PROSPECTS FOR EXPERIMENTS TO OBTAIN IMPROVED MEASUREMENTS OF QUARK MIXING ANGLES

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Among the most fundamental quantities in weak interactions is the quark mixing matrix, V, which (a) specifies how the quark mass eigenstates are mixed to form the weak gauge group eigenstates, and (b) equivalently, determines the strength of weak charged current (CC) transitions between quarks. By convention, one defines the $T_3 = +1/2$ weak eigenstates of quarks to be simultaneously mass eigenstates, and the $T_3 = -1/2$ quarks to be mixed. This mixing is determined by the transformation

$$\begin{pmatrix} d' \\ s' \\ b' \\ \cdot \\ \cdot \\ \cdot \end{pmatrix} = V \begin{pmatrix} d \\ s \\ b \\ \cdot \\ \cdot \\ \cdot \end{pmatrix} \qquad (1)$$

where $q'(q)$ denote weak (mass) eigenstates and $V^{+} = V^{-1}$. It follows that the weak charged current is

$$J_\lambda = \bar{q}_{iL}(2/3)\gamma_\lambda V_{ij}\, q_{jL}(-1/3) \qquad (2)$$

(where the charges of the quarks are indicated in parentheses), which motivates the notation

$$V = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} & \cdots \\ V_{cd} & V_{cs} & V_{cb} & \cdots \\ V_{td} & V_{ts} & V_{tb} & \cdots \\ \cdot & \cdot & \cdot & \\ \cdot & \cdot & \cdot & \\ \cdot & \cdot & \cdot & \end{bmatrix} \qquad (3)$$

In the general $n \times n$ case, the matrix V depends on $(n-1)^2$ real parameters, of which $n(n-1)/2$ are CP-conserving Euler rotation angles, and the remaining $(n-1)(n-2)/2$ are CP-violating phases. Henceforth, we shall restrict our considerations to the case n=3 unless otherwise noted. For this case a convenient parametrization of V is the one basically due to Kobayashi and Maskawa[1,2], viz.,

$$V = \begin{bmatrix} c_1 & s_1c_3 & s_1s_3 \\ -s_1c_2 & c_1c_2c_3 + s_2s_3e^{i\delta} & c_1c_2s_3 - s_2c_3e^{i\delta} \\ -s_1s_2 & c_1s_2c_3 - c_2s_3e^{i\delta} & c_1s_2s_3 + c_2c_3e^{i\delta} \end{bmatrix} \qquad (4)$$

where $c_1 = \cos\theta_1$, $s_1 = \sin\theta_1$ etc.

For the previous n = 2 case, V only depended on one angle, $\theta_{Cabibbo}$, so that the entire matrix could be determined from any one of its coefficients, and hence from just one experimental input. However, for the n=3 case, substantially more work is necessary. The determination of V begins with a precise comparison of the value of G_μ derived from μ decay, with the value of G_F derived from a set of corrected $\mathfrak{F}t$ values from very well measured superallowed $0^+ \to 0^+$ Fermi β decays. This yields $|c_1|$. Next, one performs an appropriately generalized Cabibbo fit to semileptonic hyperon decays and $K_{\ell 3}$ decays to obtain $|V_{us}|$ and thus $|s_3|$. The results of such an analysis there $|c_1| = 0.9737 \pm 0.0025$ and $|V_{us}| = 0.219 \pm 0.01$, which implies $|s_3| = {}^{+0.13}_{-0.28}$ [3].

Further bounds on quark mixing angles were obtained from an analysis of $K^o - \bar{K}^o$ mixing[4] and $K_L \to \mu^+\mu^-$ decay[5]. The latter decay provides a particularly clean method, since the hadronic matrix element is known quite reliably. The outputs from these analysis were bounds on $|s_2|$ and $|s_\delta|$. The coefficients $|V_{cd}|$ and $|V_{cs}|$ were relatively well determined from this work. It was interesting that the ratio $R_{ub} = |V_{ub}|^2/|V_{us}|^2$ was constrained to be substantially less than unity. This prediction has been confirmed by data from CESR on B-meson decays[6,7] Information about R_{ub} is obtained from measurements of the number of kaons produced,[6] and from the inclusive semileptonic spectrum[7] in B-meson decays. The latter, for example, yields the limit $R_{ub} < 0.04$.[7] This is a very important area where future experimental work shows promise of achieving a better limit on (or determination of) this ratio. The facilities which are appropriate for this research are, of course, CESR and DORIS. The potential of obtaining better knowledge of the quark mixing matrix, in particular, the b-u and b-c couplings, constitutes a strong argument for upgrading the intensity of CESR.

Further information on the mixing matrix coefficients $|V_{cd}|$ and $|V_{cs}|$ can be obtained from measurements of nonleptonic and semileptonic decays of charmed mesons. Building on the great amount of work by the SLAC-LBL and DELCO collaborations, the Mark III group will continue to amass data on these decays at SPEAR. This research effort also deserves strong support.

The question of whether high energy neutrino reactions can yield useful information on quark mixing angles, especially via measurements of b-quark production and decay, has been discussed previously.[8] This approach does not appear to be as promising as e^+e^- measurements. Some information may be gained from very high energy ep colliders, as is discussed in the section of these proceedings on this topic.

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