MEASURING α_s — THE QUANTITATIVE VERIFICATION OF PERTURBATIVE QCD

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1. Introduction

A comparison of two or more measurements of the strong coupling constant α_s would provide one of the most stringent tests of quantum chromodynamics. Agreement would be indirect but convincing verification of all of the components of the perturbation theory used in computing the measured affects — i.e. the quark and gluon spins, the gluon-gluon and gluon-quark couplings, asymptotic freedom, etc. Although there is already much qualitative evidence for QCD, definitive quantitative tests of the theory have so far proven elusive. This note briefly outlines the current status of such quantitative studies[1], and the prospects for the near future.

Since the effective coupling $\alpha_s(Q^2)$ varies with the Q^2 of the process under study, it is convenient to focus upon the QCD scale parameter Λ, defined by

$$\alpha_s(Q^2) = \frac{4\pi}{(11-\frac{2}{3}n_{\text{flavors}})\,\ln(Q^2/\Lambda^2)} + \ldots \qquad (1)$$

Given a specific definition of the effective coupling (i.e. given a "scheme"; for convenience, $\overline{\text{MS}}$ is adopted here), Λ is a fundamental constant of nature. Current data suggests $\Lambda_{\overline{MS}} \sim 0.15$ GeV, but with an uncertainty of at least ±100%. This value of $\Lambda_{\overline{MS}}$ is used for all the estimates that follow. It should be remembered that scaling violations, cross sections, etc. are related to Λ only through their dependence on $\alpha_s(Q^2)$. So even a rather crude determination of $\Lambda_{\overline{MS}}$, say ±50% leads to rather accurate predictions for the quantities being measured ($\sim$±10% when $Q^2 \sim 100$ GeV^2 and less for higher Q^2).

2. Which Energy: Radiative Corrections vs. Born Amplitudes

Beyond some point, higher energies become less and less useful for measuring Λ. This is true for two reasons. First the effect of being measured becomes smaller due to asymptotic freedom (i.e. $\alpha_s(Q^2) \to 0$ as $Q^2 \to \infty$). For example, an effect proportional to $\alpha_s(Q^2)$ is about 50% larger at $Q^2 = 100$ GeV (i.e. Υ energies) than it is at $Q^2 = 10^4$ GeV^2 (i.e. Z^0 energies). Secondly, greater precision is required at higher Q^2. This is because a fixed uncertainty in Λ (of $\pm d\Lambda$) can be achieved only by increasing the relative accuracy with which α_s is measured as Q^2 increases:

$$\frac{d\alpha_s}{\alpha_s} \sim \alpha_s(Q^2)\,\frac{d\Lambda}{\Lambda}$$
$$\to 0 \qquad \text{as } Q^2 \to \infty.$$

Thus a 50% determination of Λ requires that α_s be measured to within ±11% at $Q^2 = 100$ GeV^2, while an accuracy of ±6½% is required at $Q^2 = 10^4$ GeV^2. Taken together, these two effects suggest that it is roughly twice as hard to determine Λ at Z^0 energies than it is at Υ energies [i.e. $(\alpha_s(10^2)/\alpha_s(10^4))^2 \sim 2$].

Very high Q^2 is essential only where QCD enters first as a radiative correction to what is basically a QED (or other non-QCD) process. For example, both scaling violation in ep inelastic scattering, and gluon jets in $e\bar{e}$ annihilation are due to gluonic corrections to a QED reaction. In such cases, the QCD corrections must compete with uncalculable non-perturbative corrections to the basic QED process. Since the QCD corrections are of order $\alpha_s(Q)/\pi \lesssim 10$-20%, the non-perturbative effects somehow must be suppressed to $\lesssim$ 1-2% before even modest accuracy is attainable for Λ (i.e. $\sim$±50%). This is accomplished by going to very high energies — e.g. $Q^2 \gtrsim 100$ GeV^2 for ep where non-perturbative ("higher-twist") corrections fall as $1/Q^2$; and $s \gtrsim 10^4$ GeV^2 for $e\bar{e} \to$ jets where non-perturbative corrections fall as $1/\sqrt{s}$. The alternative to employing such high energies is to attempt to correct for higher twist, hadronization, or other non-perturbative effects by using empirical models fit to the data. This is obviously a tricky business.

The situation is much better for processes such as the inclusive decays of the Υ or η_b. For these, the leading order ("Born") amplitude is due to QCD — e.g. $\Gamma(\Upsilon) \propto \alpha_s^3(M_\Upsilon)(1+\ldots)$. Thus even though Q^2 is low and non-perturbative corrections may be of order 10-20%, a reasonably accurate measurement of α_s (and Λ) is possible.

3. Which Process?

Several processes have been proposed for measuring α_s. Some of the more important are reviewed here, with emphasis on the theoretical and experimental problems yet to be resolved for each.

a) ep, νp, ... Inelastic Scattering

Considerable data exists for the scaling violations in deep-inelastic scattering structure functions. There are scaling violations proportional to $\alpha_s(Q)$, and these can be used to determine Λ. However, the quantitative analysis of existing data has been complicated by several factors:

- perturbative scaling violations are small ($\sim\alpha_s/\pi \sim 20\%$) and must compete with uncalculable higher twist corrections of order $1/Q^2$. These higher twist corrections probably can be neglected for $Q^2 \geq 100$ GeV^2. Unfortunately, the bulk of the data is for lower Q^2.
- for a model independent extraction of Λ, it is critical that the entire x region be measured for at least some range of Q^2 (preferably $\gtrsim 100$ GeV^2). Current data tends to be deficient at very small x and very large x. Determinations of Λ rely upon assumptions for the functional dependence on x of the structure functions which extrapolate the data into these regions. The procedure is obviously not fool-proof.
- the scaling violation in F_2^{ep} (or $F_2^{\nu p}$) is determined in QCD by three coupled equations involving three separate structure functions — F_2^{NS}, F_2^S and G. Given just F_2^{ep} at a given Q^2 (and all x) it is impossible to predict its evolution with varying Q^2. Analysis of existing data again relies upon assumptions for the functional dependence on x of F_2^{NS}, F_2^S and G. The results seem to be highly model

dependent, with uncertainties in Λ ranging from 100-400%[3]. It should also be remembered that theoretical predictions become unreliable at small x (say $x \lesssim 0.1$), which is just where the scaling violations are largest. These problems are not solved by going to higher Q^2. They can be avoided by measuring $F_2^{ep}-F_2^{en}$ (or $F_3^{\nu p}$), whose evolution is determined by a single equation.

- the separation of structure functions F_1 and F_2 from the data is very difficult. This is reflected by the ambiguous situation regarding the comparison of theory and experiment for $R = \sigma_L/\sigma_T$.

Many of these issues cannot be resolved simply by going to higher Q^2. From a theoretical point of view, the most desirable measurements would be of $F_2^{ep}-F_2^{en}$ (or $F_3^{\nu p}$) for some large range of $Q^2 \geq 100$ GeV^2 and including the full x range for some Q^2. The scaling violations amount to $\sim$15-20% over a range $Q^2 = 100$-4000 GeV^2, suggesting that 1-2% accurate measurements of $F_2^{ep}-F_2^{en}$ would be necessary. QED radiative corrections, and threshold and higher twist corrections due to heavy quarks must also be understood to the 1% level.

b) $e\bar{e} \to$ Jets

Thrust distributions, energy correlations, etc. determine α_s from data for $e\bar{e} \to$ jets by measuring deviations from the basic QED process $e\bar{e} \to q\bar{q}$ ($\to$ 2 jets). Currently, QCD predictions can be compared with data only after rather substantial corrections ($\sim 1/\sqrt{s}$) have been made for the non-perturbative interactions which change quarks and gluons into hadrons, i.e. for hadronization[4]. Even with these corrections, the systematic uncertainty in the measured value of α_s has been estimated to be as large as 30-50% at PETRA energies[5]. Extrapolated to Z^0 energies (like $1/\sqrt{s}$), such uncertainties could lead to 100-200% errors in the measured Λ. Further experimental and theoretical studies of hadronization may improve this situation. Especially important is the inclusion of $1/p_\perp^4$ power-law tails in fragmentation functions; these are predicted by QCD and are analogous to higher twist corrections in ep scattering. Also important is the search for quantities which have minimal sensitivity to the assumptions concerning hadronization (e.g. asymmetries in energy correlations?)

c) $e\bar{e} \to$ Hadrons (Inclusive)

The QCD corrections to $R = \sigma(e\bar{e} \to \text{hadrons})/\sigma(e\bar{e} \to \mu\bar{\mu})$ is about 8% at $s = 100$ GeV^2. At this energy, non-perturbative corrections are probably of order $1/s \sim 1\%$. Thus a 1% measurement of R could determine Λ to within $\sim\pm50\%$. Current measurements of R at PETRA and PEP quote systematic errors of $\sim$3-6%.[6]

d) $e\bar{e} \to \psi$, T, ...

A number of hadronic or semi-hadronic decay rates and branching ratios of the ψ, T, ... are sensitive measures of α_s:[7]

$$\left.\begin{array}{l}\Gamma(T \to \text{hadrons}) \\ B_{\mu\mu}^{-1}(T)\end{array}\right\} \propto \alpha_s^3$$

$$B_{\gamma_{\text{direct}}}^{-1} \propto \alpha_s$$

$$\left.\begin{array}{l}\Gamma(\eta_b \to \text{hadrons}) \\ B_{\gamma\gamma}^{-1}(\eta_b)\end{array}\right\} \propto \alpha_s^2$$

High precision measurements of these quantities are unneeded; 10-20% is adequate. Non-perturbative corrections are far less important here, especially for the T ($\lesssim \langle v^2/c^2\rangle \sim 8\%$ for T and $\sim 24\%$ for ψ). Furthermore the nature of these corrections is much better understood here than in $ee \to$ jets or ep scattering, since we have a very complete theory of the internal structure of the heavy quark mesons. Current measurements of $B_{\mu\mu}^{-1}(T) = 0.031(5)$ already would determine Λ to $\pm$30-50%, except that QCD perturbation theory is not very convergent for this process — i.e. $B_{\mu\mu} \propto \alpha_s^3(1 - 14\,\alpha/\pi + ?)$. [Perturbation theory seems to converge quite well for all other processes discussed in this note]. This theoretical problem may be resolved some day. Another quantity of interest is the η_b-T splitting which is predicted in perturbation theory. However, the relevant Q^2 for this calculation is the typical internal momentum of the heavy quark — i.e. $Q^2 \sim 1$-2 GeV^2 — and this may be a little low for a reliable application of perturbation theory. This problem is even worse for the hadronic decay rate of p-wave states.

e) Other Processes

The photon structure function ($e\bar{e} \to e\bar{e}\gamma\gamma^* \to e\bar{e}$+hadrons) is quite sensitive to Λ at large x and Q^2. Current data (PETRA/PEP) involves only rather low Q^2 ($\lesssim 5$ GeV^2), making the interpretation difficult. Hadron-hadron scattering, producing large-$p_\perp$ particles, also measures α_s. However, high precision measurements are very difficult due to uncertainties in the many structure and fragmentation functions convoluted with the basic QCD subprocess (not to mention chronic problems with higher twist, etc.). Analogous problems plague the interpretation of large $p_\perp$ exclusive processes although many of the uncertainties can cancel if ratios are considerable — e.g. $F_\pi(Q)/|F_{\pi\gamma}(Q)|^2 \propto \alpha_s(Q)$ where F_π is measured in $e\bar{e} \to \pi^+\pi^-$ and $F_{\pi\gamma}$ in $e\bar{e} \to \pi^0\gamma$ (very high luminosity in the region $Q^2 \gtrsim 10$ GeV^2 would be necessary).

4. Conclusions

It is clearly desirable to have several completely different determinations of the QCD scale parameter Λ. While work will continue on scaling violations and jet analyses, a concerted effort, both theoretical and experimental, is necessary in the study of quarkonium properties, $R(e^+e^-)$, and other 'low' energy measurements of α_s. The theoretical basis for these latter processes is as sound or more so as the basis for the more conventional processes. Detailed studies of the perturbation theory at widely separated energies will provide important information on the Q^2 variation of the effective coupling, and they will compliment the results just now starting to come from Monte Carlo studies (on a lattice) of the very low energy structure of the theory.

Acknowledgements

The opinions expressed here are largely my own. However, they were brought into sharper focus by many discussions with members of the QCD subgroup at the Workshop, and especially with M. Tannenbaum, J. Friedman, M. Tuts and L. Brown.

References

1. A more detailed survey, and an extensive bibliography, can be found in A. J. Buras, 'A Tour of Perturbative QCD', in the Proceedings of the 1981 Symposium on Lepton and Photon Interactions at High Energies, edited by W. Pfeil (Bonn, 1981).

2. Some possible evidence for higher twist components in existing data is discussed in M. Barnett, Phys. Rev. Lett. 48, 1657 (1982).

3. See for example, M. Barnett and D. Schlatter, Phys. Lett. 112B, 475 (1982).

4. See for example P. Söding, DESY preprint #81-070 (1981).

5. S. D. Ellis, Univ. of Washington preprint #40048-19-P2 (1982); H. Behrend, talk presented at the XXI International Conference on Particle Physics (Paris, 1982).

6. See, for example, talks presented by D. Ritson and G. Heinzelman at the XXI International Conference on Particle Physics (Paris, 1982).

7. These processes are reviewed by G. P. Lepage, in the Proceedings of the 1981 SLAC Summer Institute on Particle Physics, SLAC Report 245 (1982) and in Proceedings of the Banff Summer Institute on Particle Physics, (Banff, Alberta, Canada, 1981), edited by A. Capri and A. Kamal.

STANDARD MODEL GROUP, QCD SUBGROUP - DYNAMICS
ISOLATING AND TESTING THE ELEMENTARY QCD SUBPROCESS*

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Introduction

QCD to an experimentalist is the theory of interactions of quarks and gluons. Experimentalists like QCD because QCD is analogous to QED. Thus, following Drell and others[1] who have for many years studied the validity of QED, one has a ready-made menu for tests of QCD. There are the static and long distance tests such as:

- the value of the coupling constant α_s
- the shape of the QCD potential and "onia" spectroscopy in analogy to atomic spectroscopy and tests of Coulomb's law at large distances. (One might try to imagine the QCD analogue of g-2 and the Lamb shift.)
- tests of confinement: i.e., can you break up a proton into 3 quarks?

These topics are covered by Peter LePage in the static properties group. In this report, dynamic and short distance tests of QCD will be discussed, primarily via reactions with large transverse momenta.

This report is an introduction and overview of the subject, to serve as a framework for other reports from the subgroup. In the last two sections, the author has taken the opportunity to discuss his own ideas and opinions. Other people who contributed to the QCD dynamics subgroup were:

a. ep - Structure Functions:

J. Friedman, W. Lee, T. O'Halloran, G. Tzanakos, D.H. White

b. ep, e^+e^- - Jets in Final States

M. Derrick, J. Friedman, H. Sticker

c. e^+e^- - QCD Tests in Resonance Decays

M. Tuts, H. Vogel

d. Exclusive Reactions

G. Bunce

e. Hadron-Hadron

R.L. Cool, R. Odorico, H. Sticker, M.J. Tannenbaum

The basic equations for the elementary QCD constituent subprocesses have been given by Cutler & Sivers[2] and by Combridge, Kripfganz & Ranft.[3] These are what I call "pure" QCD processes, only involving quarks and gluons, and are shown in Figure 1. Most of these processes follow directly from analogy with QED and one can recognize Moller, Bhabha and Compton scattering. However, the distinctive feature of QCD compared to QED is that gluons carry color charge whereas photons do not carry electric charge. This is illustrated by the diagrams in the dashed box which involve the gluon self coupling and have no analogy in QED.

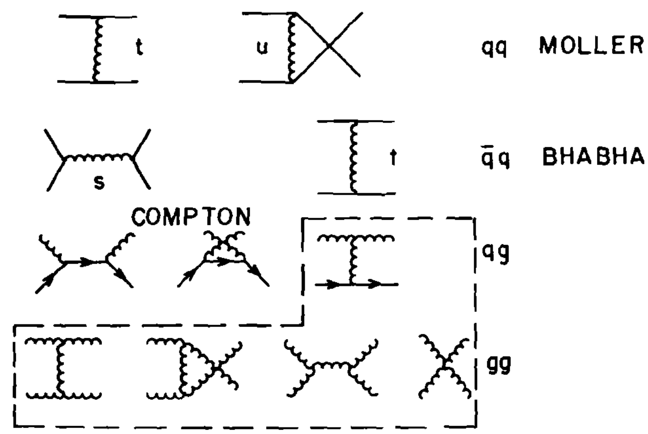

FIGURE 1

Another distinctive feature of QCD is that the coupling constant α_s changes with the momentum transfer, Q^2, of the reaction, with a scale parameter Λ. In a model with four flavors:

$$\alpha_s\,(Q^2) = 12\pi/25\ \ln\,(Q^2/\Lambda^2)\ .$$

The scattering cross sections for the constituent subprocesses are given by the formula:

$$\frac{d\sigma}{dt} = \frac{\pi\alpha_s^2(Q^2)}{s^2} \times \sum (\cos\theta^*) \qquad (1)$$

where s, t, and u are the Mandelstam variables of the subprocess; s = the total constituent c.m. energy squared; t = the invariant four-momentum-squared of the scattering, and $s + t + u = 0$. It is worthwhile to recall that in terms of the constituent subprocess scattering angle, θ^*:

$$t = -s\,\frac{(1-\cos\theta^*)}{2} \qquad u = -s\,\frac{(1+\cos\theta^*)}{2} \qquad (2)$$

and the constituent transverse momentum is

$$P_T = \frac{\sqrt{s}\,\sin\theta^*}{2}\ .$$

For 90° scattering

$$2\,P_T = \sqrt{s} \quad \text{and} \quad u = t = -2\,P_T^2. \qquad (3)$$

The angular factors $\sum(\cos\theta^*)$ are given in Table I[3] for the pure QCD processes of Figure 1.

One of the conceptual difficulties in dealing with QCD compared to QED is that experiments can not be performed directly on quarks and gluons. Thus a "standard" methodology has developed as illustrated for proton-proton collisions. The protons consist of 3 valence quarks and gluons which can scatter as constituents but can never emerge as free particles (presumably) because of a conservation law. The scattered

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