PREDICTED PROPERTIES OF THE $W^{\pm}$ AND Z^0

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Summary

Predicted properties of the $W^{\pm}$ and Z^0 bosons in the standard SU(2) × U(1) model are presented. For the current value of the weak mixing angle, $\sin^2\theta_W(m_W) = 0.215 \pm 0.014$, one finds

$$m_W = 83.0^{+3.0}_{-2.8} \text{ GeV and } m_Z = 93.8^{+2.5}_{-2.4} \text{ GeV}.$$

Implications of deviations from these predictions are discussed. Decay rates and branching ratios for the $W^{\pm}$ and Z^0 are given. Radiative corrections, higher order rare decays and exotic possibilities are described.

1. $W^{\pm}$ and Z^0 Masses

The standard SU(2) × U(1) electroweak model makes rather definite predictions regarding the masses of the $W^{\pm}$ and Z^0 intermediate vector bosons. In the case of the $W^{\pm}$, one has the following lowest order relationship

$$m_W^0 = \left[\frac{\pi\alpha^0}{\sqrt{2}\, G_\mu^0 \sin^2\theta_W^0}\right]^{1/2} \tag{1}$$

where the superscript zero signifies bare (unrenormalized) parameters. For $\alpha = 1/137.036$, $G_\mu = 1.116632 \times 10^{-5}$ GeV^{-2} (the muon decay constant) and $\sin^2\theta_W^{exp} = 0.227$ (the lowest order result from ν_μ-hadron scattering[1]), one finds

$$m_W = 78.3 \text{ GeV} \quad \text{(lowest order)}. \tag{2}$$

However, the $O(\alpha)$ radiative corrections to the above relationships are sizeable[2]. Employing the renormalized weak mixing angle[3-4] $\sin^2\theta_W(m_W)$, defined by $\overline{\text{MS}}$ (modified minimal subtraction) with $\mu = m_W$, gives to $O(\alpha)$[2-5]

$$m_W = \frac{38.5 \text{ GeV}}{\sin\theta_W(m_W)} \tag{3}$$

A two parameter fit to ν_μ and $\bar{\nu}_\mu$ scattering data yields[1,6]

$$\rho^{\text{exp.}} = 1.010 \pm 0.020 \tag{4a}$$

$$\sin^2\theta_W(m_W) = 0.236 \pm 0.030 \tag{4b}$$

where $\rho^0 = m_W^{0^2}/m_Z^{0^2}\cos^2\theta_W^0$. Combining Eqs. (3) and (4), we find $m_W = 79.3^{+5.5}_{-4.7}$ GeV. One can do better by employing the standard model's constraint $\rho^0 = 1$ and appending the $O(\alpha)$ radiative corrections to the analysis of deep-inelastic ν_μ-hadron scattering above. In that way one finds[7]

$$\rho^{(\nu_\mu;h)} \approx 0.99 \quad \text{(theoretical)} \tag{5a}$$

$$\sin^2\theta_W(m_W) = 0.215 \pm 0.014 \tag{5b}$$

which implies the rather precise prediction[7]

$$m_W = 83.0^{+3.0}_{-2.8} \text{ GeV} \tag{6}$$

The difference between Eqs. (2) and (6) is about 6%; a large shift induced by radiative corrections. We see that a precise measurement of m_W will test the standard model at the level of its quantum loop corrections.

The prediction for m_Z follows from the lowest order relation $m_Z^{0^2} = m_W^{0^2}/\cos^2\theta_W^0\,\rho^0$. Employing $\rho^0 = 1$ implies the lowest order prediction via Eq. (1)

$$m_Z = 89.0 \text{ GeV} \quad \text{(lowest order)} \tag{7}$$

However, including radiative corrections one finds[2-5]

$$m_Z = \frac{77.1 \text{ GeV}}{\sin 2\theta_W(m_W)}(1+\Delta)^{-1/2} \tag{8}$$

where Δ represents possible corrections to ρ not included in the standard model's radiative corrections (in the simplest case $\Delta = 0$). Comparing Eqs. (4a) and (5a) we find $\Delta = 0.02 \pm 0.02$ which when correlated with Eq. (4b) leads to $m_Z = 89.9 \pm 4.4$ GeV. If we assume $\Delta = 0$, i.e. nothing unexpected in ρ, then Eq. (5b) implies[7]

$$m_Z = 93.8^{+2.5}_{-2.4} \text{ GeV} \tag{9}$$

It is anticipated that m_Z will be determined to within 0.1 GeV at e^+e^- colliders[8]; such a measurement will provide a value of $\sin^2\theta_W(m_W)$ to within 0.3% via Eq. (8) but only under the assumption $\Delta = 0$. What would be interesting is to have an independent precise determination of $\sin^2\theta_W(m_W)$ which when combined with m_Z would yield Δ. For example, by precisely measuring both m_W (which determines $\sin^2\theta_W(m_W)$ via Eq. (3) and m_Z one can obtain Δ using

$$\Delta = 1 - \frac{m_W^4}{m_Z^2[m_W^2 - (38.64\ \mathrm{GeV})^2]} \tag{10}$$

A determination of m_W to within 0.5 GeV at hadron colliders combined with a 0.1 GeV measurement of m_Z at e^+e^- colliders will determine Δ to within 0.01.

An interesting example of an effect that could contribute to Δ is the possibility $m_t > m_W$ (t = top quark). [Eq. (5a) assumes $m_t \simeq 20$ GeV.] In that case, a one loop calculation gives[9]

$$\Delta = \frac{3\alpha}{16\pi \sin^2\theta_W(m_W)} \frac{m_t^2}{m_W^2} \simeq 0.002 \frac{m_t^2}{m_W^2} \tag{11}$$

Other potential contributions to Δ are additional fermion generations, higher dimensional Higgs representations, dynamical symmetry breaking effects[10] etc.

Clearly, precise measurements of m_W, m_Z, Δ, and $\sin^2\theta_W(m_W)$ by as many methods as possible should be high priority experiments at future collider facilities. Those fundamental parameters test the standard model at the level of its quantum loop corrections and provide tight constraints on the structure of electroweak interactions.

2. $W^\pm$ and Z^0 Decays

In this section we review the anticipated decay properties of the $W^\pm$ and Z^0 in the standard model. Since high luminosity colliders will produce about 10^7 $W^\pm$ and Z^0 bosons per year, one may anticipate precise measurements of branching ratios, total widths, and decay asymmetries[8]. Perhaps rare higher order decays may even be observed. In addition, if the Higgs scalar is light it should be detectable through Z^0 decays. The most exciting possibility would be a distinct deviation from the standard model, signaling new physics.

2.1 $W^\pm$ Decays

Including QCD corrections and lowest order top quark mass effects, the hadronic decay width of the W is predicted to be[11-12]

$$\Gamma(W\to\mathrm{hadrons}) = \frac{3G_\mu m_W^3}{2\sqrt{2}\pi}\left(1 - \frac{m_t^2}{2m_W^2} + \frac{1}{6}\frac{m_t^6}{m_W^6}\right)\left[1 + \frac{\alpha_s(m_W)}{\pi}\right]$$

$$\simeq 2.28\ \mathrm{GeV} \quad (\text{for } m_t \equiv 20\ \mathrm{GeV}) \tag{12}$$

for 3 generations. (Eq. (12) assumes $m_t < m_W$.) The leptonic partial widths are

$$\Gamma(W \to \ell\bar{\nu}_\ell) = \frac{G_\mu m_W^3}{6\sqrt{2}\pi} \simeq 0.25\ \mathrm{GeV}, \quad (\ell = e,\mu,\tau) \tag{13}$$

Together these imply a total width

$$\Gamma(W \to \mathrm{all}) \simeq 3.03\ \mathrm{GeV} \tag{14}$$

and leptonic branching ratios

$$\frac{\Gamma(W \to \ell\bar{\nu}_\ell)}{\Gamma(W \to \mathrm{all})} = 0.083 \tag{15}$$

The electroweak radiative corrections to these results are small, provided one measures inclusive rates i.e. including soft and hard bremsstrahlung. However, severely constraining the final state energy configurations can lead to significant QED effects (see Ref. 11).

2.2 Z^0 Decays

In the case of the Z^0, the partial decay widths are predicted to be (for three generations and $m_t \leq m_Z/2$)[11-12]

$$\Gamma(Z^0 \to \ell\bar{\ell}) = \frac{G_\mu m_Z^3(1 - 4s^2 + 8s^4)}{12\sqrt{2}\pi} \simeq 0.092\ \mathrm{GeV} \tag{16}$$

$$\Gamma(Z^0 \to \nu_\ell\bar{\nu}_\ell) = \frac{G_\mu m_Z^3}{12\sqrt{2}\pi} \simeq 0.181\ \mathrm{GeV}$$

$$\Gamma(Z^0 \to \mathrm{hadrons}) = \frac{G_\mu m_Z^3}{4\sqrt{2}\pi} F(s^2, m_t^2/m_Z^2)\left(1 + \frac{\alpha_s(m_Z)}{\pi}\right) \tag{17}$$

$$\simeq 2.20\ \mathrm{GeV}, \quad (\text{for } m_t = 20\ \mathrm{GeV})$$

where $s^2 \equiv \sin^2\theta_W(m_W) = 0.215$ and[12]

$$F(s^2, m_t^2/m_Z^2) = 5 - \frac{28}{3}s^2 + \frac{80}{9}s^4 + \left(1 - \frac{4m_t^2}{m_Z^2}\right)^{1/2}$$

$$\left[1 - \frac{8}{3}s^2 + \frac{32}{9}s^4 - \frac{m_t^2}{m_Z^2}\left(1 + \frac{16}{3}s^2 - \frac{64}{9}s^4\right)\right] \tag{18}$$

Adding these contributions, one finds

$$\Gamma(Z^0 \to \mathrm{all}) \simeq 3.02\ \mathrm{GeV} \tag{19}$$

$$\frac{\Gamma(Z^0 \to \ell\bar{\ell})}{\Gamma(Z^0 \to \mathrm{all})} \simeq 0.030\ \mathrm{GeV} \tag{20}$$

The t quark mass effect can be important, as illustrated by Eq. (18). If $m_t = 40$ GeV rather than 20 GeV, then the total width is reduced by 0.18 GeV i.e. the equivalent of 1 neutrino species. The statements made above regarding electroweak corrections to $W^\pm$ decay also apply to the Z^0.

2.3 QCD Jets

In the hadronic decays of the $W^\pm$ and Z^0, one expects to observe rather well collimated streams of jets in the final states. Indeed, it is a straightforward exercise to apply all of the QCD jet calculations originally carried out for $e^+e^- \to$ hadrons to such decays. (Only heavy quark mass effects modify the formulas.) For example, using the Sterman-Weinberg[13] criterion for a two jet event as one in which all but at most ε of the total available energy, m_Z, is emitted within a pair of oppositely directed cones of opening half-angle $\delta \ll 1$, one finds to order $\alpha_s(m_Z)$

$$\frac{\Gamma(Z^0 \to 2 \text{ jets})}{\Gamma(Z^0 \to \text{hadrons})} = \tag{21}$$

$$1 - \frac{4}{3}\frac{\alpha_s(m_Z)}{\pi}\left\{(4 \ln 2\varepsilon + 3)\ln\delta + \frac{\pi^2}{3} - \frac{5}{2}\right\}$$

(the same result with $m_Z \to m_W$ holds for $W^\pm$ decays). From this expression we learn that for $\varepsilon = 0.1$ and $\delta = 0.2$ (i.e. $\simeq 12°$), about 70% of all hadronic decays are expected to form two narrow jets with energy $\geq 0.9\ m_Z$ and opening angle $\delta \leq 0.2$. Other jet parameters such as thrust, sphericity etc, which have been analyzed in great detail for e^+e^- annihilation similarly apply to Z^0 and $W^\pm$ decays. In particular, for the decay $Z^0 \to q + \bar{q}$ + gluon which materializes as three distinct hadronic jets, one finds the familiar differential decay rate[11]

$$\frac{d^2\Gamma(Z^0 \to 3 \text{ jets})}{dxdy} = \tag{22}$$

$$\Gamma(Z \to \text{hadrons})\ \frac{2\alpha_s(m_Z)}{3\pi}\ \frac{x^2 + y^2}{(1-x)(1-y)}$$

where $x = 2E_q/m_Z$ and $y = 2E_{\bar{q}}/m_Z$, with E_q and $E_{\bar{q}}$ the energies contained in the quark and antiquark initiated jets, $0 \leq x \leq 1$ and $0 \leq y \leq 1$. In such three jet configurations, one expects the gluon jet to be somewhat broader than those initiated by quarks, because gluons carry more color and should therefore fragment more[14]. Clearly, the Z^0 and $W^\pm$ provide excellent settings for studying perturbative QCD and jet phenomenology.

2.4 Number of Neutrino Species

An anticipated use of a precise measurement of the Z^0's total width, is to determine the number of distinct neutrino species, N_ν. From the formulas in eqs. (16) - (19), we see that one might interpret deviations from $\Gamma(Z^0 \to \text{all}) \simeq 3.02$ GeV as being due to additional decays involving 4th, 5th etc. generation neutrinos. If their coupling to the Z^0 is universal, then one expects a 0.181 GeV increase in the width for each new flavor, i.e.

$$(N_\nu - 3)(0.181 \text{ GeV}) \approx \Gamma(Z^0 \to \text{all}) - 3.02 \text{ GeV} \tag{23}$$

(assuming that additional quarks and leptons are too massive to contribute to the Z^0's width). Big bang cosmology combined with the observed helium abundance in the universe implies an upper bound of 3 or 4 neutrino species[15]; it will be interesting to see whether a precise measurement of $\Gamma(Z^0 \to \text{all})$ supports that bound.

Do Z^0 propagator effects presently tell us anything about N_ν? Neutrinos contribute to the propagator via loop effects. Within the framework of the standard model too large a number of neutrinos would ruin the excellent agreement between theory and experiment observed in the $e^+e^- \to \mu^+\mu^-$ backward-forward asymmetry. A crude analysis gives $N_\nu \lesssim 10^3$; not a very impressive bound.

2.5 Higher Order Rare Decays

A variety of higher order induced decays (in addition to the three jet configuration in Eq. (22) of the $W^\pm$ and Z^0 have been studied. Most important are rare Z^0 decays, since e^+e^- colliders will produce $\simeq 10^7$ Z^0 events /yr. on resonance with little background[16]. Therefore, one will have the opportunity to observe decay modes with branching ratios as small as $\simeq 10^{-6}$.

An interesting possibility is the observation of the Higgs scalar, ϕ, in Z^0 decays. If $m_\phi \ll m_Z$, one finds[17-18]

$$B(Z^0 \to \phi^0 + \mu^+ + \mu^-) \simeq 7 \times 10^{-5} \tag{24}$$

$$B(Z^0 \to \phi^0 + \gamma) \simeq 2 \times 10^{-6} \tag{25}$$

These appear to be observable; unfortunately, the rates in Eq. (24) decreases rather rapidly with increasing m_ϕ.[8]

Gluons may be studied at the Z^0. Calculations find[19]

$$B(Z^0 \to 3 \text{ gluons}) \simeq 10^{-5} \tag{26}$$

$$B(Z^0 \to \gamma + 2 \text{ gluons}) \simeq 2 \times 10^{-6} \tag{27}$$

These rates are detectable. We remark that two body final states such as $Z^0 \to \gamma$ + glueball are highly unlikely.

It would be nice to be able to detect the $W^\pm$ in the decay products of the Z^0. In that way one could study the non-Abelian Z^0WW coupling. Unfortunately, this seems to be unlikely, since one finds[20]

$$B(Z^0 \to W^\pm + \text{anything}) \simeq 2 \times 10^{-7} \tag{28}$$

$$B(Z^0 \to W^+ + e^- + \bar{\nu}) \simeq 10^{-8} \tag{29}$$

Radiative two body decays can be reliably computed. One finds[21]

$$B(Z^0 \to \pi^0 + \gamma) \simeq 3 \times 10^{-11} \tag{30}$$

$$B(Z^0 \to \eta + \gamma) \simeq 3 \times 10^{-10} \tag{31}$$

$$B(Z^0 \to Q\bar{Q} + \gamma) \simeq 10^{-7} \tag{32}$$

where $Q\bar{Q}$ is a heavy $\simeq 50$ GeV pseudoscalar quarkonia state

Finally, one might ask: Will flavor changing decays of the Z^0 be observable? In particular, if $m_t > m_Z/2$, the decay $Z^0 \to t\bar{t}$ is kinematically forbidden; however $Z^0 \to t$ + light quark may go. Unfortunately, the branching ratio[22]

$$B(Z^0 \to t + X) \lesssim 10^{-10}, \quad (m_t > m_Z/2) \qquad (33)$$

is extremely suppressed.

It is important to measure as many decays of the Z^0 and $W^{\pm}$ as possible. In so doing, one will test the standard model; perhaps even at the level of its quantum corrections. Deviations from the expected may signal exotic new physics such as technicolor, supersymmetry etc.

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