MASS AND WIDTH OF THE Z_0 IN e^+e^- ANNIHILATION

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The mass of the Z_0 is predicted by the standard model and measurements of $\sin^2\theta_w$.[1] Whether the mass is in agreement with these predictions will almost certainly be answered at some precision long before any Z_0 factory can be constructed. Grand unified theories relate m_z to other measurable parameters such as $\overline{\Lambda}_{ms}$.[1] Precise measurement of the Z_0 mass is an important constraint on these theories.

The width of the Z_0 also has important implications for possible theories because it is sensitive to the number of fundamental constituents with mass less than one-half the Z_0 mass. For example, a $\nu\bar{\nu}$ pair contributes 160 MeV to a width of approximately 2.5 GeV.[2] Precision measurement of the Z_0 width is an important contribution of Z_0 factories.

Both the mass and width measurements are completely dominated by systematic errors. A rough model of the peak as a Gaussian indicates that 1000 nb^{-1} will be adequate to obtain statistical precision of ±60 MeV for the Z_0 width; this leads to the conclusion that systematic problems dominate. The following sources of systematic errors are considered in detail below:

a) Luminosity measurements,
b) Effects of the energy width of the beam,
c) Absolute energy measurement,
d) Radiative effects.

Several of these discussions require numerical estimates; for these purposes numbers for CESR II are used.[3]

Luminosity Measurements

The luminosity should be monitored by a high counting rate reaction which is independent of the difference between the center-of-mass energy and the Z_0 mass. Small angle Bhabha scattering is the obvious choice because of rate.

Using the quadrupole strengths and beam emittance from ref. 3, the radius of the beam pipe is determined by synchrotron radiation emitted in the horizontally focusing quadrupole at the interaction point. Considering a beam particle at 5σ in this quadrupole, the beam pipe must be 7.5cm in radius to avoid synchrotron radiation striking the pipe in the detector. Putting the luminosity monitors at 1m and 8.0cm from the beam, the minimum angle for a luminosity monitor is 80 mrad.

The SLC Workshop report[4] contains a detailed discussion of the effect of the Z_0 on the small angle Bhabha rate. Measuring from 80 to 140 mrad the cross section is 13nb (about ¼ the peak Z_0 cross section!), and the maximum effect of the Z_0 on the Bhabha rate is 1.4%.

Near the peak of the Z_0 the dominant statistical error will be from the luminosity measurement. The systematic error which affects the width, effect of the Z_0 on the Bhabha rate, should not be significant. Of course, a study of large angle Bhabha's should make it possible to correct the small angle rate, and by iteration the effect of the Z_0 on luminosity measurements ultimately should be less than 0.5%.

Energy Width of the Beam

An estimate of the effect of the energy spread of the beam on the Z_0 width can be made by treating the resonance as a Gaussian. Then the error introducted in the width of the Z due to uncertainty in the beam energy spread is:

$$\delta\Gamma_Z \cong 2.35 \, \frac{\sigma_w}{\Gamma_Z} \, \delta\sigma_w$$

where σ_w/Γ_Z is the ratio of the center-of-mass energy spread to the width of the Z_0, and $\delta\sigma_w$ is the uncertainty in the energy spread. Using Γ_Z^w = 2.5GeV and $\sigma_w = \sqrt{2} \times 96$ MeV [3], $\delta\Gamma_Z = .13\ \delta\sigma_w$. Knowledge of the energy spread to ±100% contributes only 18 MeV uncertainty to the width. This precision should be easily obtainable by synchrotron light monitors measuring the horizontal beam size in regions of finite and zero η. The difference in the horizontal size of the beam at two such stations will be 40% and should be easy to measure.

Absolute Energy Calibration

The best measurement of absolute beam energy comes from studies of beam polarization vs energy. The spin equivalent of the closed-orbit resonance occurs at spacings of 440 MeV; beam polarization is measured by a Compton scattering polarimeter. At the Z_0 energy the beam rms energy spread is ∿100 MeV and the polarization time ∿2 minutes. With this combination of short polarizing time and beam energy spread a sizeable fraction of the separation between resonances it is not clear whether the beam will be polarized. Two additional methods of measuring the energy are discussed below. One of the methods is a magnetic field measurement; the other uses an undulator. The former method relies on beam polarization at a lower energy for calibration; the latter is absolute but would benefit from a cross-check with a beam polarization measurement.

Uniformity (from magnet-to-magnet) of the field integral per ampere-turn can be controlled during manufacture to better than one part per thousand. The ratio of the field integral to the central magnetic field should be studied in several magnets before installation in the ring. With NMR probes installed in a reasonable number of randomly chosen magnets, it should then be possible to track the ring energy because:

a) From the (field integral/central field value) measurements the effective length is known as a function of the central field value

b) The average central field value is measured by the NMR probes.

The number of magnets to measure and the number of NMR probes to install would depend on experience with the magnet manufacture. Such a system of field probes should be calibrated and checked at several depolarizing resonances. It should be possible to measure the beam energy in the Z_0 range to ±0.1%.

An undulator with field

$$B = B_0 \sin \frac{2\pi z}{\lambda_w}$$

produces a quasi-monoenergetic spectrum[5]

$$\lambda = \frac{\lambda_w}{2\gamma^2 n}\left(1 + a_w^2 + \gamma^2\theta^2\right) \qquad (1)$$

where θ = emission angle

$\gamma = E/m_0$

n = integer

$$a_w = \frac{eB_0\lambda_w}{2\pi m_0 c^2} \quad \text{(cgs units)}$$

$$\cong B_0(T)\lambda_w(\text{cm})$$

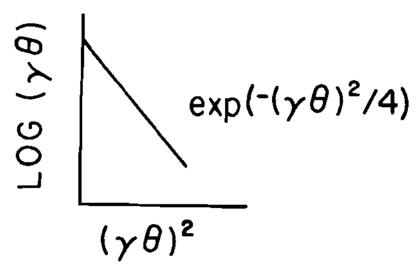
The magnetic field dependence is the same as for a Free Electron Laser and arises from a phase coherence condition.[6] The intensity for $n = 1$, $\theta = 0$ varies as (ref. 5) $n_w^2\gamma^2 a_w^2/(1 + a_w^2)$ where n_w is the number of periods of the undulator.

Even if one looks directly at the beam, $\theta \neq 0$ because of the betatron oscillations. For a betatron oscillation of amplitude a, the rms slope of the trajectories is:

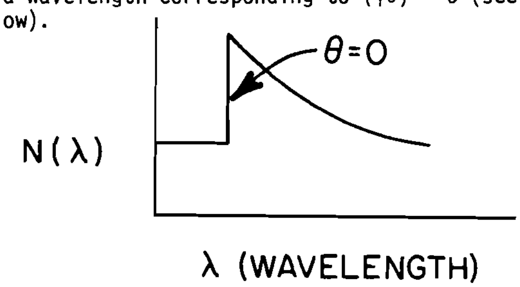
$$\theta^2 \sim \frac{a^2}{\beta} \sim \frac{\varepsilon}{\beta}$$

where β is the beta function at the emission point, and ε is the emittance. For CESR II one would want to work in the vertical plane, and assuming a coupling of order 0.1 the average θ^2 is 4×10^{-10}.

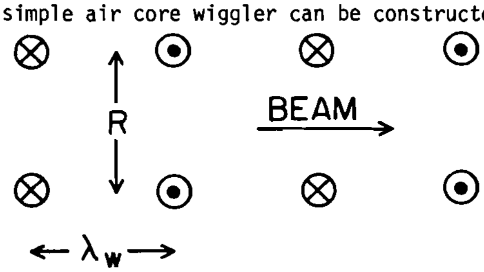
The actual distribution in $(\gamma\theta)^2$ is exponential with a slope of -1/4



and the n = 1 spectrum will have a sharp discontinuity at a wavelength corresponding to $(\gamma\theta) = 0$ (see sketch below).



A simple air core wiggler can be constructed



This magnet has dipole and sextupole fields but no quadrupole fields. R must be chosen large enough to make the sextupole fields small. Choose λ_w = 1m to put the photons generated into the X-ray region, and R=0.2m to make the sextupole components small. For purposes of estimation the peak field is equated with the fundamental Fourier Amplitude to give:

$$B_0 \cong \frac{8\mu_0 I}{2\pi R} \qquad \text{(mks)}$$

To have $a_w = 1$, B_0 = 100 Gauss, $I \cong 600$ amps.

If the $\theta = 0$ edge is sufficienty sharp, the energy can be determined directly from eq. 1. The current can be measured and from it the magnetic field calculated; the wavelength corresponding to $\theta = 0$ can be measured with negligible uncertainty.

If the edge is not sufficiently sharp to allow an unambiguous measurement of λ, a point halfway up the edge can be selected, and one can measure the variation of this wavelength with current. The slope of the wavelength for $\theta = 0$ vs current is then:

$$\frac{d\lambda}{dI^2} = \frac{8\mu_0^2\lambda_w^3}{\pi^2 R^2\gamma^2} \qquad \text{so}$$

$$\gamma = \frac{2\mu_0\lambda_w}{\pi R}\sqrt{\frac{2\lambda_w}{d\lambda/dI^2}}$$

To obtain $\sigma_E/E = \pm 10^{-3}$ should be straight-forward. The mechanical parameters λ_w and R should be controlled to better than ±0.002". Current measurements of $\pm 1 \times 10^{-4}$ are of sufficient precision and are not difficult, and X-ray wavelength measurements of this precision are possible.

Radiative Effects

Final state radiative effects have been estimated[7] and shown to be negligible provided tight cuts are not placed on the final state energy. The dominant effect is initial state radiation. The first order radiative correction to the cross section is[8]:

$$\delta\sigma = \frac{2\alpha}{2\pi}\ln\frac{s}{me^2}\int_0^1 dx\,\frac{1 + x^2}{1 - x}\left(\sigma(xs) - \sigma(s)\right)$$

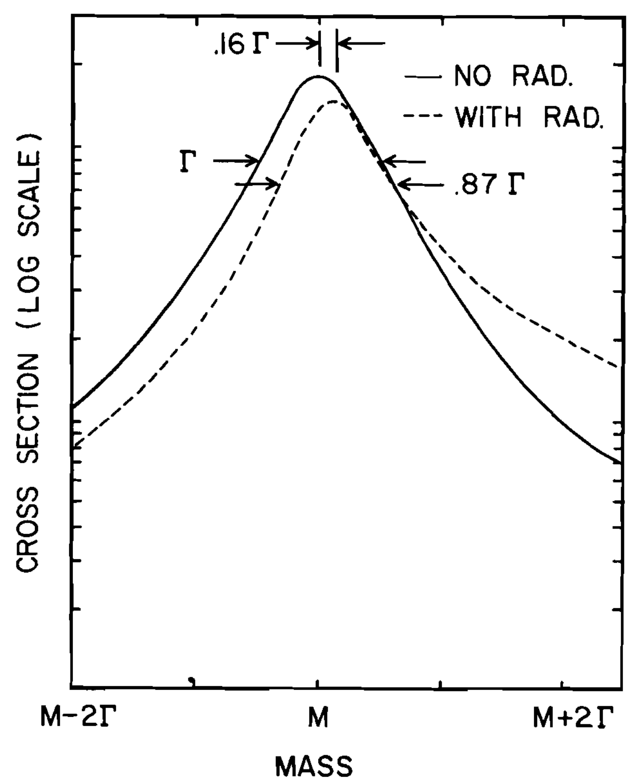


Figure 1: Z_0 cross section with and without initial state radiation.

where s is the square of the center-of-mass energy, α is the fine structure constant and m_e is the electron mass. For σ use

$$\sigma = \frac{1}{(s-M^2)^2 + \Gamma^2 M^2}$$

The resultant cross sections with and without the initial state radiative correction are shown in Figure 1. The observed mass is shifted by approximately $\frac{\Gamma}{6}$ = 420 MeV, and the observed width is decreased by 13%.

It is clear that the first order initial state radiation must be calculated more carefully and second order corrections will also be needed. The uncertainty in the mass of ± 60 MeV and in the width of ± 50 MeV can be achieved if the uncertainty in the radiative correction is ±10%. This should be possible if the second order radiative correction are calculated.

Conclusions

Measurement of the mass of Z_0 should be possible with precision of $\pm 1/10^3$. The dominant systematic problem is the absolute energy calibration. The width can be measured± 50 MeV where the initial state radiation is the dominant error. Interpretation of the width will require additional QCD and electroweak correction.

References

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