AN e-p PRIMER *

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The interest in e-p colliders has been maintained in recent times by a series of studies, frequently designed as an add on to other existing facilities. In our experience, Bjorn Wiik has promoted this field of physics for many years, culminating in a proposal for HERA. In the U.S., the Columbia Group, the Canadians and collaborators have recently proposed electron rings as a complement to proton facilities at Fermilab and Brookhaven. We have listed in References 1 - 4 the reports that marked the progress of this concept in high energy physics. The physics is sometimes held to be self-evident, an extension of the work on structure functions at SLAC and later at Fermilab and CERN, and also the elegant demonstration of the existence of the weak neutral current with polarized electrons at SLAC. Although this field is not new, the basic ideas often seem to be poorly understood by both the protagonists and antagonists of e-p physics. This paper is inspired by a task force at Brookhaven charged with a review of the feasibility of an e-p collider on that site specifically, to capitalize on the local advantages.[5]

The DPF Summer Study has also a charge to consider the physics of e-p in a more general way, and we offer this note fresh from relearning the kinematics and the specific predictions of the standard model for event rates with an e-p collider.

In reviewing the extensive literature, it became clear that simple-minded relations that allowed comparison between options were not available. Each report has concentrated on a specific combination of electron and proton energies. We have tried, therefore, to generate rules of thumb which will allow comparisons of options which may be considered in the future. We attempt, therefore, to make available some appropriate simplifications which may allow others to reach their own judgments on the merits of various combinations.

First then, we discuss the kinematics, which tend to be rather different from those at fixed target machines and symmetric colliders. Of course, we make use of invariants whenever possible, with an attempt to make clear their effect in a laboratory situation. We have selected three invariants as our prime choice to describe the scattering processes s (the c.m. energy squared), Q^2 (the negative four-momentum transfer squared to to the electron), and x, the fraction of the target momentum carried by the struck parton. We will make corrections to the other frequently used variables y, ν and W, the c.m. energy of the photon (boson), proton system.

1. Kinematics

We subscript the proton variables with a p and leave electron variables unsubscripted. The c.m. energy squared

$$s = (E + E_p)^2 - (\vec{p} - \vec{p}_p)^2 \quad ,$$

where $\vec{p}_p$ is opposite to $\vec{p}$ in the lab system; then neglecting the mass of the electron

$$= 2E\,E_p + 2E\,P_p + m_p^2$$

At the energies appropriate to this discussion, we should neglect terms ~ $(1/E_p)$ giving

$$s = 4E\,E_p + m_p^2 \sim 4E\,E_p \quad .$$

The four momentum transfer squared

$$q^2 = (E - E')^2 - (E - E' \cos\theta)^2 - E' \sin^2\theta$$

It is customary to use $Q^2 = -q^2 = 4EE' \sin^2 \theta/2$. This variable is used as the scale breaking parameter in the hadronic structure functions as we shall see below. The third variable is x, the fraction of the hadron momentum that is carried by the struck parton. Before we define x, first we define ν:

$$\nu = \frac{\mathbb{P} \cdot q}{m_p} \quad .$$

$\mathbb{P}$ is the four momentum of the proton and q is the four momentum transfer to the electron. The invariant is especially simple in fixed target physics when $\mathbb{P} = (m_p, 0)$ and then $\nu = E - E'$. In the collider,

$$\mathbb{P} \cdot q = E_p (E - E') + p_p (E - E' \cos\theta)$$

$$= 2E\,E_p - E'E_p (1 + \cos\theta) \quad .$$

This has a maximum value when $\theta = \pi$

$$\nu_{max} = \frac{2E\,E_p}{m_p} \sim s/2m_p$$

$$\text{then } y = \nu/\nu_{max} = \frac{2\,\mathbb{P} \cdot q}{s} \quad ,$$

the other variable

$$x = \frac{Q^2}{2\,P \cdot q} = \frac{Q^2}{sy} \quad .$$

Both dimensionless variables x and y go from 0 to 1.

Although we calculate rates as a function of s, Q^2, we often use cross sections expressed as a function of s, x, and y, then

$$y = Q^2/sx$$

and

$$\frac{d}{dy} = sx \frac{d}{dQ^2} \quad .$$

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Much of the physics of an ep collider is reached by measuring the outgoing electron; we now discuss this facet of the kinematics. The basic model of the interaction is that elastic scattering occurs between the electron and a parton which carries a fraction x of the incoming nucleon momentum as in Fig. 1.

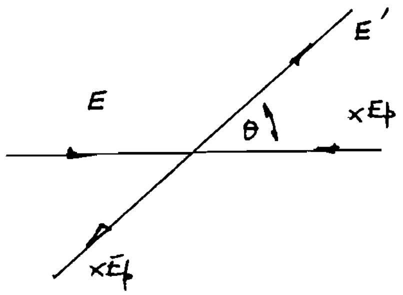

Fig. 1

We still have $Q^2 = 2EE'(1 - \cos\theta)$ and

$$y = \frac{Q^2}{sx} = \frac{2EE'(1 - \cos\theta)}{4E\ E_p\ .\ x} \quad .$$

E_p is the energy of the proton in the lab system and $E = E' = xE_p$, so

$$y = \frac{1}{2}(1 - \cos\theta)$$

$$(1 - y) = \frac{1}{2}(1 + \cos\theta) \quad .$$

We understand the significance of (1 - y) as follows. When y = 1, the electron is scattered at 180°; for a given helicity of the incident electron, the exchanged photon (boson) is completely polarized. The struck quark must change helicity by one unit to absorb the photon (boson) and so only quarks of the appropriate helicity can interact. This term (1 - y) appears in the cross sections for the part where the quark helicity is imporant.

In the laboratory system of the collider, the locus of the momentum vector is an ellipse with the interaction region at one focus as in Fig. 2.

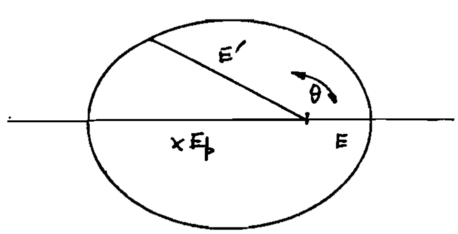

Fig. 2

When x = 1, the electron momentum is maximal for any given outgoing angle and the equation of the ellipse is

$$\frac{1}{E'} = \alpha(1 + \varepsilon\cos\theta)$$

with

$$\varepsilon = \frac{E_p - E}{E_p + E}\ , \text{ and } \alpha = \frac{E_p + E}{2EE_p} \quad .$$

When $x \neq 1$, then the locus is still an ellipse but with E_p replaced by xE_p giving

$$\frac{1}{E'} = \frac{(xE_p + E)}{2E.xE_p}\left(1 + \frac{xE_p - E}{xE_p + E}\cos\theta\right) \quad .$$

At θ = 0 and π, E' is E and xE_p as expected. U sing the ellipse equation, we can write

$$x = \frac{EE'\ (1 - \cos\theta)}{E_p\ (2E - E'(1 + \cos\theta))} \quad .$$

Since we measure θ and E', we can also write

$$\frac{dx}{x} = \frac{1}{2}\left(1 + \frac{xE_p}{E}\cot^2\theta/2\right)\frac{dE'}{E'} \quad .$$

When θ = 0, this factor becomes infinite and so we know E' and x are decoupled. Any knowledge of x comes from the current jet. As we go away from θ = 0, the term $\frac{xE_p}{E}\cot^2(\theta/2)$ is still troublesome if we have a collider with $xE_p >> E$ for interesting values of x. We can get around this with measurements of the current jet again but with reduced resolution.

In addition, all quasi photoproduction is done at low Q^2, and although E' becomes smaller than E and energy is transferred to the virtual photon, the mass of the current jet (onia) cannot be determined from the outgoing electron.

The Electromagnetic Process

The leading term in inelastic electron scattering is shown in Fig. 3.

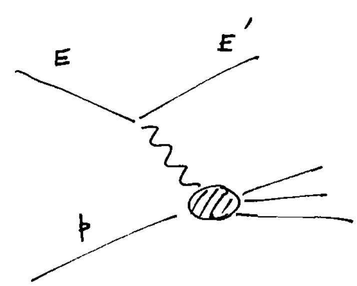

Fig. 3.

The cross section for this process is often written

$$\frac{d^2\sigma}{dxdy} = \frac{4\pi\alpha^2}{sx^2y^2}\left[(1 - y)F_2(x, Q^2) + y^2\ x\ F_1(x,Q^2)\right] \quad .$$

The Callan-Gross relation is derived from the assumption that partons (quarks) that couple to photons have spin 1/2 and gives $F_2 = 2xF_1$ and then

$$\frac{d\sigma^2}{dxdQ^2} = \frac{4\pi\alpha^2}{xQ^4}\left[1 - \frac{Q^2}{sx} + \frac{1}{2}\left(\frac{Q^2}{sx}\right)^2\right]F_2(x,Q^2) \quad .$$

The function in the square brackets is 1 when $Q^2 = 0$ and 1/2 when Q^2 is the maximum for that s and x. The variation is monotonic between these bounds and as we will see below, at the values of Q^2 that are accessible, this function varies slowly and is always close to unity. This leads us to the assertion that providing that the values of Q^2 and x are accessible to the kinematic region, then the cross section is almost independent of s.

We have the prejudice that much of the physics that is basic to electroproduction, involves the detailed measurement of distributions with reasonable precision in each bin. We have chosen bins that are constant in dQ^2/Q^2 and dx/x. We choose dQ^2/Q^2 because the scale breaking effects are logarithmic in Q^2 and so at large Q^2, we can afford to use larger bins. In contrast, the most rapid variations in the structure functions are at low x so that to observe the functions properly, we need fine bins at low x. We have chosen in our plot $dQ^2/Q^2 = 0.1$ and $dx/x = 0.1$ also. If these bins are seen to be too fine (or coarse), then the rate adjustment is trivial. We must discuss luminosities at some length, for as we see below, they are crucial, but for the moment we assume that an integrated luminosity for a single experimental configuration of 10^{38} is a reasonable number.

Then the rate in each bin is

$$\text{RATE} = \frac{4\pi\alpha^2}{Q^2}\left[1 - \frac{Q^2}{xs} + \frac{1}{2}\left(\frac{Q^2}{xs}\right)^2\right] F_2\,(x,Q^2)\,\frac{dQ^2}{Q^2}\cdot\frac{dx}{x} \quad .$$

Numerically, Q^2 is in GeV^2, as is s and then

$$4\pi\alpha^2 = 4\pi\,(1/137)^2\,0.197^2 \times 10^{-26} = 2.60 \times 10^{-31}\ \text{cm}^2\ \text{GeV}^2 .$$

For the assumed bin width and integrated luminosity of 10^{38},

$$\text{RATE} = \frac{2.60 \cdot 10^5}{Q^2}\left[1 - \left(\frac{Q^2}{xs}\right) + \frac{1}{2}\left(\frac{Q^2}{sx}\right)^2\right] F_2(x,Q^2) \quad .$$

In very approximate terms, the photon couples to the charge squared of the quark, so that it is four times as likely to see an up quark as a down. So at any finite x, we can neglect the sea and guess that $F_2 \sim 2/3$. At $Q^2 = 1000$, we get 170 events in a bin. A real calculation give 104 which is a pretty fair agreement. Frequently the rate above a given Q^2 is plotted which gives a flatter dependence on Q^2 which we feel is not especially relevant to the physics of nucleon structure. We have arbitratily assumed that 100 events in our chosen bin is our threshold of measurement credibility. We can then make a curve which traces the location of this limiting bin as a function of x and Q^2 for a given x. We show such a plot in Fig. 4. We include weak neutral current effects here, but note that the single photon exchange process falls between the e^-_L and e^+_L curves. s is chosen at 40,000 GeV^2 (25 on 400). First note that the kinematic boundary is way off scale ($Q^2_{max} = s = 40{,}000$). Our first lesson is that the region of accessible measurements is luminosity limited and not s limited. The line near x = 0 is the kinematic limit and although the low x high Q^2 region is limited by s, it is only slightly so.

In Fig. 5, we show a similar curve with x = 32,000 but with slightly improved luminosity, and we

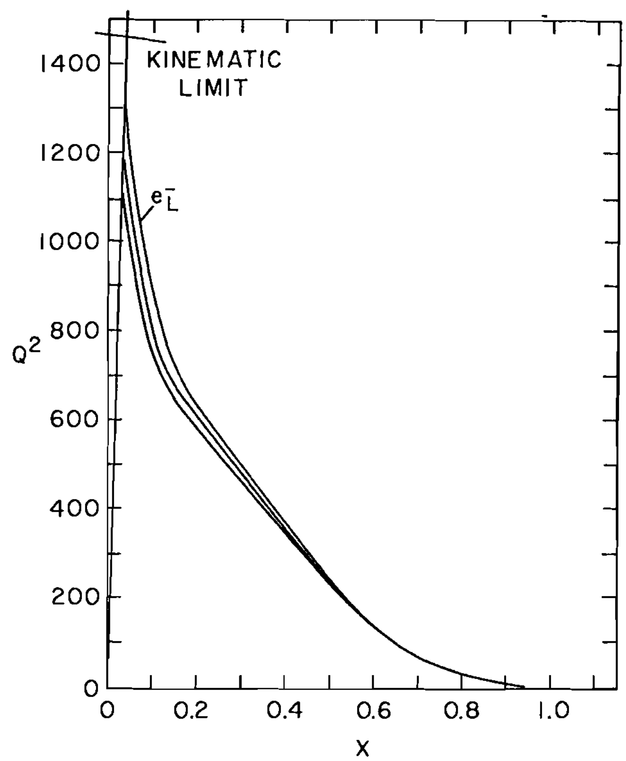

Fig. 4. The luminosity boundary for the reaction

$$e + p \rightarrow e' + x$$

for s = 40,000 GeV^2 $\sin^2\theta_W = 0.22$ and an integrated luminosity of 10^{38} cm^{-2} for each polarization state.

see that luminosity wins. It seems a shame to ignore the events above the luminosity boundary and we offer an approximate method for computation. Note first that $F_2(x,Q^2)$ varies quite slowly with Q^2 and we can ignore that variation for simple estimates. Then we have an equation

$$\text{RATE} = 1000\ Q^2_{100} \int_{Q^2_{100}}^{sx} \frac{dQ^2}{Q^4}$$

$$= 1000\ (1 - Q^2_{100}/sx) \quad .$$

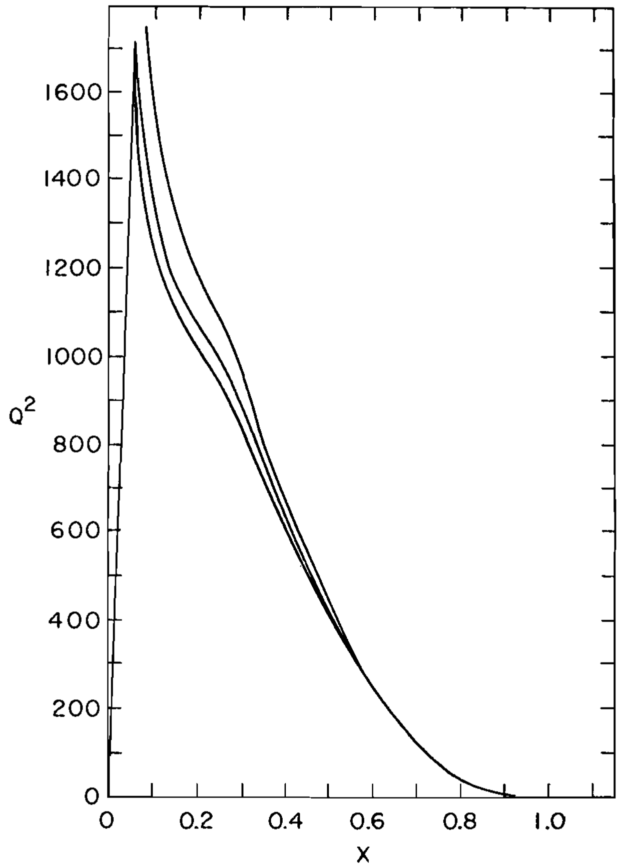

Fig. 5. The luminosity boundary for the reaction

$$e + p \rightarrow e' + x$$

for $s = 32{,}000\ \text{GeV}^2$, $\sin^2\theta_W = 0.22$ and an integrated luminosity of $2 \times 10^{38}\ \text{cm}^{-2}$ for each polarization state.

Structure Functions

As we have implied, $F_2(x,Q^2)$ is a function that describes the density of partons in x, which was independent of Q^2 in the good old days. Now we are aware that scale breaking occurs, and series of authors[6] have made parameterizations of the quark and gluon densities as a function of x. As Q^2 increase, these densities change, mostly in that the effect of QCD is to take into account the emission of gluons which in turn become quark antiquark pairs, forming an enhancement of the number of "wee" partons and a corresponding diminution of the quarks at high x. Buras and Gaemers[6] offered an early parameterization followed by some nominally more accurate versions by Owens and Reya and Baier et al. In this note, we will use Buras and Gaemers partly because of the ease of comparison with other authors concerned with ep. Buras and Gaemers parameterize the distributions as valence quarks, SU(3) symmetric sea quarks, charm quark distribution, and a gluon distribution. It is not necessary here to dwell on the niceties of the game except to say that the valence quark distributions go like

$$xV \sim x^{\eta_1} (1 - x)^{\eta_s}$$

and the others like

$$xS \sim (1 - x)^{\eta_3}$$

The coefficients and the η depend on Q^2 through a scale breaking parameter

$$\bar{s} = \ln \left(\frac{\ln Q^2/\Lambda^2}{\ln Q_0^2/\Lambda^2}\right) \quad .$$

Λ is typically 0.3 GeV and $Q_0^2 = 1.8\ \text{GeV}^2$. The value of Q_0^2 is unimportant but the value of Λ is crucial in QCD evaluations.

The η are linear in $\bar{s}$. Fits are done to experimental data on leptoproduction to get all the necessary constants. The other authors use a similar technique, but with variations in the parameterization. Fig. 5 shows the Buras and Gaemers distributions for two values of Q^2 showing the scale breaking effect.

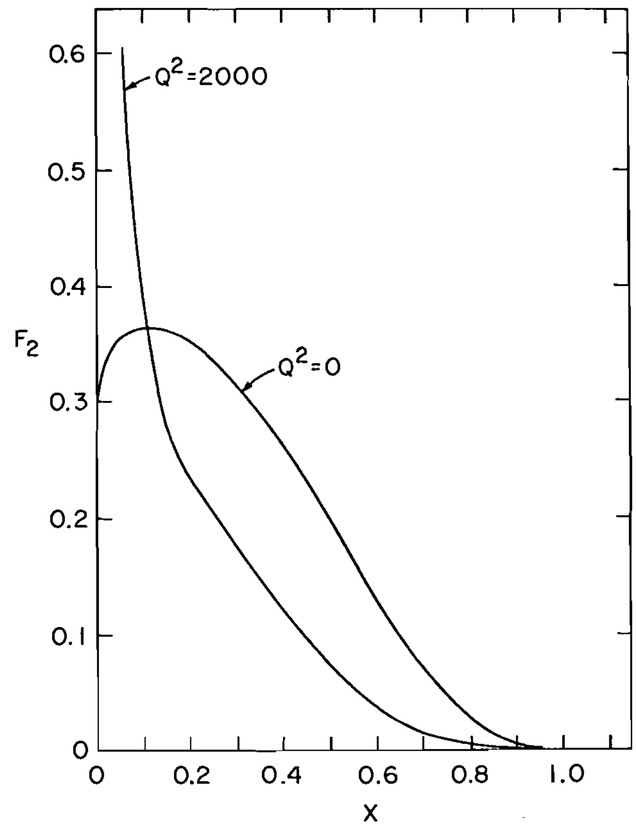

Fig. 6. Values of $F_2(x,Q^2)$ for two values of Q^2 as a function of x according to the prescription of Buras and Gaemers.

The Charged Current

The charged current cross section is easy to calculate but probably hard to measure. In terms of the usual structure functions, the cross section for the process

$$e + p \rightarrow \nu + x$$

is written

$$\frac{d^2\sigma}{dxdy} = \frac{G_F^2 s}{\pi} \left[(1 - y)\ F_2 + y^2 x F_1 - y(1 - y/2)\ xF_3\right] \; .$$

We use Callan-Gross

$$= \frac{G_F^2 s}{\pi} \left[(1 + (1 - y)^2\ F \ + (1 - y)^2) x F_3\right] \quad .$$

F_2 and xF_3 are not the same as in the photon exchange cse. For example, an incident e^- produces a

W^- only, which is capable of the transitions $u \to d$ or $\bar{d} \to \bar{u}$. If we restrict ourselves to the first two flavors then in terms of quark densities

$$F_2 = (u + c + \bar{d} + \bar{s}) \quad .$$

Also

$$xF_3 = -(u + c) + (\bar{d} + \bar{s})$$

Then if we rewrite the cross section in terms of these quark structure functions

$$\frac{d^2\sigma}{dxdy} = \frac{G_F^2}{4\pi} \cdot 2s \left((u + c) + (\bar{d} + \bar{s})\,(1 - y)^2\right) \quad .$$

because of (V-A), only quarks of a particular helicity interact, when this helicity is such as allow all ℓ_z of the incident W to conserve angular momentum then there is no $(1 - y)^2$ term. When the other helicity occurs, then the electron may not scatter at $\theta = \pi$, $y = 1$ from angular momentum considerations. Then in our preferred variables

$$\frac{d^2S}{dxdQ^2} = \frac{G_F^2}{\pi}\,\frac{1}{x}\left((u + c) + (1 - y)^2(\bar{d} + \bar{s})\right)$$

and in our usual bin structure

$$\text{RATE} = \frac{G_F^2}{\pi}\,Q^2 \left((u + c) + (1 - y)^2\,(\bar{d} + \bar{s})\right) \frac{dQ^2}{Q^2}\,\frac{dx}{x} \quad ,$$

and numerically the yield is shown in Fig. 7 for our usual luminosity and s. Notice that s does not come into this expression (against intuition), but if we ssume that the structure functions do not vary with Q^2, and we integrate over all Q^2 then

$$\text{RATE} \sim \int_0^{sx} dQ^2\,\frac{dx}{x} = s dx \quad .$$

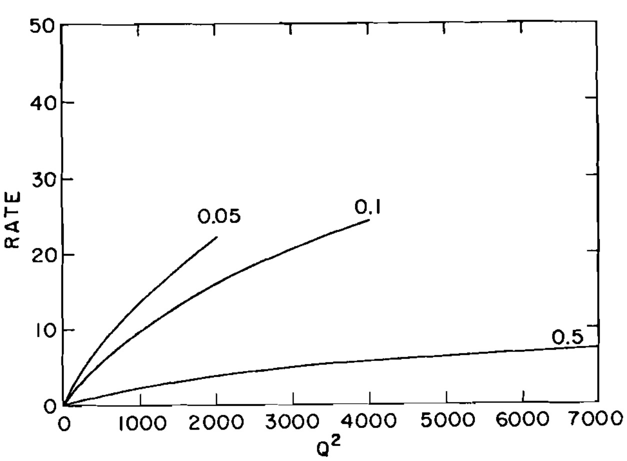

Fig. 7. The rate for the charged current reaction

$$e + p \to \nu + x$$

for $s = 40{,}000$ and $\sin^2\theta_w = 0.22$.

For a given x bin, we have a rate that is proportional to s. Again if we are interested in distributions, we do not improve the situation in any single bin by improving s, but, of course, we do get an increased range and the bin content is higher at higher Q^2. The expression above ignores the effect of the W mass, meaning that we should insert a W propagator, so

$$\frac{d^2\sigma}{dxdQ^2} = \frac{G_F^2}{\pi}\,\frac{1}{x}\,\frac{Q^2}{(1 + Q^2/m_W^2)^2}$$
$$\left((u + c) + (1 - y)^2(\bar{d} + \bar{s})\right)$$

and the effect of the propagator can be seen in the non-linear variation of rate with Q^2 in Fig. 6.

The handy rule of thumb for rate in a bin is

$$\text{RATE} = R_o\,\frac{Q^2}{(1 + Q^2/m_W^2)^2} \cdot \frac{dQ^2}{Q^2} \cdot \frac{dx}{x} \quad .$$

Then R_o for various values of x is

x	R_o
0.05	2
0.1	1.5
0.25	1
0.5	0.3

for our usual bin widths.

For an overall rate, we can integrate the expression over Q^2 and approximate the x distribution by assuming that it is flat out to x_{max} (0.3) say, and zero thereafter

$$\text{RATE} = \langle R_o\rangle\, m_W^2 \ln\left(1 + \frac{sx_{max}}{M_W^2}\right) \quad .$$

This is within 20% of the exact calculation with $\langle R°\rangle = 1.5$ and $x_{max} = 0.25$. Notice that when $sx_{max} > M_W^2$ the effect of increasing s is much reduced. When $sx_{max} = M_W^2$, and for 10^{38} total luminosity, we have 10^4 events, the error in M_W is twice that of the rate itself, so we might (very naively) say that this length of run gives ΔM_W of 2%. Real calculations with resolution, etc. believe that the error would be twice this.

Implicit in the formulae of this section is that the electrons are left-handed, the simple model assumes that the right-handed positron case, W^+ are exchanged and similar arguments give the cross section

$$\frac{d^2\sigma}{dxdQ^2} = \frac{G_F^2}{\pi} \cdot \frac{Q^2}{\left(1 + Q^2/M_W^2\right)^2}\left((\bar{u} + \bar{c}) + (1 - y)^2(d + s)\right)$$
$$\frac{dQ^2}{Q^2} \cdot \frac{dx}{x} \quad .$$

The Weak Neutral Current

The weak neutral current is sensitive to the coupling of the Z^o to the different quarks. We can in principle explore the possibilities of more than one Z^o and couplings other than those of the standard model. However, in the spirit of providing a base to compare with exotic models, we will outline the rate calculations from the standard model in some detail. Moreover, it is convenient to calculate a factor describing the effect on the one-photon exchange cross section and to use the results of the previous sections to make the neutral current effect more clear.

The cross section can be written as the sum of three terms, the single photon part, the weak part, and the interference part. In practice, the weak part is generally negligible, and it is only the interference part that is important in the ep colliders that have been considered. We write the cross section as follows:

$$\frac{d\sigma}{dxdQ^2} = \frac{d\sigma}{dxdQ^2}\Bigg|_\gamma F/\ f_+\ F_{2em}$$

where

$$F = \sum_{ij} G^j_\eta \, p^i p^j \left[f_+ F_2^{ij} + \xi_n f_- xF_3^{ij} \right]$$

for standard model i,j = 1,2 and the subscript η refers to the incoming lepton state. When i = j = 1, we have the photon part and F = 1. When i and j are both 2, we have the weak part and when the 1,2 combinations occur, we have the interference term.

The G_η refers to the couplings of the electron to the photon or Z^0 and η runs 1 through 4 for e^-_L, e^-_R, e^+_L, e^+_R. Then

$$G^1_\eta = -1, -1, -1, -1$$

$$G^2_\eta = (-1 + 2 \sin^2\theta_w),\ 2 \sin^2\theta_w,\ 2 \sin^2\theta_w,\ (-1 + 2 \sin^2\theta_w)$$

$$\xi = +1, -1, +1, -1 \quad .$$

The p^i are propagator terms

$$p^1 = 1$$

$$p^2 = \sqrt{2}\,\frac{G_F\, Q^2\, M_Z^2}{e^2(M_Z^2 + Q^2)} \quad ,$$

where $M_Z = 37.4/\sin\theta_w/\cos\theta_w$ in the standard model. The kinematic terms

$$f_+ = \frac{1}{2}(1 + (1 - y)^2)$$

$$f_- = \frac{1}{2}(1 - (1 - y)^2) \quad .$$

The structure functions $F^{11}_2 = F_{2_{em}}$ and $xF^{11}_3 = 0$.

$$F^{12}_2 = \sum_q \left(G^1_{qL}G^2_{qL} + G^1_{qR}G^2_{qR}\right)(q + \bar{q})$$

$$xF^{12}_3 = \sum_q \left(G^1_{qL}G^2_{qL} - G^2_{qR}G^2_{qR}\right)(q - \bar{q}) \quad .$$

G_q	2/3	2/3	u c
	-1/3	-1/3	d s
G^2_{qL}	$1 - 4/3 \sin^2\theta_w$	$1 - 4/3 \sin^2\theta_w$	u c
	$-1 + 2/3 \sin^2\theta$	$-1 + 2/3 \sin^2\theta_\omega$	d s
G^2_{qR}	$-4/3 \sin^2\theta_w$	$-4/3 \sin^2\theta_w$	u c
	$2/3 \sin^2\theta_w$	$2/3 \sin^2\theta_w$	d s

These couplings allows us to evaluate F for any incoming lepton state and any quark q or $\bar{q}$. We use the structure functions of the previous section to evaluate q(x, Q^2).

Numerically, the coefficient of p^1p^2 is $\frac{\sqrt{2}\, G_F}{e^2}$ = 1.75×10^{-4} GeV^{-2}, and for $Q^2 \sim 1000$ and below this term is small compared to 1, and so the weak neutral current is proportional to G_F^2 squared and is negligible. For Q^2 of this order, $Q^2 \ll m_Z^2$, and the propagator effect is small. Also f_- is small. It is also true that in crude terms, the up valence quark dominates q and $\bar{q}$ is negligible.

The factor for e^-_L is then approximately $\sim 1 + 2\,(1 - 2 \sin^2\theta_w) \cdot 1.8 \cdot 10^{-4} \cdot Q^2 \cdot 2/3\,(1 - 4/3 \sin^2\theta_w) = 1.10\, Q^2$ This compares with (1.08) when it is done properly. Of course, the effect is destructive for e^+_R when G^2_η effectively change sign. The important facts to remember are that the interference part of WNC for e^-_L is roughly proportional to Q^2 and is about 10% at Q^2 of 1000. We show a specific calculation in Fig. 8. The reason that the curves show a slight tendency to increase in slope is the scale breaking effect that more parton density occurs at low x as Q^2 increases with a consequent increase in the WNC off-setting the propagator.

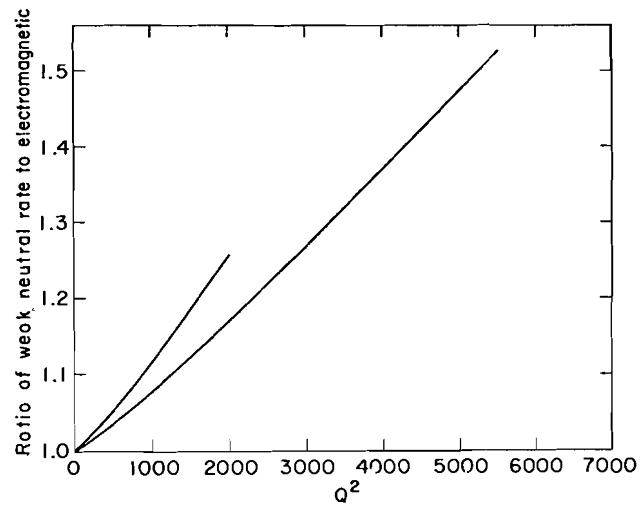

Fig. 8. Ratio of the weak neutral current cross section to the electromagnetic cross section as a function of Q^2 for x = .05 and x = 0.5.

<u>Photoproduction</u>

Vector meson production has been studied in detail using photon beams and at low Q^2 where clean signals are available in diffractive production of vector mesons. In an e-p collider, quasi-photoproduction can be studied at very high equivalent photon energies utilizing the low Q^2 (< 1 GeV^2) portion of the cross section. The photon energy (E-E') should be multiplied by the γ of the proton (E_p/m_p) to get the equivalent photon energy for the proton rest system. In the case of the 20 x 400 collider 4 TeV equivalent photon energy is easily available. The 20 x 400 effective luminosity is given by the Weizacker-Williams expression

$$d^2L_{\gamma*} = \frac{\alpha}{2\pi} L_{ep} \quad \frac{dQ^2}{Q^2} \cdot \frac{dy}{y} \left[1 + (1-y)^2 \right]$$

This luminosity is dominated by the low Q^2 region, and providing y is large enough that we are above threshold by the low y region also. We can divide the rate calculations into two parts, the virtual photon flux described above and the production cross section for heavy quark states.

$$\sigma_{\gamma*} = \sigma_o \cdot \left(\frac{m_v^2}{m_v^2 + Q^2} \right)^2 \cdot (1 - y_{th}/y) \quad .$$

The first term is a consequence of vector dominance and has been measured for ρ , ϕ and J/χ. The second term appears to fit the observed Q^2 dependence for all vector meson production from ρ to J/χ. The third term is a threshold factor, proportional to the square of the vector meson momentum in c.m. of the vector meson-proton system.

σ_o is proportional to $(q/m_q)^2$ and is approximately 0.5 μb for charm production.

At low Q^2 we can write

$$y \sim \left(1 - \frac{E'}{E}\right)$$

and since the energy squared in the c.m. of the virtual photon proton system is

$$W^2 = -Q^2 + sy + m_p^2$$

at low Q^2 again

$$y_{th} = \frac{m_v}{s}(m_v + 2m_p)$$

for any reasonable rate this is a small number, for example at 40 GeV given $y_{th} \sim .05$ on a 20 x 400 collider.

The minimum Q^2 is usually given by the minimum tagging angle for the outgoing electron θ, then for small

$$Q^2_{min} = E^2\theta^2\,(1-y)$$

when $\theta = 0$, the minimum value for no tag is $\dfrac{m_e^2 y^2}{(1-y)}$.

The yield can now be calculated

$$\text{Yield} = \frac{\alpha}{2\pi} \cdot L_{ep} \cdot \gamma_o \int_{y_{th}}^{1} \frac{dy}{y}\left(1-(1-y)^2\right)\left(1-\frac{m_v^2}{1-y}\right).$$

$$\int_{Q^2_{min}}^{Q^2_{max}} \frac{dQ}{Q^2} \cdot \left(\frac{m_v^2}{(m_v^2 + Q^2)}\right)^2$$

The yield is dominated by low Q^2 as we have said before, and then the second part of the integral is approximately $\ln\left(\dfrac{m_v^2}{E^2\theta^2(1-y)} - 1\right)$ and the first part is $\left(\ln\left(\dfrac{s}{m_v^2}\right)\right) - 1$.

It is important then to keep the tagging angle small, and $s \gg m_v^2$, although the improvement in the photon flux is only logarithmic with s/m_v^2.

For an example, take 40 GeV top in a 20 x 400 collider. The value of σ_o is 3nb, and with a tagging angle of 3° the flux integral is 5.2. The yield for an ep luminosity of 10^{38} is 1.8K events. Without tag the yield increases by about a factor of 3.

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