VECTOR MESON BACKGROUNDS AT HIGH ENERGY $e\bar{e}$ MACHINES

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This note presents cross sections for $e\bar{e} \to W^-W^+$, ZZ, $Z\gamma$ at high energies. The first of these is available in the LEP reports and elsewhere,[1,2] but the remaining two seem to be unavailable.† These cross-sections represent a background to potentially interesting physics on mass scales of order 1 TeV. They are large due to t channel lepton exchange diagrams. Higgs diagrams are neglected; they are zero if the electron mass is set to zero. The graphs of Fig. 1 give for $e^-e^- \to W^+W^-$

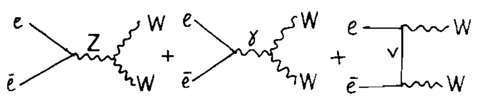

Fig. 1

$$\frac{d\sigma(e^-e^+ \to W^+W^-)}{d\Omega} = \frac{\alpha^2}{32\sin^4\theta_W}\,\frac{\beta}{s}\sum_i A_i$$

$$A_1 = \frac{2}{a} + \frac{\sin^2\theta}{2}\left[\left(\frac{s^2}{K^2}\right)^2 + \frac{1}{4a^2}\right]\beta^2$$

$$A_2 = \beta^2\left(\frac{16}{a} + \left(\frac{1}{a^2} - \frac{4}{a} + 12\right)\sin^2\theta\right)\left(\sin^4\theta_W + 2\left(\frac{s}{s-M_Z^2}\right)\right.$$

$$\left.\times \sin^2\theta_W\left(\frac{1}{4} - \sin^2\theta_W\right) + \left(\frac{s}{s-M_Z^2}\right)^2\left(\sin^4\theta_W - \frac{1}{2}\sin^2\theta_W + \frac{1}{8}\right)\right)$$

$$A_3 = \left[16(1 + M_W^2/K^2) + 8\beta^2/a + \frac{\beta^2\sin^2\theta}{2}\left(\frac{1}{a^2} - \frac{2}{a} - \frac{4s}{K^2}\right)\right]$$

$$\times \left(\left(\sin^2\theta_W - 1/2\right)\left(\frac{s}{s-M_Z^2}\right) - \sin^2\theta_W\right)$$

$$a = M_W^2/s \quad K^2 = M_W^2 - s/2 + \frac{s\beta}{2}\cos\theta$$

$\beta = \sqrt{1-4a}$ and θ is the angle between the e^+ and W^+. Those of Fig. 2 give for $e\bar{e} \to ZZ$

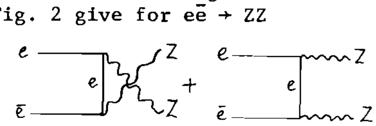

Fig. 2

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{512\sin^4\theta_W\cos^4\theta_W}\,\frac{(s/4 - M_Z^2)^{1/2}}{s\sqrt{s}\,t^2}$$

$$\times \left[1 + 6(1 - 4\sin^2\theta_W)^2 + (1 - 4\sin^2\theta_W)^4\right]$$

$$\times \left[-2t^4 - 4st^3 + 8M_Z^2t^3 - 3s^2t^2 + 8M_Z^2st^2 - 14M_Z^4t^2\right.$$

$$- s^3t + 2M_Z^2s^2t - 6M_Z^4st + 12M_Z^6t - M_Z^4s^2$$

$$\left. + 4M_Z^6s - 4M_Z^8\right]/(t + s - 2M_Z^2)^2$$

† Beware of the typographical error in LEP version. The correct formula is given in Ref. 2.

where $s = 4E^2$

$t = -2E(E - p\cos\theta) + M_Z^2$

$p = (s/4 - M_Z^2)^{1/2}$

and those of Fig. 3 give for $e\bar{e} \to Z\gamma$.

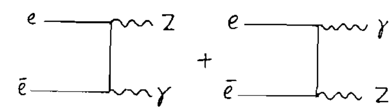

Fig. 3

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2(s - M_Z^2)\left[1 + (1 - 4\sin^2\theta_W)^2\right]}{32\sin^2\theta_W\cos^2\theta_W s^2 t(s + t - M_Z^2)}$$

$$\times \left[-2t^2 - 2st + 2M_Z^2t - s^2 - M_Z^4\right]$$

where now

$$t = -\frac{(s - M_Z^2)}{\sqrt{s}}\left[E - p_e\cos\theta\right]$$

here p_e is the electron (or positron) momentum. It is necessary to keep the electron mass (M_e) in the denominator to protect the mass singularity at $\cos\theta = \pm 1$. The cross-sections are shown in Figure 4 as a function of angle in units of the point cross section $R = \frac{4\pi\alpha^2}{3s}$ at $\sqrt{s}$ = 750 GeV and for $\sin^2\theta_W$ = 0.22. The ZZ and $Z\gamma$ cross-sections are symmetrical about $\theta = \pi/2$. The W^+W^- is not.

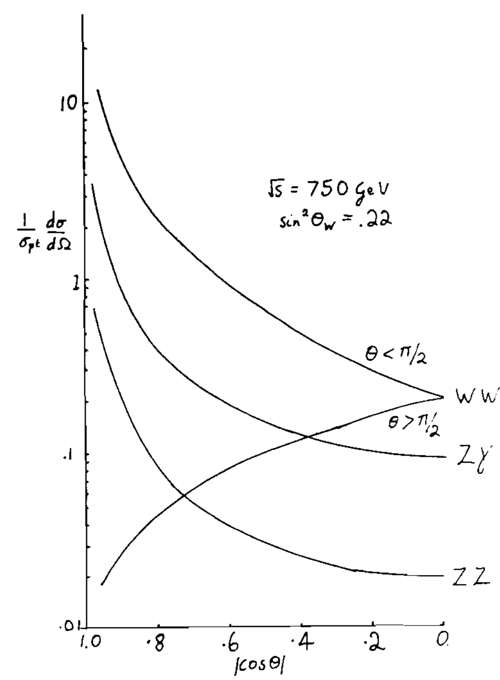

Fig. 4

The cross-section peaks near $\theta = 0$ due the presence of the t channel diagrams. The peaking is most severe for the $Z\gamma$ rate where the singularity is protected only by the electron mass.

Figure 5 the integrated cross section

$$\sigma_{INT} = \int_{-\theta}^{\theta} d\phi \; d(\cos\theta) \; \frac{d\sigma}{d\Omega}$$

is shown.

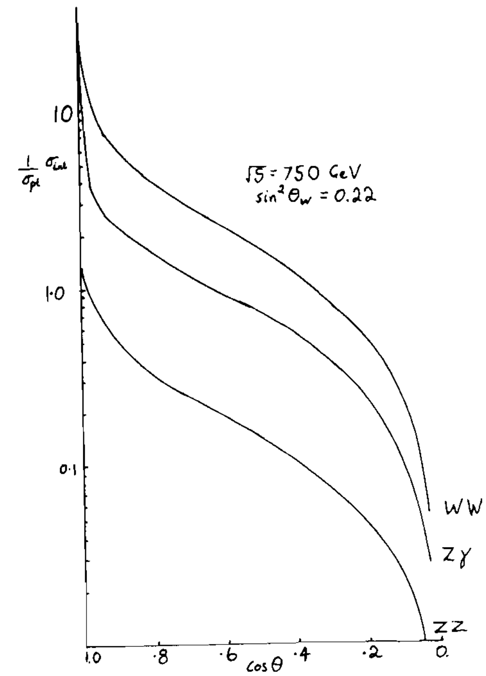

Fig. 5

The relevant formulae are

$$\sigma_{INT}(W^+W^-) = \frac{\pi\alpha^2}{16\sin^2\theta_W}\,\frac{\beta}{s}\sum_i A_i$$

$$A_1 = \frac{4\cos\theta}{a} + \beta^2\Big[\frac{\cos\theta}{4a^2}\Big(1 - \frac{\cos^2\theta}{3}\Big)$$

$$+ s^2\Big[\Big(1 - \frac{v^2}{u^2}\Big)\frac{\cos\theta}{v^2 - u^2\cos^2\theta} - \frac{\cos\theta}{u^2} + \frac{v}{u^3}L\Big)\Big]$$

$$A_2 = 2\beta^2\cos\theta\Big(\frac{16}{a} + \Big(\frac{1}{a^2} - \frac{4}{a} + 12\Big)\Big(1 - \frac{\cos^2\theta}{3}\Big)\Big)$$

$$\times\Big[\sin^4\theta_W + \Big(\frac{s}{s - M_Z^2}\Big)^2\Big(\sin^4\theta_W - \frac{1}{2}\sin^2\theta_W + \frac{1}{8}\Big)$$

$$+ 2\sin^2\theta_W\Big(\frac{1}{4} - \sin^2\theta_W\Big)\frac{s}{s - M_Z^2}\Big]$$

$$A_3 = \Big((\sin^2\theta_W - \tfrac{1}{2})\frac{s}{s - M_Z^2} - \sin^2\theta_W\Big) \times \Big[16\Big(2\cos\theta + \frac{M_W^2 L}{u}\Big)$$

$$+ \frac{16\beta^2\cos\theta_W}{a} + \beta^2\cos\theta\Big(1 - \frac{\cos^2\theta}{3}\Big)\Big(\frac{1}{a^2} - \frac{2}{a}\Big)$$

$$- 2s\beta^2\Big((1 - v^2/u^2)\cdot\frac{L}{v} + 2u\cos\theta/v^2\Big)\Big]$$

$$u = \frac{s\beta}{2}, \quad v = M_W^2 - \frac{s}{2}$$

$$L = \log\Big(\frac{v + u\cos\theta}{v - u\cos\theta}\Big), \quad a = M_W^2/s, \quad \beta = \sqrt{1 - 4a}$$

$$\sigma_{INT}(ee \to ZZ) = \frac{\alpha^2\pi}{64\sin^4\theta_W\cos^4\theta_W\, s\sqrt{s}}$$

$$\times\,[1 + 6(1 - 4\sin^2\theta_W)^2 + (1 - 4\sin^2\theta_W)^4]$$

$$\times\left\{\frac{2(M_Z^4 + s^2/4)}{(2M_Z^2 - s)\sqrt{s}}\log\left[\frac{p\sqrt{s}\,(2M_Z^2 - s) - \cos\theta[2M_Z^2 s - s^2/2]}{p\sqrt{s}\,(2M_Z^2 - s) + \cos\theta[2M_Z^2 s - s^2/2]}\right]\right.$$

$$\left. - p\cos\theta - \frac{p\cos\theta M_Z^4}{M_Z^4 - sM_Z^2 + s^2/4 + \cos^2\theta[sM_Z^2 - s^2/4]}\right\},$$

where $p^2 = s/4 - M_Z^2$.

$$\sigma_{INT}(ee \to Z\gamma) = \frac{\alpha^2\pi[1 + (1 - 4\sin^2\theta_W)^2]}{4\,s^2\sin^2\theta_W\cos^2\theta_W}$$

$$\times\left[-(s - M_Z^2)\cos\theta + \frac{(M_Z^4 + s^2)}{(s - M_Z^2)}\log\left[\frac{\sqrt{s} + 2p_e\cos\theta}{\sqrt{s} - 2p_e\cos\theta}\right]\right],$$

where $p_e^2 = s/4 - M_e^2$.

The total rates of W^+W^- and $Z\gamma$ are very large (24 and 31 units of R respectively at $\sqrt{s} = 750$ GeV), but are strongly forward peaked. A cut of $|\cos\theta| < .8$ reduces them considerably (to 4.1 and 1.6 units of R respectively). The ZZ rate is rather small. This is accidental; it is caused by $\sin^2\theta_W$ being close to 1/4. If $\sin^2\theta_W$ were $\frac{1}{2}$ the ZZ rate would increase by a factor of 8. Given the angular cuts the rates appear to be small enough not to present a serious background at high energies.

REFERENCES

1. Physics with Very High Energy $e\bar{e}$ Colliding Beams, CERN 76-18.
2. W. Alles, Ch. Boyer, A. J. Buras, Nucl. Phys. B117, 125(1977).

ACKNOWLEDGMENT

This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098.