NEUTRINO OSCILLATION SEARCH WITH COSMIC RAY NEUTRINOS

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Abstract

It is shown that a sensitive search for neutrino oscillations involving more flavors than just ν_e and ν_μ is provided by measurement of the ratio of the total interaction rates of upward- and downward-going cosmic ray neutrinos within a massive (~10 kiloton) detector. Assuming mixing between all pairs of ν_e, ν_μ and ν_τ, the experiment is capable of observing time averaged probabilities $\langle P_{e\tau}\rangle_t$ and $\langle P_{\mu\tau}\rangle_t$ of magnitude set by mixing strengths corresponding to, e.g., the d- to s-quark mixing strength, and of reaching the limit $\Delta m^2_{ij} \equiv |m_i^2-m_j^2| \approx 10^{-4}$ eV^2, where m_i, m_j are neutrino mass eigenstates, and $P_{e\tau}$ and $P_{\mu\tau}$ are the probabilities for ν_e and ν_μ, respectively, to oscillate into ν_τ after traversing a distance L $\approx$ diameter of the earth.

Introduction

Use of detectors in deep mines and under the sea to search for neutrino flavor oscillations[1] employing the atmospheric, i.e., cosmic ray, neutrino flux has been discussed extensively in recent years.[2] The idea is attractive because of the small value of neutrino mass difference that may be explored by upward-going neutrinos which traverse the earth (see Fig. 1) after their production in the atmosphere. Thus, for a full wavelength oscillation

$$\begin{aligned} \Delta m^2(\text{eV}^2) &\equiv |m_1^2-m_2^2| \approx 2.5\langle E_\nu(\text{MeV})\rangle/L(\text{meter}) \\ \Delta m^2 &\approx 2.5\times 300\ \text{MeV}/1.3\times 10^7\text{m} \le 10^{-4}\ \text{eV}^2 \quad , \end{aligned} \tag{1}$$

where m_1 and m_2 represent neutrino mass eigenstates, $\langle E\ (\text{MeV})\rangle$ is the average incident neutrino energy, and $L = 1.3\times 10^7$m is the difference in distance ($\approx$diameter of the earth) traversed by the upward- and downward-going neutrinos.

It is of interest to note that there is an additional contribution to the relative phases between different neutrino mass eigenstates besides that due to the neutrino masses and mixing, namely that due to the different forward scattering amplitudes and resultant different indices of refraction of different neutrino flavors[3]. This arises because the upward-going neutrinos traverse a large amount of matter of substantial density. The characteristic wavelength describing these matter-induced oscillations is independent of the neutrino energy E_ν and given by $\ell_o = \sqrt{2}\pi/(G_F N_e) = 9.7\times 10^{32}\,\text{cm}^{-2}/N_e$, where $N_e = \rho_E(Z/A)N_{\text{Avogadro}}$ denotes the number density of electrons. For the earth, ρ_E varies from ~2 gm/cm^3 in the mantle to ~5 gm/cm^3 in the core; taking average values, $\rho_E \approx 3$ gm/cm^3 and $Z/A \approx 0.5$, we obtain $\ell_o \approx 1.1\times 10^7$ m $\simeq D_E$, where D_E is the diameter of the earth. Oscillations due to neutrino masses and mixing alone ("vacuum oscillations") with wavelengths $\ell_{vac} = 4\pi E_\nu/|\Delta m_\nu^2| >> \ell_o$ would therefore be suppressed. However, this condition does not reduce the sensitivity to small Δm^2 of the experiment described here. Since $\ell_o \approx D_E$, it follows that if $\ell_{vac} >> \ell_o$, then also $\ell_{vac} >> D_E$, so that the experiment would not have been able to detect the vacuum oscillations anyway. If $\ell_{vac} \approx \ell_o$, the matter-induced oscillations can even enhance the effect for either ν or $\bar{\nu}$. Thus, we find that a neutrino oscillation experiment using cosmic ray neutrinos is indeed sensitive to $|\Delta m^2| \gtrsim 10^{-4}$ eV^2.

For comparison we show in Fig. 2 a summary of the limits set by present and anticipated neutrino oscillation data using accelerator and reactor neutrinos. One sees that the value of Δm^2 in eq. (1) is roughly two orders of magnitude smaller than the lowest limiting value of Δm^2 obtained from present neutrino oscillation experiments in which a search is made with reactor produced $\bar{\nu}_e$ for the "disappearance" of a fraction of the incident $\bar{\nu}_e$ beam, i.e., $\nu_e \to \bar{\nu}_\mu, \bar{\nu}_\tau, \ldots$. In addition, as will be seen, the cosmic ray neutrino oscillation search is capable of measuring strenghts of mixing between neutrino flavors about equal to that obtainable in reactor experiments, viz, similar in magnitude to the mixing between d- and s-quarks. It is of particular importance that the cosmic ray oscillation search is done primarily with ν_μ and $\bar{\nu}_\mu$ (see below) because, to our knowledge, no other neutrino oscillation experiment searching for the disappearance of ν_μ or $\bar{\nu}_\mu$ is capable of reaching such a low value of Δm^2.

The primary obstacle to realizing the sensitivity represented in eq. (1) is the low flux of atmospheric neutrinos, for which the calculated spectra[4] are shown in Fig. 3. There is an uncertainty of about a factor of two in the absolute values of the curves in Fig. 3; the rapid fall-off of flux with increasing neutrino energy is, however, reproduced in all calculations.

One type of experiment attempts to overcome the low flux obstacle by looking at upward-going muons produced by ν_μ in the large volume of rock within muon range of a buried detector. The differential neutrino flux is slightly steeper than E_ν^{-3}. The muon range R_μ is, however, proportional to $E_\mu \equiv E_\nu(1-y)$, where $y = E_H/E_\nu$ and E_H is the energy of the hadrons produced

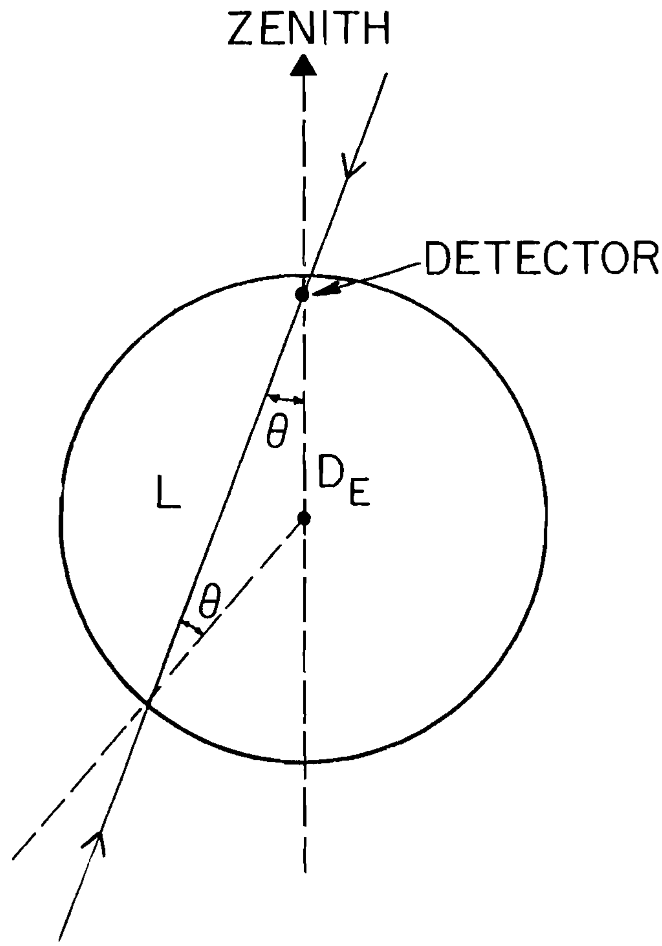

Fig. 1. Sketch of the experimental method. The neutrino detector is located as indicated roughly 1-2 km below the earth's surface. Neutrinos originate in the 10-20 km thick atmospheric shell surrounding the earth. Neutrinos from near the zenith that intersect little of the earth's matter before interacting in the detector are called down-going, N(dn). Neutrinos that have traversed a large fraction of the earth's diameter ($D_E = 1.3 \times 10^7$m) and are observed to produce upward-going interactions in the detector are called up-going, N(up). Present accelerator limits on neutrino oscillations suggest that oscillations should have a negligible effect on the down-going atmospheric neutrino flux.

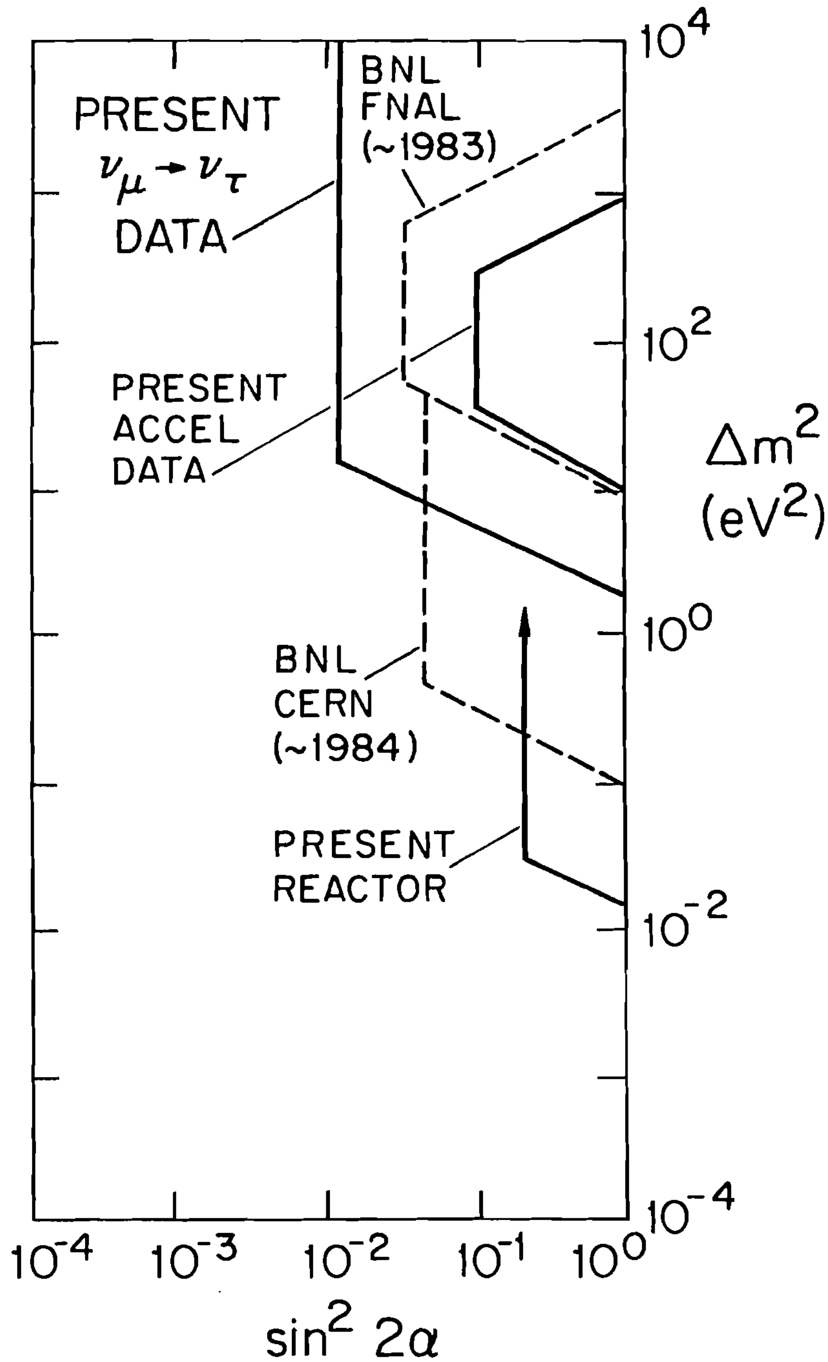

Fig. 2. Plot of the generalized mass parameter $\Delta m^2 (eV^2)$ vs the generalized mixing parameter $\sin^2 2\alpha$ showing approximate limits of present and expected neutrino oscillation data from experiments that measure the disappearance of neutrinos of a given flavor from an incident beam. Also shown is the present limit on ν_μ-ν_τ oscillations. This plot is taken from a paper by R. E. Lanou and R. E. Shrock in these Proceedings.

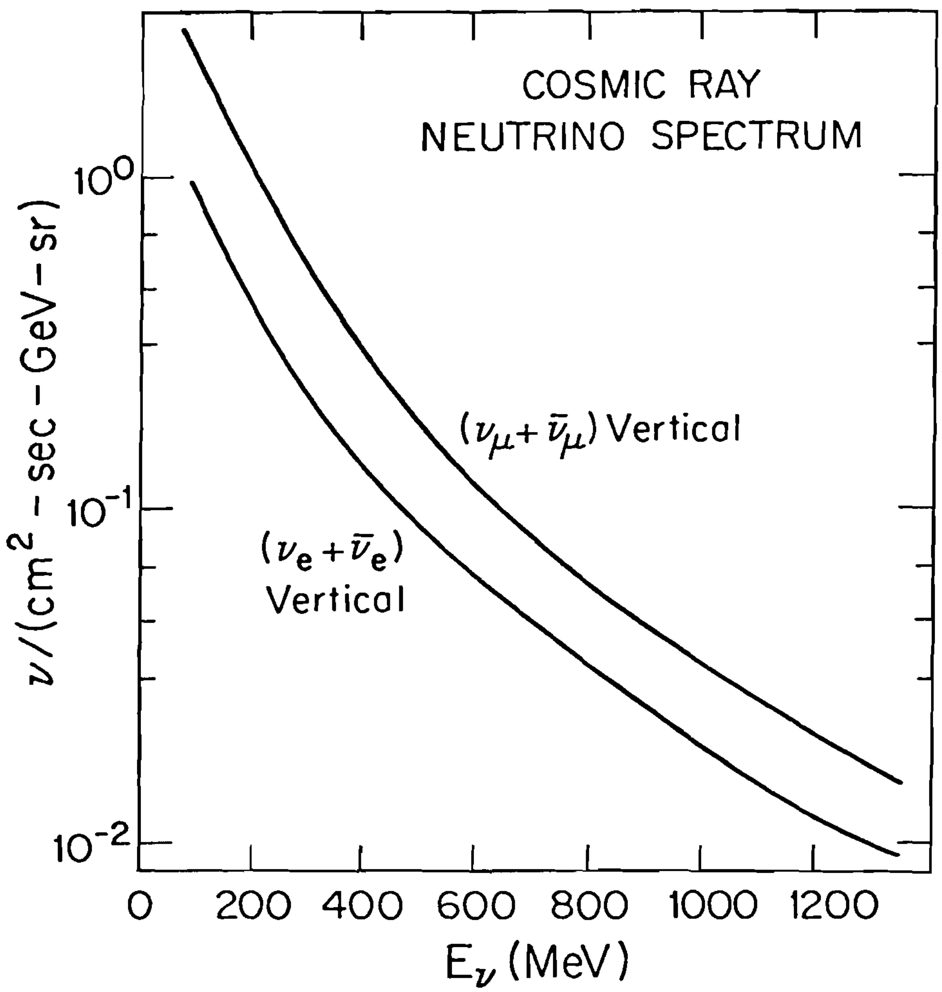

Fig. 3. Calculated cosmic ray neutrino spectra from reference 4.

in charged current interactions. Moreover, σ_ν is approximately proportional to E_ν. Hence, the differential spectrum of parent neutrinos of the observed upward-going muons is nearly proportional to E_ν^{-1} up to the TeV range where both R_μ and σ_ν cut off. This means that a very large interval of neutrino energies contributes to the muons actually observed. Consequently, the relevant neutrino energies in such an experiment are significantly higher than that in eq. (1). Preliminary results of this type of experiment have been reported recently which are consistent with no oscillation[5,6]. This conclusion depends, however, on a calculation that folds the primary cosmic ray spectrum into meson production cross sections, meson and muon decay distributions, neutrino interaction cross sections, the muon range-energy relation, and the detector acceptance, to produce an absolute value of the expected muon rate which is compared with the observed rate. A deficiency of observed ν_μ interactions would signal neutrino oscillations.

Because of the large systematic uncertainties inherent in this experimental method, it seems that observation of a significant flux of upward-going muons within a factor of about two of the expected flux has already exploited this technique as fully as possible. It is possible that the uncertainties of normalization mentioned above could be largely removed by a measurement of the angular dependence of muons arriving from below the horizon. Because of the much higher neutrino energies, however, the Δm^2 range probed would be significantly larger than in eq. (1) and correspondingly less interesting.

To obtain a conclusive result on neutrino oscillations using cosmic ray neutrinos, it is necessary to observe during an appreciable time interval interactions that occur in and are contained in a massive detector capable of event reconstruction from which the upward and downward neutrino fluxes can be extracted without ambiguity. This measurement will become possible in the next few years with a generation of detectors aimed principally at searching for nucleon decay[7], and, indeed, significantly increases the physics potential of such detectors. In this paper we discuss the nature, possible outcomes, and limitations of a class of searches for neutrino oscillations using cosmic ray neutrinos in which statistical and systematic errors are small enough to allow the sensitivity indicated in eq. (1) to be realized.

Discussion

Comparison of $(N_e/N_\mu)_{up}$ and $(N_e/N_\mu)_{down}$

In principle, it is desirable in such an experiment to measure the interaction rates of both neutrino types, ν_μ and ν_e (and $\bar{\nu}_\mu$ and $\bar{\nu}_e$), over all solid angles in the detector, and to compare the ratios $N_e(\text{up})/N_\mu(\text{up})$ and $N_e(\text{dn})/N_\mu(\text{dn})$. This procedure for atmospheric neutrinos is, however, severely limited statistically, as we show later. Furthermore, there is a serious systematic limitation to an experiment based on measurements of $(N_e/N_\mu)_{up}$ and $(N_e/N_\mu)_{down}$. Precise identification of the incident neutrino type is made difficult by low energy events in ν_μ-induced neutral current channels, e.g., $\nu_\mu + n \to \nu_\mu + n + \pi^0$, $\nu_\mu + p \to \nu_\mu + p + \pi^0$, as well as in charged current channels, $\nu_\mu + n \to \mu^- + p + \pi^0$, in which the muon range is very short. Even in a relatively sophisticated massive detector the distinction between the electromagnetic shower produced by an electron and a photon can be accomplished only with limited efficiency (roughly one photon in five is likely to be misidentified as an electron). For these reasons, both statistical and systematic, a measurement based on comparison of the ratios $(N_e/N_\mu)_{up}$ and $(N_e/N_\mu)_{down}$ is unlikely to yield a definitive result.

Comparison of $N_{tot}(\text{up})$ and $N_{tot}(\text{down})$

On the other hand, measurement and comparison of the total rates, $N_{tot}(\text{up}) = N_e(\text{up}) + N_\mu(\text{up})$ and $N_{tot}(\text{dn}) = N_e(\text{dn}) + N_\mu(\text{dn})$, has the following advantages: (i) The comparison can be made with sufficient statistical significance to allow relatively small oscillation probabilities to be observed, and (ii) the systematic uncertainties are inherently small and tractable. Thus, using the fluxes shown in Fig. 3 in the energy interval from 200 to 600 MeV, and assuming a ten kiloton fiducial mass detector exposed for one year, we find the total number of observed downward-going neutrino interactions within the angular interval $\theta_{zenith} \leq 60^\circ$ to be about 300 per year, assuming a solid angle of π sr.[8] In the comparison of $N_{tot}(\text{up})$ with $N_{tot}(\text{dn})$ precise identification of the incident neutrino type is unnecessary, thereby eliminating a principal source of systematic error that is present in comparing the ratios $N_e(\text{up})/N_\mu(\text{up})$ and $N_e(\text{dn})/N_\mu(\text{dn})$.

A straightforward geometrical construction (see Fig. 1) suffices to show that the upward and downward geometries are symmetric if the detector itself is up-down symmetric. For every downward-going neutrino of zenith angle in the interval θ to $\theta + d\theta$ there is a corresponding upward-going neutrino in the same solid angle. For each neutrino type, there is an effective, angular dependent height of production $h(\theta)$ above the level of the detector measured along the extension of the chord of length L. The acceptance a of the detector of area A is for downward-going neutrinos

$$a_{dn} = 2\pi h^2 \sin\theta d\theta (A/h^2)$$

and for upward-going neutrinos

$$a_{up} = 2\pi (h+L)^2 \sin\theta d\theta \left[A/(h+L)^2\right] .$$

Since $a_{up} = a_{dn}$ for each θ the geometry is completely symmetric, even if the zenith angle and energy dependences of ν_e and ν_μ are significantly different.

Illustrative Cases of Possible Atmospheric Neutrino Oscillations

We proceed now to consider several illustrative cases of possible neutrino oscillations that might be observed in such an experiment.

1) $\nu_\mu(\bar{\nu}_\mu) \overset{\leftarrow}{\to} \nu_e(\bar{\nu}_e)$ only. Assuming the flux of downward-going neutrinos is unaffected by oscillations, we can write immediately

$$\begin{aligned} N_e(\text{up}) &= N_\mu(\text{dn})P_{\mu e} + N_e(\text{dn})(1-P_{e\mu}) , \\ N_\mu(\text{up}) &= N_e(\text{dn})P_{e\mu} + N_\mu(\text{dn})(1-P_{\mu e}) , \end{aligned} \tag{2}$$

where $N_e(\text{up})$, $N_\mu(\text{up})$ are the numbers of upward-going $(\nu_e + \bar{\nu}_e)$- and $(\nu_\mu + \bar{\nu}_\mu)$- induced events observed in the detector; $N_e(\text{dn})$, $N_\mu(\text{dn})$ are the corresponding numbers of downward-going events observed in the detector, assumed to be equal to the number of N_e and N_μ produced by unmixed neutrinos originating in the atmospheric shell about the earth; and, e.g., $P_{e\mu} \equiv P(\nu_\mu, L|\nu_e, 0)$ is the probability that a ν_μ is present at distance L when a ν_e was present at the flux origin. Assuming

CP-invariance, eqs. (2) yield directly

$$N_{tot}(up) = N_{tot}(dn) \quad . \tag{3}$$

It follows then that there are two possible explanations for the result in eq. (3), if it is observed: either (a) there are no oscillations between ν_e and ν_μ within experimental error, i.e., $P(\nu_\mu,L|\nu_e,0) = P(\nu_e,L|\nu_\mu,0) << 1$, or (b) oscillations do occur only between ν_e and ν_μ but in such a way as to yield eq. (3). In this case only the difference between $(N_e/N_\mu)_{up}$ and $(N_e/N_\mu)_{dn}$ can demonstrate the existence of oscillations.

It is instructive to note, however, that in alternative (b), if time-averaging of $P_{e\mu} = \sin^2 2\alpha \sin^2 1.27L\Delta m^2/E_\nu$ is appropriate, then eqs. (2) become

$$\begin{aligned} N_e(up) &= [N_\mu(dn) - N_e(dn)]\tfrac{1}{2}\sin^2 2\alpha + N_e(dn) \quad , \\ N_\mu(up) &= [N_e(dn) - N_\mu(dn)]\tfrac{1}{2}\sin^2 2\alpha + N_\mu(dn) \quad , \end{aligned} \tag{4}$$

where $\sin^2 2\alpha$ is the strength of mixing between ν_e and ν_μ. From the reactor data in Fig. 2, it appears likely that $\sin^2 2\alpha \lesssim 0.1$; using that limit and expected values of $N_e(dn)$ and $N_\mu(dn)$ for a 10 kiloton detector[8], we find $N_e(up)/N_\mu(up) = 0.60$, to be compared with $N_e(dn)/N_\mu(dn) = 0.56$. The difference is less than the statistical error on either of the ratios, which bears out the earlier assertion of the improbability of reaching a definitive conclusion from measurements of the ratios N_e/N_μ.

2) $\nu_e(\bar\nu_e) \leftrightarrows \nu_\mu(\bar\nu_\mu)$, $\nu_\tau(\bar\nu_\tau)$; $\nu_\mu(\bar\nu_\mu) \leftrightarrows \nu_e(\bar\nu_e)$, $\nu_\tau(\bar\nu_\tau)$.

Again, if the flux of downward-going neutrinos is unaffected by oscillations, we may write directly

$$\begin{aligned} N_e(up) &= N_\mu(dn)\langle P_{\mu e}\rangle_t + N_e(dn)\left[1 - \langle P_{e\mu}\rangle_t - \langle P_{e\tau}\rangle_t\right] \\ N_\mu(up) &= N_e(dn)\langle P_{e\mu}\rangle_t + N_\mu(dn)\left[1 - \langle P_{\mu e}\rangle_t - \langle P_{\mu\tau}\rangle_t\right] \end{aligned} \tag{5}$$

or

$$N_{tot}(up) = N_{tot}(dn) - N_e(dn)\langle P_{e\tau}\rangle_t - N_\mu(dn)\langle P_{\mu\tau}\rangle_t \tag{6}$$

where $\langle P_{e\tau}\rangle_t \equiv \langle P(\nu_\tau,L|\nu_e,0)\rangle_{\text{time average}}$; $\langle P_{\mu e}\rangle_t \equiv \langle P(\nu_e,L|\nu_\mu,0)\rangle_{\text{time average}}$, etc. Eq. (6) shows, as expected, that the only oscillations which contribute to the diminution of the sum of the incident $\nu_e(\bar\nu_e)$ and $\nu_\mu(\bar\nu_\mu)$ fluxes are those to $\nu_\tau(\bar\nu_\tau)$, and consequently comparison of $N_{tot}(up)$ and $N_{tot}(dn)$ in a cosmic ray neutrino oscillation experiment leads to a measurement of linear combination $N_e(dn)\langle P_{e\tau}\rangle_t + N_\mu(dn)\langle P_{\mu\tau}\rangle_t$, if there are only three neutrino flavors, each capable of oscillating into the others.

To proceed further with eq. (6), i.e., to evaluate $\langle P_{e\tau}\rangle_t$ and $\langle P_{\mu\tau}\rangle_t$, requires knowledge of the lepton mixing matrix, U, which it is the aim of the experiment to determine. For our purpose here it is instructive, nevertheless, to estimate possible outcomes of the experiment for certain possible mixing matrices. We assume that there are $n = 3$ generations of leptons; the case $n > 3$ is treated in the next subsection. Existing neutrino oscillation searches (Fig. 2) and experiments attempting to detect the effects of neutrino masses and mixing in weak nuclear and particle decays[9] place constraints on the form of the matrix U. These depend, however, on the values of the relevant neutrino masses (or differences of masses squared). For sufficiently small masses, commensurately large mixings are allowed. As illustrations we give two examples of mixing matrices with large off-diagonal elements (which thus implicitly require appropriately small $|\Delta m^2|$); first[1],

$$U \equiv \begin{bmatrix} \langle\nu_e|\nu_1\rangle & \langle\nu_e|\nu_2\rangle & \langle\nu_e|\nu_3\rangle \\ \langle\nu_\mu|\nu_1\rangle & \langle\nu_\mu|\nu_2\rangle & \langle\nu_\mu|\nu_3\rangle \\ \langle\nu_\tau|\nu_1\rangle & \langle\nu_\tau|\nu_2\rangle & \langle\nu_\tau|\nu_3\rangle \end{bmatrix} = \begin{bmatrix} -1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ 1/\sqrt{6} & -2/\sqrt{6} & 1/\sqrt{6} \end{bmatrix} \tag{7}$$

from which, using for the time averaged case of interest here,

$$\begin{aligned} \langle P_{\eta\eta}\rangle_t &= \sum_j |\langle\nu_\eta|\nu_j\rangle|^4 \ ; \\ \langle P_{\eta\xi}\rangle_t &= \sum_j |\langle\nu_\eta|\nu_j\rangle|^2 |\langle\nu_j|\nu_\xi\rangle|^2 \ , \end{aligned} \tag{8}$$

where $\eta, \xi = e,\mu,\tau$, and $j = 1,2,3$, we find

$$\langle P_{e\tau}\rangle_t = 1/6 \text{ and } \langle P_{\mu\tau}\rangle_t = 1/3 \quad . \tag{9}$$

Alternatively, choosing the matrix[10] (which violates CP-invariance, unlike other choices here)

$$\langle\nu_\eta|\nu_k\rangle = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^* \\ 1 & \omega^* & \omega \end{bmatrix} \ , \ \omega = \exp(2\pi i/3) \tag{10}$$

yields

$$\langle P_{e\tau}\rangle_t = \langle P_{\mu\tau}\rangle_t = 1/3 \ . \tag{11}$$

As a third example, consider the general lepton mixing matrix, parameterized in a standard manner [11].

$$\langle\nu_\eta|\nu_k\rangle = \begin{bmatrix} \cos\theta_1 & \sin\theta_1\cos\theta_3 & \sin\theta_1\sin\theta_3 \\ -\sin\theta_1\cos\theta_2 & \cos\theta_1\cos\theta_2\cos\theta_3 + \sin\theta_2\sin\theta_3 e^{i\delta} & \cos\theta_1\cos\theta_2\sin\theta_3 - \sin\theta_2\cos\theta_3 e^{i\delta} \\ -\sin\theta_1\sin\theta_2 & +\cos\theta_1\sin\theta_2\cos\theta_3 - \cos\theta_2\sin\theta_3 e^{i\delta} & +\cos\theta_1\sin\theta_2\sin\theta_3 + \cos\theta_2\cos\theta_3 e^{i\delta} \end{bmatrix} \tag{12}$$

The values of $\langle P_{e\tau}\rangle_t$ and $\langle P_{\mu\tau}\rangle_t$ resulting from four sets of angle choices which are CP-conserving and involve rotations all of magnitude of the Cabibbo angle are given in Table I.

For the CP-violating mixing matrix of Wolfenstein[10], one finds

$$1 - \frac{N_{tot}(up)}{N_{tot}(dn)} = \frac{1}{3} \ , \tag{13}$$

while for the general Kobayashi-Maskawa matrix with Cabibbo-sized rotation angles[11] this quantity (Table I) can be as large as

$$1 - \frac{N_{tot}(up)}{N_{tot}(dn)} = \frac{1}{5} \tag{14}$$

Eqs. (13) and (14) suggest that, if either of these matrices is a reasonable approximation to the actual neutrino mixing matrix, the cosmic ray neutrino oscillation search described here (10 kt-yr exposure) might

Table I. Values of the time averaged probabilities $\langle P_{e\tau}\rangle_t$ and $\langle P_{\mu\tau}\rangle_t$ calculated for four sets of possible mixing angles from the general lepton mixing matrix in eq. (13). Note that the angle δ is taken to conserve CP.

θ_1	θ_2	θ_3	δ	$\langle P_{e\tau}\rangle_t$	$\langle P_{\mu\tau}\rangle_t$	$1-\frac{N_{tot}(up)}{N_{tot}(dn)}$
11°	13°	15°	0°	4.3×10^{-3}	2.4×10^{-3}	3.1×10^{-3}
11°	13°	15°	180°	1.1×10^{-2}	3.3×10^{-1}	.22
13°	13°	13°	0°	5.0×10^{-3}	1.8×10^{-4}	1.9×10^{-3}
13°	13°	13°	180°	1.4×10^{-2}	3.0×10^{-1}	.20

yield a positive result with statistical significance between about 2.5 and 5 standard deviations. An additional year's exposure might then provide a statistically definitive result.

It is worth noting that the Kobayashi-Maskawa matrix also yields $\langle P_{e\mu}\rangle_t = 0.083$, assuming, e.g., $\theta_1 = \theta_2 = \theta_3 = 13^\circ$, $\delta = \pi$, which, in conjunction, with $\langle P_{e\tau}\rangle_t = 0.014$, tends to keep $N_e(\mathrm{up}) \approx N_e(\mathrm{dn})$. On the other hand, the relatively large value of $\langle P_{\mu\tau}\rangle_t$ tends to diminish $N_\mu(\mathrm{up})$, so that in this particular example the ratio $(N_e/N_\mu)_{\mathrm{up}}$ is statistically different from $(N_e/N_\mu)_{\mathrm{down}}$, as is $N_{tot}(\mathrm{up})$ from $N_{tot}(\mathrm{dn})$.

3) Oscillations among four neutrino flavors, each capable of oscillating into the others. It is sufficient here to observe that a general result applicable to this case (but with CP-conservation assumed) has been obtained implicitly in a paper by Frampton and Glashow[2]. They form, for atmospheric neutrinos, the two-dimensional plot (see Fig. 4), $N_\mu(\mathrm{up})$ vs $N_e(\mathrm{up})$, and identify the following quantities in that plot: (a) a given line as the locus of values of $N_\mu(\mathrm{up})$ and $N_e(\mathrm{up})$ corresponding to 2-flavor ν_e-ν_μ mixing; and (b) a given area as containing all paired values of $N_\mu(\mathrm{up})$ and $N_e(\mathrm{up})$ corresponding to 3-flavor ν_e-ν_μ-ν_τ mixing. Paired values of $N_\mu(\mathrm{up})$ and $N_e(\mathrm{up})$ that lie outside the 3-flavor area indicate the existence of a fourth neutrino flavor. The region outside the 3-flavor area can, in turn, be subdivided into two parts: one part, approximately 20 percent of that outside region, in which the values of $N_\mu(\mathrm{up})$ and $N_e(\mathrm{up})$ are correlated such that $2/3 \leq N_{tot}(\mathrm{up})/N_{tot}(\mathrm{dn}) \leq 1.0$; and a second part, about 80% of that outside region (shown shaded in Fig. 4), in which the correlation between $N_e(\mathrm{up})$ and $N_\mu(\mathrm{up})$ satisfies $N_{tot}(\mathrm{up})/N_{tot}(\mathrm{dn}) \leq 2/3$.

It follows then that 4-flavor mixing, if it exists at all, might (4 to 1 odds) yield a statistically significant, detectable reduction in the ratio $N_{tot}(\mathrm{up})/N_{tot}(\mathrm{dn})$ of magnitude equal to or greater than that possible in the 3-flavor case.

Summary

Neutrino oscillation experiments fall into two classes: (i) "appearance" experiments in which a search is made for the appearance of a given neutrino flavor in an incident flux which initially did not contain that flavor except possibly as a small contamination, and (ii) "disappearance" experiments in which a suitably normalized measurement is made of the flavor content of a neutrino beam after it has traversed a given distance to provide a search for the disappearance of a fraction of a given neutrino flavor originally present at zero distance. In the latter class, experiments sensitive to small values of Δm^2 are done with $\bar{\nu}_e$ from reactors at the level $\Delta m^2 \gtrsim 10^{-2}$ eV^2 (see Fig 2), and may be done with ν_e from the sun at the level $\Delta m^2 \gtrsim 10^{-11}$ eV^2. The strength of neutrino mixing that is accessible in such disappearance experiments is of magnitude $\sin^2 2\alpha \gtrsim 0.1$.

In this paper we have explored a complementary disappearance experiment that may be carried out with $\nu_e + \bar{\nu}_e$ and $\nu_\mu + \bar{\nu}_\mu$ from cosmic ray sources at the level of $\Delta m^2 \gtrsim 10^{-4}$ eV^2 and $\sin^2 2\alpha \gtrsim 0.1$. The experiment has the following advantages: (1) it is the only experiment of which we are aware that is capable of searching for the disappearance of ν_μ and $\bar{\nu}_\mu$ at the limiting value $\Delta m^2 \gtrsim 10^{-4}$ eV^2; (2) because it measures the quantities $N_{tot}(\mathrm{up})$ and $N_{tot}(\mathrm{dn})$ the experiment is relatively insensitive to systematic errors; (3) the experiment is capable of observing time averaged probabilities $\langle P_{e\tau}\rangle_t$ and $\langle P_{\mu\tau}\rangle_t$ of magnitude set by mixing strengths corresponding to, e.g., the d- to s-quark mixing strength; (4) although the experiment relies on the upward-going neutrinos (see Fig. 1) traversing a substantial fraction of the earth's diameter, its sensitivity is not limited by matter-induced oscillations. The principal disadvantage of the experiment is that it requires a very massive (~10 kiloton) detector in which the neutrino interactions must occur and be contained; the detector must also be well enough instrumented to distinguish clearly upward-going from downward-going neutrinos. A lesser disadvantage is that the detector must be located underground (~2000 meters of water equivalent) so that the number of interactions initiated by cosmic ray muons is reduced to a value substantially less than that on the earth's surface. To obtain sufficient statistical precision, the data-taking period must be at least one year.

It seems probable that the very massive detectors intended to search for proton and bound neutron decay, which are now or will soon be in operation deep underground, will be able to carry out this experiment in the next few years. If the limit of sensitivity of which the experiment is capable is achieved, the result will place an upper bound on Δm^2 about three orders of magnitude lower than the upper bound that can be obtained from any corresponding experiment using accelerator produced neutrinos.

References

1 For reviews of neutrino oscillations, see, e.g., A. K. Mann and H. Primakoff, Phys. Rev. D 15, 655 (1977); S. M. Bilenky and B. Pontecorvo, Phys. Rep. 41, 225 (1978).

2 B. Cortez and L. R. Sulak in Unification of Fundamental Particle Interactions (ed. by S. Ferrara, J. Ellis, P. vanNieuwenhuizen, Plenum, New York and London) 1981, p. 661; P. Franpton and S. L. Glashow, Phys. Rev. D 25, 1982 (1982); see also P. V. Ramana Murthy, Proc. 17th International Cosmic Ray Conference (Paris), (Centre d'Études Nucleaires, Saclay) 13, 381 (1981).

3 L. Wolfenstein, Phys. Rev. D 17, 2369 (1978); V. Barger et al., Phys. Rev. D 22, 2718 (1980).

4 J. L. Osborne, S. S. Said and A. W. Wolfendale, Proc. Phys. Soc. (London) 86, 93 (1965); E. C. M. Young in Cosmic Rays at Ground Level (ed. by A. W. Wolfendale, Inst. of Physics Press, London and Bristol, 1973, p. 105. L. V. Volkova, Sov. J. Nucl. Phys. 31, 1510 (1980) has recently computed the flux of muon and electron type neutrinos and anti-neutrinos above 1 GeV. See also L. V. Volkova, Proc. DUMAND Workshop 1980, p. 75. At 1 GeV the calculation of Volkova is about 70% higher than the earlier calculation.

5 M. M. Boliev et al., in Proc. 17th International Cosmic Ray Conference, (Paris), 7, 106 (1981).

6 M. R. Krishnaswamy et al., report at Neutrino-'82, Balaton, Hungary, 1982.

7 For a recent survey of present and proposed detectors, see the Proceedings of the 1982 Summer Workshop on Proton Decay Experiments, June 7-11, 1982, Argonne National Laboratory report ANL-HEP-PR-82-24 (ed. by D. S. Ayres).

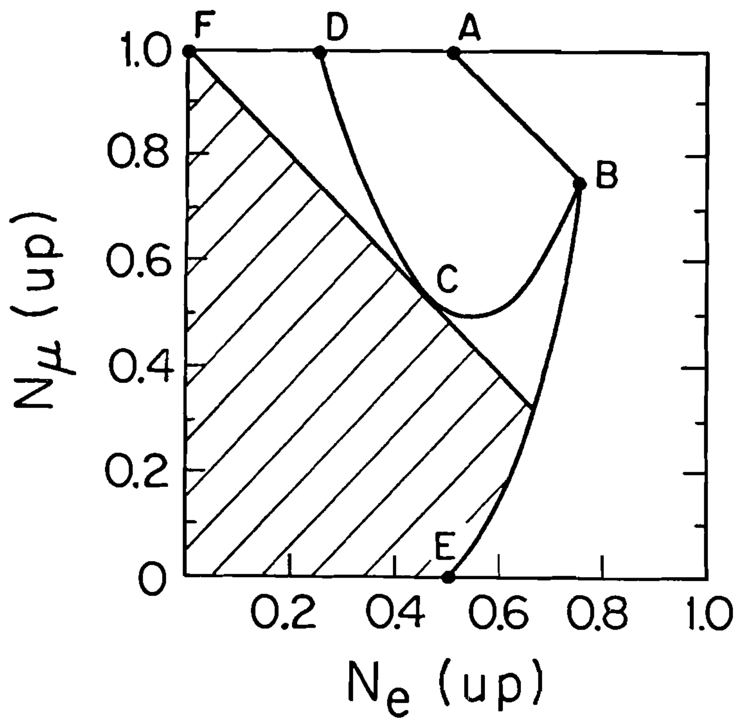

Fig. 4. Plot of N_μ(up) vs N_e(up) after Frampton and Glashow (reference 2). The ratio $N_\mu(dn)/N_e(dn)$ is taken to be 2:1, and $N_{tot}(dn)$ to be 1.5 (see reference 8). The four allowed regions of interest are as follows. (I) The point A corresponds to no mixing of ν_μ or ν_e. Departure from this point requires a nonzero neutrino mass. (II) The line AB corresponds to N = 2 flavor ν_μ-ν_e mixing. Departure from this line signals a third neutrino flavor. (III) The region ABCDA corresponds to N = 3 flavor ν_μ-ν_e-ν_τ mixing. Departure from this region reveals the existence of a fourth neutrino flavor. (IV) The allowed domain ABEOFA corresponds to arbitrary mixing of any number ($N \to \infty$) of neutrino flavors. This limit follows from the Schwarz inequality and unitarity.

The region outside the 3-flavor area may be subdivided into two parts, one of which (shown cross-hatched) contains paired values of N_μ(up) and N_e(up) that all satisfy the condition $N_{tot}(up)/N_{tot}(dn) \lesssim 2/3.^\mu$

8 We obtain for the number of observed events per 10 kiloton-yr: $N_{\mu^-}(dn) = 144$, $N_{\mu^+}(dn) = 48$, $N_{e^-}(dn) = 72$, $N_{e^+}(dn) = 36$; hence $N_\mu(dn) = 192$, $N_e(dn) = 108$. For a detailed breakdown of reaction channels see Table 8 of the Soudan 2 proposal, Minnesota-Argonne-Oxford, 1981. A similar estimate of the expected neutrino interaction rate is given by B. Cortez and L. R. Sulak in reference 2.

9 R. E. Shrock, Phys. Lett. 96B, 159 (1980); Phys. Rev. D 24, 1232, 1275 (1981); for reviews, see T. Yamazaki in the Proceedings of the Neutrino-82 Conference and International Conference on High Energy Physics, Paris, 1982; and R. E. Shrock, these Proceedings.

10 L. Wolfenstein, Phys. Rev. D 20, 2634 (1979).

11 M. Kobayashi and K. Maskawa, Prog. Theor. Phys. 49, 652 (1973); Particle Data Group Review of Particle Properties, Phys. Lett. 111B, 1 (1982).

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