MEASUREMENT OF ν and $\bar{\nu}$e ELASTIC SCATTERING AS A TEST OF THE STANDARD MODEL

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Abstract

Various tests of standard SU(2)XU(1) model of weak interactions which can be performed by measurements of electron and muon neutrino-electron elastic scattering are reviewed. Electron neutrino-electron elastic scattering has both a neutral current part as well as a charged current part, and therefore offers a unique place to measure the interference of these two amplitudes. A measurement of the y-dependence of neutrino-electron elastic scattering can separately measure g_V and g_A as well as test for the presence of S, P, or T terms. Several measurable quantities involving cross sections and the interference term are derived from the standard model. Various design considerations for an experiment to determine the NC-CC interference term and the y-dependence of muon neutrino-electron elastic scattering are discussed.

I) Introduction:

In this note we discuss various tests of the standard SU(2)XU(1) gauge model of weak interactions which involve the elastic scattering of electron and muon neutrinos off electrons. These experiments are difficult owing to the extreme smallness of the purely leptonic cross section, but are well predicted by the standard model and as such offer clean tests of the theory.

There have been several measurements of $\overset{(-)}{\nu}_\mu e$ elastic scattering and one measurement of $\bar{\nu}_e e$ elastic scattering[1]. But all of these experiments have been with limited statistics and (or) of limited resolution. Hence it is worthwhile to envision extensions of the present experimental technique to obtain a high statistics measurement of these rare interactions. In Section II we review the tests of the standard model which can be done with a measurement of ν-e elastic scattering at high energy. A method for measuring $\overset{(-)}{\nu}_e$-e elastic scattering is discussed in Section III. A discussion of the test of the standard model which can be obtained from a measurement of the differential cross sections $d\sigma/dy$ ($\overset{(-)}{\nu}_\mu e$) is covered in Section IV. The conclusions are given in Section V. Several design considerations for a beam of electron neutrinos is discussed in the Appendix.

II. The ν-e Scattering Cross Sections

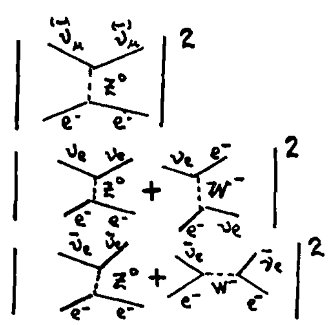
A) General Consideration

The Feynman diagrams for ν-e elastic scattering in the standard model are the following:

$\sigma(\overset{(-)}{\nu}_\mu e)$ ~

$\sigma(\nu_e e)$ ~

$\sigma(\bar{\nu}_e e)$ ~



We note that only the NC amplitude contributes to $\overset{(-)}{\nu}_\mu e$ scattering and that both the NC and CC contribute to $\overset{(-)}{\nu}_e e$ scattering.

The neutrino-electron elastic scattering cross sections are all very small compared to σ_T (ν-nucleon) namely $\sigma(\nu_e)/\sigma_T(\nu N) \sim 10^{-3}$ to 10^{-4}) since the center of mass energy is intrinsically small. Furthermore extremely good angular resolution on the recoil electron direction and small angular divergence of the incident neutrino beam are required to measure the scaling variable $y = E_e/E_\nu$ in a broad band neutrino beam. For these reasons, only $\sigma_T = \int_0^1 (d\sigma/dy)\, dy$ has to date been measured.

The total cross sections in the standard model are given by (neglecting m_e/E_ν terms)

$$\sigma_T(\nu_e + e \to \nu_e + e) = \frac{G^2 m_e E_\nu}{2\pi} \left((g_V + g_A + 2)^2 + \frac{1}{3} (g_V - g_A)^2 \right)$$

$$\sigma_T(\bar{\nu}_e + e \to \bar{\nu}_e + e) = \frac{G^2 m_e E_\nu}{2\pi} \left((g_V - g_A)^2 + \frac{1}{3} (g_V + g_A + 2)^2 \right) \qquad (2)$$

$$\sigma_T(\nu_\mu + e \to \nu_\mu + e) = \frac{G^2 m_e E_\nu}{2\pi} \left((g_V + g_A)^2 + \frac{1}{3} (g_V - g_A)^2 \right)$$

$$\sigma_T(\bar{\nu}_\mu + e \to \bar{\nu}_\mu + e) = \frac{G^2 m_e E_\nu}{2\pi} \left((g_V - g_A)^2 + \frac{1}{3} (g_V + g_A)^2 \right)$$

where:
$$g_V = -\frac{1}{2} + 2\sin^2\theta_W$$

$$g_A = -\frac{1}{2} \quad .$$

Note that the $\overset{(-)}{\nu}_e e$ cross sections are related to the $\overset{(-)}{\nu}_\mu e$ cross sections by replacing g_V by $g_V + 1$ and g_A by $g_A + 1$. The additional terms in $\sigma(\overset{(-)}{\nu}_e e)$ are the CC contribution and the NC-CC interference term. From these expressions of σ_T, it is apparent that the one helicity amplitude which is modified in going from $\nu_\mu e$ to $\nu_e e$ elastic scattering corresponds to $\nu_L\ e_L \to \nu_L\ e_L$, or $\bar{\nu}_R\ e_L \to \bar{\nu}_R\ e_L$. That is, only the left-handed component of the neutral current can interfere with the charged current amplitude.

B) Electron neutrino-Electron Scattering:

In the standard model of electroweak interactions, electron neutrinos may interact with electrons by both a neutral current as well as a charged current. A measurement of the interference between these two interactions tests:

1. The identity of the outgoing neutrino in the neutral current interaction. If the interference term is present, then $\nu_{out} = \nu_e$, at least some of the time.

2. The nature of the NC coupling to electrons. A non-zero interference term implies a coupling to left-handed electrons.

3. The helicity properties of the NC interactions. A finite interference term states that at least some of the time the NC interaction preserves the electron helicity, and at least part of the interactions has V and A terms, rather than a S, P, T structure which flips the electron helicity.

4. The μ-e universality hypothesis. A comparison of ν_e-e interactions with ν_μ-e interactions is an important test of this hypothesis.

In the standard SU(2)XU(1) model, the NC-CC interference term in $\overset{(-)}{\nu}_e$-e scattering is proportional to

$$I_3\ (e_L) + \sin^2\theta_W$$

where $I_3(e_L)$ is the third component of the weak isospin of the left-handed electron. For the standard assignment of $I_3(e_L) = -1/2$ and for $\sin^2\theta_W \sim 0.25$ the interference term above will be destructive (negative). Variants to the standard Weinberg-Salam model have been examined by Kayser, Fischback, Rosen, and Spivack[2]. They find these models all predict a destructive interference term in ν_e-e scattering, or are ruled out by inconsistencies with various measurements. Therefore a measurement of the sign and magnitude of the interference term within the standard Weinberg-Salam model is an important test of the whole theoretical framework.

Carrying these relations between cross sections described above further, we may write:

$$\sigma_T(\nu_e e) = \sigma_T(\nu_\mu e) + 4\,\sigma_o\ (1 + g_V + g_A)$$
$$\qquad (3)$$
$$\sigma_T(\bar{\nu}_e e) = \sigma_T(\bar{\nu}_\mu e) + \frac{4}{3}\,\sigma_o\ (1 + g_V + g_A) \quad ,$$

where $\sigma_o \equiv G_F^2 m_e E_\nu/2\pi$ and the terms additional to $\sigma_T(\nu_\mu e)$ are the charged current and the interference terms. If $\sin^2\theta_W \sim 0.25$, then the $\overset{(-)}{\nu}_e e$ cross sections will have large contributions from the CC amplitude. In the standard model, the CC contribution to $\sigma_T(\nu_e e)$ is $4\,\sigma_o$ and is thus ~ 12 times larger than the neutral current part. The interference term is roughly 46% of the NC + CC contributions. For $\sigma_T(\bar{\nu}_e e)$, the CC part $(4\sigma_o/3)$ is ~ 4 times the NC part and the interference term is ~ 40% the NC + CC contribution. Thus the interference terms make a sizeable contribution to the $\overset{(-)}{\nu}_e e \to \overset{(-)}{\nu}_e e$ cross sections, and can therefore be measured[3]. Table I lists the relative cross sections $\sigma_T(\overset{(-)}{\nu}_e e)/\sigma_o$ for the standard model prediction of the interference term (destructive), for no interference, and for a constructive intereference term for $\sin^2\theta_W = 0.25$. Hence a 25% measurement of $\sigma_T(\overset{(-)}{\nu}_e e)$ is sufficient to determine the presence and the sign of the interference term $(g_V + g_A)$.

Table I

	$\sigma_T(\nu_e e)/\sigma_o$	$\sigma_T(\bar{\nu}_e e)/\sigma_o$
With Standard Model Interference (destructive	2.33	1.00
With No Interference	4.33	1.66
With Opposite Sign Interference (constructive)	6.33	2.33

To extract the interference term from measured quantities; we can compute:

$$g_V + g_A = \frac{\sigma_T(\nu_e e) - \sigma_T(\nu_\mu e)}{4\sigma_o} - 1$$

$$g_V + g_A = \frac{\sigma_T(\bar{\nu}_e e) - \sigma_T(\bar{\nu}_\mu e)}{\frac{4}{3}\sigma_o} - 1 \tag{4}$$

Assuming that $\sigma_T(\nu_e e)$ can be measured to 10% and that the errors of $\sigma_T(\nu_e e)$ and $\sigma_T(\nu_\mu e)$ are uncorrelated, the interference term $g_V + g_A$ can be measured to 12% for neutrinos and 16% for antineutrinos if $\sin^2\theta_w \sim 0.25$. (In the standard model, $g_V + g_A = -1 + 2\sin^2\theta_w \approx -1/2$.) Notice that if $\sin^2 \sim 0.25$, then the cross sections are in the ratio

$$\sigma_T(\nu_e e) : \sigma_T(\bar{\nu}_e e) : \sigma_T(\nu_\mu e) : \sigma_T(\bar{\nu}_\mu e) = 7 : 3 : 1 : 1 \quad ,$$

and thus the error in determining the interference term is most strongly dependent on the error in $\sigma_T(\overset{(-)}{\nu}_e e)$.

From general considerations[2], the interference term should satisfy the relation

$$|I| \le \sqrt{N_c}\sqrt{C_c} \quad , \tag{5}$$

where in the standard model $N_c = (g_V + g_A)^2$ and $C_c = 4$ are neutral current and charged current terms, respectively, and the interference term $I = 2(g_V + g_A)$. But in general, the equality holds only where the S, P, T terms to the cross sections vanish, and thus this relation gives a method of testing for the presence of these helicity flipping terms.

The μ-e universality hypothesis can be tested by checking the relation

$$\sigma_T(\nu_e e) - \sigma_T(\nu_\mu e) = 3\left(\sigma_T(\bar{\nu}_e e) - \sigma_T(\bar{\nu}_\mu e)\right) \tag{6}$$

which is another important test of the standard model from ν-e elastic scattering.

C) Muon Neutrino-Electron Scattering:

All experiments on $\overset{(-)}{\nu}_\mu e$ elastic scattering have measured $\sigma_T(\overset{(-)}{\nu}_\mu e)$ and thus are sensitive to only the combination

$$\left((g_V + g_A)^2 + \frac{1}{3}(g_V - g_A)^2\right), \text{ or } \left((g_V - g_A)^2 + \frac{1}{3}(g_V + g_A)^2\right). \tag{7}$$

By combining measurements of both $\sigma_T(\nu_\mu e)$ and $\sigma_T(\bar{\nu}_\mu e)$, the values of $(g_V - g_A)^2$ and $(g_V + g_A)^2$ may be separately extracted, thereby determing $\sin^2\theta_w$. But this procedure inevitabely introduces systematic errors from the neutrino flux normalization.

Further tests of the standard model are obtainable by measuring the $\overset{(-)}{\nu}_\mu e$ elastic differential cross sections. In this context, and neglecting terms of order Me/Eν:

$$\frac{d\sigma}{dy}(\overset{(-)}{\nu}_\mu e) = \sigma_o' E_\nu \left((g_V \mp g_A)^2 + (g_V \pm g_A)^2 (1-y)^2\right) \tag{8}$$

where $\sigma_o' = \frac{G_F^2 m_e}{2\pi} = 4.3 \times 10^{-42}$ cm^2/GeV .

We note that a measurement of dσ/dy for neutrino, or antineutrino scattering separately determines $(g_V + g_A)^2$ and $(g_V - g_A)^2$ and thus provides a further test of the standard model. In addition, the y-dependence of the cross section has a sensitivity to the presence of S, P, T helicity flipping terms, which would produce a term proportional to (1-y).

III. Experimental Considerations-Electron Neutrino-Electron Elastic Scattering

There are several important considerations in designing an experiment to measure $\overset{(-)}{\nu}_e\, e^-$ elastic scattering. Since the cross sections are so small the neutrino detector must be massive, and fine grained enough to have an angular resolution sufficient to discriminate against various background processes. Of no less importance, is the neutrino beam, which must be intense and normalizable. Several massive neutrino detectors which are well suited for $\overset{(-)}{\nu}_e$-e elastic scattering studies have recently been built[3], and have been described in detail elsewhere. Therefore in this discussion we will concentrate on the design of the neutrino beam and the treatment of the data.

a) Beam

There are several sources of electron neutrinos which can be used for a measurement of $\overset{(-)}{\nu}$-e elastic scattering.

Since the ν-e elastic scattering cross sections are so small, it is important that the source be intense. Since a cross section is to be determined, the flux of neutrinos must be normalized. A few possible sources are: (a) beam dump, (b) K_L^0 beams, (c) $\mu^\pm$ storage ring, and (d) π^+ beam stop.

Of these possibilities, the $\mu^\pm$ storage ring is the most attractive*, (with no consideration of money) since it has the following properties:

1. The spectrum of neutrinos is easily calculable from the μ decay process.

2. Both μ^+ and μ^- can be stored giving all flavors of neutrinos.

3. Over a limited range, the energy scale of the neutrino spectrum can be varied, giving important experimental checks.

4. The flux of neutrinos can be precisely determined by monitoring the circulating muon current in the ring.

The spectrum for μ-decay $\mu^+ \rightarrow e^+ \bar{\nu}_\mu \nu_e$ or $\mu^- \rightarrow e^- \nu_\mu \bar{\nu}_e$ in the muon rest frame is shown in Fig. 1. The flux of neutrinos in that frame has the form:

$$\phi_{\mu^+}(\nu_e) = \phi_{\mu^-}(\bar{\nu}_e) = 2NE_\nu^2(W - E_\nu)$$
$$\phi_{\mu^+}(\bar{\nu}_\mu) = \phi_{\mu^-}(\nu_\mu) = N\,E_\nu^2(W - \tfrac{2}{3}E_\nu)\ , \qquad (9)$$

where $W=(m_\mu^2 + m_e^2)/2m_\mu \approx \frac{1}{2}m_\mu$ (neglecting neutrino masses). The neutrino energy spectrum in the lab frame is then given by $\phi_{Lab} = \phi_{Com}\ 1/2\gamma_\mu$, where $\gamma_\mu = P_\mu/m_\mu$. Since $\sigma_T(\nu e) \sim E_\nu$, a given experiment measuring an integrated rate, will have slightly different neutrino energies contribution from the slight difference of $\bar{\nu}_e$ and ν_μ spectra. This difference is $\langle E_\nu\rangle_\mu/\langle E_\nu\rangle_e \approx 1.17$ and is easily correctable.

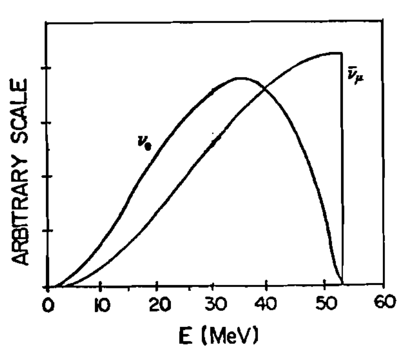


Figure 1: The energy spectrum of the neutrinos emitted in μ decay in the μ rest frame.

b. Energy Scale

An important design consideration is the choice of the energy scale of the incident neutrino beam. There are two main factors which determine this choice: (1) background rejection, and (2) event rate. Here we see how the two factors scale with energy.

The background rejection depends on two things: The intrinsic ratio of the ν-e signal to background, and the resolution in the kinematic identifier $E_e\theta_e^2 = 2m_e(1-y) \leq$ 1 MeV.

There are many sources of backgrounds in νe elastic scattering, but one of the more troublesome is the quasi-elastic process $\nu_e n \rightarrow p + e^-$ ($\bar{\nu}_e p \rightarrow p + e^+$), where at low Q^2 the recoil nucleon is not detected. These cross sections are roughly constant with energy and thus the ν-e signal to this background will improve with increasing neutrino energy. This then argues for higher neutrino energy.

The resolution in the $E_e\theta_e^2$ kinematic identifier also improves with increasing neutrino energy. There are two sources in this μ-storage ring experiment to the smearing in $E\theta^2$. One source comes from the intrinsic angular divergence of neutrinos from μ decay and because the event rate has to be maximized, this cannot be eliminated by moving the neutrino detector further away thereby losing neutrino flux. Another source comes from the angular resolution of the recoil electron direction as measured in the neutrino detector.

The μ-decay kinematics gives the limiting value of $E_e\theta_e^2$ $\langle E_\nu\theta_\nu^2\rangle$. Thus,

$$E_e\theta_e^2\ \langle E_\nu\theta_\nu^2 \approx E^*_{max}\ \ell(\theta^*)\ \left(\mathrm{Tan}^{-1}\left(\frac{\sin\theta^*}{\gamma_\mu(\cos\theta^* + \beta_\mu)}\right)\right)^2 \sim \frac{1}{P_\mu}$$

where $\ell(\theta^*) = (\gamma^2(\cos\theta^* + \beta)^2 + \sin^2\theta^*)^{1/2}$ and therefore the angular divergence of the neutrino beam improves as the energy of the ring increases.

The angular resolution of the ν-detector improves with increasing neutrino energy. The angular resolution of the recoil showers typicall goes as:

$$\sigma_{\theta_e} \sim \frac{b}{E_e}$$

for small E_e. Thus the resolution

$$\sigma(E_e\theta^2_e) \sim \frac{b^2}{E_e} \sim \frac{1}{P_\mu}$$

and improves with increasing muon energy.

In summary, the signal to background improves as the energy of the ring increases, and the resolution in the kinematic identifier $E_e\theta_e^2$ improves with increasing energy.

The second consideration in the choice of the energy scale is the neutrino-electron scattering event rate. The various factors governing this are:

1. The neutrino-electron cross section: $\sigma_T(\nu_e) \sim E_\nu \sim P_\mu \sim P_\pi$.
2. $\pi^\pm$ production cross section:

$$\frac{\frac{d\sigma_\pi}{dP_\pi}}{p\pi\; d\Omega_\pi} \sim A\; P_\pi^2$$

for small P_T and $x_R \equiv E^*/E^*_{max}$.

3. $\pi^\pm$ decay for a fixed decay pipe length ℓ_d :

$$f_{decay} \approx \frac{\ell d\; m\pi}{c\tau_o}\; \frac{1}{P_\pi} \sim \frac{1}{P_\mu}\;.$$

4. Accelerator beam power: Assuming that the power in the proton beam which produces the $\pi^\pm$ mesons which decay into $\mu^\pm$ to fill the storage ring is limited, we have:

$N_P\; P_O \sim K_A$ (which ~ 1.2×10^{15} protons-GeV/sec at FNAL).

Putting these factors together, the neutrino electron event rate should scale as:

$N(\nu\text{-}e)$ ~ (ν-e cross section) x ($\pi^\pm$ production) x ($\pi^\pm$ decay)

x (beam power)

$$\sim P_\mu \times P_\mu^2 \times \frac{1}{P_\mu} \times \frac{1}{P_\mu} \sim P_\mu$$

and thus improves as the muon momentum increases. We have assumed that $E_\nu \propto P_\mu \propto P_\pi \propto P_O$, ($P_O$ = the primary proton beam momentum), and have neglected multiple target effects which tend to reduce this correlation.

For a prototype experiment we will take the energy of the storage ring to be ~ 10 GeV which is to be fed by a ~ 150 GeV beam (generated at FNAL for example). At this energy the resolution effects discussed above are well controlled.

C) <u>Event Rates:</u>

The N(νe) event rate is given by

$$N(\nu e) = N_T \int \phi(E_\nu)\, \frac{d\sigma}{dy}\,(E_\nu)\; \epsilon(y)\; dy dE_\nu \;. \qquad (10)$$

Taking P_μ ~ 10 GeV with ~ 3×10^9 stored $\mu^\pm$/sec, with ~ 1/4 of the $\mu^{(\pm)}$ decays giving ν_μ, $\bar{\nu}_e$ in the neutrino detector and $N_T \sim 4.5 \times w^{27}/cm^2$ (roughly ~ 10^3 tons), we estimate for the decay efficiency $\epsilon(y)$ ~ 1 the followng event rates given in Table II.

Table II

Six months run* for $\mu^\pm$ each

$N_{\nu e}$	22/day	972
$N_{\bar{\nu}_e}$	9/day	423
N_{ν_μ}	3.6/day	162
$N_{\bar{\nu}_\mu}$	3.6/day	162

1 Day = 24 hours

*where for six months have included an overall efficiency factor of ~ 1/4.

These event rates would allow a good determination of the interference term and the value of $\sin^2\theta_w$. This will be discussed in the following section.

d) <u>Treatment of the data</u>

1. <u>Determination of the Interference Term</u>
Since the decaying $\mu^\pm$ neutrino beam contains both muon and electron type neutrinos, the $\overset{(-)}{\nu}_\mu$ contribution to the measured ν-e event rate has to be subtracted. This can be done by estimating $N(\nu_\mu e)$ from other measurements or by measuring $N(\nu_\mu e)$ from the prompt ν_μ from $\pi \to \mu + \overset{(-)}{\nu}_\mu$ decay. Since the observed ν-e event rate will be dominated by ν_e-e interactions (if $\sin^2\theta_w \approx 0.25$), this subtraction procedure does not introduce large errors. (See the discussion on Section II.) For the statistics here, we would have the error in the interference term ΔI = 4% for ν_ee and ΔI = 8% for $\bar{\nu}_e$e.

Alternatively, a subtraction independent test for the presence of the interference term can be obtained by computing the ratio of ν-e (all flavor neutrinos) events for stored μ^+ to that for stored μ^-. Many experimental effects would cancel in this ratio and would therefore reduce the inevitable systematic errors.

The measured ν-e event rate for stored μ^- is given by:

$$N^{\mu^-} \sim (\sigma_T(\nu_\mu e) + \eta\sigma_T(\bar{\nu}_e e))K$$

and for stored μ^+:

$$N^{\mu^+} \sim (\sigma_T(\bar{\nu}_\mu e) + \eta\,\sigma_T(\nu_e e))K$$

where K is an experimental constant (depending on the $\mu^\pm$ integrated beam current, etc.) and $\eta \approx 1.17$ is the correction factor for the difference between the ν_e and the ν_μ energy spectra. Taking the ratio

$$R \equiv \frac{N^{\mu^-}}{N^{\mu^+}} \cong \frac{\sigma_T(\nu_\mu e) + \sigma_T(\bar{\nu}_e e)}{\sigma_T(\bar{\nu}_\mu e) + \sigma_T(\nu_e e)} \tag{12}$$

where we have neglected the 17% correction of η differing from 1. In the standard model.

$$R \cong \frac{(g_V + g_A)^2 + (g_V - g_A)^2 + (g_V + g_A) + 1}{(g_V + g_A)^2 + (g_V - g_A)^2 + 3(g_V + g_A) + 3} \tag{13}$$

This ratio is plotted in Fig. 2 as a function of $\sin^2\theta_W$. Here we see that if the interference term $(g_V + g_A) = 0$, then this ratio would change by 20% for $\sin^2\theta_W \sim 0.25$. Hence an independent measure of $\sin^2\theta_w$ allows a determination of the interference term.

For the statistics of a six-month run, the presence of the interference term is easily found. To estimate the error in the interference term $g_V + g_A$, we take the statistics of Table II and $\sin^2\theta_w \sim 0.25$. With these conditions R can be measured to ~ 5% thereby giving an error in $(g_V + g_\eta)$ of 17%. This statistical error is larger than the subtraction method error, but is less sensitive to systematics.

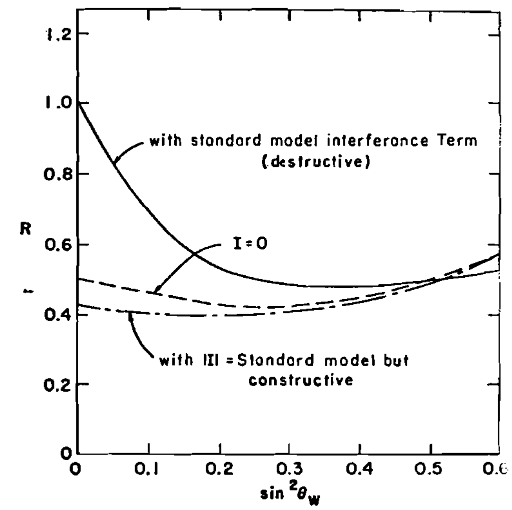


Figure 2: The ratio $R=N^{\mu^-}/N^{\mu^+}$ of event rates for a stored μ^- beam to that of μ^+ beam versus $\sin^2\theta_w$. The three cases for the interference term I>0, =0, <0 are indicated.

2. Determination of $\sin^2\theta_w$

It is an interesting experimental question to test e-μ universality by determining $\sin^2\theta_w$ from $(\overset{(-)}{\nu}_\mu e)$ scattering and comparing it to the value derived from $\overset{(-)}{\nu}_e e$ scattering. Hung and Sakurai[1] have noted that in the Bjorken approach, certain factors make these two values of $\sin^2\theta_w$ different.

In the context of the standard model, the value of $\sin^2\theta_w$ may be inferred from a measurement of $\nu_\mu e$ and $\bar{\nu}_\mu e$ elastic scattering. A clean way of extracting this parameter is to compute the ratio

$$R_{\nu_\mu} = \frac{N_{\nu\mu e}}{N_{\bar{\nu}\mu e}} = \frac{\sigma_T(\nu_\mu e)}{\sigma_T(\bar{\nu}_\mu e)} = \frac{(g_V + g_A)^2 + \frac{1}{3}(g_V - g_A)^2}{(g_V - g_A)^2 + \frac{1}{3}(g_V + g_A)^2}, \tag{14}$$

where many experimental systematic errors cancel. This ratio as a function of $\sin^2\theta_w$ is plotted in Fig. 3. Assuming a 3% measurement error on both cross sections (these measurements would be done in a $\pi^\pm$ decay beam), we have a determination of $\sin^2\theta_w$ to ± 0.005. The number of $\overset{(-)}{\nu}_\mu e$ events needed to give a 5% statistical error depends on the amount of background. For no background, $N(\overset{(-)}{\nu}_\mu e) \sim 100$ events, but for a 30% background, we require 200 events.

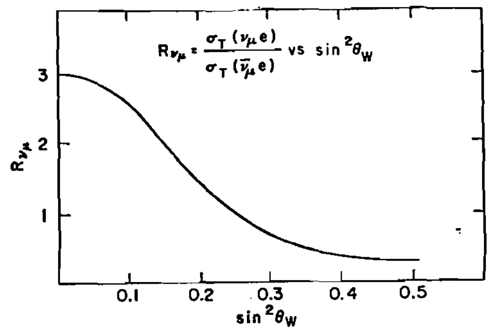


Figure 3: The ratio $R_{\nu_\mu} = \sigma_T(\nu_\mu e)/\sigma_T(\bar\nu_\mu e)$ as a function of $\sin^2\theta_w$.

To determine $\sin^2\theta_w$ in the $\overset{(-)}{\nu}_e e$ case, we compute the same ratio:

$$R_{\nu_\mu} = \frac{N(\nu_e e)}{N(\bar\nu_e e)} \simeq \frac{\sigma_T(\nu_e e)}{\sigma_T(\bar\nu_e e)}$$

$$= \frac{(g_V + g_A)^2 + \frac{1}{3}(g_V - g_A)^2 + 4\,(1 + g_V + g_A)}{(g_V - g_A)^2 + \frac{1}{3}(g_V + g_A)^2 + \frac{4}{3}\,(1 + g_V + g_A)} \qquad (15)$$

This ratio is plotted in Fig. 4 as a function of $\sin^2\theta_w$. For a 5% determination of $\sigma_T(\overset{(-)}{\nu}_e e)$, we find $\delta \sin^2\theta_w \sim \pm 0.07$. To achieve this cross section 5% error depends on backgrounds, but is feasible with a dedicated experiment of the statistics calculated here.

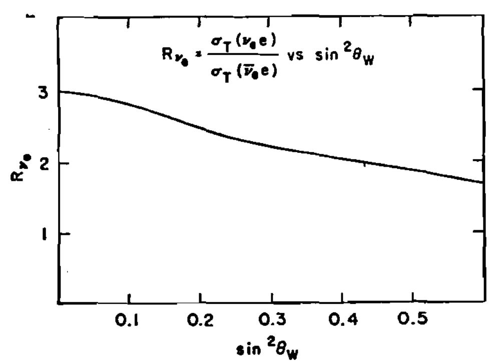


Figure 4: The ratio $R_{\bar\nu_\mu} = \sigma_T(\nu_e e)/\sigma_T(\bar\nu_e)$ versus $\sin^2\theta_w$.

IV. Experimental Considerations: Muon Neutrino-Electron Elastic Scattering

As we noted in Section II, a further test of the standard model follows from a measurement of the differential cross section $d\sigma/dy$ $(\overset{(-)}{\nu}_\mu e)$, where $y = Ee/E\nu$, which separately determines $(g_V+g_A)^2$ and $(g_V-g_A)^2$ and thus $\sin^2\theta_w$. Figure 5 shows $d\sigma/dy(\overset{(-)}{\nu}_\mu e)$ versus y for various $\sin^2\theta_w$ values. We note the relation between $d\sigma/dy(\nu_\mu e)$ and $d\sigma/dy(\bar\nu e)$ involves an interchange of the constant and the $(1-y)^2$ slope term in the differential cross section. Hence a comparison of the neutrino wht the antineutrino cross sections is very sensitive to $\sin^2\theta_w$.

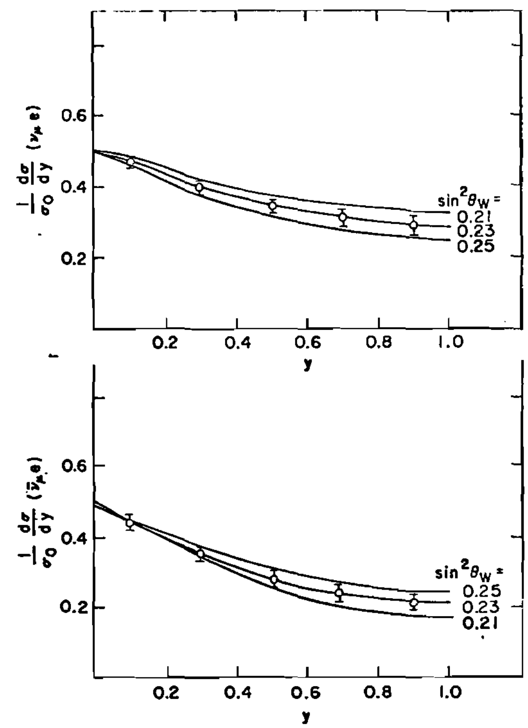


Figure 5: The differential cross section $d\sigma/dy$ $(\overset{(-)}{\nu}_\mu e \to \overset{(-)}{\nu}_\mu e)$ versus y for various values of $\sin^2\theta_w$.

a) The y-Resolution:

The most difficult experimental problem is to achieve an adequate resolution of the scaling variable $y = E_e/E_\nu$. By the elastic scattering kinematics:

$$y = \frac{E_e}{E_\nu} = 1 - \frac{2E_e}{Ym_e}\sin^2\frac{\theta_e}{2} \simeq 1 - \frac{E_e\theta_e^2}{2m_e}$$

where E_ν is the incident neutrino energy, E_e is the recoil electron energy and θ_e is the recoil electron angle. The kinematics of elastic scattering is shown in figure 6. Thus a measurement of the recoil energy and angle is sufficient to reconstruct the kinematics. Alternatively, a measurement of E_ν and E_e is also sufficient to determine the kinematics and thus the scaling variable y.

From figure 6 we see that the measurement of y by E_e and θ_e requires very good angular resolution of the recoil electron direction which makes stringent demands on the neutrino detector, while the E_e/E_ν method requires a narrow band beam of feasible energy resolution, but results in a large loss of event rate from the lower neutrino flux. Thus the experimental problem is either: 1) achieve good angular

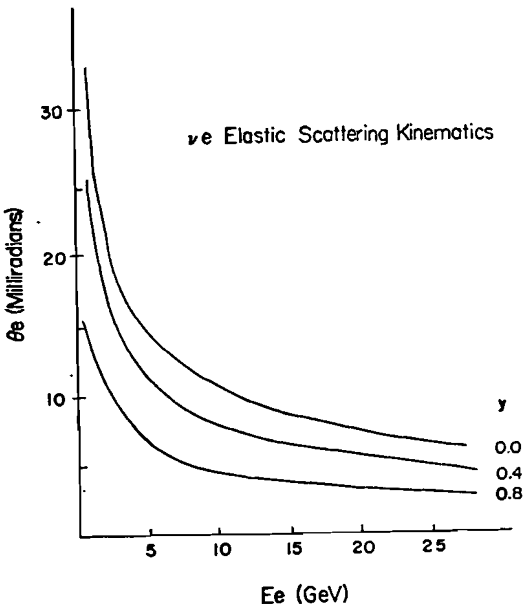


Figure 6: The kinematics of νe elastic scattering.

resolution $\sigma(\theta_e)<1$ mr at 25 GeV (see figure 6), or 2) design a super intense narrow band beam. We adapt the E_e, θ_e reconstruction method and compute the resolution of E_e and θ_e required. The y resolution is given by

$$\Delta y = (1-y)\sqrt{\left(\frac{\Delta E_e}{E_e}\right)^2 + 4\left(\frac{\Delta\theta_e}{\theta_e}\right)^2} \approx 2(1-y)\left(\frac{\Delta\theta_e}{\theta_e}\right) . \qquad (16)$$

where we have assumed that the angular resolution dominates the uncertainty in y. In the case that the contribution to the $\Delta\theta_e$ from the spacial resolution of tracking devices in the neutrino detector are small compared to the multiple Coulomb effect we have:

$$\Delta\theta_e \approx \frac{.015}{\sqrt{3}\, E_e}\sqrt{\frac{L}{L_o}} ,$$

where L/L_o is the length of the region where angle measurement is done measured in terms of radiation lengths. The corresponding y resolution is then:

$$\Delta\theta_e \approx \frac{.015}{\sqrt{3}}\sqrt{\frac{2(1-y)}{m_e E_e}\ \frac{L}{L_o}} .$$

Figure 7 shows Δy at $y=1/2$ plotted aganst E_e for two cases of L/L_o. The (1-y) dependence of Δy is rather fortunate because $d\sigma/dy$ is more sensitive to the coupling constants of larger y.

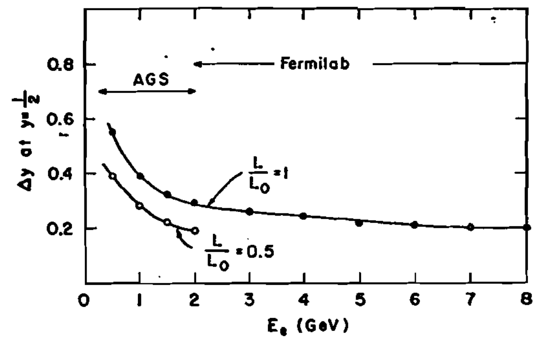


Figure 7: The y-resolution Δy at $y=1/2$ versus E_e for two values of L/L_o.

The y resolution gets better as the electron energy increases provided that we have a detector with $L/L_o=1$. Below 2 GeV we require a smaller L/L_o ratio.

To simulate the y-resolution in a real experiment, a prototype experiment has been designed by taking $L/L_o=1$ for the recoil electron angular resolution and $\sigma(E_e)/E_e=0.10/\sqrt{E_e}$ for the electron energy resolution. We have taken the neutrino beam to be the 400 GeV broad band spectrum at FNAL and have required an energy cut $E_e \geq 5$ GeV. The resulting y-resolution is shown in figure 8. Thus a resolution of $\sigma(y)\sim 0.09$ can be achieved, which is adequate for determing $d\sigma/dy$. However, even with this y-resolution, the measured y distribution is distorted by resolution effects (lower y-resolution at small y) which have to be unfolded.

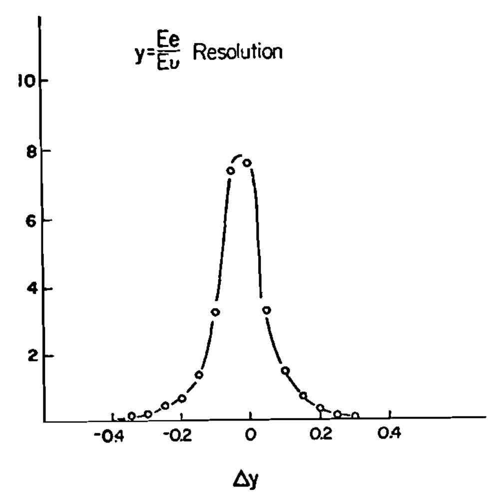


Figure 8: The y-resolution for a prototype experiment in a 400 GeV broad band beam.

b) Treatment of the Data:

By equation (8) we have seen that the neutrino-electron scattering cross section has the form:

$$\frac{d\sigma}{dy} = \sigma E_\nu\ (\ \alpha + \beta\ (1-y)^2) \qquad (17)$$

where $\alpha = (g_V \pm g_A)^2$ and $\beta = (g_V \mp g_A)^2$ for ν_μ or $\bar{\nu}_\mu$ respectively. Hence in a broad band neutrino beam the measured y-distribution will be given by:

$$\frac{dN_{\nu e}}{dy} = \int \phi_\nu(E_\nu) \frac{d\sigma}{dy} \epsilon \, dE_\nu \qquad (18)$$

where $\phi(E_\nu)$ is the neutrino flux dN_ν/dE_ν and ϵ is the detection efficiency-resolution function containing all experimental effects. Assuming that $\epsilon \cong 1$ (this can be calculated for a given experiment) we have:

$$\frac{dN_{\nu e}}{dy} = A + B(1-y)^2 \qquad (19)$$

where: $A = \int E_\nu \phi_\nu (E_\nu) dE_\nu \; \alpha \; \sigma_o$

and $\quad B = \int E_\nu \phi_\nu (E_\nu) dE_\nu \; \beta \; \sigma_o$

By fitting dN/dy we determine A and B. From the expression of cross section we have:

$$R = \frac{A}{B} = \frac{\alpha}{\beta} = \frac{(g_A \pm g_V)^2}{(g_A \mp g_V)^2} \qquad (20)$$

For neutrinos, antineutrinos, respectively.

Thus we determine $\sin^2\theta_w$ without having to normalize the $\overset{(-)}{\nu}_\mu e$ cross section and thereby avoid a large source of systematic error. Notice that neutrino-electron elastic scattering is related to antineutrino electron elastic scattering by $R_{\nu_\mu} = 1/R_{\bar{\nu}_\mu}$ giving an important check of the theory.

The value of $\sin^2\theta_w$ is determined by R in the standard model using the following-

$$\sin^2\theta_w = \frac{1}{2} \frac{1}{(1 + \sqrt{R})} \qquad (21)$$

To explore the sensitivity of this method of determining $\sin^2\theta_w$, we plot the ratio R versus $\sin^2\theta_w$ in figure 9. We see that the ratio R_{ν_μ} is very sensitive to $\sin^2\theta_w$ at small $\sin^2\theta_w$ and $R_{\bar{\nu}_\mu}$ is sensitive at large $\sin^2\theta_w$. The error in $\sin^2\theta_w$ is given by:

$$\sigma \sin^2\theta_w = \sin^4\theta_w \frac{1}{\sqrt{R}} \sigma_R$$

and is tabulated in Table III for various number of events for $\sin^2\theta_w = 0.225$. Hence approximately 1000 events are required to determine $\sin^2\theta_w$ to 4%. For an expeiment with a massive neutrino detector (500 tons) and a dedicated run in a wide band beam at FNAL, roughly 1000 $\nu_\mu e$ events are feasible.

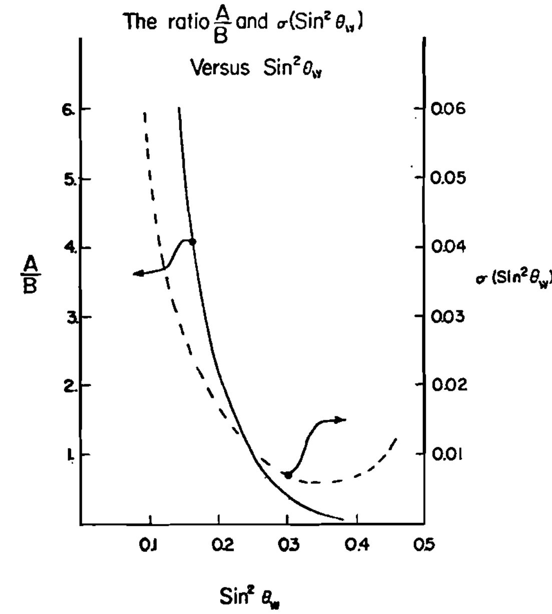


Figure 9: The ratio of intercept to slope for $dN^{(\bar{\nu})}_\mu/dy$ for various values of $\sin^2\theta_w$. The corresponding error is indicated by the dashed line.

Table III

Error in $\sin^2\theta_w$

Number of events	$\sin^2\theta_w$
50	0.24±0.06
100	0.24±0.04
500	0.23±0.01
1000	0.229±0.005

V) Conclusions:

A study of neutrino-electron elastic scattering offers many tests of the standard model. Several important points are worth noting:

1. The masurement of the total $\overset{(-)}{\nu}_e e$ elastic scattering cross section allows a determination of the interference term between NC and CC amplitudes and thereby incisively tests the standard model. It is feasiable to measure the interference term to ~ 4%. The best beam to do these measurement is a $\mu^\pm$ storage ring beam.

2. A determination of the y-dependence of $d\sigma/dy$ ($\overset{(-)}{\nu}_\mu e$) allows a determination of $\sin^2\theta_w$ independent of the

cross section normalization and as such avoids an important source of systematic errors. The considerable demands on the resolution of the recoil electron energy and angle require that this measurement be performed at electron energies $E_e > 5$ GeV.

3. A high quality $\overset{(-)}{\nu}_e$ beam would be of great use in testing μ-e universality, as well as many other confrontations of the theory of weak interactions. Such a beam sould be built.

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2. B. Kayser, et al., Phys. Rev. 20D, 87, (1979).

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Appendix:

Design of Storage Ring

The storage ring has to have a large momentum and solid angle acceptance. Such devices have been built with these properties[3] which could be adapted for the purpose of a $\overset{(-)}{\nu}_e$ source. Figure A1 shows the layout of the 10-GeV storage ring and accompanying neutrino detector. Roughly 21% of the decaying μ will radiate neutrinos into the ν-target. This factor could be increased to 33% with superconducting bends. The ν beam flux is monitored by measuring the $\mu^{\pm}$ beam current with RF cavities, or beam current transformers. The stored $\mu^{\pm}$ will decay with $\tau_{1/e} \sim 0.2$ msec and thus the ring has to be fed with single term extraction beam if FNAL is used. Then the prompt ν_μ from π decay are well separated in time from those of $\mu^{\pm}$ decay. The ring could be fed from a horn focussed $\pi^{\pm}$ beam, with some focussing devices to increase the $\mu^{\pm}$ flux into the ring. The ring is relatively small and would fit in the end of the neutrino line at FNAL. For the ring source of neutrinos to be viable, roughly 10^9 $\mu^{\pm}$ should be stored/sec.