TOPICS IN NON-PERTURBATIVE QCD

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1. INTRODUCTION

We believe that QCD is the correct theory for the hadronic world not only when it is probed at a high energy accelerator (by some external current), but when it is treated much less violently; when masses, widths and other spectroscopical data are measured. These are non-perturbative question in a $d = 4$, relativistic quantum field theory.

The past in particle physics is not very glorious in this respect. In classical statistical physics, however, the analogous problem - namely extracting universal properties of a system approaching a continuous phase transition point - has a respectable history. It is the most natural thing to try to borrow the beautiful theoretical ideas and powerful numerical methods from this field.

Using the path integral formulation, a Wick rotation $x_0 \to -ix_4$ transforms the quantum field theoretical problem into a problem in the statistical mechanics of fields in Euclidean space. Of course, it is not an easy problem either, for instance, if it is plagued by the same ultra-violet divergences as the original one. It is natural to use a lattice regularization in this respect.

QCD regularized on a lattice is the only formulation which at present is able to give quantitative predictions. A large part of my talk will be related to the developments in lattice QCD, including the problems and first results on fermions.

In recent years, nice qualitative ideas have been advanced for the driving mechanism concerning confinement. Unfortunately, none of them has been transformed into a systematic, quantitative method. Some of these ideas slowly fade away, not because they are disproved, but because they cannot be developed any further. Certain subjectiveness is unavoidable in discussing the matter.

There are interesting developments related to the realization of chiral symmetry in QCD. Largely as a by-product of the search for models where the chiral symmetry is unbroken, it is now strongly believed that if QCD confines then the final chiral symmetry pattern of the theory is that observed in nature : $SU(n_f)\times SU(n_f)\times U(1)$ is broken spontaneously to $SU(n_f)\times U(1)$ producing n_f^2-1 Goldstone bosons along this way. This topic is discussed in detail by Peskin[1] at this Conference, therefore I will mention only those problems which are closely related to the lattice formulation.

2. CONFINEMENT (QUALITATIVE)

2.1 Bag Model

Apart from its phenomenological successes, the bag model[2] provides for a qualitative picture of confined quarks.

The real vacuum is assumed to be highly non-perturbative, which cannot support the propagation of coloured objects. To create a bubble with the phase inside, where quarks and gluons propagate in the ordinary manner (~Fock vacuum), requires energy which is proportional to the volume (and surface) of the bubble.

Consider a heavy quark-antiquark system in a colour singlet state. In this case, the description offered by the bag model is very close to the picture one usually has about heavy quarks in QCD. The vacuum pressure is balanced locally by the outward pressure of the confined Coulomb electric field. The condition of local equilibrium governs the motion and shape of the bubble which in turn determines a potential for the quarks[3].

Of course, the bag model cannot explain quark confinement (it is assumed), and, unfortunately, it fails on the other important issue, namely on the problem of chiral symmetry breaking.

2.2 Hadron Propagation in Configuration Space

Though we usually draw Feynman diagrams in momentum space, it might be useful to stay in configuration space, where we have a better intuition. Consider the propagation of a colour singlet meson from the point x to y. The process is described by the propagator

$$\langle \bar{q}(x)\, \Gamma q(x) \quad \bar{q}(y)\, \Gamma q(y) \rangle \tag{1}$$

where Γ is a matrix in Dirac and flavour space.

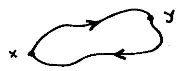
The action is built up from the gluon and quark fields in the usual way :

$$S = \int d^4x \left[\frac{1}{2g^2}\,\mathrm{Tr}\, F^2_{\mu\nu} - \bar{q}(x)\,\gamma_\mu(\partial_\mu + iA_\mu(x))\,q(x) - \bar{q}(x)\mathcal{M}\,q(x) \right] \tag{2}$$

Let me be formal, forget about regularization for a while. The action is quadratic in the quark fields, the functional integral over these fields can be done. By using an old trick due to Schwinger[4], the resulting determinant $\det(\partial_\mu + iA + \mathcal{M})$ (and derivatives thereof) can be related to the propagation of a spinning, coloured quantum mechanical particle in the external field A_μ. Neither the details of the derivation nor the exact form of the final result are interesting here. The meson propagator has the following suggestive general form[5] :

$$\langle \bar{q}\Gamma q(x)\; \bar{q}\Gamma q(y)\rangle \sim \int (\text{measure}) \cdot k(C,\Gamma)\, \langle \mathrm{Tr}\, P\, e^{i \oint_{C_{xy}} A_\mu dx_\mu} \rangle_{S_{eff}} \tag{3}$$

over all closed paths C_{xy}
connecting the points x and y



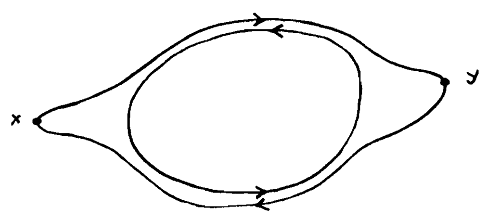
where $k(C,\Gamma)$ is a number obtained by performing the traces in Dirac and flavour space. The main weighting factor for a path C is the expectation value of the Wilson loop[6] over C. The expectation value is calculated with the effective action

$$S_{eff} = \int d^4x\, \frac{1}{2g^2}\,\mathrm{Tr}\, F^2_{\mu\nu} + \int (\text{measure})'\; k(C)\, (\mathrm{Tr}\, P\, e^{i \oint A_\mu dx_\mu}) \tag{4}$$

over all closed paths C
in four-dimensional Euclidean space

The interpretation is clear. The quark and antiquark of the meson ("valence quarks") propagate from x to y in an environment created by the gluons and virtual quark loops ("sea quarks").

Observe the basic rôle of the Wilson loop. Confinement dictates that large valence loops must be strongly suppressed in Eq. (3), except when local singlets are formed with the help of the sea quarks :



It is generally believed that the essence of confinement can be understood in a pure gauge theory with external sources. In this case the Wilson loop expectation value is an order parameter concerning confinement. It is expected to follow an area law

$$\langle W(C)\rangle \sim e^{-\sigma \cdot \mathrm{Area}(C)} \tag{5}$$

for a large planar loop C.

As I mentioned, in discussing the different qualitative ideas on confinement, one has to make a subjective choice. There is a long list of works in the literature on this subject, especially on the ideas of the Princeton people and the Copenhagen group[7],[8]. Here I want to say a few words about the mechanism suggested a few years ago by 't Hooft and Mandelstam[9].

2.3 Confinement as a Dual Meissner Effect

One would like to have a better understanding of the confining environment created by the gluons. It was suggested by 't Hooft and Mandelstam[9] that the physics behind is a dual (electric) Meissner effect.

A) The magnetic Meissner effect is well known in the physics of superconductors. Due to supercurrents an external magnetic field is repelled from the bulk of the superconducting material. If the magnetic field is strong enough it can force its way through the superconductor, but it is squeezed into narrow flux tubes with a quantized magnetic flux.

The situation is well described by the Abelian Higgs model

$$\mathcal{L} = -\tfrac{1}{4} F_{\mu\nu}F_{\mu\nu} + |(\partial_\mu - ig\, A_\mu)\phi|^2 - V(|\phi|^2) \qquad (6)$$

where V describes the self-interaction of the charged field ϕ. This Lagrangian has a $U(1)$ symmetry, which is spontaneously broken if the minimum of the potential V is away from zero. In this case $\langle 0|\phi|0\rangle = F \neq 0$, the photon acquires a mass $\frac{1}{2}g^2F^2$. Due to the photon mass, the magnetic field dies away exponentially in the superconductor, which is the Meissner effect.

The formation of narrow, quantized flux tubes is also described by this model : these are the famous Nielsen-Olesen vortices[10]. The vortex is a topologically stable solution of the classical equations of motion. It is a narrow tube of a quantized magnetic flux.

The flux tube can be closed by magnetic monopoles of magnetic charge $q = \pm(2\pi/g)n$ $(n = \pm 1, \pm 2, \ldots)$. Clearly, these magnetic monopoles would be confined by a linear potential.

Now, electromagnetism with electric and magnetic charges is symmetric under the interchange of electric and magnetic notions. It is quite natural to assume that the dually opposite can also happen : the condensation of magnetic monopoles resulting in electric confinement ; the dual Meissner effect.

B) Consider now a non-Abelian theory, but do not discard the Higgs yet. To be explicit, consider the Georgi-Glashow model[11]. It is an $SU(2)$ gauge model which is broken down to $U(1)$ by an isovector Higgs field. This model exhibits a striking classical solution, the 't Hooft-Polyakov monopole[12].

Concerning the long-distance properties, this theory is a compact $U(1)$ theory, which on his part is equal to usual electromagnetism (non-compact $U(1)$) plus monopoles. Now we can again introduce a charged Higgs field. Then we arrive at the Abelian Higgs model discussed before, with the exception that now the monopoles need not be introduced by hand.

If the Higgs field forms a condensate, these monopoles are confined by Nielsen-Olesen vortices. By changing the couplings in the Lagrangian, the monopole mass can be decreased and the confining string can be made weaker and weaker. In the ground state of the system monopole pairs will be easily created, and they can propagate over long distances due to the weakness of the confining force. It is natural to believe that by tuning the couplings they might form a condensate, giving charge confinement.

C) At the place of the monopole the symmetry is restored. By densely populating the vacuum the symmetry is restored everywhere. The Higgs fields are not important anymore and one feels we should be able to get rid of if.

Take a pure gauge theory based on the compact group $SU(N)$. How to exhibit the presence of monopole degrees of freedom explicitly ?

It is a difficult problem and it is not completely solved yet. What we dream of is a clear kinematical separation of the relevant degrees of freedom, like for instance in the XY model in two dimensions.

The XY model is a spin model. The spins are coupled ferromagnetically and can rotate in a plane. The model is described originally by a set of angle variables $\{\phi_i\}$, $\phi_i \epsilon (0,2\pi)$. By standard transformations[13] ϕ_i can be replaced by a non-compact variable $\xi_i \epsilon(-\infty,\infty)$ (describing a simple massless scalar field, called spin waves in this case) plus additional degrees of freedom representing the Kosterlitz-Thouless vortices of the model[14]. These topological excitations are analogous to the above monopoles.

The procedure is independent of the exact form of the spin-spin interaction, and correctly enumerates the degrees of freedom independently, whether the vortices are really present in bound states or form a condensate.

In a recent work, 't Hooft suggested a procedure to achieve the same goal in $SU(N)$ gauge theories[15]. He argues that by partially fixing the gauge the $SU(2)$ gauge theory goes over a theory with a $U(1)$ gauge freedom $(SU(N) \to U(1)^{N-1})$. In this gauge fixing process, the compact $U(1)$ is automatically dismantled into the $U(1)$ gauge field, charges and monopoles. An explicit solution of this gauge fixing procedure would give the form of the interaction between these degrees of freedom. The technical difficulties seem to be formidable.

Though this picture is nice and intuitive, it is far from ready for a quantitative calculation. On the other hand the lattice formulation which I am going to discuss now is able to give quantitative results. One has to admit, however, that the powerful numerical methods - especially the Monte Carlo simulation - did not help to obtain a better physical understanding, a deeper insight into the theory.

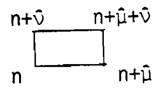
3. PURE GAUGE THEORY ON THE LATTICE; FORMULATION

3.1 Lattice Regularization

On the lattice the gauge field is described by the link variable $U_{n\mu}$ associated with the link with end-points n and $n+\hat{\mu}$. The variable $U_{n\mu}$ is an element of the gauge group. The action is built up from gauge-invariant combinations: the trace of the product of U matrices along closed curves. In the action originally suggested by Wilson[6),16)], these closed loops run around the elementary plaquettes of the lattice:

$$S_W = \frac{1}{g^2} \sum_{\text{plaquettes}} (\mathrm{Tr}\, U_p + \mathrm{Tr}\, U_p^+)$$

$$U_p = U_{n\mu}\, U_{n+\hat{\mu},\nu}\, U^+_{n+\hat{\nu},\mu}\, U^+_{n\nu}\ ; \qquad p: \tag{7}$$



In the formal $a \to 0$ limit, the usual continuum action is recovered by expanding $U_{n\mu} = e^{igA_{n\mu}}$ in terms of the vector potentials. This requirement - namely that the lattice classical theory should be identical to the usual one - is not very restrictive. Different types of loops and/or functional forms can be chosen. In terms of the vector potentials, these actions differ by terms proportional to the lattice distance, and are expected to give identical continuum theories (in different renormalization schemes).

3.2 Continuum Limit

Consider the action in Eq. (7). The only parameter it contains is the dimensionless coupling g and, of course, the lattice distance a implicitly. The lattice gives a cut-off π/a in momentum space. The dimension of masses and lengths is carried by a, the coefficients depend on g:

$$m = f(g) \cdot \frac{1}{a}\ , \qquad \xi = h(g) \cdot a\ . \tag{8}$$

In the continuum limit the cut-off must be much larger than the physical masses. Also the correlation length is much larger than the lattice distance. In the language of statistical physics, the system should approach a continuous phase transition point. This has to be arranged by tuning the coupling g. We want to find an asymptotically free theory in this limit. In an asymptotically free theory the bare coupling goes to zero as the cut-off is sent to infinity. On the lattice, g is the bare coupling. Therefore, the continuum theory should be recovered by approaching the $g=0$ point.

3.3 Dimensional Transmutation, Λ^{latt} Parameter

In the continuum limit, physical quantities should become independent of the cut-off. For instance:

$$\frac{d}{da} m = 0, \quad (a \to 0) \tag{9}$$

which is just the requirement of renormalizability. By using standard arguments, Eq. (9) implies that every physical dimensional quantity can be expressed in terms of a single, renormalization group invariant, mass parameter Λ^{latt}:

$$m = C_m\, \Lambda^{\text{latt}}, \quad \xi = C_\xi\, \Lambda^{\text{latt}}, \ \ldots \tag{10}$$

Λ^{latt} is defined in complete analogy to the Λ parameters of the continuum formulation[17)]. The non-perturbative content of the theory is carried by the constants C_m, C_ξ,...

3.4 Strong Coupling Limit

Gauge theories can be solved exactly in the other extreme limit $g \to \infty$. Quark sources are confined at $g = \infty$. However, the lattice is coarse grained when g is large; the model has not much to do with a continuum field theory. The strong coupling limit is important only as a starting point for certain non-perturbative methods.

3.5 Non-Perturbative Methods on the Lattice

I shall discuss the results obtained by strong coupling expansions and Monte Carlo simulations. Other methods such as real space renormalization techniques, variational calculations, or generalized mean field methods have not yet become quantitative.

The strong coupling expansion is a systematic expansion in $1/g^2$ ($1/g^4$ in the Hamiltonian formulation) starting from the exact, confining solution at $g=\infty$ [6),16),18)]. The resulting power series is extrapolated towards the $g=0$ continuum point. The success of the procedure largely depends on whether the region $g\epsilon(0,\infty)$ (more generally, nearby regions in the complex g plane) is free of singularities. As was discussed before, this property is influenced by the form of the action.

The Monte Carlo method is a direct way of numerically evaluating the functional integrals on a finite lattice[19)]. The size of the lattice and the numerical precision are restricted by the memory size and the speed of the computers.

4. PURE GAUGE THEORY ON THE LATTICE; RESULTS

4.1 String Tension

The string tension σ is the strength of the linear potential between two heavy quark sources. It is an order parameter concerning confinement. Its experimental value is also known from the slope of Regge trajectories and from heavy quark spectroscopy: $\sigma_{exp} \sim (400\text{ MeV})^2$.

4.1.1 Monte Carlo results

Large Wilson loop expectation values are expected to behave as

$$\langle WL\rangle \sim e^{-(A+B\cdot\text{perimeter}+\sigma\cdot\text{area})}, \tag{11}$$

where σ can be extracted by taking the appropriate ratios of different loop expectation values. Large loop means: large compared to the actual correlation length.

The first results are due to Creutz[20)]. Since then several other groups have repeated the calculation[21)].

For the qualitative question concerning the persistence of confinement in the continuum limit, one might say : there is no sign of a deconfining phase transition in SU(2) and SU(3).

The connection between the tension and Λ^{latt} is predicted to be [a)]:

$$\Lambda_W^{latt} = (0.013 \pm 0.002)\sqrt{\sigma}, \qquad SU(2),$$

$$\Lambda_W^{latt} = (0.005 \pm 0.0015)\sqrt{\sigma}, \qquad SU(3).$$

At first sight these numbers are strange. They imply that Λ_W^{latt} is only a few MeV. However, as is well known, Λ is scheme-dependent[17),22)]. There is a large, calculable connecting factor between Λ_W^{latt} and the Λ parameters of the usual continuum schemes[23)]:

$$\Lambda_{\alpha=1}^{mom} = 57.5\,\Lambda_W^{latt}, \qquad SU(2),$$

$$= 83.5\,\Lambda_W^{latt}, \qquad SU(3). \tag{13}$$

The controversy concerning these numbers is over; they have been confirmed[24)-26)]. Using the connecting factor for SU(3), the Monte Carlo result would imply $\Lambda_{\alpha=1}^{mom} = (180\pm60)$ MeV, which is quite reasonable. However, one must remember that both $\sqrt{\sigma})_{exp}$ and $\Lambda_{\alpha=1}^{mom})_{exp}$ contain fermionic corrections, whilst the above calculation is for pure gauge theory.

An important question is how close are these numbers to the real, continuum content of the model considered? A possible internal check is to compare results obtained by using different lattice actions. The corresponding scales (Λ-s) can be exactly connected by a perturbative calculation (in the $g\to 0$ limit) and we can check whether the Monte Carlo result follows the prediction. The results[27)] in Table 1 referring to two alternative actions (Manton's action[28)] and the heat kernel action[29)]) versus Wilson's action signal some discrepancy beyond the statistical errors. Clearly, it would be necessary to go deeper into the continuum regime to investigate this problem.

Table 1

	Theory 26),27)	Monte Carlo 27)
$\Lambda^{latt}_{Manton}/\Lambda^{latt}_{W}$	3.07	5.13 ± 1
$\Lambda^{latt}_{Manton}/\Lambda^{latt}_{hk}$	1	1.25 ± 0.05

4.1.2 Strong coupling results

The strong coupling results for the tension and the β function obtained by Kogut, Pearson and Shigemitsu in SU(3) Hamiltonian formulation[30] were very important in raising interest and hopes in lattice gauge theories. Since then an additional order has been calculated[31], and there is a series available for SU(2) and SU(3) in the Euclidean formulation also[32]. However, the programme is faced with unpleasant difficulties at this moment. The difficulties are related to unexpected singularities in the complex coupling constant plane. I shall return to this question; let me now give only the results. In the Euclidean case, consistency is claimed with the Monte Carlo results. The method is to take the series as it is and to make a direct comparison with the Monte Carlo points. However, in the region where the string tension should be extracted from the theory, the series is clearly not convergent.

In the Hamiltonian formulation, the procedure is similar. The prediction is[31]:

$$\sqrt{\sigma} = (69 \pm 15)\ \Lambda^{latt}_{Ham}\ , \qquad SU(3)\ . \tag{14}$$

In order to compare this with the Monte Carlo result, Eq. (12), the connection between Λ^{latt}_{W} and Λ^{latt}_{Ham} is needed. Using the results obtained by Gross[33]:

$$\Lambda^{latt}_{Ham}\ /\ \Lambda^{latt}_{W} = 3.07, \tag{15}$$

consistency is obtained. However, a recent calculation gave a completely different value[25]:

$$\Lambda^{latt}_{Ham}\ /\ \Lambda^{latt}_{W} = 0.91 \tag{16}$$

which would spoil this picture. Clarification is needed.

4.2 Glueball Mass

Correlations (for instance, plaquette-plaquette correlations) die away exponentially. The rate of this decay defines the correlation length ξ, the inverse of which can be considered as the mass of the lightest glueball excitation.

4.2.1 Monte Carlo results

It is a difficult analysis, because by increasing the distance between the plaquettes the signal rapidly disappears in the noise. No quantum number separation is made; some average value of the lowest excitations is measured. One should consider the following numbers with reservation:

$$\begin{aligned} m_g &= (3.7 \pm 1.2)\ \sqrt{\sigma}, \quad SU(2)\ ^{34)} \\ m_g &= (3 \pm 1)\ \sqrt{\sigma}, \quad SU(2), \text{ Bhanot and Rebbi}^{21)}. \end{aligned} \tag{17}$$

More recent analyses give a systematically lower value for m_g [35].

4.2.2 Strong coupling results

The series derived in Hamiltonian formulation[36] is short, and only mass ratios were formed. The Euclidean series is rather erratic, giving[37]:

$$\begin{aligned} m_g &= (1.8 \pm 0.8)\ \sqrt{\sigma}, \quad SU(2), \\ m_g &\approx 3\sqrt{\sigma}, \quad SU(3) \end{aligned} \tag{18}$$

4.3 Two Additional Quantities

There are two additional quantities investigated in pure gauge theories which, however, raised some controversy, and the question is not yet settled.

The first is the evaluation of

$$\langle \frac{\alpha}{\pi} \sum_{a,\mu,\nu} F^a_{\mu\nu} F^a_{\mu\nu} \rangle \; ^{38)}.$$

It is an important parameter in the phenomenological analysis of Shifman, Vainstein and Zakharov[39], and via the trace anomaly of the energy momentum tensor it might also be related to the volume tension of the bag model.

The second quantity is the topological susceptibility[40]:

$$A = \langle \int d^4x \; q(x) \; q(0) \rangle,$$

where $q(x)$ is the topological charge density. A non-zero A would be relevant in the resolution of the $U(1)$ problem[41].

In both cases the main problem is how these quantities are defined. Concerning the second quantity, there is an extra problem: how to define the topological charge density on the lattice. In recent interesting papers, Berg and Lüscher[42] and Martinelli, Petronzio and Virasoro[43] discussed this question in the two-dimensional σ model. They suggested a definition which really has a topological meaning. Therefore A is free from perturbative ultra-violet divergences. Unfortunately, it is plagued by non-perturbative ultra-violet divergences and this fact spoils our hopes that a fast progress can be made in this field.

4.4 QCD at High Temperature; Thermal Quark Liberation

There has been a long-standing conjecture that a phase transition must take place between the high-temperature and low-temperature phases of a non-Abelian gauge field theory[44]. In the high-temperature phase the quarks are free.

It has been shown[45] that lattice gauge theories undergo this thermal quark liberation in the strong coupling limit, and there are Monte Carlo calculations indicating that this phenomenon persists also in the continuum limit[46]. The critical temperature is predicted to be

$$T_c = (0.35 \pm 0.05) \sqrt{\sigma}, \quad SU(2), \tag{19}$$

which is quite reasonable.

In the high-temperature region the force between quark sources is a screened Coulomb law, and a recent Monte Carlo study[47] indicates that the magnetic flux is also screened.

At very high temperatures the energy density is expected to follow the Stefan-Boltzmann law: $\varepsilon \sim T^4$. This offers a unique possibility to check how the continuum limit is approached in the Monte Carlo simulation. The reason is that contrary to the previous examples, not only the renormalization group behaviour is known, but the absolute normalization also:

$$\varepsilon = \frac{N^2 - 1}{15} \pi^2 T^4 , \qquad SU(N) . \tag{20}$$

The Monte Carlo analysis for $SU(2)$ is in surprising quantitative agreement with this prediction[48].

4.5 Annoying Phase Transitions

The lattice is a regularization. A cut-off is introduced temporarily, which is removed at the end. When the cut-off is large ("a" is small), the theory should behave like a decent continuous theory: continuous QCD with certain expected properties. If, however, we make excursions into region where the cut-off is small (the lattice is coarse grained), there the model has not much to do with the final theory we are looking for, and we might meet surprises.

Many of the new results of the last year are related to the phase transitions in these intermediate regions; phase transition as the number of colours goes to infinity ($N \to \infty$, [49)-52)]), as the flux tube connecting two heavy quarks becomes rough (roughening, [53)-59)]), and different types of bulk phase transitions[60)-63),b)]. Clearly, gauge theories have a much more complicated phase structure than we naively thought. The presence of these singularities is related to the difficultie of the strong coupling expansions.

5. LATTICE QCD (INCLUDING FERMIONS)

The fate of this whole programme largely depends on our ability to int... fermions and to calculate something in this extended theory. There are theoretical and technical problems. The theoretical problems are centered around the question of chiral symmetry breaking, while the technical problems are related to the fact that it is difficult to find an effective Monte Carlo procedure for this extended theory.

Before discussing these questions let me make a general remark concerning the introduction of matter fields, indepently of whether they are fermions or bosons. If the matter field is in the fundamental representation of the gauge group then long flux tubes are broken by pair creation, large Wilson loops will follow a perimeter law. The Wilson loop is not an order parameter anymore. The parameter region where the model is "Higgs-like" is analytically connected to the region where it is "confinement-like"[64]. There is no phase transition between them. Recent Monte Carlo studies confirmed this expectation in different models without any surprises[65].

5.1 Theoretical Problems of Introducing Fermions on a Lattice

5.1.1 The naive way

The naive way of putting fermions on the lattice follows the usual recipe. One starts from the continuum action of a free Dirac particle:

$$S = \int d^4x \left[-\tfrac{1}{2}\, \bar{q}(x)\, \gamma_\mu \overleftrightarrow{\partial}_\mu\, q(x) - m\bar{q}(x)\, q(x) \right], \tag{21}$$

and by replacing derivatives by differences one arrives to

$$S = a^4 \sum_n \left[\sum_\mu \frac{1}{2a} (\bar{q}(x)\, \gamma_\mu\, q(n+\hat{\mu}) - \bar{q}(n+\hat{\mu})\, \gamma_\mu\, q(n)) - m\, \bar{q}(n)\, q(n) \right] \tag{22}$$

By inserting the gauge field $U_{n\mu}$ in the usual way (to preserve gauge invariancy) one obtains the lattice version of QED

$$S = a^4 \sum_n \left[\sum_\mu \frac{1}{2a} (\bar{q}(n)\, \gamma_\mu\, U_{n\mu}\, q(n+\hat{\mu}) - \bar{q}(n+\hat{\mu})\, \gamma_\mu\, U^+_{n\mu}\, q(n)) - m\, \bar{q}(n)\, q(n) \right] + S_{\text{gauge field}}. \tag{23}$$

For $m = 0$ the model is invariant under the transformations:

$$q \to e^{i\alpha} q \quad \text{and} \quad q \to e^{i\alpha\gamma_5} q. \tag{24}$$

The symmetry group is $U(1)\otimes U(1)$. This symmetry is exact for any value of the lattice constant, therefore it is there even in the $a\to 0$ continuum limit.

But then we realize that something went wrong here. Adler's theorem claims[66] (under very general conditions) that there is no regularization which would respect both of these symmetries. In the quantum theory the $U(1)$ chiral symmetry is necessarily explicitly broken. The axial vector current is not conserved, its divergence receives a non-zero contribution via the Adler-Bell-Jackiw anomaly[67].

The above construction seemingly contradicts this theorem. The resolution of this paradox is that although we wanted to describe the interaction of a single fermion with the electromagnetic field, the action in Eq. (23) contains 16 fermion species. S describes 16 massless fermions, and, as it was shown by Karsten and Smit[68], their contribution to the axial anomaly alternates in sign and adds up to zero.

Really, Eq. (23) gives the fermion propagator:

$$D(p) \sim \frac{1}{\sum_\mu \gamma_\mu \sin p_\mu}, \quad p_\mu \in (-\pi,\pi), \tag{25}$$

and as $\sin p = 0$ at $p = 0$ and $p = \pi$, $D(p)$ contains $2^4 = 16$ poles altogether. This is quite general: $U(1)$ axial symmetry implies species doubling[69].

Of course, we do not want to work with 16 identical fermions from the beginning.

5.1.2 What we want

Consider QCD with n_f light (massless) quarks. The classical action has a $U(n_f)\times U(n_f)$ symmetry. In quantum theory the flavour singlet axial $U(1)$ is explicitly broken due to the triangle anomaly. Therefore the symmetry in the quantum theory is:

$$SU(n_f)_{\text{vector}} \times SU(n_f)_{\text{axial}} \times U(1)_{\text{baryon number}} \tag{26}$$

Finally, we want the chiral $SU(n_f)$ to be realized in a spontaneously broken way. The axial symmetry should be broken by the ground state, producing massless Goldstone bosons.

5.1.3 What we have; suggested solutions

As we discussed before on an $U(1)$ example, chiral symmetry implies species doubling on the lattice. If we want to describe a single fermion, the $U(1)$ axial symmetry must be explicitly broken. We accept this fact, it cannot be otherwise due to the Adler theorem.

Similarly, for the general case, we accept that the flavour singlet $U(1)$ axial symmetry is explicitly broken. This is independent of the lattice. However, we would like to keep the $SU(n_f)$ axial symmetry. That is the point where the solutions suggested until now are not completely satisfactory.

Wilson's method[6),70),68)]

In order to avoid species doubling, Wilson suggested to add a new term to the action with the following properties:

- it gives large (~ cut-off) masses to the 15 unwanted fermions,
- it goes to zero in the formal $a\to 0$ continuum limit, therefore, hopefully, it will not affect the behaviour of the remaining single fermion at the end.

The action has the following form: (1 flavour, $a = 1$ is taken)

$$\begin{aligned} S = \sum_n \{ -\bar{q}(n)\, q(n) &+ K \sum_\mu \bar{q}(n)\, (1-\gamma_\mu)\, U_{n\mu}\, q(n+\hat{\mu}) \\ &+ K \sum_\mu \bar{q}(n+\hat{\mu})\, (1+\gamma_\mu)\, U^+_{n\mu}\, q(n) \; \} \\ &+ \frac{1}{g^2} \sum_{\text{plaquettes}} (\mathrm{Tr}\, U_p + \mathrm{cc}). \end{aligned} \tag{27}$$

Here $U_{n\mu} \in SU(N)$, K is called the hopping parameter, $K = \frac{1}{8}$ in the classical continuum limit. It is easy to write down the action describing n_f flavours. Unfortunately, S is not chiral invariant. It is hoped that in the continuum limit the axial symmetry will be recovered (and additionally it will be realized in a spontaneously broken way[c)]).

Susskind's method[73),74)]

It starts with a one-component fermion field ϕ, and the extra species are used to build up the four components of q. The degeneracy is decreased by a factor of 4.

It also has the advantage that some remnants of the chiral symmetry are preserved. The resulting action is invariant under certain discrete chiral transformations. It prevents the occurrence of mass counter terms which is the main advantage of this method.

The solution suggested by the SLAC group[75)] results in a non-local action and has several problems[76)].

5.2 Technical Problems of Introducing Fermions in a Monte Carlo Simulation

In a path integral formulation the fermion fields are represented by anticommuting c numbers, by Grassmann variables. There is no sensible way to represent them on a computer.

On the other hand the action is only quadratic in the fermion fields. In a concise notation it has the form

$$S = \Sigma\, \bar{q}_i\, \Delta_{ij}\, (U)\, q_j + S_{\text{gauge}} \tag{28}$$

As we discussed in Section 2.2, one can integrate over the fermion fields in the vacuum functional or in any expectation value of fermion fields. One is left with a problem in pure gauge theory but with a new effective action

$$S_{\mathbf{eff}} = S_{\text{gauge}}(U) - \mathrm{Tr}\,\ell n\, \Delta(U) \tag{29}$$

One might try to do Monte Carlo simulations with this new action. However, in the updating process, when at a given link $U \to U + \delta U$, the difference $S_{eff}(U+\delta U) - S_{eff}(U)$ should be calculated, and it is very time consuming to calculate such a non-local expression as $\mathrm{Tr}\ell n\Delta(U)$.

To avoid this difficulty, Fucito, Marinari, Parisi and Rebbi[77] suggested to calculate $\mathrm{Tr}\ell n\Delta(U)$ by Monte Carlo, by introducing an auxiliary complex scalar field ϕ.

Consider a small change at a given link in the updating process: $U \to U + \delta U$ $(\alpha \to \alpha + \delta\alpha)$

$$S_{eff}(U + \delta U) - S_{eff}(U) = \Delta(S_{gauge}) - \sum_{i,j} \Delta^{-1}_{ij}(U) \frac{\delta}{\delta\alpha} \Delta(U)_{ij} \cdot \delta\alpha. \tag{30}$$

On the other hand:

$$\Delta^{-1}_{ij}(U) = \langle\bar{\phi}_i\phi_j\rangle = \frac{\int D\phi D\bar{\phi}\ \bar{\phi}_i\phi_j\ e^{-\Sigma\bar{\phi}\Delta(U)\phi}}{\int D\phi D\bar{\phi}\ e^{-\Sigma\bar{\phi}\Delta(U)\phi}}, \tag{31}$$

where ϕ is a bosonic field. The procedures followed by Scalapino and Sugar[78], and Weingarten and Petcher[79] are closely related to this method. Seemingly, however, even this method is rather time consuming. There are interesting results in one and two dimensions[77),78),80)], but it is rather far from $d = 4$ QCD. Weingarten and Petcher[79] worked on 2^4 lattice in SU(2) with two flavours. Measuring the one-plaquette and the thermal-loop expectation values with an accuracy of ~10% required of about 50 hours of equivalent CDC computer time.

To improve upon this situation it has been suggested[81),82)] to combine the Monte Carlo simulation with a hopping parameter expansion[70),83)] in Wilson's method. The starting point is Eqs. (3) and (4) generalized on the lattice. The main physical idea is that loops, which are larger than the characteristic length of the problem, are irrelevant. A truncation is made in the length of the quark loops (the expansion in the length of the loops is made analytically) and then the resulting local action is analyzed by Monte Carlo. Although the results of Ref. 79) could be reproduced in a few minutes of computer time with this method[81], it remains to be shown that the method works for more realistic problems.

During this conference, I heard several rumours that certain groups are just on the threshold of obtaining the hadron spectrum. Presumably these rumours are largely unfounded even if we are very generous on what we mean by "obtaining the hadron spectrum". But they reflect something I strongly believe: there will be exciting progress in this field in the near future.

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Footnotes

a) The scale parameter corresponding to the action in Eq. (7) is denoted by Λ_W^{latt}. The quoted results are taken from Ref. 20).
b) For a summary, see the last reference in 16).
c) General Theorems in the continuum formulation give definite hope for that[71]. There are certain strong coupling results on the lattice also pointing in this direction[72].

Discussion

K.H. Becks, Gesamthochschule Wuppertal: Theorists start to give statistical errors. Can you give also a systematical error on Λ^{MOM}?

P. Hasenfratz: Actually yes, because the systematical error was reflected in the table which I showed. Taking different kinds of formulations you can calculate theoretically the expected result and you can compare it with the Monte Carlo result. The Monte Carlo result contains a statistical error and beyond the statistical error you observe a discrepancy and that is characteristic for the systematical error which is due to the fact that you are not in the continuum limit in the Monte Carlo calculation. It is not included, of course, because there is no way. But I think that by doing a more careful job on the Stefan-Boltzmann law one can find a way to get the systematical errors also.

S.D. Drell, SLAC: I believe there is at present no fully satisfactory scheme for putting fermions on a lattice. The approach we have used in the SLAC group is to retain chiral symmetry on the lattice and to retain correct counting of fermion states by defining the gradient to couple sites on the lattice beyond nearest neighbour (i.e. non-local on the lattice and avoiding no-go theorems). J. Rabin has confirmed the satisfactory character of this scheme (to be published). However, fermions on a lattice are still not simple.