IMPLICATIONS OF NEUTRINO MASSES AND MIXING FOR THE MASS, WIDTH, AND DECAYS OF THE Z

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Abstract

We point out that if heavy neutrinos exist, their contributions to the width of the Z depend on whether they are of Dirac or Majorana type. Prospects for determining the charge conjugation properties of neutrinos via Z production and decay are noted. Finally, certain indirect effects of neutrino masses and mixing on the mass of and width of the Z are discussed.

The observation of the Z boson and the measurement of its properties will clearly provide a great amount of information about the structure and particle content of the standard SU(3) x SU(2) x U(1) model. In this model, the contribution to the width of the Z from the decay $Z \to f\bar{f}$, where f is an arbitrary, non self-conjugate fermion with weak isospin component T_{3L} and charge Q is[1]

$$\Gamma(Z \to f\bar{f}) = \frac{G_o m_Z^3 c}{6\pi\sqrt{2}}\left[(g_V^2+g_A^2)+ 2\delta_f^Z(g_V^2 -2g_A^2)\right](1-4\delta_f^Z)^{1/2} \tag{1}$$

where

$$\delta_f^Z = \frac{m_f^2}{m_Z^2} \tag{2}$$

$$g_V = T_{3L} - 2Q \sin^2\theta_W \tag{3}$$

$$g_A = - T_{3L} \tag{4}$$

$$c = \begin{cases} 3 & \text{for } f = \text{quark} \\ 1 & \text{for } f = \text{lepton} \end{cases} \tag{5}$$

and

$$G_o = 2^{1/2}\left(\frac{g^2}{8m_W^2}\right) = 2^{1/2}\left(\frac{g^2+g'^2}{8m_Z^2}\right) \tag{6}$$

Given the upper limits on the masses of the three known neutrinos,[2] it is certainly reasonable to neglect their possibly nonzero values in calculating the rate for the $Z \to \nu_i\bar{\nu}_i$ modes, as has been done in the literature. (Here, ν_i denotes a neutrino mass eigenstate and $i = 1,\ldots n$, where n is the number of fermion generations.) However, in assessing the physics possibilities at the Z, it is interesting to consider the general case where m_{ν_i}/m_Z is not negligibly small. In this case, in contrast to the one in which $m_{\nu_i} = 0$, one must address the issue of whether the neutrino is a Dirac (D) or Majorana (M), i.e., self-conjugate, neutrino. For Dirac neutrinos the general formula gives

$$\Gamma(Z \to \nu_i^D \bar{\nu}_i^D) = \frac{G_o m_Z^3}{12\sqrt{2}\,\pi}\left(1 - \delta_{\nu_i}^Z\right)\left(1 - 4\delta_{\nu_i}^Z\right)^{1/2} \tag{7}$$

We have calculated the corresponding rate for Majorana neutrinos in the standard model and find

$$\Gamma\left(Z \to \nu_i^M \bar{\nu}_i^M = \nu_i^M \nu_i^M\right) = \frac{G_o m_Z^3}{12\sqrt{2}\pi}\left(1 - 4\delta_{\nu_i}^Z\right)^{\frac{3}{2}} \tag{8}$$

A plot of these decay rates, normalized by their $m_{\nu_i} = 0$ values, is shown in Fig. 1. Thus, assuming equal D and M neutrino masses for the comparison,

$$\frac{\Gamma\left(Z \to \nu_i^M \nu_i^M\right)}{\Gamma\left(Z \to \nu_i^D \bar{\nu}_i^D\right)} = \frac{\left(1 - 4\delta_{\nu_i}^Z\right)}{\left(1 - \delta_{\nu_i}^Z\right)} \tag{9}$$

$$\leq 1$$

As was true of pure leptonic decays,[3] charged and neutral current reactions,[4] and electromagnetic properties,[5] the results (7) and (8) again illustrate the theorem that massless chiral Dirac and Majorana neutrinos are equivalent.

As in the case of other two-body decays, such as leptonic decays of pseudoscalar mesons involving massive neutrinos, the matrix element squared is a constant with respect to the phase space integration. As a consequence, the rate consists of a simple product of a factor from the matrix element squared and one from the phase space integral. The latter is, of course, the same for both the Dirac and Majorana cases, and is proportional to $(1-4\delta_\nu^{\,Z})^{\frac{1}{2}}$. However, the matrix element squared is proportional to $(1-\delta_{\nu_i}^{\,Z})$ for the D case but to $(1-4\delta_{\nu_i}^{\,Z})$ for the M case.[1] It is interesting that as a result, as m_{ν_i} approaches the kinematic limit of $m_Z/2$, the D matrix element approaches a constant, whereas the M matrix element vanishes.

If indeed, there exist neutrinos which are heavy enough for these differences to be significant, then they are likely to decay in the detector, and their decays will provide another tool for the determination of whether they are D or M particles. The leptonic decay channels are especially clean. Both the spectral distributions and total rates will differ for Dirac versus Majorana neutrinos, as has been studied in detail elsewhere.[3] It is worth emphasizing that just as one should keep an open mind concerning the possibility of very heavy neutrinos, so also one should be alert to the fact that the mass pattern observed up until now, according to which $m_{\nu_i} < m_{\ell_a}$ for $i = 1,2,3$ and $\ell_a = e, \mu, \tau$, respectively, may be reversed for one or more higher-lying generations. A model in which this situation occurs was studied previously;[6] one of the main consequences is that the decay of the heavy neutrino is strongly inhibited, since it can proceed only through mixing.

Finally, as we have noted before,[7] there is a very fundamental issue pertaining to the prediction of the mass and width of the Z (and also W), and the comparison of these predictions with forthcoming experimental measurements. The mass of the Z, as usually predicted, is

$$(m_Z)_{pred.} = \left(\frac{\pi\alpha}{2^{\frac{1}{2}}G_\mu}\right)^{\frac{1}{2}} \frac{1}{\sin\theta_W \cos\theta_W} \left(1 + \frac{\Delta r}{2}\right) \qquad (10)$$

where $\Delta r/2$ denotes the one-loop electroweak radiative corrections.[8,9] In order to evaluate this formula, one uses, perforce, the <u>observed</u> value of the muon decay constant, $G_\mu = (1.16632 \pm 0.00004)\times 10^{-5}$ obtained in the standard way[10] (with pure electromagnetic radiative corrections extracted) from the measured muon decay rate. In fact, however, if there are neutrino masses and mixings, then the observed value of G_μ would <u>differ</u> from the true value, defined to lowest order in Eq.(6). Specifically,

$$G_\mu = \kappa^{\frac{1}{2}} G_o \qquad (11)$$

where

$$\kappa = \sum_{i,j} |U_{2i}^* U_{1i}|^2 \, \bar{\Gamma}\left(\frac{m(\nu_i)}{m_\mu}, \frac{m(\nu_j)}{m_\mu}, \frac{m_e}{m_\mu}\right)_{[c]} \Big/ \bar{\Gamma}\left(0, 0, \frac{m_e}{m_\mu}\right)_{[c]} \qquad (12)$$

where U is the lepton mixing matrix, $\bar{\Gamma}$ is the reduced rate for the decay $\mu \to \nu_i e \bar{\nu}_j$ (normalized to unity for all particles in the final state massless), [c] denotes the fact that, in accord with convention, the pure electromagnetic radiative corrections[11] are extracted, and, for reference,

$$\bar{\Gamma}(0,0,x)_{[c]} = \left[(1-8x^2+x^4)(1-x^4) + 24x^4 \ln(1/x)\right] \times \left(1 + \frac{\alpha}{2\pi}\left(\frac{25}{4} - \pi^2\right)\right) \qquad (13)$$

Note that $\kappa \leq 1$. Thus

$$(m_Z)_{true} = \kappa^{\frac{1}{4}} (m_Z)_{pred.} \leq (m_Z)_{pred.} \qquad (14)$$

i.e., the true Z mass would like lower than the value predicted in the conventional manner, with one-loop radiative corrections included. A crucial intermediate step in our analysis is that the value of $\sin^2\theta_W$ can be measured in a way that is independent of the neutrino masses and mixing (by polarized eN scattering at several values of y). Given the renormalization scheme of Ref.8, where the relation $m_W = m_Z \cos\theta_W$ holds to all orders of perturbation theory, it follows also that

$$(m_W)_{true} = \kappa^{\frac{1}{4}} (m_W)_{pred.} \qquad (15)$$

Similarly, the true widths of the Z and W would differ from the conventionally predicted widths by the factor κ :

$$(\Gamma_{Z,W})_{true} \simeq \kappa^{\frac{1}{4}} (\Gamma_{Z,W})_{pred.} \qquad (16)$$

as well as by a factor due to kinematic differences in decay rates. It is thus important to determine how large an effect this could be, and whether it could seriously complicate the use of the measured values of m_Z and Γ_Z to test the standard model and probe its particle content. In order to do this, one must calculate the rates the μ-decay involving massive neutrinos, taking into account the upper bounds that have been established on lepton mixing as a function of neutrino masses. We have performed this analysis and have found that the upper bound on $(1-\kappa)$ is sufficiently small that the induced changes in m_Z and Γ_Z are not likely to be detected in anticipated experiments. This is, in a sense, fortunate, since otherwise there would have been a significant complication in testing the standard model with massless or light neutrinos using Z production and decay.

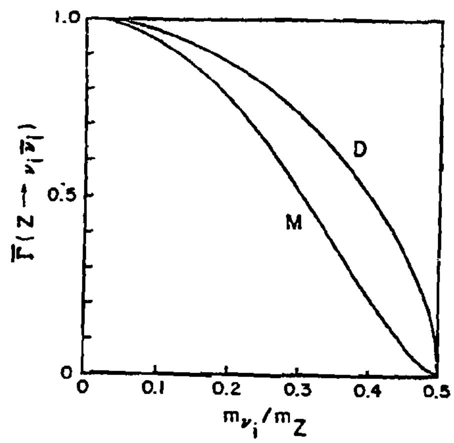

Fig. 1

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Figure Caption

Fig.1. Plot of $\bar{\Gamma}(Z \to \nu_i\bar{\nu}_i) \equiv \Gamma(Z \to \nu_i\bar{\nu}_i)/\Gamma(Z \to \nu_i\bar{\nu}_i\ ;\ m_{\nu_i} = 0)$

For Dirac (D) and Majorana (M) neutrinos, as a function of m_{ν_i}/m_Z.