A NON-STANDARD GRAND UNIFIED MODEL

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Introduction

This is a report on some work done in collaboration with Prof. G. Feldman of the Johns Hopkins University. It is interesting to include it in the proceedings of the DPF summer study because it illustrates that a model different from the standard model can incorporate all present data, predict a number of testable results concerning presently known particles as well as other predictions which will be testable in the near future. The model includes all presently known flavors (including the t-quark) in a grand unified structure, the gauge group being SO(14). For an account of the motivation, the fermion quantum number assignments and other results see ref. 1. The problems of fermion mass relations and b-decay phenomenology are discussed in detail in ref. 2.

Structure of the Model

The fermions are placed into the complex, anomaly free, irreducible spinor representation of SO(14). This representation is 64 dimensional and has the SO(10) content

$$\underline{64} = 2(\underline{16}) + 2(\underline{\overline{16}}). \tag{1}$$

Hence, there are two standard families and, what appear to be, two V+A families. This contradicts τ-lepton data[3] which indicates that, for any value of m_{ν_τ}, V+A weak interactions for the τ system are ruled out (in fact V-A is favored). Hence, in order to be able to use SO(14) (and not to have to go to SO(18) with fermion representation R_F equal to the $\underline{256}$), we must modify the usual charge assignments. Up to a relabelling of particles there is only one charge assignment consistent with experimental data. In the SU(7) basis of SO(14), it is given by

$$Q = \mathrm{diag}\ (-1/3,-1/3,-1/3,1,0,-1,1) \tag{2}$$

Note that the first five entries are those of the SU(5) or SO(10) charge operator; standard charge assignments would require that all other entries be zero. The consequences of this charge assignment are as follows:

(a) The top and bottom quarks, t, b exist but are not weak I-spin doublets. They are I-spin singlets while b^c, (t^c) is part of an I-spin doublet with partner $x^c(y^c)$, a quark of charge +4/3 (-5/3). This has some implications on b-decays which shall be explored in further detail later on.

(b) There exist new leptons T, M of charges -2 and -1 respectively such that T^c, and M^c form an I-spin doublet and T,M form I-spin singlets.

(c) The value of $\sin^2\theta_w$ at the unification mass, M_1, is

$$\sin^2\theta_w(M_1) \equiv \frac{\mathrm{Tr}\ I_3^2}{\mathrm{Tr}\ Q^2} = \frac{3}{20} = 0.15. \tag{3}$$

This means that the symmetry breaking pattern must be chosen so as increase $\sin^2\theta_w$ as opposed to the standard charge case (in this case $\sin^2\theta_w(M_1) = 3/8$). This fact allows us to classify all possible symmetry breaking patterns of SO(14) consistent with the data on $\sin^2\theta_w$ and the strong coupling constant, α_s. We find[1] that intermediate symmetries must be present at energies less than or equal to 10^5 GeV.

The model examined here is the SO(14) generalization of the SU(7) model of J.E. Kim[4]. However, due to the fact that SO(14) contains more subgroups than SU(7), we will have more patterns of symmetry breaking, more intermediate symmetries and more fermion mass relations (this is also the case when one goes from SU(5) to SO(10)). The model of ref. 4 is also incomplete since it did not address the problem of b-decays.

Predictions of the Model

A. Exotic Particles

We have seen that in order to accomodate the τ lepton system, exotic particles must exist in the theory. Examination of the group theoretical properties of exotic mass terms shows that these particles can only acquire a mass once the electroweak group is broken down to $U_{EM}(1)$. Hence, their masses must be less than or equal to 100-300 GeV so that they are not superheavy. If it turns that all these exotics are accessible to foreseeable e^+e^- machines, we would be able to see an increase in the R value of 20 units. However, this is clearly not the easiest way to test this model.

B. Symmetry Breaking

Using the renormalization group equations for the strong coupling constant α_s and the Weinberg angle, we were able to classify all possible symmetry breaking patterns of SO(14) consistent with the known low-energy values of α_s and $\sin^2\theta_w$.[5] We found that intermediate symmetry groups must appear at energies $\lesssim 10^5$ Gev. These groups are $SU_c(4) \times SU_F(3)$ and/or $SU_c(4) \times SO_F(6)$, where, under $SU_c(4) \times SO_F(6)$, the fermion representation decomposes as follows:

$$\underset{(6,4)}{\begin{bmatrix} u & t^c \\ d & y^c \\ y & d^c \\ t & u^c \end{bmatrix}} + \underset{(4,\bar{4})}{\begin{bmatrix} s & M^c \\ c & T^c \\ x & \nu_\mu{}^c \\ b & \mu^c \end{bmatrix}} + \underset{(\bar{4},\bar{4})}{\begin{bmatrix} b^c & u \\ x^c & \nu_\mu \\ c^c & T \\ s^c & M \end{bmatrix}} + \underset{(1,4)}{\begin{bmatrix} \nu_e \\ e \\ \tau^c \\ \nu_\tau \end{bmatrix}} + \underset{(1,4)}{\begin{bmatrix} \nu_\tau \\ \tau \\ e^c \\ \nu_e^c \end{bmatrix}} \tag{4}$$

The particles above the dotted line belong to $SU_F(3)$ triplets or antitriplets.

U(1) symmetries only appear at energies $< 10^5$ GeV so that the corresponding U(1) monopoles must have masses $\lesssim 10^7$ GeV.

C. b-Decay

Since the b is not part of a doublet (although b^c is) we would expect that the b-decay theorem of Kane and Peskin[6] would rule out our model on the basis of the di-lepton branching ratio. However, the b can also decay via $SU_F(3)$ currents:

$$b \rightarrow \begin{matrix} c + e + \bar{\nu}_\tau \\ c + \tau + \bar{\nu}_e \end{matrix} \tag{5}$$

If there were no b mixing, the ratio of e to μ's in b decays would be 3 to 1 (the τ branches equally into e's and μ's). There would also be no pure hadronic decays. Hence, we must allow b-mixing large enough to satisfy constraints on the ratio of e, μ branching ratios and on pure hadronic decay branching ratios, but still small enough to suppress unwanted contributions to the K_o-K_o mass difference. In order to evade the Kane-Peskin result, we must also allow $SU_F(3)$ currents to mediate decays at a reasonable rate. If we let the (b,s) mixing angle be ~ 1/10 and the mass of the $SU_F(3)$ gauge boson be $\sim 3M_W$ we find:

$$\frac{B(b \rightarrow eX)}{B(b \rightarrow \mu X)} \sim 1.7. \tag{6a}$$

$$b(b \rightarrow \text{hadrons only}) \sim 52.7\% \tag{6b}$$

$$\frac{B(b \rightarrow X\, \ell^+ \ell^-)}{B(b \rightarrow X\, \ell\, \nu)} \sim 8\text{-}9\% \tag{6c}$$

The numbers above are consistent with known data[7] and perhaps in a year or so there will be enough data to rule out (or not!) the model.

D. Proton Decay

In the SO(14) model, nucleon decay can occur via the standard SU(5) and SO(10) currents, which give rise to the standard decay modes. However, the gauge bosons of the intermediate symmetry groups $SU_c(4) \times SU_F(3)$, $SU_c(4) \times SO_F(6)$ can also mediate proton decay as shown in fig. 1

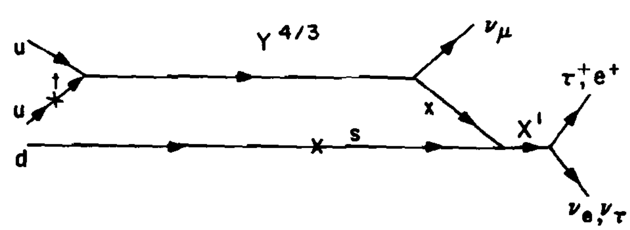

Fig. 1: Nucleon decay via u-t, d-s mixing and intermediate symmetry gauge bosons.

We can use the lower limits on the proton lifetime τ_p to find constraints on the amount of u-t mixing allowed. The rate for the process in fig. 1 is given by:

$$\Gamma \sim \alpha^4 \left(\frac{m_p^2}{M_X M_Y}\right)^4 m_p\, \theta_{d-s}\, \theta^2_{ut}. \tag{7}$$

We know that M_X, M_Y must be less than or equal to 10^5 GeV. Hence $\Gamma \gtrsim \alpha^2 m_p (10^{-53} \theta^2_{ut})$, which implies that $\theta^2_{ut} < 10^{-15}$. Note that the standard unified models do not place any such bounds on mixing angles. We have not yet investigated the naturality of the above condition.

E. Exotic Particle Decays

The exotic particles t,x,y,M,T can be made in e^+e^- or ep collisions. The cross sections for their creation are given by the standard cross-sections for these process where the exotic mass can run from 20 GeV to $\sim 10^2$ GeV. We now consider some of their decays. Since the charge conjugates of these particles are $SU_L(2)$ doublets, one of the partners in the doublet will be able to decay into the other via $SU_L(2)$ currents. The gauge bosons of the intermediate symmetry groups can also mediate these decays. The rate for these processes is

$$\Gamma_{int} \sim \alpha^2_{int} \frac{m^5_{exotic}}{(M_{gauge\ boson})^4} \tag{8}$$

Now, we have the bounds:

$$20 \text{ GeV} \lesssim m \text{ exotic} \lesssim 10^2 \text{GeV}. \tag{9a}$$

$$3M_W \lesssim M_{\substack{\text{gauge} \\ \text{boson}}} \lesssim 10^2 M_W. \tag{9b}$$

Hence, we may compare Γ_{int} to the charm decay rate to find

$$10^{-7} \lesssim \frac{\Gamma_{int}}{\Gamma_{charm}} \lesssim 10^{8} \tag{10}$$

We now consider some of the exotic decay modes:

$$\bar{x} \rightarrow \bar{b} + \begin{matrix} \bar{e} & \bar{\nu}_e, u\bar{d} \\ \bar{\mu} & \bar{\nu}_\mu, c\bar{s} \\ \bar{\tau} & \bar{\nu}_\tau, \end{matrix} \text{ via } SU_L(2) \text{ currents} \tag{11}$$

$$t \rightarrow \bar{b} + \bar{u} + \bar{\mu} \quad \text{via } SU_c(4) \text{ currents} \tag{12a}$$

$$t \rightarrow d + \begin{matrix} \bar{e}\, \nu_\tau \\ \bar{\tau}\, \nu_e \end{matrix} \quad \text{via } SU_F(3) \text{ currents} \tag{12b}$$

Note that the first of the top decays violates baryon number!

$$y \rightarrow u + \begin{matrix} \bar{\tau}\, \nu_e \\ \bar{e}\, \nu_\tau \end{matrix} \quad \text{via } SU_F(3) \text{ currents} \tag{13a}$$

$$y \rightarrow \bar{d} + \bar{b} + \bar{\mu} \quad \text{via } SU_c(4) \text{ currents} \tag{13b}$$

This decay also violates baryon number.

$$M \rightarrow \nu_\mu + \begin{matrix} e + \nu_\tau \\ \tau + \nu_e \end{matrix} \quad \text{via } SU_F(3) \text{ currents} \tag{14a}$$

$$M^c \rightarrow s + \bar{b} + \bar{u} \quad \text{via } SU_C(4) \text{ currents} \tag{14b}$$

$$T \rightarrow \mu + \begin{matrix} \tau + \bar{\nu}_e \\ e + \bar{\nu}_\tau \\ \bar{b} + c \end{matrix} \quad \text{via } SU_F(3) \text{ currents} \tag{15}$$

There are also other decay modes available, but these depend on the relative masses of the exotic particles.

F. Baryon Asymmetry

In order to see whether the SO(14) model can generate sufficient baryon asymmetry, we use the theorem of Haber, Segre' and Soni[8]. This theorem states that if the group theoretical charge conjugation operator C is broken at a scale V, then the baryon asymmetry is suppressed by a factor $(\frac{V}{M})^2$ where M is the mass of the particles whose out-of-equilibrium decays generate the baryon asymmetry. In our case C is broken when SO(14) breaks down to $SU_C(4) \times SU_F(3)$. Hence, if we make this the first stage of symmetry breaking, then there will be no suppression. Since there are intermediate symmetry gauge bosons which can generate a baryon asymmetry, V can actually be as low as 10^5 GeV so that we may break the symmetry via $SU_c(4) \times SO_F(6)$. Thus we see that the model can generate the required asymmetry.

G. Anomalous Currents

The SO(14) model contains anomalous currents. An example of this can be found in the $SU_F(3)$ gauge boson mediated process

$$\bar{\tau} \to \nu_e + \bar{\nu}_\tau + \bar{e}. \tag{16}$$

This gives a V+A contribution to τ decay. These currents also mediate the V+A b-decay process

$$\begin{array}{l} \bar{b} \to \bar{c} + \bar{\nu}_e + \bar{\tau} \\ \qquad\quad \bar{\nu}_\tau + \bar{e} \end{array} \tag{17}$$

The rates of these processes are given by:

$$\Gamma_A \sim \alpha_3^2 \frac{m^5}{M_3{}^4}, \tag{18}$$

where m is the τ or b mass, M is the $SU_F(3)$ gauge boson mass and α_3 is the $SU_F(3)$ fine structure constant. We have seen that b-decay considerations demand that $M \sim 3M_W$ so that $\alpha_3 \sim \alpha_2$, where α_2 is the $SU_L(2)$ fine structure constant. Hence the ratio of Γ_A to the $SU_L(2)$ rate Γ_2 is $(\frac{M_W}{M_3})^4 \sim 0.01$. There is also an anomalous neutral current mediated by a gauge boson $\hat{B}^0_\mu$ of mass $\sim 3M_W$. Its coupling to the fermions is given by

$$\bar{\psi}_L \gamma_\mu [g_3 \frac{Y_3}{2} \cos\alpha + g_4 \frac{Y_4}{2} \sin\alpha] \psi_L, \tag{19}$$

where $g_3 (g_4)$ is the $SU_F(3)$ $(U_{Y_4}(1))$ coupling constant, α is a mixing angle which depends on how the symmetry in broken and Y_3 (Y_4) is a diagonal generator contained in $SU_F(3)$ $(SU_C(4))$. This current is an isoscalar current since both Y_3 and Y_4 are orthogonal to $SU_L(2)$. Since $M_B \sim 3M_W$, $g_3 \sim g_4 \sim g_2$ so that

$$\frac{\Gamma_A}{\Gamma_{SU(2)}} \cong (\frac{M_W}{M_B})^4 \times (\text{either } \cos^2\alpha, \sin^2\alpha) \times 10^{-1} < 10^{-3} \tag{20}$$

Hence these anomalous currents effects will be hard to see. However interference effects are possible between the $\hat{B}^0$ and the Z° current (since they both lead to $e^+e^- \to \mu^+\mu^-$). Hence the strength of the anomalous effects relative to the standard ones can actually be bigger than 10^{-3} (i.e. about 10^{-1}). Hence better measurements of asymmetry effects in $e^+e^- \to \mu^+\mu^-$ can be a good test of the model.

H. Neutrino Masses

Our model predicts non-zero neutrino masses. The Dirac-neutrino-quark mass relations are:

$$m_{\nu_e} \sim m_u, \; m_{\nu_\mu} \sim m_c, \; m_{\nu_\tau} \sim m_t, \tag{21}$$

where ~ means either equal to or equal to 1/3 times the quark mass. Using the results of the symmetry breaking scale analysis and the group theoretical tranformations properties of the fermion mass terms, we find that the right-handed neutrino mass scales are

$$M_{\nu_e} \sim 10^{15} \text{ GeV}, \; M_{\nu_\mu} \sim 3M_W, \; M_{\nu_\tau} \sim 10^5 \text{ GeV}. \tag{22}$$

Hence, in the absence of mixing, the neutrino mass hierarchy in

$$m_{\nu_e} << m_{\nu_\tau} << m_{\nu_\mu} \tag{23}$$

which is somewhat different than in the standard models (the electron and the tan Majorana masses may be interchanged, giving an electron neutrino mass ~ 10 ev). When mixing is put in, we lose predictive power in the neutrino sector, since we have little control over the mixing terms. If we neglect effects of the right handed neutrino mass matrix, we would say that the mixing is approximately Cabibbo-like.

I. Z° Mass Shifts

The Z° can mix with the neutral boson $\hat{B}^0$ discussed above via vacuum polarization diagrams. The corrections are proportional to the square of the mass of the intermediate fermion pair, m_f. Recall that M_f can be, at most, as large as m_W. The shift also depends on the mixing angles in Eq (19). These can such that the mixing is ~ $\frac{1}{10}$%. Hence the observation of Z° mass shifts in consistent with the model.

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References

1. G. Feldman and R. Holman, JHU-HET Preprint 8201; to be published in Journal of Physics G.
2. R. Holman, JHU-HET Preprint 8205; to be published in Journal of Physics G.
3. M. Perl, Ann. Rev. Nucl. Part. Sci. 30, 299 (1980).
4. J. E. Kim, Phys. Rev D 23 2706 (1981).
5. J. E. Kim, P. Langacher, M. Levine and H. H. Williams, Rev. Mod. Phys. 53 211 (1981); P. Langacker Phys. Rep. 72, 185 (1981).
6. G. Kane and M. Peskin, University of Michigan Preprint UM, He 81-51 (1981).
7. CLEO Collaboration Preprint 81/06 (1981).
8. H. E. Haber, G. Segre' and S. K. Soni, Phys Rev D 25, 1400 (1982).