BEAM STRAHLUNG EFFECTS IN e-p COLLIDER*

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Introduction

The electromagnetic fields produced by one beam in an interaction point of a colliding beam facility cause to the emission of synchrotron radiation by the other beam. This effect, the beam strahlung, for the e^+e^- colliders has been considered by several authors,[1,2] and they have pointed out that the effect is very important consideration at very high energy e^+e^- colliders.

At the first glance, the beam strahlung effect can play an important role in the e-p collision due to the fact that the circulating currents in the collider are much higher than those of the e^+e^- machine. However the detailed study shows that is not the case because of the collision goemetry involved.

What follows in this note is the beam strahlung derivations using the method previously used by Hofmann and Keil.[1] The difference between this note and that of Hofmann and Keil is that in the case of e^+e^- collider, equal mass particles are involved in the consideration and, in the e-p case, the electrons radiates and the protons provide the electromagnetic fields.

Beam-Beam Synchrotron Radiation

At first we consider the geometry of the proton bunch which causes the electrons to emit the radiation. We assume an elliptical proton bunch which has a uniform transverse charge density. Then the three dimensional charge density depends only on the longitudinal coordinate z. That is to say, on the z-axis the beam has a Gaussian distribution. The current density can be expressed

$$\overset{p}{\lambda}(z) = \left[\frac{I(z)}{\pi abc}\right]_{proton} = \left[\frac{I(z)}{2\pi c\sigma_x\sigma_y}\right]_{proton}$$

with local current I(z), which is

$$I(z) = \left[\frac{I_o\sqrt{2\pi}\,R}{k_b\,\sigma_z}\exp\left(-\frac{z^2}{2\sigma_z^2}\right)\right]_{proton}$$

where a, b and c are the beam size expressed along the semi-axes of the ellipse, σ_x, σ_y abd σ_z are the rms sizes, R is the radius of the ring, k_b is the number of bunches in the ring, and I_o is the circulating current.

We now consider an electron passing through the proton bunch described above. The electron would experience radial kick due to the field created by the proton bunch, and the curvature of the additional motion can be expressed

$$\left[\frac{1}{\rho(x,y,z)}\right]_{el} = \left[\frac{4}{\sqrt{\pi}}\frac{re}{\gamma}\right]_{el}\cdot\left[\frac{Ne^{-z^2/2\sigma_z^2}}{k_b\,\sigma_z\,(\sigma_x+\sigma_y)}\right.$$

$$\left.\cdot\sqrt{\frac{x^2}{2\sigma_x^2}+\frac{y^2}{2\sigma_y^2}}\right]_{proton}$$

where $[N]_p$ is the total number of protons in the ring, and r_e is the classical radius of electron. Above curvature has a maximum at z = 0 on the surface of the proton beam ellipse

$$\frac{x^2}{2\sigma_x^2}+\frac{y^2}{2\sigma_y^2} = 1 \quad ,$$

$$\left|\frac{1}{\rho}\right|^{max}_{el} = \frac{4}{\sqrt{\pi}}\left(\frac{r_e}{\gamma}\right)_{el}\cdot\left[\frac{N}{K_b\,\sigma_z\,(\sigma_x+\sigma_y)}\right]_{proton}$$

The bending angle of this curvature can be obtained by integrating the curvature

$$\Psi)_{el} = \frac{1}{2}\int_{-\infty}^{+\infty}\frac{1}{|\rho|}\,dz$$

$$= \left[\frac{2\sqrt{2}\,r_e}{\gamma}\right]_{el}\cdot\left[\frac{1}{k_b}\cdot\frac{N}{(\sigma_x+\sigma_y)}\sqrt{\frac{x^2}{2\sigma_x^2}+\frac{y^2}{2\sigma_y^2}}\right]_{proton}$$

From this bending angle, we can calculate the linear tune shift of the electron

$$\Delta\nu_x\Big]_{el} = \left(\frac{\beta_x^*}{4\pi}\cdot\frac{\Psi}{x}\right)_{el}$$

$$= \left(\frac{r_e\,\beta_x^*}{2\pi\gamma}\right)_{el}\left[\frac{N}{k_b\,(\sigma_x+\sigma_y)\,\sigma_x}\right]_{proton}$$

This detivation checks with the standard equation.

We now consider the synchroton radiation integral of the beam-beam field in one crossing point.

$$I_2(x,y) \equiv \frac{1}{2}\int_{-\infty}^{+\infty}\left[\frac{dz}{\rho^2}\right]_{el}$$

$$= \frac{8}{\sqrt{\pi}}\left[\frac{r_e}{\gamma}\right]_{el}\left[\frac{1}{\sigma_z}\left(\frac{N}{k_b\,(\sigma_x+\sigma_y)}\right)\right]_{proton}$$

Above integral depends on the transverse coordinates, x and y, and we take the average by using the transverse particle distribution

$$\langle I_2\rangle = \frac{\int I_2\,(x,y)\,\lambda\,(x,y)\,dxdy}{\int\lambda\,(x,y)\,dxdy}$$

$$= \frac{4}{\sqrt{\pi}}\left[\frac{r_e}{\gamma}\right]_{el}\left[\frac{1}{\sigma_z}\left(\frac{N}{k_b\,(\sigma_x+\sigma_y)}\right)^2\right]_{proton}$$

$$= 16\pi\sqrt{\pi}\left(\frac{\Delta\nu_x\,\Delta\nu_y}{\beta_x^*\,\beta_y^*}\right)_{el}\left[\frac{\sigma_x\,\sigma_y}{\sigma_z}\right]_{proton}$$

The average energy loss of an electron per crossing due to the beam-strahlung is

$$\langle \delta u \rangle_{bb} = \frac{2}{3} [r_e \, m_o \, C^2 \, \gamma^4]_{el} \cdot \langle I_2 \rangle$$

$$= \cdot \frac{4}{\sqrt{\pi}} \cdot \frac{2}{3} [r_e^2 \, m_o \, C^2 \, \gamma^3]_{el}$$

$$\cdot \left[\frac{1}{\sigma_z} \left(\frac{N}{k_b (\sigma_x + \sigma_y)}\right)^2\right]_{proton}$$

The average power radiated in an IR due to the beam-strahlung is

$$\langle p_\gamma \rangle_{bb} = Ie \, \frac{\langle \delta u \rangle_{bb}}{e} \quad .$$

Conclusion

Inspectin of above $\langle \delta u \rangle_{bb}$ equation reveals that the dominant factors involved for the beam strahlung in the e-p collision is the geometrical sizes of the proton bunch, which are orders of magnitutide larger compare to the e^+e^- case, and accordingly the beam-strahlung effect should not play an important role for the e-p. For example, a 20 ter protons colliding with 120 GeV electrons with the proton geometry of $\sigma_x = \sigma_y = 0.025$ mm, $\sigma_z = 0.5$ m and $I_p = 75$mA and 2000 bunches would emit and average proton energy of 7.8 KeV/IR, which is negligible compared to the regular synchrotrom radiation.

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Reference

1. A. Hofmann and E. Keil, "Effects of the Beam-Beam Synchrotron Radiation", LEP Note 122 (Nov. 1978).

2. Group I Report, Proceedings of the Second ICFA Workshop on Possibilities and Limitations of Accelerators and Detectors (Oct. 1979).