Synchrotron Radiation in Multi-TeV Proton Synchrotrons*

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Synchrotron radiation by protons of several TeV in magnetic guide fields of several Tesla becomes large enough to be a consideration in the design of future proton synchrotrons. This question has been considered by R.R. Wilson[1] and by Courant and Pellegrini[2]; those papers include the relevant formulae and conclusions. The purpose of this note (beyond education of the author) is to summarize and tabulate the numerical values for a matrix of possible accelerators and to suggest a possible solution to the problem of the power dissipation of synchrotron radiation in a 4K magnet and vacuum structure. In a 20 TeV, 10 Tesla storage ring, the luminosity will grow with a time constant of about 5 hours due to radiation damping of horizontal and vertical oscillations, however the power radiated by a quarter-ampere circulating beam will be over 40 kw.

The formulas of refs. 1 and 2 may be recast to a parameterization as articulated below. If the following definitions and units are used:

E in TeV

B in Tesla

C the circumference factor, R/ρ (ratio of machine radius to radius of curvature),

the characteristic damping time constant for the energy in vertical synchrotron oscillations, τ, is given by:

$$\tau = 3\times10^{7}\ C/B^{2}E \text{ seconds,} \tag{1}$$

or

$$\tau = \frac{C}{B^{2}E} \text{ years} \tag{2}$$

The critical energy, E_c, of synchrotron radiation is in the soft x-ray region;

$$E_c = 0.107\ BE^{2} \text{ eV}$$

The power radiated in synchrotron radiation is given by

$$P = 1.6\times10^{-7}\ \frac{NE}{\tau} = 5.63\times10^{-15}\ \frac{NE^{2}B^{2}}{C} \text{ watts} \tag{3}$$

where N is the number of total protons in the ring. The power dissipated per unit length in the ring, dP/dz, is:

$$\frac{dP}{dz} = 4.8\times10^{-5}\ \frac{PB}{E}\ \frac{\text{watts}}{\text{m}}. \tag{4}$$

In order to give numerical examples, we will assume C=1.25 and $N = 2\pi R(\text{km}) \times 5\times10^{12}$ protons, corresponding to a circulating current of 0.24 amperes. This corresponds to the performance of the Fermilab 400 GeV synchrotron.

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The phase space area (energy) damping time constant is half the time constant for damping vertical oscillations amplitude. Radial and synchrotron radiation damping are interrelated through the choice of machine parameters; with suitable parameters the energy in each of these degrees of freedom will damp with τ and the radial amplitude with 2τ. The luminosity for colinear colliding beams will hence grow with τ because of the shrinkage of beam diameter (in x and y). In practice the luminosity will be limited by various instabilities and beam-beam effects, although the synchrotron damping will surely improve the situation over the undamped case.

Courant and Pellegrini have looked at quantum fluctuations[2], and Wilson considered coherent radiation; in neither case was there any evidence of a problem or reason to modify the above formulae.

One serious question was posed in connection with the examples with large radiated power. If this power were absorbed by a vacuum wall at 2-4K, the cryogenics requirements become much more severe. As it costs 300-1000 watts of electric power to remove 1 watt of heat at 4K, the extreme examples (45 kw of synchrotron radiation) would require an additional 15-50 Mw to power the refrigerator. Of course a warm bore magnet as in the Palmer BNL design would avoid the problem, and in a cold bore system a small gaseous helium line along the outer radius of the vacuum chamber could carry this heat away. In either case, such cooling and heat transport would cost aperture.

An alternative solution would be to make use of the phenomenon of "total" external reflection of x-rays from metallic surfaces. When the radiation frequency exceeds the plasma frequency of the conduction electrons, the index of refraction falls below unity, and the reflection from a metal surface becomes very high for grazing angles (within some critical grazing angle). Unfortunately, the reflection is not total (100%) as the radiation absorption in the metal results in a complex index of refraction, nevertheless the reflection can exceed 90%.[3] This is used in astronomical imaging x-ray telescopes, such as the Einstein Observatory satellite, and in x-ray optics in conjunction with electron synchrotron radiation facilities.

This principle could be used in a 10-20 TeV, 10 Tesla synchrotron to pipe the radiation through a shiny aluminum vacuum tank in the bending magnets to straight sections where nitrogen-cooled baffles or roughened surfaces would absorb the radiation at an economical temperature. The vacuum pipe would thus function just as a curved plastic rod does for visible light, except for the above mentioned imperfect nature of total external reflection.

The synchrotron radiation leaves the beam with a characteristic angle of mc^2/E, or 10^{-4} radians at 10 TeV. The angle of incidence of this radiation against the chamber wall depends on the ratio of the major to the minor radii of the vacuum tank. With a tank radius of r and a radius of curvature of ρ, the grazing angle θ is given by

$$\theta = \sqrt{2r/\rho} \tag{5}$$

For a 20 TeV, 10 Tesla machine with r=3 cm and ρ=6 km, $\theta \cong 3$ mr, comfortably within the critical angle.

A more serious study of the parameters of the total external reflection for particular machine designs has not been done here for lack of time to collect the details of this process, however the electron synchrotron radiation facility people are very familar with this subject, and adequate design data are available.

References

1. R.R. Wilson, Proceedings of the "Workshop on Producing High Luminosity High Energy Proton-Antiproton Collisions" March 27-31, 1978; Lawrence Berkeley Laboratory, Fermi National Accelerator Laboratory, LBL-7574, UC-34c, CONF-780345 (1978).

2. E.D. Courant and C. Pellegrini, Proceedings of the Second ICFA Conference on Possibilities and Limitations on Future Accelerators, Les Diablerets, Switzerland, E. Amaldi, editor, CERN October, 1979.

3. C.H. Pruett, private communication.

TABLE

Machine E (TeV)	Field B (Tesla)	τ (a) (Seconds)		E_c (eV)	P (b) (watts)	dP/dz (watts/m)	N (Proton)
2	2	4.7×10^6	(1.8 mo)	.86	8.9	4.3×10^{-4}	1.3×10^{14}
	5	7.5×10^5	(1-1/4 wk)	2.15	22.5	2.7×10^{-3}	5.2×10^{13}
	10	1.9×10^5	(2.2 day)	4.3	44	11×10^{-3}	2.6×10^{13}
5	2	1.9×10^6	(3.2 wk)	5.4	134	2.6×10^{-3}	3.2×10^{14}
	5	3×10^5	(1/2 wk)	13.4	346	1.7×10^{-2}	1.3×10^{14}
	10	7.5×10^4	(20 hr)	16.9	693	6.6×10^{-2}	6.5×10^{13}
10	2	9.4×10^5	(1.6 wk)	21.5	1106	10.6×10^{-3}	6.5×10^{14}
	5	1.5×10^5	(1.7 day)	53.7	2775	6.6×10^{-2}	2.6×10^{14}
	10	3.8×10^4	(10 hrs)	107	5475	2.6×10^{-1}	1.3×10^{14}
20	2	4.7×10^5	(5.4 day)	86	8850	4.3×10^{-2}	1.3×10^{15}
	5	7.5×10^4	(20 hrs)	215	22,190	2.6×10^{-1}	5.2×10^{14}
	10	1.9×10^4	(5 hrs)	430	43,790	10.5×10^{-1}	2.6×10^{14}

(a) damping time for emittance, or oscillation energy. Oscillation amplitude will damp with 2τ.

(b) per beam of 0.24 amperes circulating current.