A LASER DRIVEN GRATING LINAC*

Robert B. Palmer

Brookhaven National Laboratory, Upton, NY 11973

1. INTRODUCTION

In his introductory talk, A. Salam challenged us to aim for an accelerator in the 100 TeV range. I would like to take this challenge seriously and consider a 50 TeV on 50 TeV collider. Even a hadron machine with such an energy seems unrealistic with current technology. Magnetic fields higher than 10 Tesla are difficult and at this field the circumference would be 120 kM! I conclude that only a high gradient Linac could be practical and that one should aim for 10 GeV/meter so as to keep the total length down to the order of 10 km. Currently it is only plausible to obtain such fields using the very high energy densities produced by lasers.

The luminosity is another issue. I will aim for 10^{33} to 10^{34} but I am conscious that higher luminosities than even these are really desired, especially for an e^+e^- machine. I will in fact tend to assume that the machine is an e^+e^- machine but we should remember that it will also accept hadrons

2. ACCELERATION THEORY

The use of a laser to accelerate particles was first proposed by K. Shimoda[1] in 1962. He noted that high values of acceleration per meter could be obtained if velocity matching and mode selection were achieved. These requirements are, however, not easy to obtain.

Fields in free space, far from all sources, consist of a sum of all possible traveling electromagnetic waves. Provided the particles to be accelerated are traveling less than the velocity of light, acceleration[2] can occur. Once the velocity approaches that of light only waves traveling in the same direction as the particles remain in phase with the particles. Unfortunately, since free radiation is transversely polarized, no continuous acceleration is possible. In the presence of a magnetic field, the particle's direction can be perturbed in such a way as to allow continuous acceleration[3] but this too decreases as the particle's momentum increases and significant perturbations become impractical.

---

*Work performed under the auspices of the U.S. Department of Energy.

Acceleration has also been proposed in a vacuum close to a periodic structure. In particular, two papers have attempted[4] to employ the inverse Purcell effect[5] by illuminating a grating with plane parallel light and passing the particles over the surface of the grating at right angles to the lines (Fig. 1a). Unfortunately, it has been shown by Lawson[6] that these geometries also fail to accelerate relativistic particles. In Lawson's paper consideration is also given to the field between two parallel plane structures. It is shown in this case finite acceleration of relativistic particles is possible. Such a geometry seems to be little more practical than a scaled down conventional Linac. It does, however, show that there is no fundamental reason why a solution cannot be found. Indeed if we simply rotate the grating by an angle ϕ with respect to the beam (see Fig. 1b), then acceleration is indeed possible. In this case surface waves may be induced whose velocity c/K will be lower than the velocity of light, but these waves can remain in phase with a highly relativistic particle because of the angle ϕ between the wave and particle directions.

An alternative to a skew grating is to employ a skew initial wave (Fig. 2c). In this case although the grating lines are perpendicular to the particle beam, nevertheless the induced surface waves can still be at an angle to the beam and acceleration can again be obtained.

In order to consider diffraction in this skew condition, it will be convenient to introduce the following modified vector notation. Three-dimensional vectors (A_x, A_y, A_z) will be described by the two-dimensional vector (A_x, A_y) together with the z component A_z. The two-dimensional vectors will be shown $\tilde{A}$, the corresponding z component would then be shown as A_z. We will be considering the fields above a grating placed nominally at z = 0. Any such fields can be parameterized by:

$$\tilde{E} = \sum_{n=-\infty}^{n=+\infty} \tilde{A}_n \, e^{j(p_n z + \tilde{K}_n \cdot \tilde{R} - \omega t)}$$

where (1)

$$E_z = \sum_{n=-\infty}^{n=+\infty} \frac{-\tilde{A}_n \cdot \tilde{K}_n}{p_n} \, e^{j(p_n z + \tilde{K}_n \cdot \tilde{R} - \omega t)}$$

where

$$\tilde{K}_n = \tilde{K} + n\tilde{G}$$

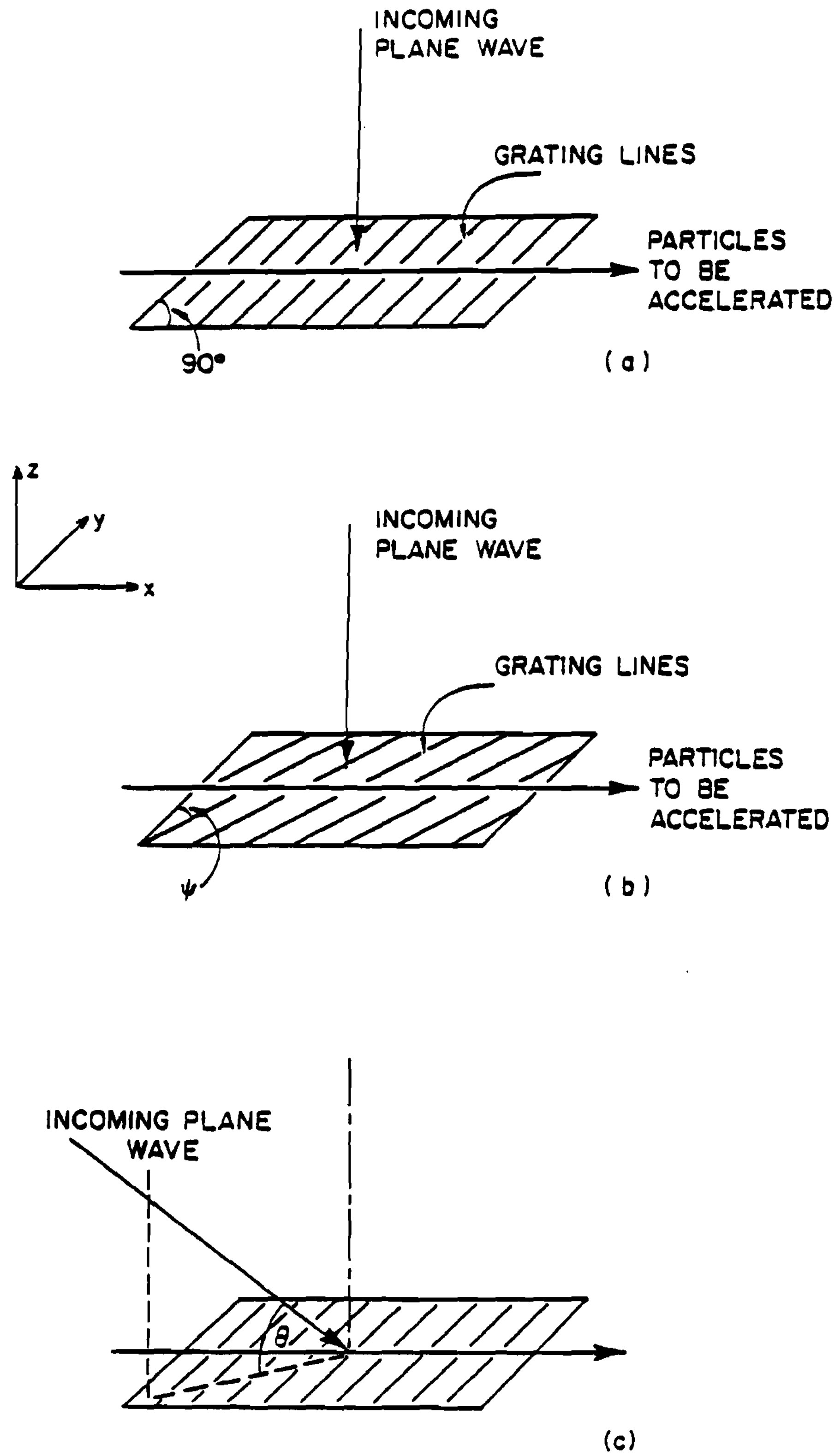


Fig. 1. Geometries of Grating Accelerators:
a) as proposed by Takeda and Matsui;
b) with skew grating to allow acceleration of relativistic particles;
c) with skew initial wave as alternative to b).

$$p_n = \pm \sqrt{-|\tilde{K}_n|^2 + k_o^2} \quad , \quad k_o = \frac{2\pi}{\lambda}$$

$$|\tilde{G}| = 2\pi/S \ .$$

The n is the order of diffracted waves. $\tilde{A}_n$ is a set of two dimensional complex vectors (A_x, A_y) describing the amplitudes of the modes polarized in the two directions. $\tilde{G}$ is a vector pointing along the surface perpendicular to the grating lines, and whose amplitude is as given. $\tilde{K}_n$ is a vector along the surface perpendicular to the wave fronts of the mode. $\tilde{K}$ is this vector for the incoming wave.

When $|\tilde{K}_n| < k_o$, then p_n is real and the mode is a free propagating wave either approaching (p_n negative) or leaving (p_n positive) the surface. Only the initial wave with n = 0 is incoming with p_n negative. All others have p_n positive and are the various diffracted modes. To distinguish between the amplitude of the single incoming (p negative) and outgoing (p_o positive) wave, the former will be given without subscript ($\tilde{A}$) and the latter with subscript $\tilde{A}_o$. The sum in Eq. (1) covers both the incoming $\tilde{A}$ and the set of outgoing waves A_n ($n = -\infty$ to $+\infty$). When $|\tilde{K}_n| > k_o$, the p_n is positive and complex and the mode is a surface of evanescent wave that falls off exponentially from the surface. The surface velocity of these waves have magnitude $c\ k_o/\tilde{K}_n$ and direction $\tilde{K}_n$. If the particle has a direction and velocity $\tilde{\beta}c$, then the condition that the particles remain in phase with a particular mode n is

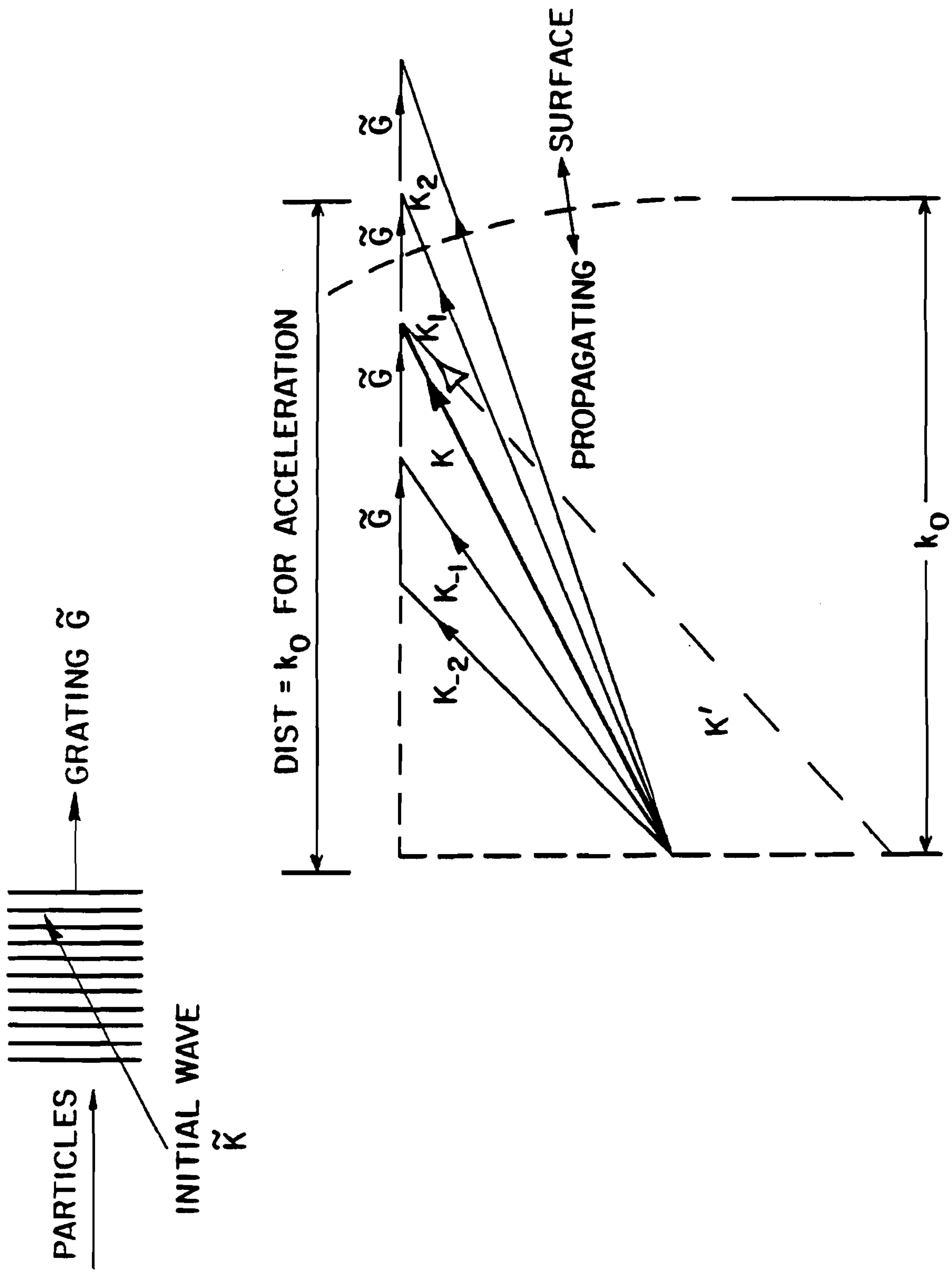
$$\tilde{K}_n \cdot \tilde{\beta} = k_o \ . \tag{2}$$

The case illustrated in Fig. 1c is when $\tilde{\beta} \parallel \tilde{G}$, i.e., when the particles are traveling perpendicular to the grating lines. For $\beta = 1$ this implies that the projection of the $\tilde{K}_n$ vector onto the vector $\tilde{G}$ have the length k_o. This conditions is shown in Fig. 2 for n = +1. We may now note that there is an infinite set of initial waves $\tilde{K}'$ whose first mode will satisfy the condition (2). It can then be shown that the angle ξ between such initial rays and the beam axis is given by

$$\beta \cos\xi = 1 - \frac{n\tilde{G} \cdot \tilde{\beta}}{k_o} \ . \tag{3}$$

The set of all such rays form a half cone about the beam axis analogous with a Cerenkov cone (see Fig. 3). This fact turns out to be very advantageous since the sum of all such waves will form a line image on the

Fig. 2. Graphical representation of diffracted waves (K_n) induced by a skew initial wave K.



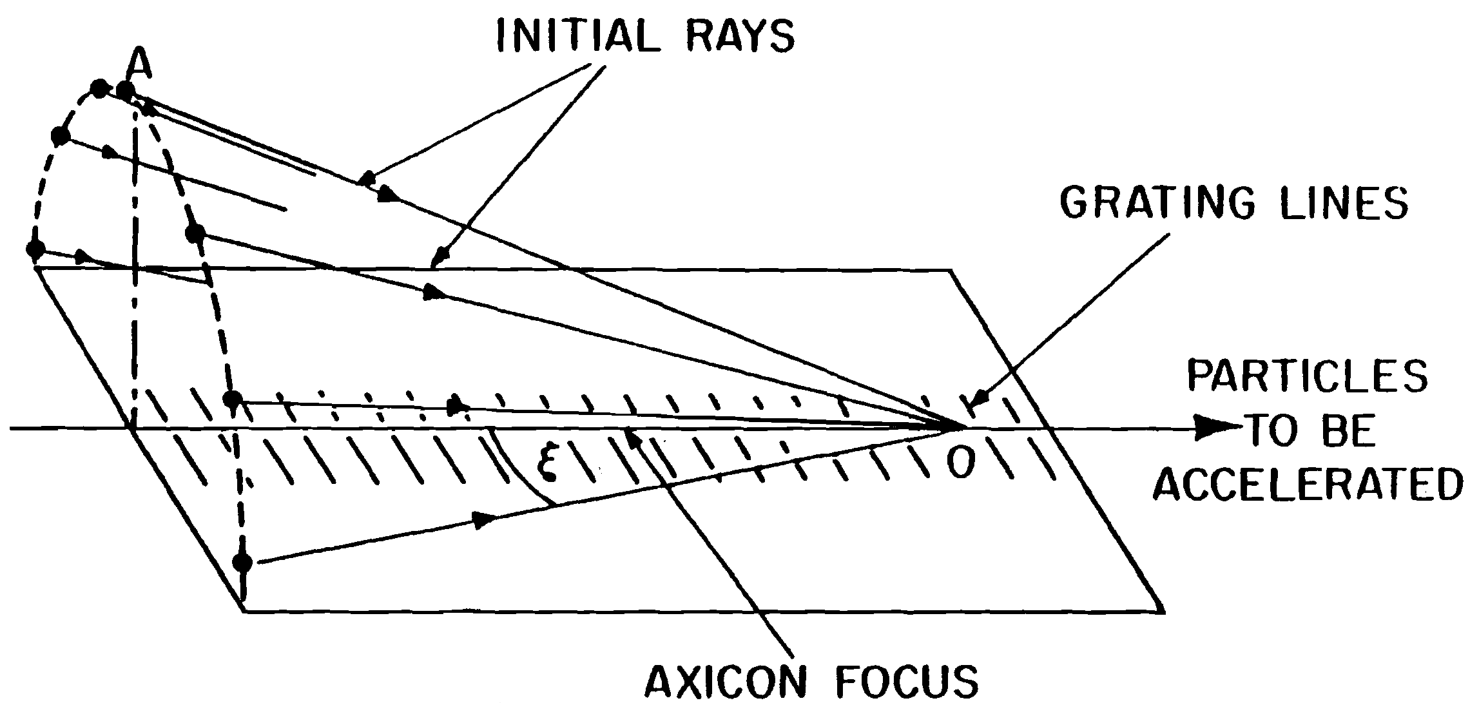


Fig. 3. General geometry of initial wave inducing acceleration of particles over a grating.

grating such that the direction of the line points along the particle direction. The narrowness of such a line image ($\sim \lambda$) will then assure the maximum local field for a given electromagnetic energy, and thus represents an efficient laser accelerator.

It should be noted that the one initial ray that is perpendicular to the grating lines (AO in Fig. 3) cannot, by Lawson's argument, induce acceleration. In practice rays near to this case would probably be omitted.

It remains now to determine the actual magnitude of the acceleration for given incident electromagnetic energy. This I will do for a particular case.

There are two fundamentally different approaches to obtaining numerical Eigen solutions to the fields above a surface boundary condition. The first and more common is to define the boundary and then adjust the amplitudes of all possible modes until the boundary condition is satisfied. An alternative that I will follow here is to pick a suitable combination of modes and then find the boundary condition (i.e., grating shape) that is consistent with the resulting fields. This approach is particularly easy if incident waves are chosen such that all resulting modes form standing waves. The field lines for these waves can then be drawn and any surface that is perpendicular to these lines is an acceptable shape for the grating.

For simplicity, I will restrict myself to special cases with the following character: the incoming rays will be chosen to be perpendicular to the grating lines. Such a case is illustrated in Figs. 4 and 5. The only variables in describing the incoming wave are its angle ϕ to the normal and its state of polarization, which will be taken to be in the beam direction (x), i.e.,

$$\tilde{K} \cdot \tilde{\beta} = 0 \quad , \quad \tilde{K} \cdot \tilde{G} = 0$$

$$\tilde{A} \cdot \tilde{B} = 0 \, . \tag{4a}$$

If we further require that the grating shape by symmetric with respect to a reversal of the beam direction then

$$\tilde{A}_n = \tilde{A}_{-n} \tag{4b}$$

and the number of free parameters is reduced by two. If we consider β = 1 and require acceleration for n = 1, then the condition for acceleration (Eq. 2) reduces to:

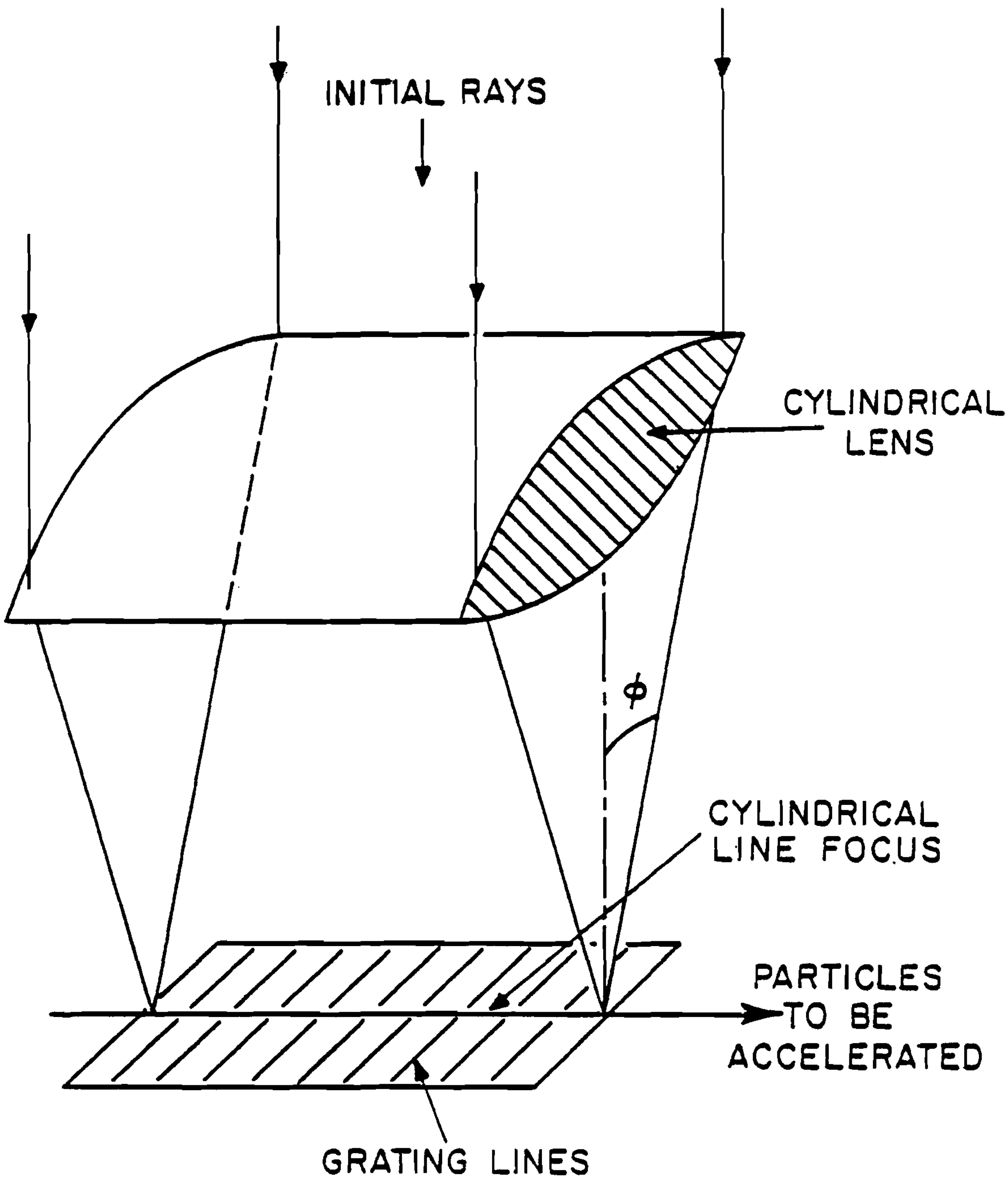
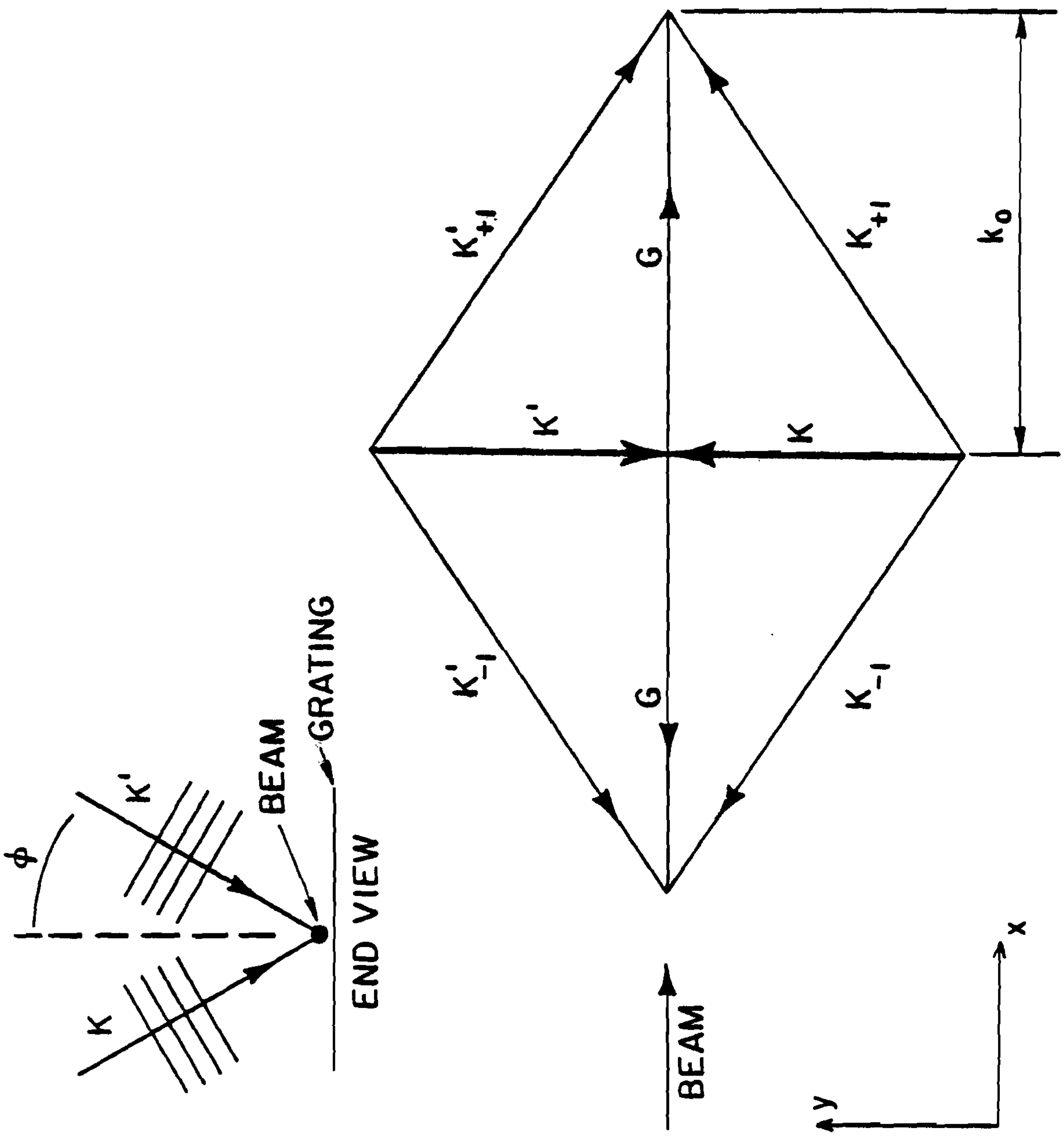


Fig. 4. Special case of the geometry shown in Fig. 3 where $\theta = 90^{\circ}$.

Fig. 5. Graphical representation of first diffracted waves (K_1) induced by initial waves in the geometry of Fig. 4.



$$n|\tilde{G}|\beta = k_o \tag{4c}$$

$$\text{or } S = \lambda\beta n$$

but $\beta = 1$, $n = 1$, so $S = \lambda$ where S is the grating spacing. Finally, I will consider fields induced by two equal and simultaneous incoming waves, A and A', whose angles to the normal ϕ and ϕ' are equal and opposite. Two sets of induced waves will then be present denoted by $\tilde{A}_n$ and $\tilde{A}'_n$ with the condition that:

$$\tilde{A}_n = \tilde{A}'_n \; . \tag{4d}$$

Condition 4c assures that all modes other than n = 0 are surface waves. Energy conservation thus requires that the amplitudes of the outgoing reflected waves (n = 0) be equal and opposite to the incoming waves:

$$\tilde{A}_o = -\tilde{A}, \; \tilde{A}'_o = -\tilde{A}' \; . \tag{4e}$$

With all these conditions (4a-e), the resulting fields are assured to be standing waves and the only variables are the set of scalar amplitudes A_n for n = 1 to ∞.

The fields are now given by:

$$E_x = \cos(\omega t)\cos(Ky)\left\{B\sin(pz) + \sum_{n=1}^{\infty} B_n e^{-q_n z}\cos(nk_o x)\right\}$$

$$E_y = 0 \tag{5}$$

$$E_z = -\cos(\omega t)\cos(Ky)\left\{0 + \sum_{n=1}^{\infty} B_n (nk_o/q_n) e^{-q_n z}\sin(nk_o x)\right\}$$

where

$$B = 4j|\tilde{A}|$$

$$B_n = 4|\tilde{A}_n|$$

$$p = \sqrt{k_o^2 - K^2}$$

$$K = |\tilde{K}|$$

$$q_n = jp_n = +\sqrt{K^2 + k_o^2 (n-1)^2}$$

Note $q_1 = K$.

B and B_n are now real numbers. All waves vary in the same way with both time and y position. Clearly maximum acceleration is obtained at y = 0 and at values of y spaced at intervals of $2\pi/K$. The first term inside the curly bracket is that due to the incoming and outgoing waves. It is only in the x direction, varies sinusoidally with distance above the grating, and is constant along the direction of acceleration. The second term in the brackets includes all the surface waves that fall off exponentially with height above the grating and vary periodically with position in x. The average acceleration of a particle traveling in the x direction at a height (h) above the surface depends only on the mode n = 1 and is

$$\left(\frac{dW}{dx}\right) = \frac{B_1}{2} e^{-Kh} . \tag{6}$$

It is convenient to express this mean accelerating field as a fraction (ε) of the peak field (B/2) that would be present in the absence of the grating. Thus

$$\begin{aligned} \left(\frac{dW}{dx}\right) &= \varepsilon \frac{B}{2} \\ \varepsilon &= \frac{B_1}{B} e^{-Kh} \end{aligned} \tag{7}$$

All fields vary in the same way with y. Thus, if a line is found that is perpendicular to the fields at one y, the same line will be perpendicular at all other values of y. In other words, we will have found a surface with a cross section independent of y, i.e., a grating. It remains then to consider some individual cases, examine the pattern of x,z fields at y = 0 and find lines perpendicular to these fields, thus defining Eigen solutions to the problem.

We are searching for a solution in which the ratio of the accelerating mode to the incoming mode is as large as possible. It is relevant, therefore, to ask why this ratio cannot be infinite. In other words, ask whether there is an Eigen solution with surface modes, including an accelerating mode, and no free propagating waves at all. In such a solution the grating is behaving like a cavity containing accelerating fields which would, if there were no losses, remain indefinitely without the application of any external field. First we can examine the accelerating mode (n = 1) alone. This is shown in Fig 6b. Any surface perpendicular to these field lines contain cuts that extend to infinite depth; clearly not a practical solution. If, however, we add the mode (n = 3) with opposite phase, then at once a solution becomes possible. Consider for instance $K = 0.2\ k_o$, $B_3 = -0.025\ B_1$. The field pattern obtained is shown in Fig. 6c where a surface perpendicular to all lines is indicated. This

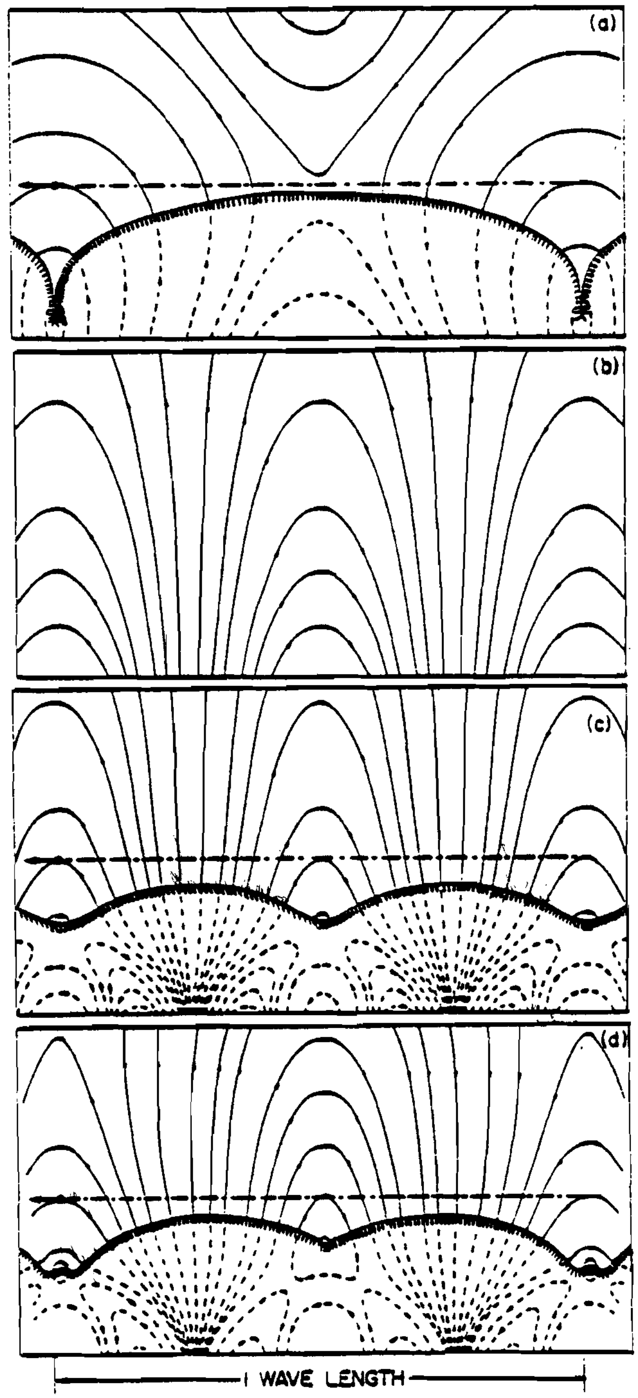


Fig. 6. The electric field patterns produced by different combinations of modes, together with the shape of the grating surfaces that will support these combinations: a) case with initial wave (n=0) and the accelerating modes (n=±1) only; b) field lines for the accelerating (n=±1) modes alone, there is no grating surface that will support this mode alone; c) case with accelerating mode (n=±1) and a small addition of the third mode (n=0), a strong accelerating mode (n=±1) and a small addition of the third mode (n=±3), this solution couples to the initial wave and provides good acceleration.

then is a solution which, if excited, would accelerate and would remain without radiating away its energy. However, since it is not coupled to the incoming wave, it would in fact not be excited in the first place. What is required is a solution similar to the above but with a small admixture of the incoming mode. For instance $K = 0.2\ k_o$, $B_3 = -0.025\ B_1$, $B = -0.5\ B_1$, which gives the field pattern of Fig. 4d. It may be noted that the uncoupled grating, Fig. 6c, had periodicity with half the wave length, and it can be shown that such a periodicity cannot couple to the incoming waves. This new solution is similar but has a small component of one wave length periodicity. It is this component that provides the coupling.

The acceleration at the surface of this grating is given by $\varepsilon = 5.0$ and is thus considerably larger than the peak field present without the grating! This result is not surprising when compared to a conventional accelerating RF cavity. If the "Q" of the cavity is higher, then the accelerating fields for given RF power are also higher. The realizable accelerating field is set when the losses in the cavity approach the RF power applied.

In the grating case the losses at the surface can be calculated if they are due purely to resistive effects. They are then given approximately by

$$f = \frac{S_{losses}}{S_{incoming}} \approx \left(\frac{k_o}{K}\,\varepsilon\right)^2 \cdot \frac{1}{4}\left(\frac{c}{\lambda\sigma}\right)^{1/2} . \qquad (8)$$

For a copper grating ($\sigma = 1/1.5\ 10^{-6}\ \Omega\ \mathrm{cm} \equiv 6\ 10^{17}\ \mathrm{sec}^{-1}$), wave length of 10μ, $K = 0.2\ k_o$, and $\varepsilon = 5.0$, we obtain the fractional loss $f \equiv 100\%$. Thus the value of $\varepsilon = 5$ represents the highest value possible. It would be more realistic to limit the fractional loss to approximately 25% and thus ε to 2.5. This value will be used for the following examples.

3. PRACTICAL CONSIDERATIONS

3.1 Grating Survival

Two quite different limits must be considered here. Firstly: up to what power level will the grating survive such that it can be used for subsequent pulses. Secondly: up to what power level will the grating survive in the sense that acceleration will still occur above its surface. The second limit is appropriate for a grating whereas quite inappropriate for a conventional LINAC. Only a narrow band (~ 25μ wide) would be destroyed and in fact only a layer of the order of a micron thick would be evaporated. For R&D purposes the grating could be displaced between pulses and eventually replaced. For a real accelerator, it is possible that one could use ripples on a liquid metal surface such as mercury or potassium or allow a thin surface layer (e.g., of evaporated ice) to be blown off a permanent copper base and replaced between pulses.

The limits clearly depend, and the pulse duration as well, on the instantaneous power level and frequency. If a CO_2 LASER were employed, then the shortest pulse obtainable would be about 30 psec and for such a pulse the first "few pulse" limit corresponds to an acceleration of only 300 MeV/meter, (based on a surface heating calculation). The second "one pulse" limit is far harder to estimate. A calculation on plasma growth by P. Channell[7] suggests a limit on acceleration of 10 GeV/meter. Experiment is required to confirm this, but I will use these numbers for the subsequent discussion. It is interesting to note that we are here accelerating in the presence of a periodic plasma. Allowing the surface of a metal grating to be destroyed is only one way of producing such a periodic plasma. It may eventually be that other methods are more practical and this proposal may become more and more similar to the beat wave accelerator.

3.2 Power Requirements

The total electromagnetic energy in the 10 km of grating "cavity" is only 200 joules. The reason it is so small is simple: despite the high fields present the volume is tiny (about 1 cc!). The optimum "fill time" is equal to the field decay time in the cavity and this, assuming the resistive losses of copper is 0.3 psec. Unfortunately, we do not know how to make a laser with an pulse length of 0.3 psec. If a 30 psec pulse is used the laser energy required is 20,000 joules, but even this is a very low value compared with conventional cavities with such high fields. It may also be that the losses above a plasma surface are in fact lower than those calculated for a copper grating and the fill time will be longer and the total laser energy less.

3.3 Injection

One of the objections raised to such laser driven accelerators was that the phase space accepted was so small that a negligible number of particles could be accelerated. This appears not, in fact, to be the case when the phase space density of the proposed SLAC single pass collider is considered. For our example we will assume that the specific emittance ε/N is the same as that in the SLC proposal.

3.4 Stability and Focussing

Kroll and Kim[8] have shown that horizontal stability can be obtained if the phase of the grating is alternately advanced and retarded with respect to the bunches.

Longitudinal stability is not obtained in this example but is found not to be needed since the synchrotron wave length is longer than the entire accelerator. A special buncher would be required at the front end that would employ magnetic fields to lower the synchrotron wave length.

3.5 Beam Loading

P. Wilson and M. Tigner[9] concluded that for single bunch operation the fraction of electromagnetic energy that could be transferred to the bunch would be similar to that in a conventional linac (i.e., about 5-10%). All wake field effects being independent of wave length. In practice however it seems more reasonable to operate the grating accelerator in a multibunch mode for which even higher efficiencies may be possible. A debuncher consisting of a single magnetic wiggle would be used prior to the final focus.

3.6 Luminosity

It is clear that since the total laser energy per pulse is small, yet the energy achieved is high, that the number of particles accelerated per pulse is small (of the order of 10^8) and it might be thought that the resulting luminosities obtained must be low. This is not obviously the case.

Technical problems aside, luminosities will probably be limited by the power consumption. From the Les Diablerets meeting[10] we have:

Luminosity $$L = \frac{1}{4\pi} \frac{N^2 f}{\sigma^2} k \qquad (9a)$$

Power $$P = E_e \, f \, \gamma \, N \, \eta \qquad (9b)$$

Disruption $$D = \frac{r_o}{4} \frac{d\ N}{r\ \sigma^2} \qquad (9c)$$

Beamstrahlung $$\delta = r_o^{\ 3} \frac{2}{3\sqrt{3}} \frac{r\ N^2}{d\ \sigma^2} \qquad (9d)$$

emittance $$\varepsilon = \frac{\sigma^2 \gamma}{\beta^*} \qquad (9e)$$

where

- k = luminosity enhancement due to beam beam interaction
- f = bunch repetition rate
- γ = final beam gamma
- N = particles/bunch
- σ = beam diameters at intersection
- d = bunch length
- β^* = focus parameter
- E_e = rest energy of the electron
- η = efficiency of energy transfer to the beam
- r_o = classical electron radius

and I assume equal horizontal and vertical σ.

These expressions can be rearranged to give the power vs. luminosity relation in terms of the spot size σ eliminating f, N and d; these values then also being given in terms of σ:

$$\frac{P}{L} = 4\pi\ E_e \left\{\frac{r_o^{\ 4}}{6\sqrt{3}}\right\}^{1/3} \frac{\gamma\sigma^{2/3}}{(D\delta)^{1/3}} \frac{1}{k\eta} \qquad (10a)$$

$$f = 4\pi \left\{\frac{r^4}{6\sqrt{3}}\right\}^{2/3} \frac{L}{(D\delta)^{2/3}\ \sigma^{2/3}\ k} \qquad (10b)$$

$$N = \left\{\frac{r_o^{\ 4}}{6\sqrt{3}}\right\}^{-1/3} (D\delta)^{1/3}\ \sigma^{4/3} \qquad (10c)$$

$$d = \left(\frac{32\ r_o}{3\sqrt{3}}\right)^{1/3} \frac{D^{2/3}}{\delta^{1/3}} \gamma\ \sigma^{2/3} \qquad (10d)$$

$$\frac{\varepsilon}{N} = \frac{r_o}{4} \frac{1}{D} \frac{d}{\beta^*} \qquad (10e)$$

We see from 10a that the power can be reduced for fixed luminosity without limit so long as σ can be reduced. As σ is reduced we see that 1) f the pulse frequency goes up, 2) the bunch length d goes down, 3) the specific emittance stays constant and finally, 4) the number of particles per bunch N goes down. We can thus write:

$$\frac{P}{L} \propto \gamma\ N^{1/2}$$

i.e., the power is reduced if the bunches have small members of particles and a conventional wavelength Linac becomes inappropriate.

I now give the parameters of a very hypothetical 50 + 50 TeV grating laser accelerator

Energy		2 × 50 TeV
Length		2 × 5 km
Gradient		10 GeV/m
Specific emittance		3 10^{-12} radian meters/particle
Disruption	D	2.3
Luminosity enhancement	k	5
Beamstrahlung	δ	0.1
Number of particles/bunch	N	3.2 10^7
Spot Size	σ	10 A° = 10^{-7} cm
Bunch length	d	1 cm
Focus	β^*	1 cm
Frequency	f	24 Khz
Beam power	P	2 × 6 10^6 watts
Luminosity	L	10^{33} cm^{-2} sec^{-1}

The beam power of 12 MW implies that a very high efficiency in the laser and acceleration mechanism are required if the luminosity of 10^{33} is to be obtained for reasonable total power consumption.

Higher luminosities or lower power would be achieved if: 1) D could be increased. This, from Eq. (10e) is only possible with a decrease in the specific emittance. 2) if flat beams or charge cancellation is employed. 3) if σ is still further reduced with a consequent reduction in d and the requirement of even shorter laser pulse lengths. Note incidentally that the use of flat beams also implies a reduction in d as can be seen by the relation:

$$\frac{P}{L} = \pi\ E_e\ r_o\ \frac{d}{Dk\eta}$$

Note: d is in any case much larger than the wave length and contains many individual RF bunches.

The question of whether a 10 A° spot is realistic remains to be answered. The β^* is not unreasonable in itself nor is the ratio of this spot size to the wave length. The beam size in the accelerator itself is only 500 A° at 50 TeV if grating errors do not blow up the emittance. One should not therefore reject such a spot size out of hand.

CONCLUSION

I conclude that acceleration in fields generated over a grating surface is possible and that very high gradients may be feasible. More detailed study including experimental tests should be carried out, but unless such work reveals a major flow the proposed scheme should be taken as a serious candidate for the next generation of accelerators.

REFERENCES

1. K. Shimoda, "Proposal for an Electron Accelerator using an Optical Maser", Applied Optics 1, 33, (1962).
2. E.M. McMillan, Phys. Rev. 79, 498 (1950); Yau Wa Chan, Phys. Lett. 35A, 305 (1971).
3. For axial fields:
   A.A. Kolomenskii, A.N. Lebedev, Dokl. Akad. Nauk., SSSR 145, 1259 (1962) [Sov. Phys. Doklady 7, 745 (1963)].
   W.B. Colson, S.K. Ride, Applied Physics 20, 61 (1979).
   For helical or "wiggler" fields:
   R.B. Palmer, J. Appl. Phys., 43, 3014 (1972).
4. Y. Takeda and I. Matsui, "Laser Linac with Grating", Nucl. Instr. and Meth. 62, 306 (1968).
5. S.J. Smith, E.M. Purcell, "Visible Light from Localized Surface Charges Moving across a Grating", Phys. Rev. 92, 1069 (1953).
6. J.D. Lawson, "Laser Accelerators?" Rutherford Lab. Report RL-75-043, (1975).
7. P. Channell, "Laser Acceleration of Particles" (Proceedings of Los Alamos Workshop), AIP Conference Proceedings No.91, p244 (1982).
8. K-J. Kim and N.M. Keil, as ref 7, p140.
9. P.B. Wilson, as ref 7, p199.
10. E. Keil et al, Proc. 2nd ICFA Workshop on Possibilities and Limitations of Accelerators and Detectors, Les Diablerets, (ed. U.Amaldi) p3.

DISCUSSION

Hand. I am worried about the problem of tolerances; one is talking about 1/10 micron in a kilometre, and the earth is not stable to that level. SLAC did a study showing misalignments of order of a micron when a truck goes by.

Palmer. You can periodically rebunch the beam, as in an FEL. Putting 'wiggles' in can give the required longitudinal softness. The accelerator would be built on granite slabs, as long as possible, with a re-buncher on each. Another possibility is to have the granite blocks on long period vibration mountings and servo their position using a master laser. One can hold a fraction of a micron at such distances, but this is expensive at the moment.

Participant. What sort of densities in transverse phase space do you require for your example where you have a luminosity of 10^{31}?

Palmer. They are the same order as at SLAC. They talk about getting a beam of diameter about 1 micron with β of about 5 mm. We need ε about ten times smaller, but 10^8 particles instead of 10^{11}, so a collimator alone would suffice. The value of ε/n is smaller than at SLAC.