THE VALUE OF EXPERIMENTS ON DECAYS OF THE TYPE $K \to \pi$ + MISSING NEUTRAL(S)

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In this note we shall assess the physics information which can be obtained from experimental searches for decays of the type $K \to \pi$ + missing neutral(s). Since one has better control over the beam in the case of an initial charged K, and since charged pions are easier to detect and reconstruct than the two photons from a π^o, we anticipate that charged K's would be more amenable for experimental purposes. Furthermore, since most decay experiments utilize stopped K's, it is preferable to use K^+'s in order to maximize the decay yield relative to kaon interactions. The most interesting decays of this form are $K \to \pi\nu_i\bar{\nu}_i$, where ν_i denotes a neutrino mass eigenstate, $K \to \pi$ + axion, and $K \to \pi\tilde{\gamma}\tilde{\gamma}(=\tilde{\gamma}\tilde{\gamma}^c)$ where $\tilde{\gamma}$ denotes a photino, the supersymmetric (Majorana) partner of the photon. Since one does not detect the neutrinos emitted in the first decay, an observed rate would be the sum $\sum_i \Gamma(K \to \pi\nu_i\bar{\nu}_i)$, of all $\nu_i\bar{\nu}_i$ pairs that can be emitted. For simplicity of notation, we shall henceforth define "the" decay "$K \to \pi\nu\bar{\nu}$" as the above sum of decays. In addition to the experimental reasons noted before, charged kaons are preferable for these decays because, for fixed $\Gamma(K \to$ final state (f.s.)), $B(K_S^o \to (\text{f.s.})^o) \simeq 10^{-2}B(K^+ \to (\text{f.s.})^+)$. Further, in the quark model approach used here $B(K_L^o \to \pi^o\nu\bar{\nu}) << B(K_S^o \to \pi^o\nu\bar{\nu})$ and similarly, $B(K_L^o \to \pi^o\tilde{\gamma}\tilde{\gamma}) << B(K_S^o \to \pi^o\tilde{\gamma}\tilde{\gamma})$.

We begin by analyzing the decay $K \to \pi\nu\bar{\nu}$. This is one of several one-loop induced flavor-changing (here, strangeness-changing) processes, others being $K^o - \bar{K}^o$ mixing, $K_L^o \to \mu^+\mu^-$, $K \to \pi e^+e^-$, and $K_L^o \to \gamma\gamma$. Historically, the great suppression of $K^o - \bar{K}^o$ mixing, as indicated by the ratio $[m(K_L) - m(K_S)]/m_K \simeq 0.7 \times 10^{-14}$, led to the requirement that there be no flavor-changing neutral currents at tree level, and hence to the GIM mechanism in electroweak gauge theories.[1] From studies of one-loop induced strangeness-changing processes, it was found that the GIM mechanism also works to suppress most such processes below the possible level $\sim(\alpha/\pi)G_F$ of a one-loop graph, to the level $\sim(\alpha/\pi)G_F(\Delta m_q^2/m_W^2)\varepsilon$ where Δm_q^2 denotes a generic difference of quark masses squared, and ε denotes a product of mixing matrix coefficients.[2] (Exceptions include $K^+ \to \pi^+e^+e^-$, $K_S^o \to \pi^oe^+e^-$, and $K_L^o \to \gamma\gamma$, which are logarithmically GIM-suppressed.) General conditions for the natural suppression of one-loop hadronic,[3] semileptonic,[3] and leptonic[4] flavor-changing processes were later formulated. The resulting branching ratio in the n = 2 case was estimated to be $B(K^+ \to \pi^+\nu_i\bar{\nu}_i) \sim 10^{-10}$ (and correspondingly, $B(K_S^o \to \pi^o\nu\bar{\nu}) \sim 10^{-12}$).[2] In the present standard electroweak theory with n = 3 generations, the t-quark contribution could increase this branching ratio significantly, just as was the case for $K_L^o \to \mu^+\mu^-$.[2]

The quark graphs that contribute to the decay $K \to \pi\nu_i\bar{\nu}_i$ at the lowest (one-loop) level are shown in Fig. 1. (Only U-gauge graphs are shown, but it is understood that the calculations are done in an R_ξ gauge with $\xi \neq 0$.) In the standard $SU(2)_L \times U(1)$ electroweak theory with two generations, the resulting amplitude for $K^+ \to \pi^+\nu_i\bar{\nu}_i$ is[2,5] (retaining electroweak, but neglecting strong, interaction logarithms)

$$\text{Amp}(K^+ \to \pi^+\nu_i\bar{\nu}_i) \sim \frac{3\alpha}{\pi}\frac{G_F}{\sqrt{2}}\csc^2\theta_W\left[\frac{m_c^2}{m_W^2}\ln\left(\frac{m_W^2}{m_c^2}\right) - \frac{m_u^2}{m_W^2}\ln\left(\frac{m_W^2}{m_u^2}\right)\right]\sin\theta_C\cos\theta_C\langle\pi^+|\bar{d}_L\gamma^\lambda s_L|K^+\rangle[\bar{\nu}_{iL}\gamma_\lambda\nu_{iL}] \qquad (1)$$

where $\langle\pi^+|\bar{d}_L\gamma^\lambda s_L|K^+\rangle = (1/2)[f_+(q^2)(P_K+P_\pi)^\lambda + f_-(q^2)q^\lambda]$ and $q = p_K - p_\pi$. Here, the term $\propto m_u^2$ is negligible compared to m_c^2 and is retained only to show the effect of the GIM mechanism. In the present standard theory with three generations, the amplitude will involve a factor

$$\sum_{j=1}^{3} V_{jd}^* V_{js}\left(\frac{m_{q_j}^2}{m_W^2}\right)\ln\left(\frac{m_W^2}{m_{q_j}^2}\right) \qquad (2)$$

where the sum is over all charge 2/3 quarks and the approximation $m_t^2 << m_W^2$ is made, rather than the form given in Eq. (1). Here V is the quark mixing matrix, defined by

$$\begin{bmatrix} d' \\ s' \\ b' \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} = V\begin{bmatrix} d \\ s \\ b \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} \qquad (3)$$

where d', s', etc. (d, s, etc.) are the weak (mass) eigenstates. From the requirements that the real and imaginary parts of the K^o-$\bar{K}^o$ transition amplitude, and the $K_L^o \to \mu^+\mu^-$ decay rate be predicted correctly, correlated constraints on the elements of V as functions of the t-quark mass, m_t, have been derived.[6]

The most recent experiment on $K^+ \to \pi^+$ + missing neutrals is that of a KEK-Osaka-Tokyo collaboration at KEK which has reported the limit $B(K^+ \to \pi^+\nu\bar{\nu}) \equiv \sum_i B(K^+ \to \pi^+\nu_i\bar{\nu}_i) < 1.4 \times 10^{-7}$ (90% CL).[7] The previous world limit was $B(K^+ \to \pi^+\nu\bar{\nu}) < 6 \times 10^{-7}$ (90% CL).[8] More precisely, these upper limits apply to the sum of all decay modes of the form $K^+ \to \pi^+$ + neutral, weakly interacting particles which do not decay or interact in the detector, whence the label "missing". Indeed, as will be explained below, it is possible that an observed signal might be due to other decay modes as well as $K^+ \to \pi^+ \nu\bar{\nu}$.

We believe that it would be worthwhile to improve that limit on $K^+ \to \pi^+$ + missing neutrals. We do not state this on the grounds that such a search would yield any precise test of the standard model; on the contrary, it would not. Moreover, it would not provide as stringent a constraint on induced strangeness-changing neutral currents or quark mixing angles as the K^o-$\bar{K}^o$ transition amplitude and $K_L^o \to \mu^+\mu^-$ decay rate already do.[6] It would be useful as another experimental number characterizing one-loop induced strangeness-changing neutral current processes, and perhaps, more importantly, as a test for supersymmetric decay modes.

Before discussing the latter topic, it is necessary to address the question of whether a $K \to \pi$ + missing neutrals experiment can determine the number of neutrino types. We shall show this is not possible, even in principle. To give the experiment the greatest benefit of the doubt, we shall assume the standard $SU(2)_L \times U(1)$ electroweak theory with neutrinos of zero or negligible masses (of Dirac of Majorano type[5]). The basic point is that the test is completely circular; if one assumes the premise that the number of neutrinos is unknown, that the number, masses and weak couplings of the additional charge 2/3 heavy quarks thereby implied are, of course, also unknown. Hence, one cannot calculate the graphs of Fig.1, since each involves a sum over the contributions of all such charge 2/3 virtual quarks. Indeed, in graph 1(c), one does not know the number or masses of the additional charged heavy leptons implied by the premise stated above, and this graph consists of a sum of the contributions of all such leptons, as well as a sum over the quark contributions. Note that the terms in the amplitude arising from a given quark, $q(2/3)_i$, are not in general all of the same sign, so that there could be destructive interference. This, in turn prevents one from bounding the number of neutrinos by use of inequalities on individual terms in the amplitude. Thus, a measurement of the $K \to \pi\nu\bar{\nu}$ branching ratio cannot determine the number of neutrino types.

Let us then consider two other possibilities. First, one might think that the lack of knowledge of the additional heavy quark and lepton numbers, masses, and, for the quarks, weak couplings, could be eliminated by measuring the decay $K \to \pi e^+e^-$ and analyzing the ratio of branching ratios

$$\frac{B(K^+ \to \pi^+\nu\bar{\nu})}{B(K^+ \to \pi^+e^+e^-)} \quad . \tag{4}$$

This, however, is incorrect, for several reasons. The graphs that contribute to the decay $K \to \pi e^+e^-$ are shown in Fig.2. The dominant contribution comes from the non-diagonal charge radius term in graph $2(a)_\gamma$,[2] just as was the case with the analogous leptonic decay $\mu \to e e\bar{e}$.[4] Now, the analytic structure resulting from this graph has a very different form from that of the $K \to \pi\nu\bar{\nu}$ amplitude; in particular, the GIM mechanism operates logarithmically rather than multiplicatively. Hence, the quark mass and mixing angle dependences do not divide out in the ratio of branching ratios. There are, in addition, the unknown number and masses of charged leptons in graph 1(c) for the decay $K \to \pi\nu\bar{\nu}$, which do not enter in the amplitude for the decay $K \to \pi e^+e^-$; the uncertainty resulting from these would obviously not cancel out in the ratio (4). Moreover, graph 2(a) has infrared logarithms which indicate manifestly that it is not well short-distance dominated and hence not reliably calculable. Thus, even the ratio $B(K\to\pi\nu\bar{\nu})/B(K\to\pi e^+e^-)$ does not give information on the number of neutrino types.

Finally, let us consider the weakest claim: that one could at least test the premise that there are n=3 types of neutrinos, and thus three generations of fermions in the standard electroweak model. This is quite unlikely, because one does not at present known the mass of the t-quark, and even when this is determined (assuming that such a quark exists), one will not have sufficiently accurate knowledge of the corresponding mixing matrix coefficients V_{ts} and V_{td}.[6] As was true of arbitrary n, these uncertainties would not divide out in the ratio (4), because the amplitudes have different analytic structures. Extensions of the standard model to include right-handed currents (e.g., as in $SU(2)_L \times SU(2)_R \times U(1)$ theories) massive neutrinos, lepton mixing, etc. only increase the theoretical uncertainties in the calculation of the $K \to \pi\nu\bar{\nu}$ amplitude and hence strengthen our negative conclusions.

This is not, however, to say that such experiments are without value. First, they may serve, like other induced neutral strangeness-changing processes, as a useful constraint on technicolor models, which have the hope of explaining the electroweak mass scale of ~300 GeV as the result of dynamical symmetry-breaking.[9]

Second, if one measures the π^+ energy distribution near the endpoint, one obtains a limit on the two-body decay $K^+ \to \pi^+ + \text{axion}$.[10] Thus, the KEK experiment has reported the limit $B(K^+ \to \pi^+ \text{ axion}) < 3.8 \times 10^{-8}$(90% CL).[7] This upper limit is substantially below most authors' calculations of the expected branching ratio[11] and constitutes one of the strongest pieces of evidence against the standard axion. Since the axion mass is[11]

$$m_a \sim 150 \text{ KeV}\left(\frac{n}{3}\right)\left(\frac{x + x^{-1}}{2}\right)$$

where n is the number of generations and x is the ratio of vacuum expectation values of the two neutral Higgs fields in the Peccei-Quinn model,[10] for moderate values of x, the two-body peak in dN/dE_π will be at the endpoint of the spectrum. If one considers extremely small or large values of x, then this peak would move down into the three-body distribution, but an experiment would still retain sensitivity to the peak for reasonable values of the axion mass. Thus, data from a recent KEK experiment[12] (whose main purpose was to search for peaks in the $K_{\mu 2}$ muon momentum distribution due to heavy neutrinos[13]) can yield a very good upper bound on the decay $K^+ \to \pi^+ a$ for a heavy axion, since this would cause a peak in the pion momentum distribution for the $K^+ \to \pi^+$ + missing neutral mode, which was also measured.[14]

Moreover, since an experiment on $K^+ \to \pi^+$ + missing neutral(s) would require an extremely good photon detection capability in order to reject backgrounds such as $K^+ \to \pi^+\pi^o$ and $K^+ \to \pi^+\pi^o\pi^o$, it would also be able to search for the direct decay $K^+ \to \pi^+\gamma\gamma$ and the cascade decay $K^+ \to \pi^+ a$; $a \to \gamma\gamma$. Indeed, the KEK experiment of Ref. 7 on $K^+ \to \pi^+\nu\bar{\nu}$ also obtained the upper limits $B(K^+ \to \pi^+\gamma\gamma) \le 0.84 \times 10^{-5}$ and $B(K^+ \to \pi^+ a; a \to n\gamma\text{'s}) < 1.4 \times 10^{-6}$ (both at the 90% CL) for $m_a < 100$ MeV and $\tau_{a \to n\gamma\text{'s}} < 10^{-9}$ sec.[15]

A very exciting possibility is that the decay $K^+ \to \pi^+\tilde{\gamma}\tilde{\gamma}$ might yield an observable signal in a $K^+ \to \pi^+$ + missing neutrals experiment. Here we recall that the photino, $\tilde{\gamma}$, is the supersymmetric partner of the photon. This decay has recently been analyzed by Suzuki.[16] In general, the dominant contribution arises from the tree-level graphs shown in Fig. 3. The virtual intermediate particles in this graph are the scalar supersymmetric partners of the Q = -1/3 quarks. More precisely, in a supersymmetric field theory, to each chiral component of a Dirac fermion there corresponds a complex scalar partner; we label these as $\phi_{q_{iL}}$ and $\phi_{q_{iR}}$. Even for n = 1 generation, these two scalars are not in general mass eigenstates, but rather linear combinations thereof. In addition, there is mixing between different generations, so that the weak eigenstates can be written in terms of the mass eigenstates according to the unitary transformation

$$\begin{bmatrix} \phi_{dL} \\ \phi_{sL} \\ \cdot \\ \cdot \\ \cdot \\ \phi_{q(-1/3)_{nL}} \\ \phi_{dR} \\ \phi_{sR} \\ \cdot \\ \cdot \\ \cdot \\ \phi_{q(-1/3)_{nR}} \end{bmatrix} = \mathcal{U} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \cdot \\ \cdot \\ \cdot \\ \phi_n \\ \phi'_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \phi'_n \end{bmatrix} \qquad (5)$$

The non-degeneracy of $\phi_{q_{IL}}$ and $\phi_{q_{iR}}$ is not important for our calculation. (In any case, it is constrained to be small.[17]) However, the generation mixing is important. Thus let us assume that

$$m_{\phi_{q_{iL}}} = m_{\phi_i} = m_{\phi'_i} = m_{\phi_{q_{iR}}} = m_{\phi_{q_i}} \qquad (6)$$

and write

$$\begin{bmatrix} \phi_{dL} \\ \phi_{sL} \\ \cdot \\ \cdot \\ \cdot \\ \phi_{q(-1/3)_{nL}} \\ \phi_{dR} \\ \phi_{sR} \\ \cdot \\ \cdot \\ \cdot \\ \phi_{q(-1/3)_{nR}} \end{bmatrix} = \begin{bmatrix} \bar{\mathcal{U}} & 0 \\ 0 & \bar{\mathcal{U}} \end{bmatrix} \begin{bmatrix} \phi_1 \\ \cdot \\ \cdot \\ \cdot \\ \phi_n \\ \phi'_1 \\ \cdot \\ \cdot \\ \cdot \\ \phi'_n \end{bmatrix} \qquad (7)$$

Then, the amplitude for the quark process is

$$\text{Amp}(s \to d\tilde{\gamma}\tilde{\gamma}^c) = \frac{2}{9} e^2 \sum_{j=1}^{n} \frac{V^*_{id} V_{is}}{(q^2 - m^2_{\phi_{q_i}})} \qquad (8)$$

$$\times \{[\bar{u}_{\tilde{\gamma}R} u_{sL}][\bar{u}_{dR} v_{\tilde{\gamma}^c L}] + [\bar{u}_{\tilde{\gamma}L} u_{sR}][\bar{u}_{dL} v_{\tilde{\gamma}^c R}] - (\tilde{\gamma} \leftrightarrow \tilde{\gamma}^c)\}$$

where q denotes the momentum transfer to the $d\tilde{\gamma}^{(c)}$ pair. We may crudely approximate the prefactor as

$$\sim \frac{2}{9} e^2 \epsilon \frac{\Delta m^2_{\phi_q}}{m^4_{\phi_q}} \qquad (9a)$$

where ϵ denotes a generic product of $\phi_{q_i\chi}$ mixing matrix coefficients, (χ = L or R and will be suppressed in the notation hereafter) which we will take as comparable in size to $\sin\theta_c \cos\theta_c \sim 0.2$ in analogy with the mixing matrix factor for the n = 2 case, $\sin\theta_c \cos\theta_c \simeq .2$, and the momentum dependence of the ϕ_{q_i} propagators has been dropped because $m^2_{\phi_{q_i}}$ is constrained to be $>>(q^2)_{max} = (m_K - m_\pi)^2$. The quantity $\Delta m^2_{\phi_q}$ represents a generic difference of squared m_{ϕ_q} masses. If, as is reasonable by analogy with quark mixing, ϕ_{q_i} mixing is hierarchical, then the ϕ_{q_i} mass factor is approximately

$$\frac{2}{9} e^2 \frac{\epsilon(m^2_{\phi_s} - m^2_{\phi_d})}{m^2_{\phi_s} m^2_{\phi_d}} \quad . \qquad (9b)$$

The K^o-$\bar{K}^o$ transition amplitude yields the constraint that, if $\epsilon \simeq 0.2$ and $m_{\phi_{q_i}} \sim 100$ GeV, then $(\Delta m^2_{\phi_{q_i}} / m^2_{\phi_{q_i}}) <$ few $\times 10^{-3}$.[16,18] Inserting these numerical values into Eq.(9), and assuming that $m_{\tilde{\gamma}} << m_K - m_\pi$, **we find** the result

$$B(K^+ \to \pi^+\tilde{\gamma}\tilde{\gamma}) \quad \lesssim 10^{-7}. \qquad (10)$$

This is very interesting because the upper bound may be significantly larger than $B(K^+ \to \pi^+\nu\bar{\nu})$ and moreover, is not much smaller than the present upper limit on $B(K^+ \to \pi^+$ + missing neutrals).[7] (Our result is in agreement with the analysis by Suzuki, who used this decay among others, to put upper limits on $\epsilon(\Delta m^2_{\phi_q} / m^4_q)$.[12]

It has been stressed by M. Peskin[19] that one can construct supersymmetric models in which the mass eigenstates of the quarks and their supersymmetric scalar partners are rotated in exactly the same way to form weak eigenstates. In this case $\mathcal{U}_{ij} = \delta_{ij}$, and the graphs of Fig. 3 would not occur. Instead, the decay would proceed via graphs such as those of Fig.4. An analysis of these graphs, again assuming $m_{\phi_q} \sim m_{\tilde{W}} \sim 10^2$ GeV, leads to the conclusion that even if they are responsible for the decay rather than the tree level graphs of Fig.3, $B(K^+ \to \pi^+\tilde{\gamma}\tilde{\gamma})$ can still be comparable to $B(K^+ \to \pi^+\nu\bar{\nu})$.

Let us pursue the question of the photino mass further. The assumption made in obtaining the estimate (10) is quite reasonable; the mass $m_{\tilde{\gamma}}$ vanishes at tree level in a spontaneously broken supersymmetric theory, since in such a theory the Lagrangian must be exactly supersymmetric, and the photon mass vanishes. In certain types of theories the photino remains massless at the one-loop level but in any case if it picks up mass from loop contributions, this is proportional to powers of α, and therefore it is reasonable to infer that the resulting mass may not be very large. Further guidance concerning $m_{\tilde{\gamma}}$ is available from an astrophysical analysis, which implies that $m_{\tilde{\gamma}} \lesssim 30$ eV or $m_{\tilde{\gamma}} \gtrsim 0.3$ MeV.[20] If the photino is sufficiently heavy, it will decay with an observable lifetime. Its main decay mode is $\tilde{\gamma} \to \gamma G$, where G denotes the Goldstino which results from spontaneous breaking of supersymmetry. We assume here that any explicit breaking supersymmetry is absent or small so that the Goldstino is massless or light (in particular, lighter than the photino). The resulting lifetime is[20]

$$\tau_{\tilde{\gamma}} = 1.7 \text{ sec} \left(\frac{\Lambda_{SS}}{100 \text{ GeV}}\right)^4 \left(\frac{1 \text{ MeV}}{m_{\tilde{\gamma}}}\right)^5 \qquad (11)$$

where Λ_{SS} denotes the scale of supersymmetry breaking, which is expected to be of order 10^2 GeV is supersymmetric theories are to fulfill their role of explaining the gauge hierarchy problem in grand unified theories. Evidently, for a large range of photino masses, the $\tilde{\gamma}$ will escape from the detector before decaying. In this case an experiment would not be able to distinguish between the decay modes $K \to \pi\nu\bar{\nu}$ and $K \to \pi\tilde{\gamma}\tilde{\gamma}$. It is conceivable, however, that $m_{\tilde{\gamma}}$ is sufficiently large that $\tilde{\gamma}$ decays might be observed. To be capable of such an observation, an experiment would require very good photon detection over as large a solid angle as possible. The signature would obviously be a photon which converts (and originates) at a significant distance from the K decay point and whose reconstructed momentum does not point directly back to this location (because of the missing Goldstino).

The most important backgrounds relevant to an experiment searching for $K^+ \to \pi^+$ + missing neutrals are $K^+ \to \pi^+\pi^o$ and $K^+ \to \mu^+\nu_\mu\gamma$ accidentals. The former can be avoided by considering only $|\vec{p}_{\pi^+}| \gtrsim 205$ MeV. The latter is only a problem if the photon detectors fail to observe the associated γ. As has been demonstrated most recently by the KEK experiment,[7] these backgrounds do not present a serious obstacle to a high sensitivity search.

We have also analyzed other decays of the type $K \to \pi$ + missing neutrals involving supersymmetric particles in the final state, but they appear to have quite small branching ratios. For example, consider the decay $K^+ \to \pi^+ \tilde{\gamma} G$. The graphs which could possibly give the main contribution to this decay are shown in Fig.5. The parts of these graphs involving the W loop are the same as the corresponding part of the graph that gives the main contribution to the decay $K \to \pi e^+ e^-$. The dominant term is a non diagonal electromagnetic charge radius term.[2] The resulting quark amplitude is

$$\mathrm{Amp}(s \to d\tilde{\gamma}G) \sim \frac{\alpha}{\pi}\left(\frac{2}{3}\right) \frac{G_F}{\sqrt{2}} \sum_{i=1}^{n} V_{id}^{*} V_{is} \ell n\left(\frac{m_W^2}{m_{q_i}^2}\right) \tag{12}$$

$$\times [\bar{d}_L \gamma_\lambda s_L]\left(\frac{m_{\tilde{\gamma}}}{2\Lambda_{SS}^2}\right)\{[\tilde{\gamma}\sigma_{\lambda\mu} q^\mu G] - (\tilde{\gamma} \to \tilde{\gamma}^c, G^c \to G)\}.$$

where the simplifying assumption $m_q^2/m_W^2 << 1$ is made.[21] The second term, corresponding to the crossed graph of Fig. 5, is added because the photino and Goldstino are self-conjugate, so that $|\tilde{\gamma}G^c>$ and $|\tilde{\gamma}^c G>$ constitute the same final state (which we denote as $|\tilde{\gamma}G>$). This is analogous to the situation in a pure leptonic decay of the form $\mu \to e\nu_i\nu_j^c$ with Majorana neutrinos.[22] And, just as was the case with such a decay, in the framework of general Lorentz structure, the tensor current vanishes identically.[22] Hence, what would normally be the dominant contribution to the $K^+ \to \pi^+ \tilde{\gamma} G$ amplitude vanishes identically, and the actual amplitude is much smaller. We estimate very roughly that

$$B(K^+ \to \pi^+ \tilde{\gamma} G) \sim \left(\frac{m_{\tilde{\gamma}} m_K}{\Lambda_{SS}^2}\right)^2 B(K^+ \to \pi^+ \nu\bar{\nu})$$

$$\lesssim 10^{-11} B(K^+ \to \pi^+ \nu\bar{\nu}) \tag{13}$$

where we have taken $m_{\tilde{\gamma}} \sim 350$ MeV the maximum value relevant for this decay, and $\Lambda_{SS} \sim 100$ GeV.

It is appropriate, finally, to address the question of which facilities are advantageous for this type of experiment. Since one is searching at very low values of branching ratio, it is obvious that a crucial requirement is a kaon beam with a very high flux. Furthermore, since the K^+'s are usually stopped in a precision decay experiment, one does not need a very high energy kaon beam. In the U.S., Brookhaven has such a high intensity, relatively low energy K^+ beam. Its intensity could be increased further in the hypothetical upgrading of the AGS which is being studied in this workshop. The competition for such a U. S. experiment could come from two sources. First a CERN-Geneva collaboration (M. Ferro-Luzzi, spokesman) has submitted a proposal for a search for $K^+ \to \pi^+$ + missing neutral(s) to the CERN PS.[23] (The proposal is actually entitled "Measurement of the Rare Decay $K^+ \to \pi^+\nu\bar{\nu}$", but in view of our analysis above, it is clear that such an experiment is sensitive not only to this mode.) In addition, KEK has a high flux kaon beam and groups which have just completed K decay experiments.[7,12,14,15] It is possible that these groups might attempt to reduce the upper limit on $K^+ \to \pi^+$ + missing neutral(s) in the future. Finally, if the LAMPF II project is built, it would also be capable of such an experiment, although this would presumably occur only several years after a CERN, BNL, or new KEK experiment could be launched.

In summary, we have found that a search for a K decay of the type $K^+ \to \pi^+$ + missing neutral(s) is a worthwhile experiment which would provide a sensitive probe of several different physics questions of current interest.

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20. G.Farrar and P.Fayet, Phys.Lett.105B,155 (1981 and references therein.
21. Our conclusions would not change significantly if this assumption were relaxed.
22. R.E. Shrock, op. cit., Ref. 5.
23. M.Ferro-Luzzi et al., CERN Proposal "Measurement of the Rare Decay $K^+ \to \pi^+\nu\bar{\nu}$", CERN/PSCC 82-24.

Figure Captions

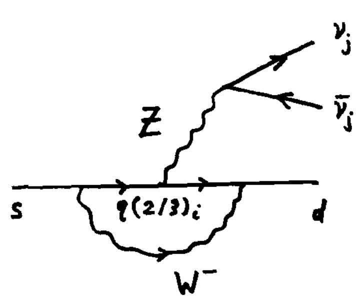
Fig.1 Graphs contributing to $K \to \pi\nu_j\bar{\nu}_j$. Each graph represents a sum over i from i = 1 to i = n, where n is the number of generations. Further, graph 1(c) represents a sum over a form a = 1 to a = n. (Only the a = j term contributes if there is no lepton mixing.) For Majorana neutrinos, one must also add the crossed graphs with $\nu_j \leftrightarrow \nu_j^c$.

Fig.2 Graphs contributing to $K \to \pi e^+e^-$. Each graph represents a sum over i from i = 1 to i = n, where n is the number of generations. Graph (c) also involves a sum over all a from 1 to n. (Only a = 1 contributes if there is no lepton mixing.)

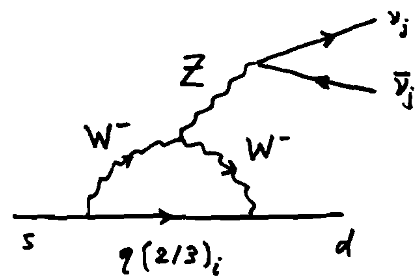
Fig.3 Tree-level graphs contributing to the decay $K \to \pi\tilde{\gamma}\tilde{\gamma}$. The graphs represent sums over i from i = 1 to i = n, where n is the number of generations. The crossed graph must be added since $\tilde{\gamma}$ is a Majorana fermion.

Fig.4 One-loop graphs contributing to the decay $K \to \pi\tilde{\gamma}\tilde{\gamma}$. $\tilde{w}$ denotes the spin 1/2 supersymmetric partner of the W boson.

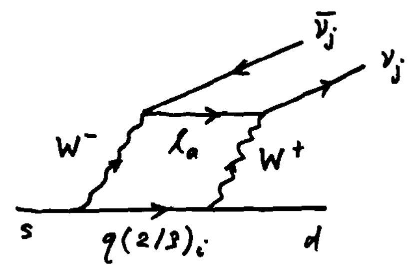
Fig.5 Graph for $s \to d\tilde{\gamma}G$.



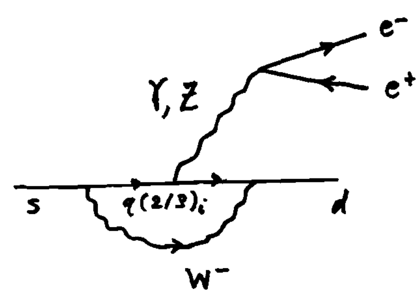
1(a)



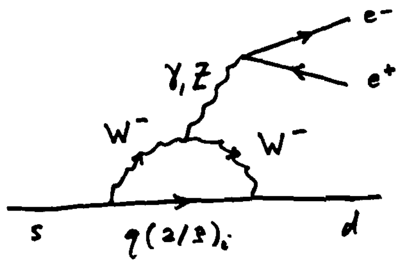
1(b)



1(c)



$2(a)_{\gamma,Z}$



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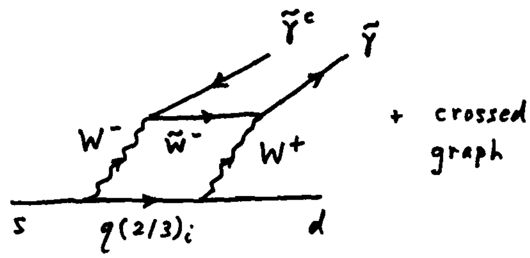
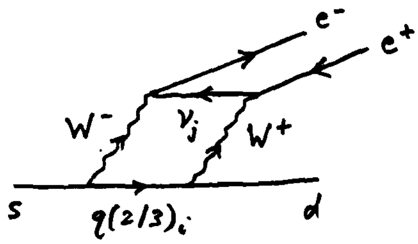
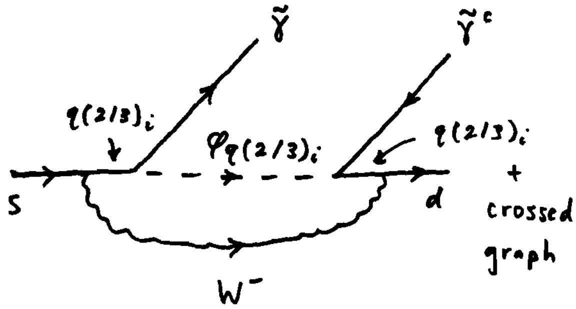


4(a)



2(c)



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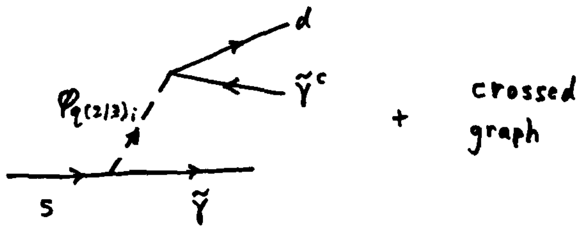
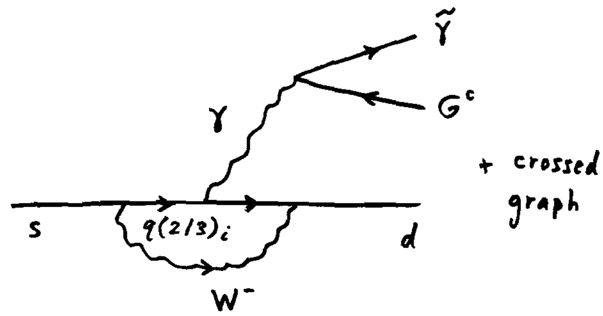


(3)



(5)