MASS LIMITS FOR THE MUON NEUTRINO

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Summary

The possibility of improving the present limit on the mass of the muon neutrino is discussed. It is found that decays of muons and pions are not useful means to significantly improve this limit. On the other hand, the decays $K_L^0 \to \pi^{\pm}\mu^{\mp}\nu_\mu$ and $K^+ \to \pi^0\mu^+\nu_\mu$ appear to be quite promising. Possible experiments are discussed.

Introduction

One of the central issues in particle physics and astrophysics is the question of whether or not neutrinos are massive. There have been two main approaches used to search for evidence of neutrino masses in laboratory experiments. One method is to search for neutrino oscillations. The existence of neutrino oscillations requires both a difference in the masses of two types of neutrinos and lepton-flavor violation to permit coupling between the two neutrino types. These experiments are sensitive to very small masses (typically $\lesssim 1$ eV/c^2) but are subject to the condition that the lepton-flavor violation must be large enough to render the oscillations observable.

The second approach is to search for kinematic effects that result from nonzero neutrino masses. In the decay of a particle into a final state that includes a neutrino, the maximum energies of the other final-state particles decrease as the neutrino mass increases. This method does not depend upon the existence of lepton-flavor violation. Unfortunately, these studies have only been sensitive to relatively large neutrino masses (≈ 10 eV/c^2 for ν_e, 500 keV/c^2 for ν_μ, and 250 MeV/c^2 for ν_τ).

It is the purpose of this report to examine the sensitivity of several decays to the mass of the muon neutrino. We find that three-body kaon decays hold the promise of improved mass limits.

The Present Limit

The present upper limit for M_{ν_μ} is derived from two measurements of the process $\pi^+ \to \mu^+\nu_\mu$, one at rest and the other in flight. For decays at rest, the neutrino mass is related to the muon momentum by

$$M_{\nu_\mu}^2 = M_\mu^2 + M_\pi^2 - 2M_\pi\left(P_\mu^2 + M_\mu^2\right)^{1/2} \quad . \tag{1}$$

M. Daum et al.[1] measured $P_\mu = 29.7877 \pm 0.0014$ (an accuracy of 47 ppm!). Combining this value with the measured masses, $M_{\pi^\pm} = (139.5675 \pm 0.0009)$ MeV/c^2 (Ref. 2) and $M_\mu = (105.65946 \pm 0.00024)$ MeV/c^2 (Ref. 3)* yields

$$M_{\nu_\mu}^2 = (0.102 \pm 0.120)\ (\mathrm{MeV/c^2})^2 \tag{2}$$

or

$$M_{\nu_\mu} < 0.52\ \mathrm{MeV/c^2}\ (90\%\ \mathrm{C.L.}) \quad . \tag{3}$$

The fact that Eq. (1) depends upon $M_{\nu_\mu}^2$ implies that the quantities on the right-hand side of this equation must be determined with 4 times better accuracy to reduce the upper limit for M_{ν_μ} by a factor of 2. It is obviously extremely difficult to achieve significant improvement in sensitivity to M_{ν_μ} by this method. The existing errors in M_π and M_μ imply that a perfect measurement of P_μ would result in an upper limit of 0.28 MeV/c^2 for M_{ν_μ}.

The other experiment[4] studied decays of 350-MeV/c π^+, where the μ^+ was emitted in the forward direction. This method has the advantage that the determination of M_{ν_μ} is nearly independent of M_π and M_μ. The result obtained is

$$M_{\nu_\mu}^2 = (-0.14 \pm 0.20)(\mathrm{MeV/c^2})^2 \tag{4}$$

or

$$M_{\nu_\mu} < 0.50\ \mathrm{MeV/c^2}\ (90\%\ \mathrm{C.L.}) \quad . \tag{5}$$

Even though the error in $M_{\nu_\mu}^2$ in this experiment [Eq. (4)] is nearly twice that in the experiment at rest [Eq. (5)], the limit on M_{ν_μ} is slightly better. This is because the central value for $M_{\nu_\mu}^2$ in Eq. (4) is negative; one must regard this as a statistical fluke. The experiment at rest has a higher precision. (If the central value for $M_{\nu_\mu}^2$ were zero with the experiment in flight, the limit would be $M_{\nu_\mu} < 0.57$ MeV/c^2.) Nevertheless, it appears that this method could be pushed to higher precision.* The statistical precision could certainly be improved (the experiment collected data for roughly 6 weeks). In addition, the largest systematic errors (namely, errors in the π-μ decay angle and the effect of chamber nonlinearities) could be decreased. With a tremendous effort (a group of at least six to eight people full time for about 4 years), these errors could conceivably be reduced by as much as a factor of four. This would result in an upper limit of 0.3 MeV/c^2 (assuming a central value for $M_{\nu_\mu}^2$ of zero). A big disadvantage of such an effort is that the apparatus is not suited for any other measurement; finding people willing to devote so much time to this single measurement will be difficult.*

* The most accurate measurement is by D. Casperson (Ref. 3); the world average was compiled by the Particle Data Group (Ref. 3).

* This information was received from P. G. Seiler in a private conversation on 8/23/82.

Three-Body Decays

Consider the three-body decay $A \to B + C + \nu$. If q_i is the four-momentum of the i^{th} particle, energy-momentum conservation implies

$$q_A = q_B + q_C + q_\nu \quad . \tag{6}$$

In the center of mass of particle A, $q_A = (M_A, 0)$ and so

$$(q_A - q_B)^2 \equiv (q_A - q_B)_\mu \cdot (q_A - q_B)^\mu$$

$$= (q_C + q_\nu)^2 \quad , \tag{7}$$

or

$$M_A^2 + M_B^2 - 2M_A E_B = M_{C\nu}^2 \quad , \tag{8}$$

where $M_{C\nu}$ is the invariant mass of particle C and the neutrino. Rearranging Eq. (8) we find

$$E_B = \frac{1}{2M_A} \left(M_A^2 + M_B^2 - M_{C\nu}^2\right) \quad . \tag{9}$$

The maximum E_B occurs when $M_{C\nu}$ is a minimum; this happens when particle C and the neutrino have the same velocity, i.e., there exists a Lorentz frame in which particle C and the neutrino are both at rest. Then

$$M_{C\nu}^{min} = M_C + M_\nu \quad , \tag{10}$$

so that

$$E_B^{max} = \frac{1}{2M_A} \left[M_A^2 + M_B^2 - (M_C + M_\nu)^2\right] \quad . \tag{11}$$

If M_C is not zero, Eq. (11) implies that the end-point energy, E_B^{max}, depends linearly on M_ν (ignoring the small term proportional to M_ν^2). The fractional rate of change of the endpoint energy with M_ν is given by

$$\frac{1}{E_B^{max}} \frac{\partial E_B^{max}}{\partial M_\nu} = \frac{2(M_C + M_\nu)}{M_A^2 + M_B^2 - (M_C + M_\nu)^2} \quad . \tag{12}$$

This equation shows that the most favorable situation occurs when M_C is comparable to M_A. If both M_B and M_C are small (as in $\mu^+ \to e^+ \nu_e \bar{\nu}_\mu$), the sensitivity is very poor. For muon decay

$$\frac{1}{E_e^{max}} \frac{\partial E_e^{max}}{\partial M_{\nu_\mu}} = (1.8 \times 10^{-4})\, M_{\nu_\mu} \left(\frac{\text{MeV}}{c^2}\right)^{-1} \quad . \tag{13}$$

It would require a measurement of the e^+ end-point energy with an accuracy of one part in 10^4 to equal the present limit on M_{ν_μ}.

For the decay $\pi^+ \to \mu^+ \nu_\mu \gamma$, we have

$$\frac{1}{E_\mu^{max}} \frac{\partial E_\mu^{max}}{\partial M_{\nu_\mu}} = 6.6 \times 10^{-5}\, M_{\nu_\mu} \left(\frac{\text{MeV}}{c^2}\right)^{-1} \tag{14}$$

and

$$\frac{1}{E_\gamma^{max}} \frac{\partial E_\mu^{max}}{\partial M_{\nu_\mu}} = 2.5 \times 10^{-2} \left(\frac{\text{MeV}}{c^2}\right)^{-1} \quad . \tag{15}$$

The latter appears to be an attractive candidate. Unfortunately, the branching ratio for $\pi^+ \to \mu\nu_\mu\gamma$ where E_γ is within 1 MeV of the end-point is $\approx 3 \times 10^{-9}$ (Ref. 5), making it an extremely difficult measurement.

For $K^+ \to \pi^0 \mu^+ \nu_\mu$,

$$\frac{1}{E_\mu^{max}} \frac{\partial E_\mu^{max}}{\partial M_{\nu_\mu}} = 1.14 \times 10^{-3} \left(\frac{\text{MeV}}{c^2}\right)^{-1} \quad , \tag{16}$$

where $E_\mu^{max} = 240.48$ MeV, assuming $M_{\nu_\mu} = 0$. The advantage here is that the branching ratio near the end-point is not so small.

Another possible reaction is $K_L^0 \to \pi^\pm \mu^\mp \nu_\mu$. The problem here is that it is difficult to know the K_L^0 momentum with precision. Clark et al.[6] circumvented this problem by rewriting Eq. (9) as

$$E_\nu = \frac{1}{2M_K} \left(M_{K^0}^2 + M_\mu^2 - M_{\pi\mu}^2\right) \quad , \tag{17}$$

where E_ν is the neutrino energy in the K^0 rest system. The advantage to this method is that $M_{\pi\mu}$ can be measured in the laboratory and so the K^0 momentum need not be known. A nonzero neutrino mass alters the extreme low end of the neutrino energy spectrum (or high $M_{\pi\mu}$ region).

Three-body decays of heavier particles (such as $\Lambda \to p\mu\nu_\mu$ or $\Sigma^- \to n\mu\nu_\mu$) are not promising means to measure M_{ν_μ}. First, the total branching ratios are small ($\sim 10^{-4}$). Second, the direction of the hyperon at decay is not known to high precision, resulting in an additional dilution of the sensitivity to M_ν. For the Σ^- decay, the hyperon energy at decay is unknown, even for $K^-p \to \Sigma^-\pi^+$ as the Σ^- slows down before decay.

The rest of this report is devoted to a discussion of the two possible $K_{\mu 3}$ experiments.

$K_{\mu 3}$ Experiments

A. $K_L^0 \to \pi^\pm\mu^\mp\nu_\mu$

This reaction was used by Clark et al.[6] to set a limit $M_{\nu_\mu} < 0.65$ MeV/c^2. Their apparatus detected π's and μ's in a magnetic spectrometer designed to render these tracks parallel when E_ν was small. The K_L^0 beam at the Bevatron had a momentum range 0.8-3.2 GeV/c and an intensity of 6×10^5 K_L/pulse. They collected $\sim 10^6$ $K_L \to \pi^+\pi^-$ and $\sim 10^4$ $K_L^0 \to \pi\mu\nu_\mu$ with $E_\nu < 5$ MeV. The invariant mass resolution was measured to be 2.2 MeV (FWHM) from the $K_L^0 \to \pi^+\pi^-$ mode; the use of this mode as a calibration decreases the sensitivity of the neutrino-mass determination to the uncertainties in the charged-particle masses to a negligible level.

To improve the limit of Ref. 6 would require a larger statistical sample or an improved invariant mass resolution, or both. It will be difficult to significantly improve the mass resolution, as they used low-mass (5×10^{-4} radiation length) chambers and helium throughout the apparatus to reduce multiple scattering. With more modern data-acquisition methods, the statistical sample could be increased, especially if a more intense well defined K_L^0 beam was available. A serious systematic background came from $K_L^0 \to \pi^+\pi^-$, where one of the pions decayed into a muon before the first detector. These events, which tend to peak at a π-μ invariant mass near 480 MeV with a tail towards higher masses, resulted in a systematic shift in the neutrino-mass limit as the range of π-μ invariant masses used in the analysis was varied. This background could be reduced if a small beam spot and low neutron contamination would permit the upstream detectors to lie closer to the end of the decay volume. A smaller beam spot itself also helps reduce the background from muon decay as the intersection of the pion and muon trajectories is required to lie within the cross-sectional area of the beam.

Without a detailed Monte Carlo program, it is difficult to estimate how well the neutrino-mass limit could be improved. The method appears capable of being pushed to a sensitivity to M_{ν_μ} of $\lesssim 0.2$ MeV/c^2 with a reasonable effort. The systematic errors from the $K_L^0 \to \pi^+\pi^-$ backgrounds entered at a level of $\simeq 0.1$ MeV/c^2 in Ref. 6. Thus progress beyond this level would require significant improvements.

A disadvantage of this process is that the neutrino mass limit depends somewhat on assuming the correct matrix element for the decay (V-A is assumed). However, any deviation from the V-A form would be extremely interesting even if it could not be unambiguously ascribed to a massive neutrino. An advantage of this process is that the apparatus can be used to study many other phenomena such as $K_L \to \mu^+\mu^-$, $K_L \to \pi^\pm\mu^\mp\nu_\mu$ charge asymmetry, etc.

B. $K^+ \to \pi^0\mu^+\nu_\mu$

To our knowledge, this reaction has never been investigated as a means of detecting neutrino mass. From Eq. (11) we find

$$E_\mu^{max} = 239.69 - 0.27\, M_{\nu_\mu}\ (\text{MeV}) \quad . \tag{18}$$

The obvious experimental difficulty arises from the decay $K^+ \to \pi^+\pi^0$, followed by the process $\pi^+ \to \mu^+\nu_\mu$. These processes yield a μ^+ with $151.4 < E_\mu < 238.9$ MeV; a small contamination from this background might lead one to conclude that a non-zero neutrino mass had been detected. However, one can use the energy of the π^0 to reduce this background. For the $K^+ \to \pi^+\pi^0$ decay, the energy of the π^0 is unique and is equal to 245.55 MeV. At the muon end-point in $K^+ \to \pi^0\mu^+\nu_\mu$, $E_{\pi^0} = 254.0$ MeV. Using NaI(Tl), with an energy resolution of 5% (FWHM) for 120-MeV γ's, these π^0 energies differ by about 2.25 standard deviations. Experimentally, one would select events with $E_{\pi^0} > 250$ MeV, thereby rejecting $K^+ \to \pi^+\pi^0$ by more than a factor of twenty. Notice that if some of the γ energy escapes from the NaI, some potentially good events will be rejected, but no $K^+ \to \pi^+\pi^0$ will be accepted.

Consider then an experiment with a target in which K^+'s stop, the NaI detectors on one side and a magnetic spectrometer to measure P_μ on the other. The branching ratio[*,7]

$$\frac{\Gamma(K^+ \to \pi^0\mu^+\nu_\mu,\ E_\mu^{max} - E_\mu < 3\ \text{MeV})}{\Gamma(K^+ \to \text{all})} = 1 \times 10^{-5} \quad . \tag{19}$$

TABLE I

NUMBER OF $K^0 \to \pi^0\mu^-\nu_\mu$ EVENTS VS. $(E_\mu^{max} - E_\mu)$

$E_\mu^{max} - E_\mu$ (MeV)	Number of Events
0 - 1 MeV	7000
1 - 2 MeV	30000
2 - 3 MeV	63000
3 - 4 MeV	100000
4 - 5 MeV	200000

If we assume 10^4 K^+/s stopped in a thin target and an apparatus acceptance of 10% of 4π, 10^5 events with $E_\mu^{max} - E_\mu < 3$ MeV would be collected in a run of 10^7 seconds. The number of events vs $E_\mu^{max} - E_\mu$ is presented in Table I. Even allowing for a sizable loss of events from the π^0 energy cut, a large statistical sample would be collected. The absolute momentum scale of the muon spectrometer is calibrated with $K^+ \to \mu^+\nu_\mu$ and $K^+ \to \pi^+\pi^0$ (momenta of 236 and

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* The results for the muon energy spectrum come from integrating expressions from Calpacas, Ref. 7, assuming the form factors are unity. The corresponding equation in Marshak et al. (also Ref. 7) has several errors.

205 MeV/c, respectively). These momenta are quite insensitive to the neutrino mass.

Major backgrounds are from $K^+ \to e^+\pi^0\nu_e$ and $K^+ \to \pi^+\pi^0$ followed by $\pi^+ \to \mu^+\nu_\mu$. The K_{e3} can be eliminated with a Cerenkov counter detecting the e^+. The other background is more troublesome. The size of the background, to be compared with Eq. (19), is given by

$$\frac{\Gamma(K^+ \to \pi^+\pi^0)}{\Gamma(K^+ \to \mathrm{all})} \times (\pi^+ \text{ decay probability})$$

$$\times \text{ (solid angle for } \pi^+ \to \mu^+)$$

$$\times (\pi^0 \text{ energy cut}) \quad .$$

Assuming that the π^+ must decay within 10 cm of the target to be a problem, and that we are concerned with only those π^+ decays forward of 20^o in the π^+ rest system, the size of the background is

$$(0.21) \times (9 \times 10^{-3}) \times (3 \times 10^{-2}) \times (1/20) = 3 \times 10^{-6} \quad .$$

Thus, the signal-to-background ratio is 3/1. This ratio is uncomfortably low, but could be improved by a tighter cut on E_{π^0}. The size and shape of the background can be studied either by selecting the appropriate π^0 energy or by moving the muon spectrometer further from the stopping target, thus increasing the allowed π^+ decay path. The limit on M_{ν_μ} that can be achieved by this reaction will be determined by how well the background can be understood. The contribution from the statistical uncertainties in the signal is quite small. A limit on M_{ν_μ} of several hundred keV/c^2 may well be possible.

Conclusions

The present upper limit for the mass of the muon neutrino is $M_{\nu_\mu} < 0.50$ MeV/c^2. It appears to be very difficult to significantly improve this limit by studying pion or muon decays. On the other hand, the reactions $K_L^o \to \pi^\pm\mu^\mp\nu_\mu$ and $K^+ \to \pi^0\mu^+\nu_\mu$ appear capable of leading to significantly improved sensitivities.

For completeness, we should mention the reaction

$$\mu^- + {}^6\mathrm{Li} \to {}^3\mathrm{H} + {}^3\mathrm{H} + \nu_\mu \quad ,$$

which has been suggested as a means of measuring M_{ν_μ} (Ref. 8). The capture rate for all $E_\nu < 10$ MeV is $\sim 2 \times 10^{-3}$ s^{-1}, which implies that < 1 in 10^8 stopping muons result in neutrinos with less than 10 MeV. This small rate, coupled with the experimental requirement of a thin ^{6}Li target to minimize energy losses by the tritons, implies that the event rate is probably too small to be useful in improving the measurement of M_{ν_μ}.*

* A group at SIN led by R. Frosch has reached a similar conclusion according to a conversation with P. G. Seiler.

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$K^+ \to \pi^0\mu^+ \nu$