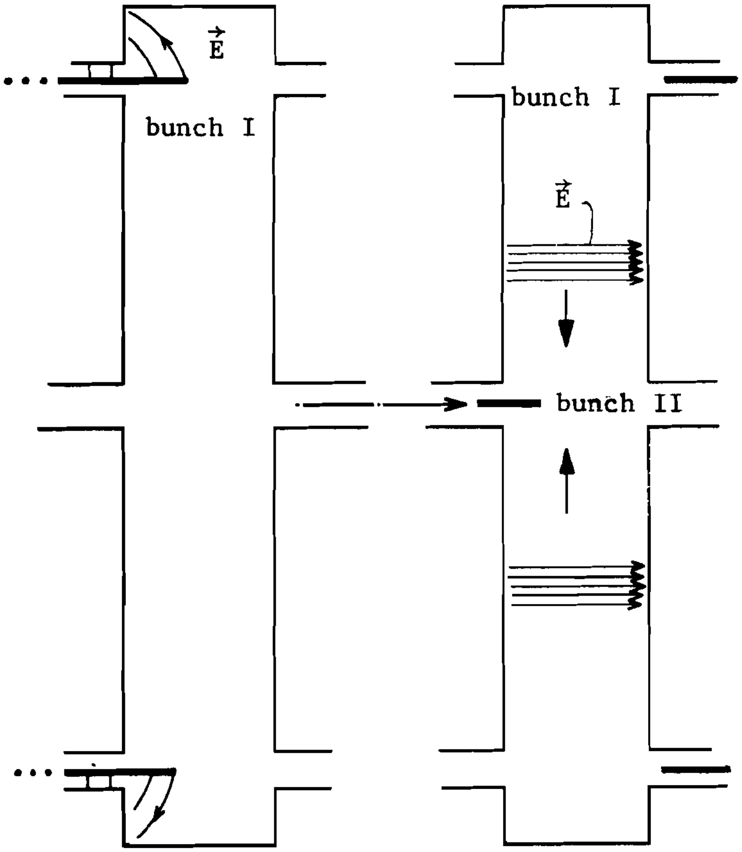


Figure 3: Simple wake field transformer consisting of a pill box with a central beam hole for the high energy bunch and a slot near the outer boundary for the hollow ring bunch.

Fields are radiated into the outer cavity and subsequently focussed towards the center where a high gradient is generated.

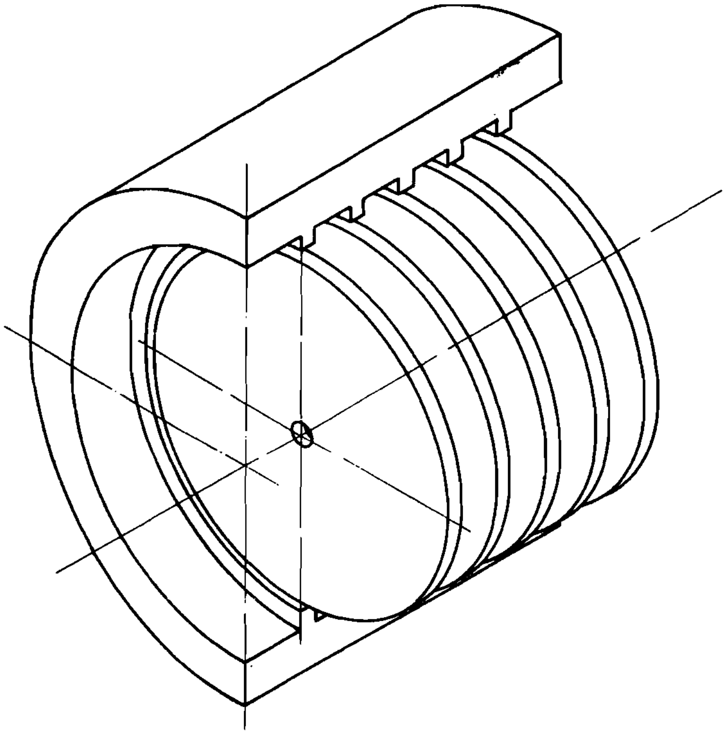


Figure 4: Stack of plates in a wake field transformer section.

Each subsection represents a short pill box. The outer diameter is of the order of 6 cm or less.

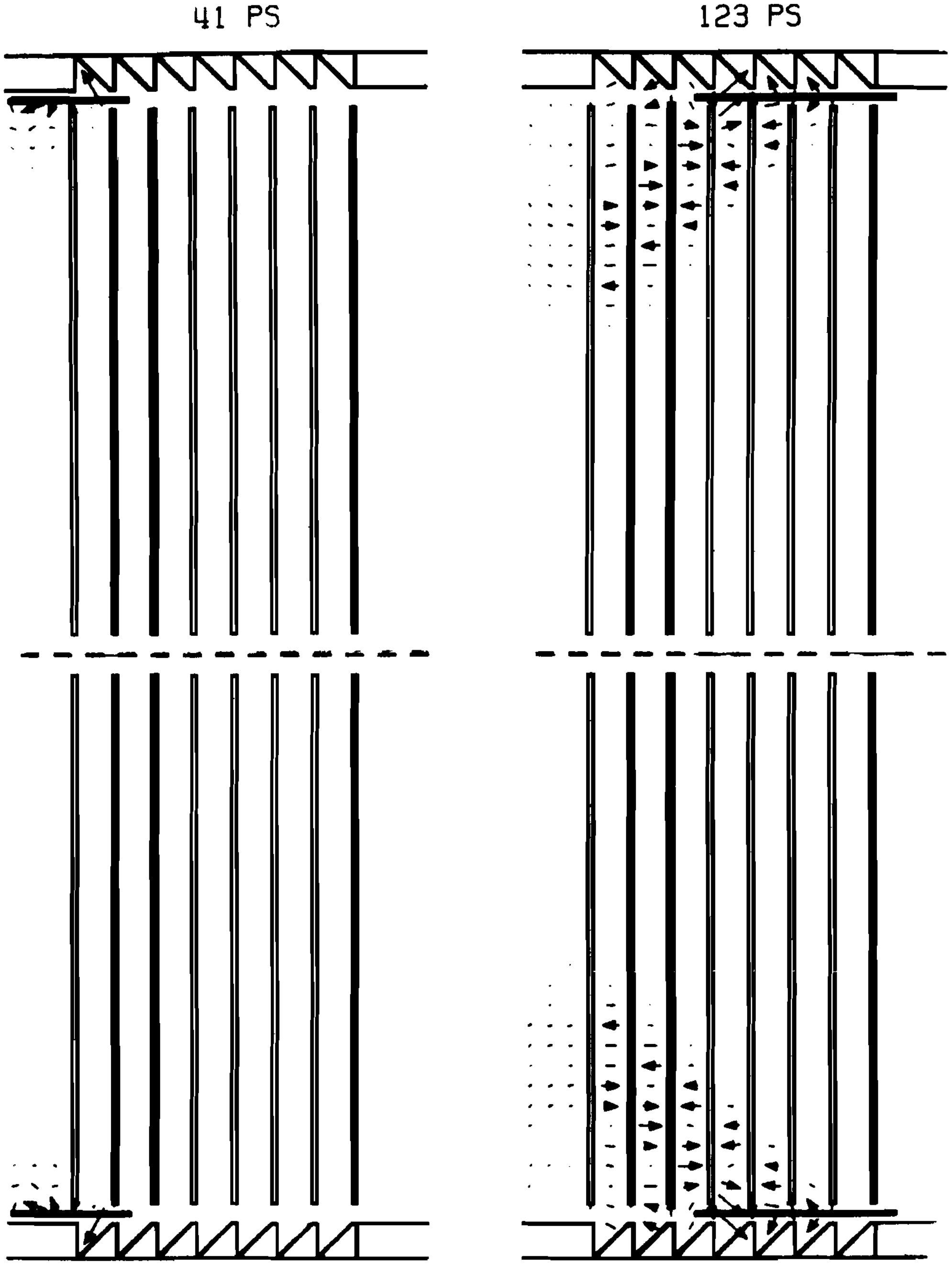


Figure 5: Electric field at eight subsequent time points generated by a hollow beam passing a seven cell wake field transformer. All dimensions are given in figure 6. On top of each picture the real time is given in picoseconds.

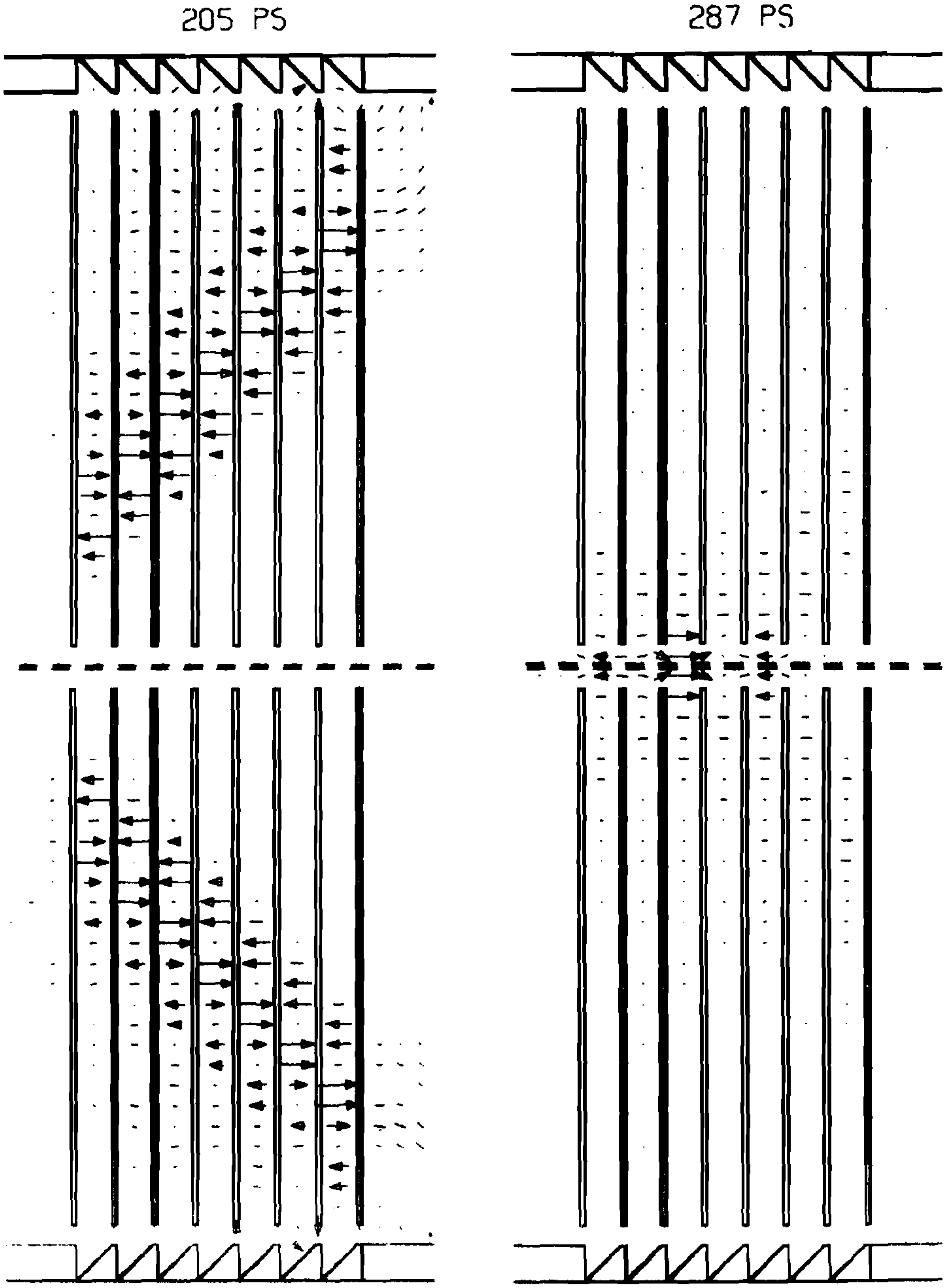


Figure 5: contd.

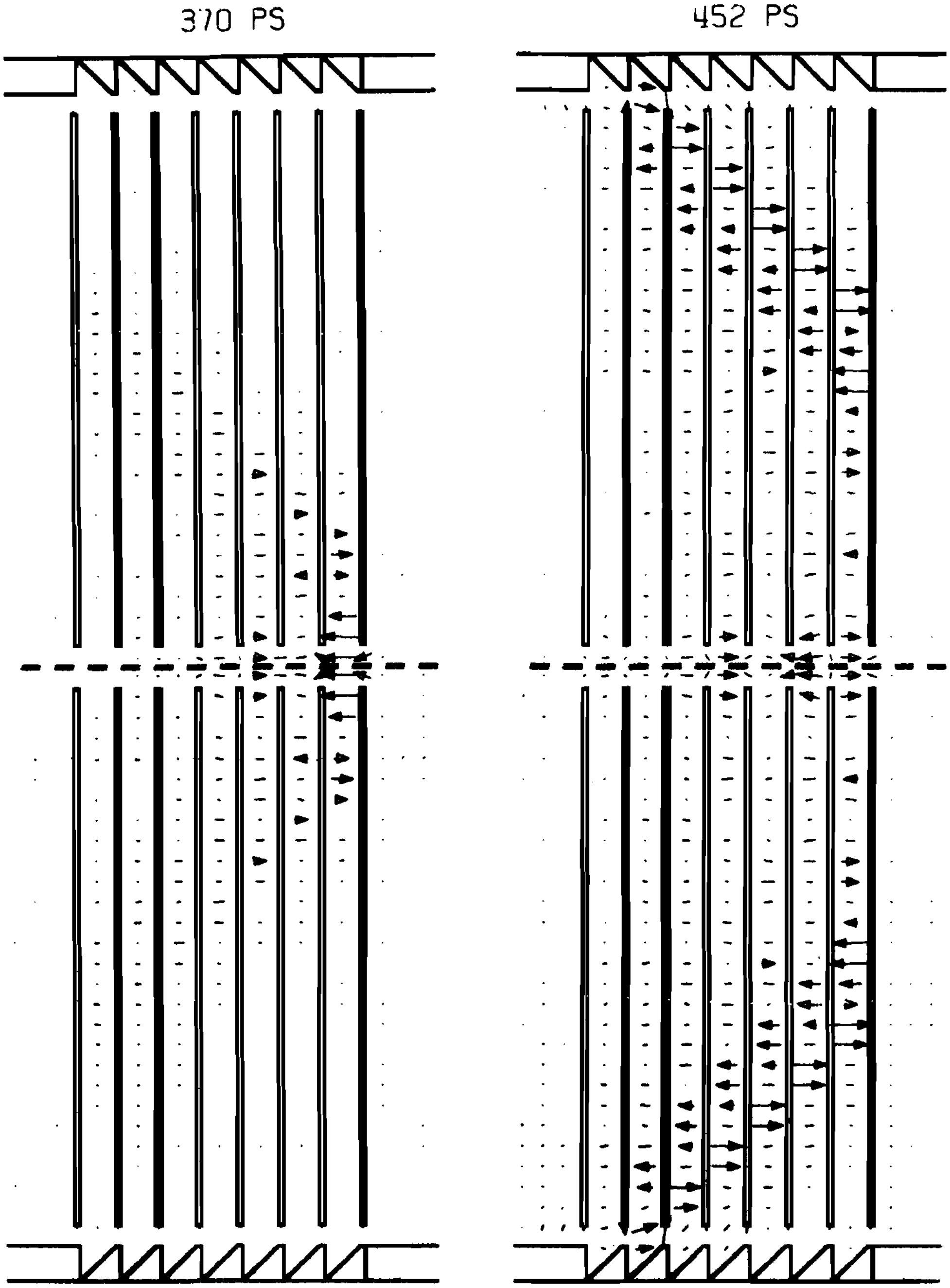
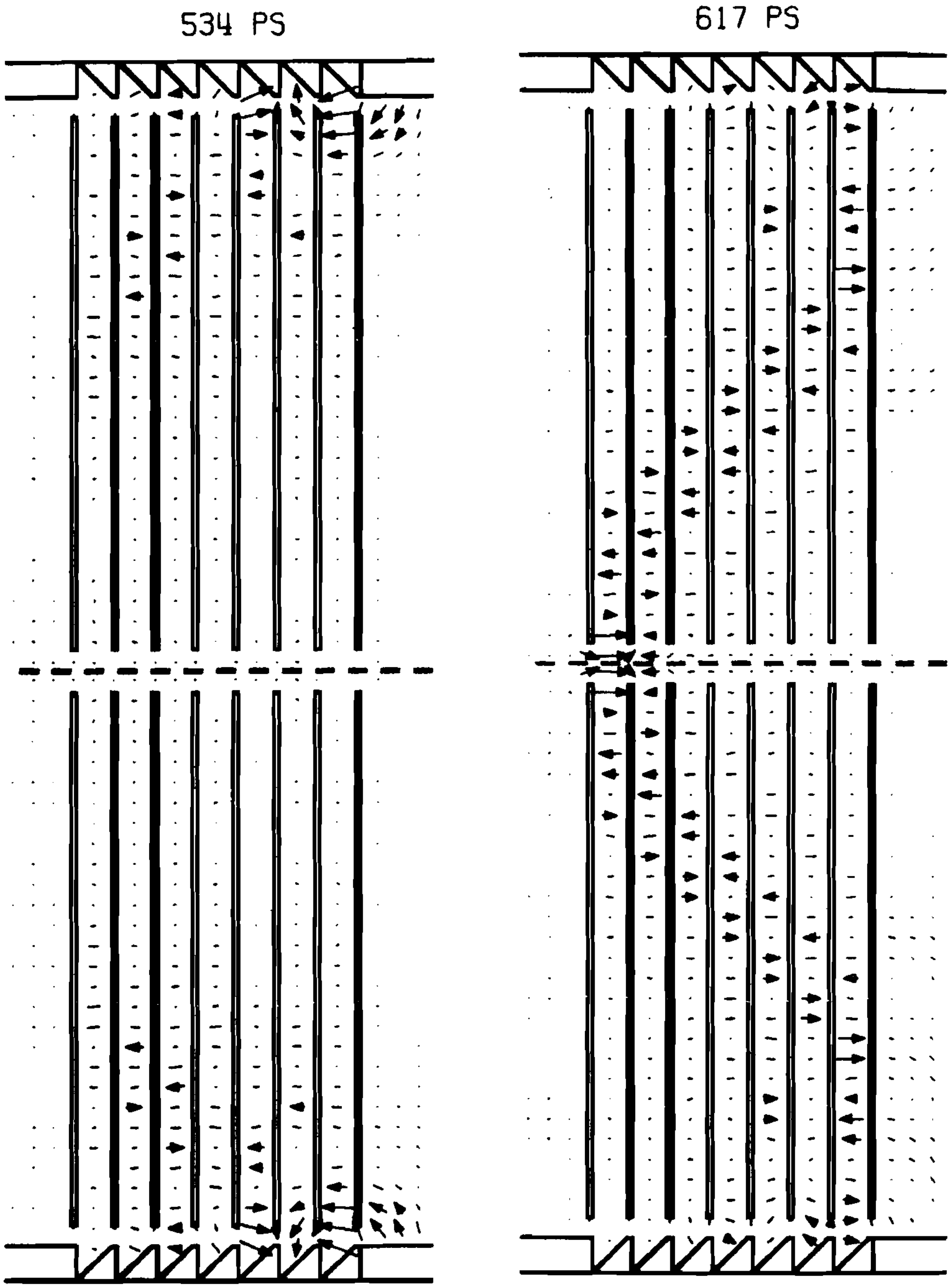


Figure 5: contd.



534 PS

617 PS

Figure 5: contd.

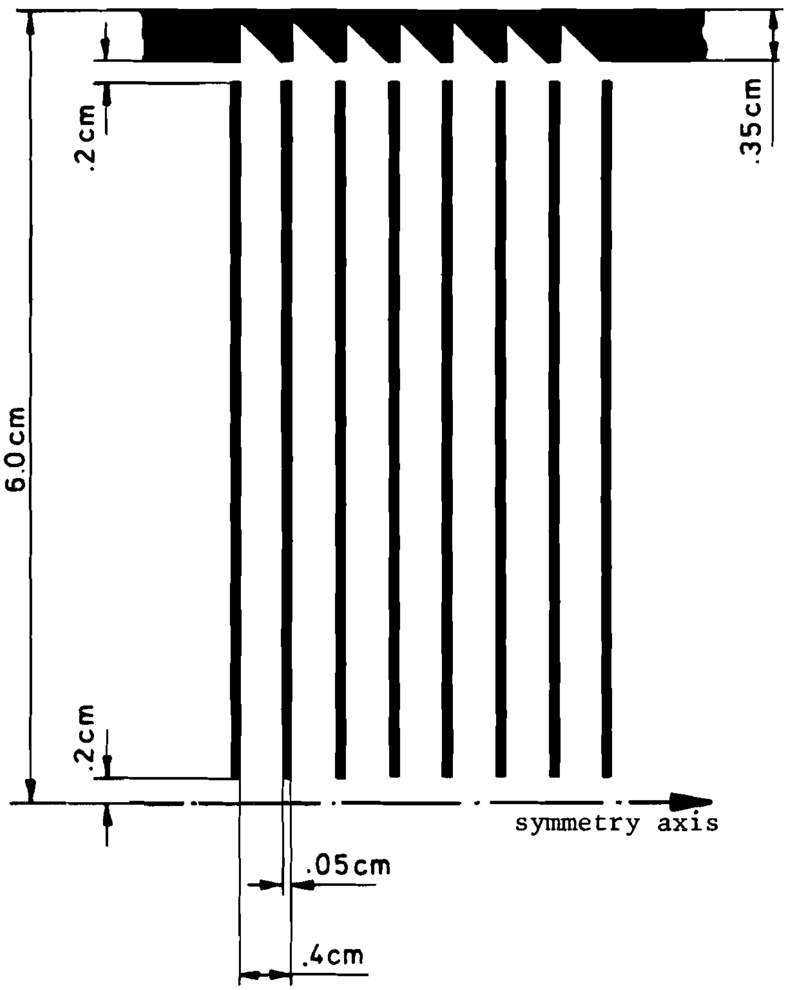


Figure 6: Dimensions of wake field transformer used for most of our calculations.

The triangular teeth are used only for the electron accelerator. The positron version using the second pulse reaches a better transformation ratio with simple rectangular teeth as shown in figure 3.

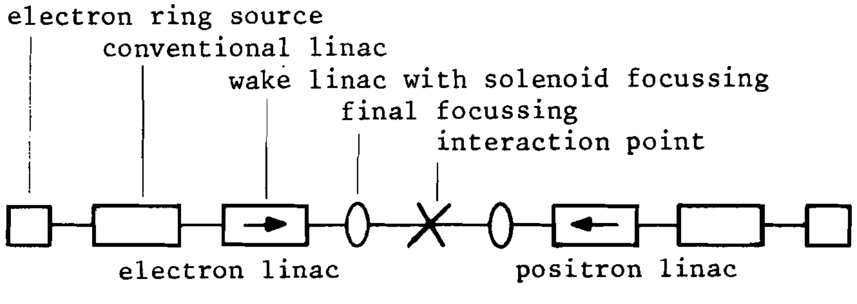


Figure 7: Principle layout of a 50 GeV colliding linear wake field accelerator.

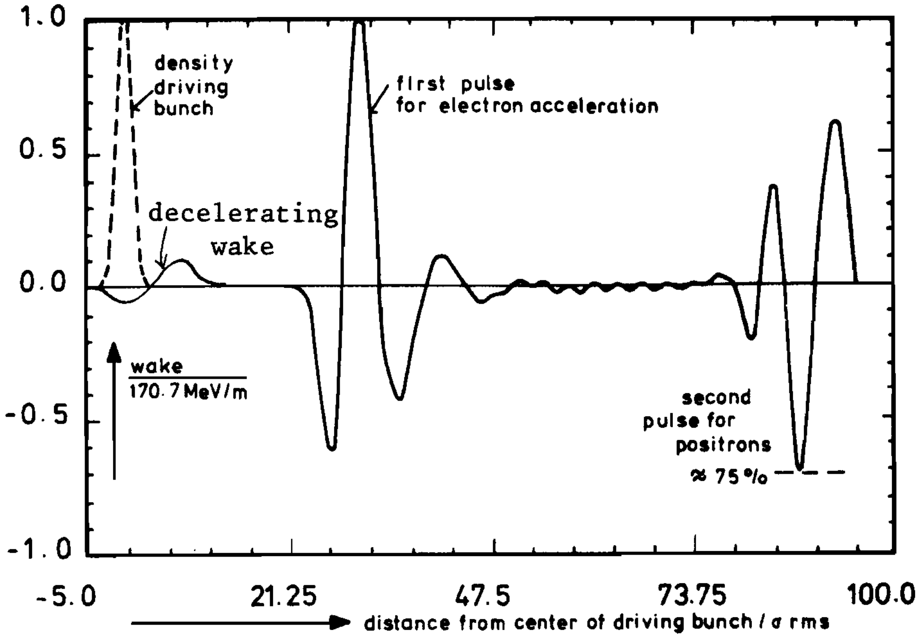


Figure 8: Wake potential at outer driving beam and at the center generated by a hollow driving beam. (The second pulse has the inverted sign necessary for positron acceleration but has only 75 percent magnitude.)

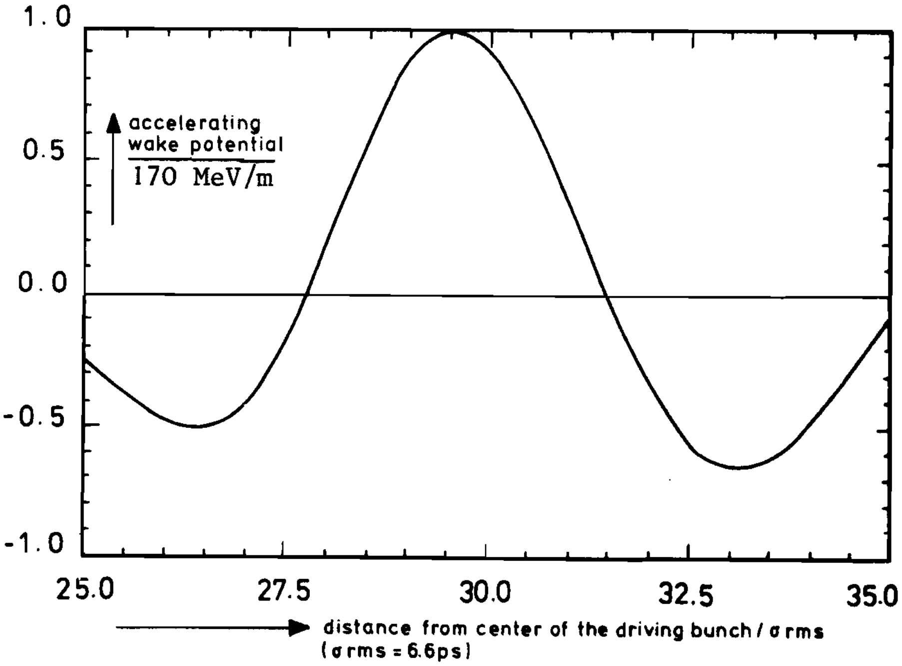


Figure 9: Deatiled graph of accelerating pulse as seen at the center. (The full length of the positive pulse is about 25 ps.)

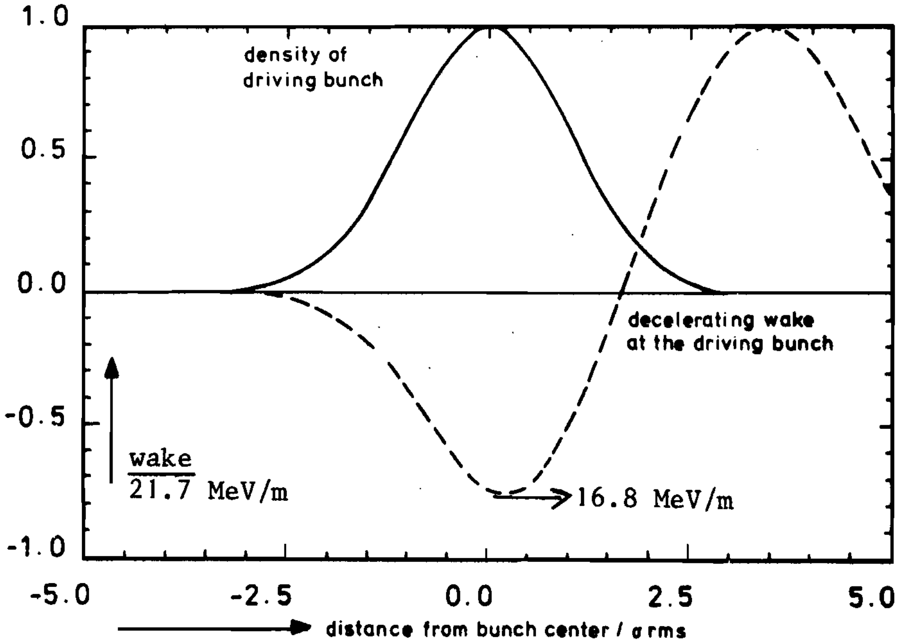


Figure 10: Longitudinal wake potential in the driving beam

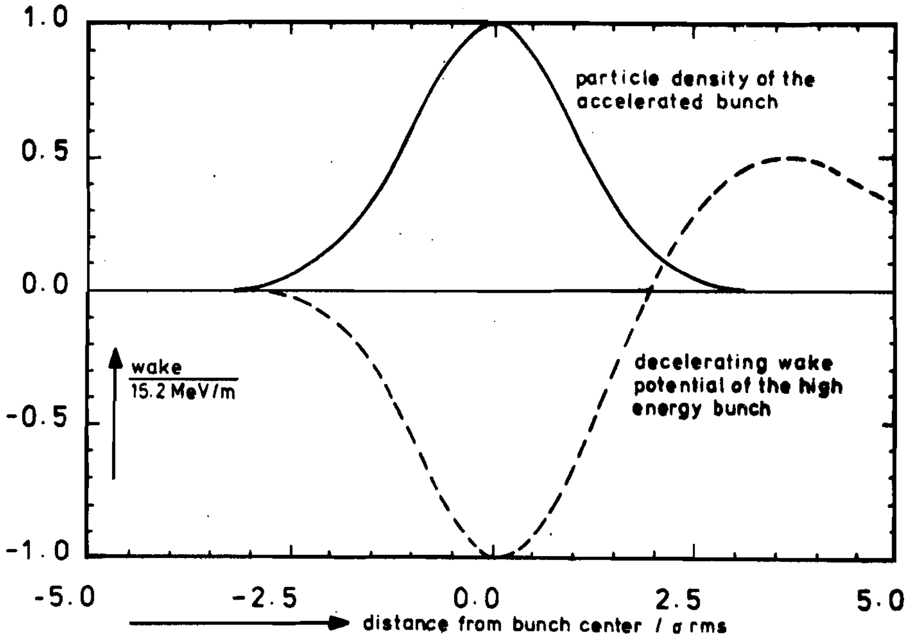


Figure 11: Longitudinal wake potential in the accelerated beam

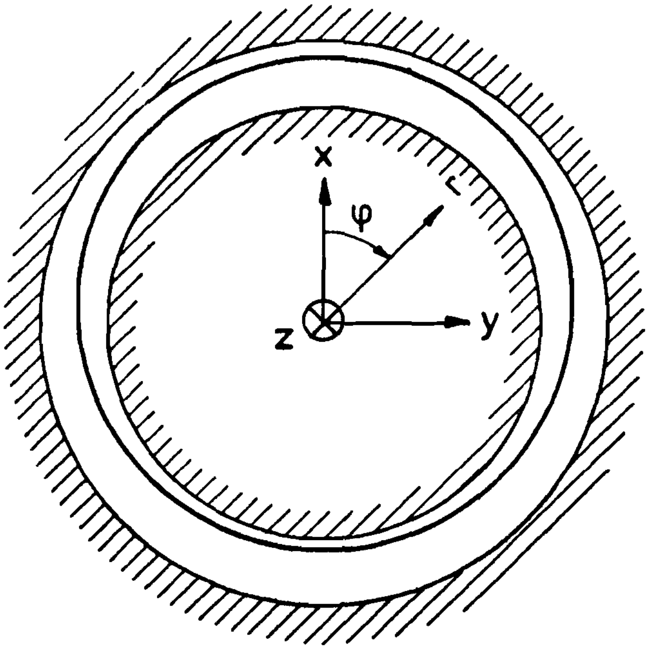
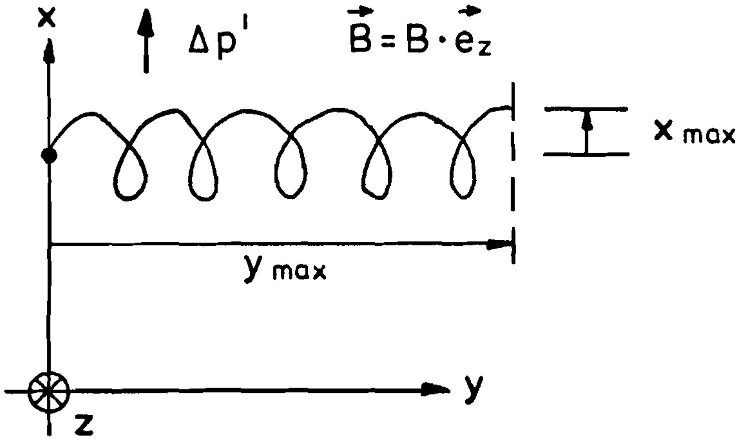


Figure 12: Radially displaced hollow bunch and definiton of coordinate systems for the wake forces

Figure 13: Particle motion in a solenoid field



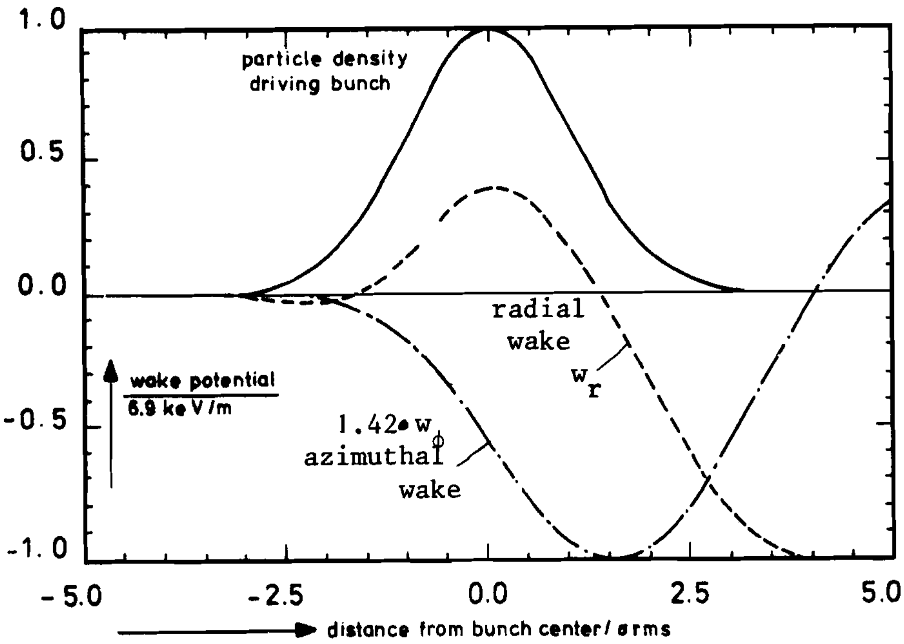


Figure 14: Transverse kick distribution along the bunch for the particles in the driving beam.
Note: the azimuthal force is different from the radial one in shape and amplitude (scaling factor 1.42).

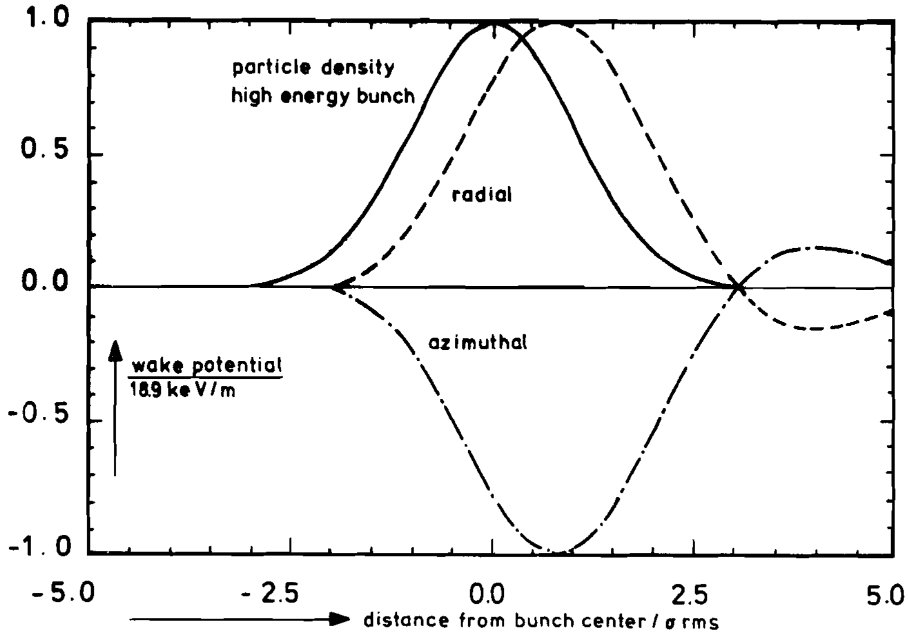


Figure 15: Transverse kick distribution along the bunch for particles in the accelerated high energy bunch.
Note: The azimuthal force is just the inverse to the radial one.

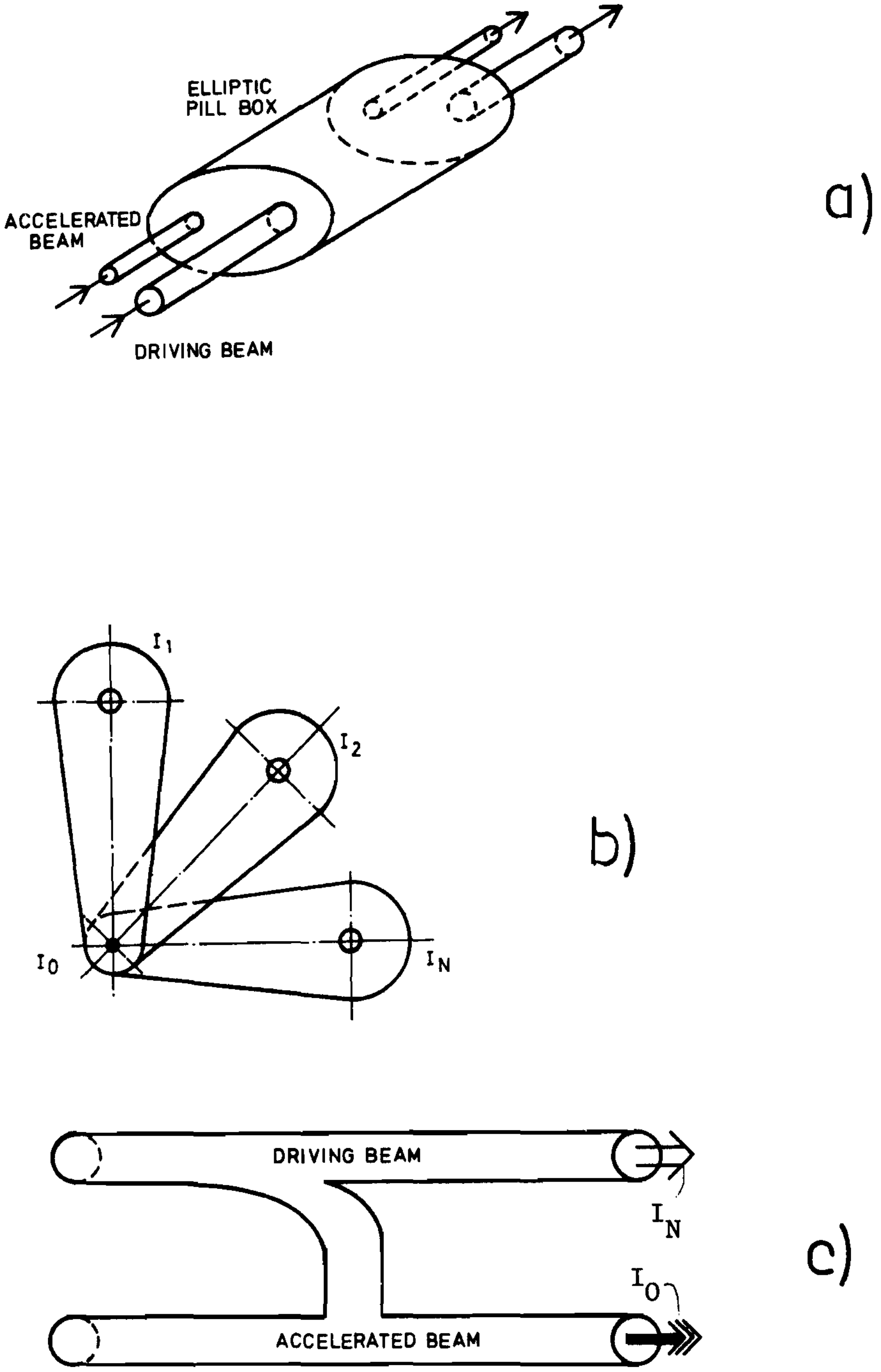


Figure 16: Elliptical wake transformer (a) with two seperated single beams and a multi beam star transformer (b and c)

DISCUSSION

Participant. Have you looked at electron emission from the high field regions?

Weiland. No

Winterberg. What is the maximum electric field you can get in this scheme?

Weiland. If you neglect breakdown and other physical things there is no limit.

Winterberg. But what is the practical limit including breakdown?

Weiland. It is hard to answer, but it will be delayed with this scheme.

Reiser. The accelerating bunches also produce wake fields. Have you looked at how they affect the driving bunches, (by feedback).

Weiland. The second bunch comes 180 psec behind the first, so the wake fields can never catch up.

Lawson. The electrons in different parts of the ring are likely to have rather different energies due to the action of the wake. Futher, the ring will be Lorentz contracted, and greater in radial than axial extent, whereas you have shown the length greater than the radial width.

Weiland. I have taken the radial width and axial extent about equal.

Richter. The ring cannot be of the same form as in the ERA, since this has rotation. If it has rotation then as it loses energy, its radius in a solenoidal field changes.

Voss. The ring initially has rotation, which is stopped after acceleration by reversing the direction of the axial magnetic field. The particles then follow the lines of magnetic field.

Motz. Have any such large fields as you are describing been observed?

Weiland. No, up to now this is just theory.

Keefe. There is in fact some experimental evidence of imploding amplification because the impedance is changing as you come in. This was achieved with a pulsed power device, in which a number of cables were arranged on the outside, and just a spark gap trigger. You can indeed see the amplification as the voltage pulse comes in.

Have you looked at how the bunch length changes in axial length through the accelerator? There will be a Lorentz contraction factor of ten, and secondly there may be some dispersion from the different velocities induced in the ring.

Weiland. The beam comes out with energy spread of zero to peak energy. This is just something we have to see from our code.

Participant. What is the energy conversion efficiency from primary to secondary beam?

Weiland. The charge ratio was 60 and the energy ratio was 10, giving 16%. Varying parameters we find it is between 7% and 18%.