LUMINOSITY OF CONTINUOUS BEAMS WITH CROSSING ANGLE

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Since it appears difficult to reach a luminosity of even 10^{32} cm^{-2} sec^{-1} with $(p, \bar{p})$ in a single ring if the number of events per bunch collision should not exceed unity, it seems appropriate to ask what could be done with two continuous beams in independent rings, ISR style. This subject was treated at some length in the first ICFA report by Keil and King[1]), based on an optimization procedure developed earlier by Keil[2]). In this note, a different approach is taken by considering the length of the interaction diamond and the luminosity to be of primary importance and relating the other parameters to them.

Luminosity Formulas

In Reference 2, an expression is given for the luminosity of two continuous round beams with gaussian transverse distribution colliding at an angle, α:

$$\mathcal{L} = \frac{c\lambda^2}{\pi\sigma_0^2}\,\beta_0^2 \int_0^{\ell_i/2} \frac{ds}{s^2+\beta_0^2}\, e^{-\frac{\eta^2}{4}\frac{s^2}{s^2+\beta_0^2}} = \frac{c\lambda^2\beta_0}{\pi\sigma_0^2}\int_0^{\tan^{-1}\frac{\ell_i}{2\beta_0}} d\theta\; e^{-\frac{\eta^2}{4}\sin^2\theta} \tag{1}$$

where:
- λ = no. protons per unit length in each beam
- σ_0 = r.m.s. beam radius at crossing point
- β_0 = β-function at crossing point
- $\eta = \frac{\beta_0\alpha}{\sigma_0}$
- ℓ_i = total length in which 95% of the events occur; i.e., $\mathcal{L}(\ell_i) = .95\mathcal{L}(\infty)$
- c = velocity of light

The parameter, η, has a geometric meaning. As a function of distance, s, from the crossing point, the beam centers are displaced from each other by a distance,

$$\Delta = \alpha s.$$

At the same time the r.m.s. beam radius increases with s according to:

$$\sigma = \sigma_0\sqrt{1+\frac{s^2}{\beta_0^2}} \sim \frac{\sigma_0 s}{\beta_0} \text{ for } s > \beta_0$$

then $\frac{\Delta}{\sigma} \sim \frac{\beta_0\alpha}{\sigma_0} = \eta$; that is, η measures the separation of the beams in units of beam radius. For $\eta \gtrsim 5$, the beams are said to be well separated.

Table 1 gives the value of the dimensionless integral (I) in equation (1) and the corresponding value of $\frac{\ell_i}{2\beta_0}$ for a range of values of η. It can be seen that small values of η lead to high luminosity, but only because the length of the interaction diamond is large. The luminosity per unit diamond length is larger for higher η since the beams are more compact where they interact. Furthermore, since ℓ_i should be in the range of 1-2 meters for the sake of the detectors and we are considering in this workshop $\beta_0 \geq 2$ meters, $\frac{\ell_i}{2\beta_0}$ should be less than $\sim\frac{1}{2}$, corresponding to $\eta \gtrsim 6$ (the well separated regime). For large η, the integral in equation (1) can be approximated by an error function, leading to the simple relations:

$$\alpha = 5.56\,\frac{\sigma_0}{\ell_i}, \tag{2}$$

$$\mathcal{L} = .535\,\frac{c\lambda^2}{\sigma_0\alpha} = 1.72\,\frac{c\lambda^2\gamma\ell_i}{\epsilon\beta_0}, \tag{3}$$

where the normalized emittance, ϵ, is defined as:

$$\epsilon = 6\pi\gamma\sigma_0^2/\beta_0{}^{*} \tag{4}$$

and the numerical coefficients are combinations of 2, $\sqrt{\pi}$, etc.

Table I

η	1	2	3	4	5	6
I	1.32	.96	.65	.46	.35	.29
$\frac{\ell_i}{2\beta_0}$	11.0	7.6	3.2	1.3	.74	.55

Beam-Beam Tune Shift

An expression for the tune shift in the plane perpendicular to the plane of crossing is given in reference (2) as:

$$\Delta\nu = \frac{2}{\pi}\frac{\lambda r_p}{\gamma\alpha^2}\int_0^{\frac{\ell_s}{\beta_0}} dx\,\frac{(1+x^2)}{x^2}\left[1 - e^{-\frac{\eta^2}{2}\frac{x^2}{1+x^2}}\right] \tag{5}$$

which, for large η and $\frac{s}{\beta_0} \gtrsim 1$ is approximately:

$$\Delta\nu = \sqrt{\frac{2}{\pi}}\,\frac{\lambda r_p\beta_0}{\alpha\sigma_0\gamma}\left[1+\sqrt{\frac{2}{\pi}}\frac{\ell_s}{\beta_0}\frac{\sigma_0}{\beta_0\alpha}\right] = 2.69\,\lambda r_p\,\frac{\ell_i}{\epsilon}\left[1+.142\,\frac{\ell_s\ell_i}{\beta_0^2}\right] \tag{6}$$

*In references (1) and (2), $\epsilon = 4\pi\gamma\sigma_0^2/\beta_0$.

where $r_p = \dfrac{e^2}{m_p c^2} = 1.535 \times 10^{-18}$ m

and ℓ_s is the distance from the crossing point to the nearest point where the beams can either be shielded electromagnetically from each other or deflected more rapidly away from each other by a bending magnet (for pp collisions). The second expression for $\Delta\nu$ is obtained by using equations (2) and (4). The second term in the bracket in equation (6) is the long range contribution; the tune shift in the crossing plane is approximately the negative of this term. Although the beams are well separated in the sense of $n \gg 1$, it will turn out that the absolute separation is of the order of a millimeter at reasonable distances, so that termination by a bending magnet sounds more practical than a shielding pipe.

Numerical Examples

By solving equation (3) for line density in terms of luminosity, the tune-shift (equation 6) can be written in yet another way:

$$\Delta\nu = 2.05 \left[\frac{\ell_i}{\varepsilon}\,\frac{\beta_0 \mathcal{L} r_p^2}{\gamma c}\right]^{1/2} \left[1 + .142\,\frac{\ell_s \ell_i}{\beta_0^2}\right]$$

$$= 7.25 \times 10^{-3} \left[\frac{\ell_i \beta_0}{\varepsilon_0} L\right]^{1/2} \left[1 + .142\,\frac{\ell_s \ell_i}{\beta_0^2}\right]$$

where $\gamma = 2 \times 10^4$ (20 TeV)

$L = \mathcal{L}/10^{34}$

$\varepsilon_0 = \varepsilon/\pi \times 10^{-6}$

and ℓ_i, β_0 and ε_0 are expressed in meters. It is assumed that $\Delta\nu$ should not exceed 5×10^{-3}. This workshop did not lead to agreement on a realistic value of ε_0; guesses ranged from 1 to 30 meter-radians, with 10 most favored. The total diamond length, ℓ_i, probably should not exceed 2 meters; according to (3), any reduction beyond what is necessary requires more protons per ring for the same luminosity. It was agreed that β_0 could not be less than 2 meters, and that that value would be difficult to achieve. Finally, the distance from interaction point to the nearest separating magnet, ℓ_s, probably must be at least 5-10 meters; that is, well clear of experimental equipment.

The first conclusions to draw are that $\mathcal{L} = 10^{32}$ ($L = 10^{-2}$) appears easily achievable for the other parameters in the ranges discussed above and that for $\mathcal{L} = 10^{34}$ ($L = 1$), $\varepsilon_0 = 1$ is too small, unless ℓ_i is reduced to ~20 cm. Table II lists all the parameters discussed for a 20 TeV machine with $\mathcal{L} = 10^{33}$, $\varepsilon_0 = 10$, $\beta_0 = 2$ and 4 meters and $\ell_i = 1$ and 2 meters. N is the total number of protons per ring, assuming a circumference of 60 km. (10T magnets).

At $\mathcal{L} = 10^{33}$, the $\Delta\nu$ requirement does not impose a severe restriction on ℓ_s. Instead of listing a maximum ℓ_s for $\Delta\nu = 5 \times 10^{-3}$, the table includes the separation of beam centers, Δ, at ℓ_s = 10 meters.

Caveats

Note that σ_0 in Table II is 10-20 microns. This implies that the beams in the independent rings must be steered to an accuracy considerably better than 10 microns to achieve the luminosity and avoid possible disaster from the beam-beam interaction. Also, synchrotron radiation in this configuration could be very troublesome. There is not only the problem of getting rid of the heat generated in the cryogenic environment, but the energy loss to the beam has to be made up in some way. The simplest solution would be a modest r.f. system, but then the luminosity would be modulated from zero to maximum at the r.f. frequency. Finally, and this would be true also for (p, $\bar{p}$) bunched beam operation, radiation damping would change the beam characteristics fast enough to be annoying (5-10 hour damping times) but not fast enough to wait for the beams to reach equilibrium after filling. Perhaps an emittance spoiling scheme to maintain the initial configuration would be needed.

References

1) Proceedings of the Workshop on Possibilities and Limitations of Accelerators and Detectors (ICFA sponsored) Fermi Lab, April, 1979, p. 117.

2) E. Keil, Nucl. Inst. and Meth. 113, p. 333 (1973). See also E. Keil, Perspectives on Colliding Beams. Proc. 8th Inter. Conf. on High Energy Accelerators, Stanford, p. 660 (1974).

Table II

β_0(m)	ℓ_i(m)	λ(m^{-1})	N	α(μ-rad)	σ_0(μm)	Δ(mm)	n
2	1	0.78×10^{10}	4.7×10^{14}	72	13	0.72	11.1
2	2	0.55×10^{10}	3.3×10^{14}	36	13	0.36	5.5
4	1	1.10×10^{10}	6.6×10^{14}	72	18	0.72	16
4	2	0.78×10^{10}	4.7×10^{14}	36	18	0.36	8