GRAND UNIFIED THEORIES

Paul Langacker

Department of Physics, University of Pennsylvania

Philadelphia, Pennsylvania, 19104-3859, U.S.A.

I. Introduction

One of the most exciting advances in particle physics in recent years has been the development of grand unified theories[1] of the strong, weak, and electromagnetic interactions. In this talk I discuss the present status of these theories and of their observational and experimental[2] implications.

In section II, I briefly review the standard $SU_3^c \times SU_2 \times U_1$ model of the strong and electroweak interactions. Although phenomenologically successful, the standard model leaves many questions unanswered. Some of these questions are addressed by grand unified theories, which are defined and discussed in Section III. The Georgi-Glashow SU_5 model[2] is described, as are theories based on larger groups such as SO_{10}, E_6, or SO_{16}. It is emphasized that there are many possible grand unified theories and that it is an experimental problem not only to test the basic ideas but to discriminate between models.

Therefore, the experimental implications are described in Section IV. The topics discussed include: (a) the predictions for coupling constants, such as $\sin^2\theta_W$, and for the neutral current strength parameter ρ. A large class of models involving an $SU_3^c \times SU_2 \times U_1$ invariant desert are extremely successful in these predictions, while grand unified theories incorporating a low energy left-right symmetric weak interaction subgroup are most likely ruled out. (b) Predictions for baryon number violating processes, such as proton decay or neutron-antineutron oscillations. The implications of a small value[3] (100-200 MeV) for the QCD $\Lambda_{\overline{MS}}$ parameter are discussed. For most theories with an $SU_3^c \times SU_2 \times U_1$ invariant desert, the proton lifetime is predicted to be short enough for nucleon decay to be easily observable in forthcoming experiments. The aspects of nucleon decay that are most useful for distinguishing between models are also described. Other topics discussed in Section IV are: (c) Cosmological implications, such as predictions for the baryon number and lepton number of the universe and for the density of superheavy magnetic monopoles. (d) Predictions for neutrino masses and lepton number violation, and (e) predictions for the b quark mass.

Theoretical issues and problems are considered in Section V. Topics discussed include attempts to understand the fermion spectrum, models in which grand unification is combined with supersymmetry, CP violation, other cosmological issues, and naturalness problems (i.e. the problem of understanding such small quantities as the strong CP parameter , the ratio of weak and lepto-quark gauge boson masses, or the observed cosmological constant).

A summary of predictions, successes, and unsolved problems in grand unification is given in Section VI.

II. The Standard Model[4]

The standard model is the union of quantum chromodynamics (QCD) with the electroweak theory of Glashow, Weinberg, and Salam. It is a gauge theory based on the group $G_s \equiv SU_3{}^c \times SU_2 \times U_1$ with gauge couplings g_s, g, and g' for the three factors. The model involves twelve gauge bosons: the eight gluons G^α_β ($\alpha,\beta=1,2,3$ with $G^\alpha_\alpha = 0$) of QCD and the four bosons $W^\pm$, Z, γ of the electroweak model. The minimal model involves a single complex doublet $(\phi^+\ \phi^o)^T$ of Higgs fields (three of the four real Higgs components are eaten to give masses to the $W^\pm$ and Z). One can extend the model to involve n_H complex Higgs doublets if desired.

The fermions are arranged in $F\geq 3$ families that are identical with respect to their gauge interactions. The SU_2 doublet and singlet assignments for the $F = 3$ case are (neglecting mixing):

$$\underbrace{\begin{pmatrix} u \\ d \end{pmatrix}_L \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L}_{\text{doublets}} \qquad \underbrace{u^c_L \qquad d^c_L \qquad e^+_L \qquad (\nu^c_{eL})}_{\text{singlets}}$$

$$\begin{pmatrix} c \\ s \end{pmatrix}_L \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \qquad c^c_L \qquad s^c_L \qquad \mu^+_L \qquad (\nu^c_{\mu L}) \tag{2.1}$$

$$\begin{pmatrix} t \\ b \end{pmatrix}_L \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L \qquad t^c_L \qquad b^c_L \qquad \tau^+_L \qquad (\nu^c_{\tau L})$$

The superscripts c in (2.1) refer to anti-particles, so that, for example, u_L and u^c_L are the fields associated with the left-handed up quark and anti-up quark, respectively. I have adopted the useful convention of specifying the representations of the left-handed fields ψ_L and ψ^c_L. The right-handed fields ψ^c_R and ψ_R are related to ψ_L and ψ^c_L by CP transformations, and are therefore not independent[4]. For example,

$$\gamma_o\, \psi^c_L \underset{CP}{\leftrightarrow} \psi_R \equiv C\, \bar{\psi}_L{}^{cT} , \tag{2.2}$$

where C is a Dirac matrix. The left-handed antineutrino fields $\nu^c_L \leftrightarrow \nu_R$ in (2.1) are not present in the minimal model, but can be included if desired.

The standard model is remarkably successful in that it is a mathematically consistent theory of the elementary particle interactions that is compatible with all known facts. However, it leaves too many questions unanswered to be considered a serious candidate for the ultimate theory. For example, the pattern of groups and representations is complicated and arbitrary, and there is no explanation for the repetition of fermion families. Moreover, there are an uncomfortably large number of free parameters: the minimal model with $F = 3$, $n_H = 1$ has 19 (26 if the ν^c_L are included). Even more serious is the lack of any explanation of electric charge quantization. For example, the simple relations such as $q_{e^-} = 3q_d$

between quark and lepton charges must be put into the model by hand. Finally, the standard model does not incorporate gravity.

One of the major motivations for grand unified theories was the desire to constrain some of these arbitrary features of the standard model.

III. Grand Unification[4]

Grand unified theories are defined as theories in which G_s is embedded in a larger gauge group G with a single running coupling constant g_G. This requires that G is either simple (SU_n, SO_n, Sp_{2n}, G_2 (which is actually too small), F_4, E_6, E_7, or E_8) or that $G = G_1 \times \ldots \times G_1 \times D$, where the identical gauge factors G_1 are simple and D is a discrete symmetry that interchanges the factors (so that the fermion and Higgs representations and the gauge couplings are the same). Some of the general features of grand unified theories are: (a) the strong, weak, and electromagnetic interactions are all combined in an underlying unified theory. They only emerge as separate interactions at low energy because of the pattern of spontaneous symmetry breaking. (b) The coupling constants g_s, g, and g' of the low energy theory are all related to g_G and therefore to each other. (c) Quarks (q), antiquarks (q^c), leptons (ℓ), and antileptons (ℓ^c) are fundamentally similar. They are often placed together in representations, which usually implies charge quantization. (d) There are new interactions which connect q, q^c, ℓ, and ℓ^c. These usually lead to proton decay and may explain the cosmological baryon asymmetry (the apparent excess of baryons over antibaryons in the present universe).

I will first give a brief description of the simplest realistic grand unified theory, the Georgi-Glashow SU_5 model[2], and will then indicate a few of the many possible extensions, modifications, and alternatives.

A. The Georgi-Glashow SU_5 Model[2,4]

The SU_5 gauge group involves 24 gauge bosons A^a_b, a, b, = 1, ..., 5 ($A^a_a = 0$), of which twelve are the bosons of the standard model subgroup. In addition there are an SU_2 doublet of bosons (X_α, Y_α) which have electric charges $q_X = 4/3$, $q_Y = 1/3$ and which carry color (they are color anti-triplets). Finally, there are their antiparticles ($\bar{X}^\alpha$, $\bar{Y}^\alpha$).

The fermion representations are still complicated. Each family transforms as a reducible sum of 5^* and 10 dimensional representations. For F = 3 the assignments are (neglecting mixing)

$$
\begin{array}{ccc}
\overset{X,Y}{\leftrightarrow} & \overset{X,Y}{\leftrightarrow}\qquad\quad\overset{X,Y}{\leftrightarrow} & \\
\begin{pmatrix} \nu^e & \\ & d^c \\ e^- & \end{pmatrix}_L & \begin{pmatrix} & u & \\ e^+ & & u^c \\ & d & \end{pmatrix}_L & \updownarrow W^\pm \\
\begin{pmatrix} \nu_\mu & \\ & s^c \\ \mu^- & \end{pmatrix}_L & \begin{pmatrix} & c & \\ \mu^+ & & c^c \\ & s & \end{pmatrix}_L & \updownarrow W^\pm \\
\begin{pmatrix} \nu_\tau & \\ & b^c \\ \tau^- & \end{pmatrix}_L & \begin{pmatrix} & t & \\ \tau^+ & & t^c \\ & b & \end{pmatrix}_L & \updownarrow W^\pm \\
5^* & 10 &
\end{array}
\tag{3.1}
$$

The notation in (3.1) is that fields arranged in columns transform as SU_2 doublets (i.e. they are transformed into each other by the emission or absorption of W^+ or W^- bosons), while fields in adjacent columns are transformed into each other by the emission or absorption of the new SU_5/G_s bosons X and Y. There is no room for left-handed antineutrinos ν_L^c (i.e. right-handed neutrinos ν_R) in the 5* + 10 assignment, but they could be added as SU_5 singlets if desired.

The new bosons in the SU_5 model can mediate proton decay. A typical diagram is shown in Figure 1. For $M_X \simeq M_Y >> m_p$ (the proton mass) one expects a proton (or bound neutron) lifetime

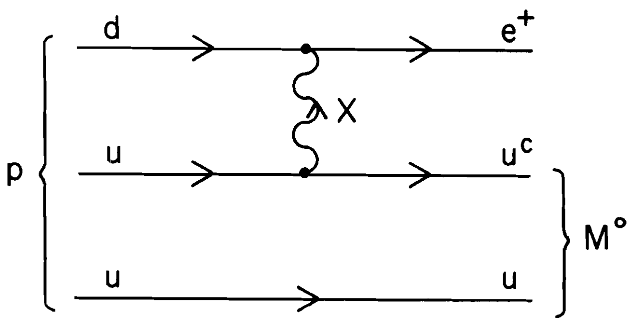

Figure 1. A typical diagram for proton decay. M^o represents any combination of neutral mesons (π^o, ρ^o, η, ω, $\pi^+\pi^-$, etc.) The upper (lower) vertices are referred to as lepto-quark (diquark) vertices.

$$\tau_p \sim \frac{1}{\alpha_5^2} \frac{M_X^4}{m_p^5} , \tag{3.2}$$

where $\alpha_5 \equiv g_5^2/4\pi$ is the SU_5 fine structure constant. For $\alpha_5 \sim \alpha$ and $\tau_p > 10^{30}$yr (the experimental limit) one has $M_X > 10^{14}$ GeV, more than twelve orders of magnitude larger than $M_{W,Z}$! It is remarkable that $M_X \gtrsim 10^{14}$ GeV is predicted independently from the observed values of α/α_s and $\sin^2\theta_W$ at low energies, implying a proton lifetime near the present limit.

In the Georgi-Glashow model SU_5 is spontaneously broken in two steps down to $SU_3^c \times U_1^{EM}$

$$\underset{M_X \simeq M_Y}{SU_5 \rightarrow} SU_3^c \times SU_2 \times \underset{M_W}{U_1 \rightarrow} SU_3^c \times U_1^{EM} \tag{3.3}$$

The dominant breaking down to G_s is induced by an adjoint (24) representation Φ_b^a of Higgs fields. As a perturbation on this picture, a second (five dimensional) Higgs representation

$$H^a = \begin{pmatrix} H^\alpha \\ \phi^+ \\ \phi^o \end{pmatrix} , \tag{3.4}$$

is introduced. H^a contains the standard model doublet (which breaks $SU_2 \times U_1$ down to U_1^{EM}) along with a color triplet H^α of Higgs particles. The H^α can also mediate proton decay and must therefore be superheavy. The uninteresting region between M_W and M_X, which contains no thresholds, is referred to as the desert. It should be mentioned that the incredibly tiny ratio $(M_W/M_X)^2 \lesssim 10^{-24}$ does not occur naturally: it must be adjusted or fine-tuned by hand. This is known as the gauge hierarchy problem.

B. Other Models and Options[4]

The SU_5 model is only one out of many possible grand unified theories, some of which have very different features. Let me briefly discuss a few of the possible alternatives.

The $SU_5 \subset SO_{10} \subset E_6$ Chain

One interesting possibility is the chain of successively larger groups $SU_5 \subset SO_{10} \subset E_6$. We have seen in (3.1) that in SU_5 each of the F families is placed in a reducible 5* + 10 representation. In the SO_{10} model[5,4] each family is assigned to an irreducible 16 which decomposes as $16 \to 5^* + 10 + 1$ under the SU_5 subgroup. The new SU_5 singlet can be interpreted as a left-handed antineutrino $\nu_L^c \leftrightarrow \nu_R$. Because of the presence of this field the neutrinos will almost certainly acquire masses at some level in SO_{10} (and in larger models). There are 45 gauge bosons in SO_{10}: the 24 bosons of the SU_5 subgroup; an SU_2-doublet, color-triplet of bosons (X',Y') with charges 2/3 and -1/3, which can also contribute to proton decay; an SU_2-singlet color-triplet X_s with $q = 2/3$, which is associated with the extension of SU_3^c to SU_4^c. The X_s does not contribute to nucleon decay except through negligible mixing effects. The X', Y', and X_s bosons and their antiparticles mediate transitions between the SU_5 5* and 10 representations and between the 10 and 1. Finally, there are three new bosons associated with the larger left-right symmetric weak subgroup $SU_{2L} \times SU_{2R} \times U_1'$ that occurs in SO_{10}. However, in most versions of the model the SU_{2R} bosons must be much too large ($M_{W_R} \gtrsim 10^9$ GeV) to be of phenomenological interest.

In the still larger E_6 model[6,4], each family of fermions is assigned to a 27. The 27 decomposes as 16 + 10 + 1 under the SO_{10} subgroup, where the 10 and 1 represent new heavy or superheavy fermions. (There is a different identification of the fermions in an alternate, topless version[6,4] of E_6 which assigns two charge 2/3 quarks (u,c), four charge -1/3 quarks (d,s,b,h), four charged leptons (e^-, μ^-, τ^-, M^-), and ten two-component neutrinos to two 27's. Such models appear to be ruled out by recent CESR data on b decay[7]). E_6 contains new bosons associated with transitions between the 16, 10, and 1.

Horizontal Symmetries

Another type of extension is to add a family or horizontal symmetry[8], in which additional bosons are associated with transitions between different families (which are often displayed in a horizontal arrangement). For example, Wilczek and Zee[9] have described an $SO_{16} \supset SO_{10} \times SO_6$ model which contains an SO_6 family group. The fermions are placed in a single irreducible 128^+ spinor representation which decomposes as $(16^+, 4^+) + (16^-, 4^-)$ under $SO_{10} \times SO_6$. This implies the existence of four ordinary families as well as four heavy conjugate families with V + A weak interactions.[10]

Low Mass Scale Models

To illustrate a model with a low unification mass, consider the SU_4^4 model of Pati and Salam.[11,4] The left and right-handed fermions of two families are assigned to separate representations f_L and f_R defined by

$$f_{L,R} \equiv \begin{pmatrix} u^R & u^G & u^B & \nu_e \\ d^R & d^G & d^B & e^- \\ s^R & s^G & s^B & \mu^- \\ c^R & c^G & c^B & \nu_\mu \end{pmatrix}_{L,R} \tag{3.5}$$

where R, G, B refer to the three colors. Two of the SU_4 factors form an $SU_{4L} \times SU_{4R}$ chiral weak group which acts vertically. The other two form an extended chiral color group. The lepto-quark X boson associated with SU_4^c/SU_3^c can mediate such decays as $K_L \to e^-\mu^+$. Hence, $M_X \gtrsim 10^4$-10^6 GeV. In the fractional charge quark (FCQ) version of the model the proton can only decay through high order Higgs exchange diagrams, so $M_X \sim 10^4$-10^6 GeV is acceptable. There is also an integer charge quark (ICQ) version in which quarks can decay into $\nu + \pi$'s, etc., through W-X mixing. Proton decays such as $p \to 3\nu + \pi^+$ occur via the (highly suppressed) simultaneous decay of three quarks.

Alternatives

These examples illustrate some of the many options and possibilities available in grand unification. Models can differ in choice of group, in fermion and Higgs representation, and in the pattern of spontaneous symmetry breaking (gauge boson masses). In particular, models with large or small[12] M_X or with multiple thresholds (i.e. no desert) are possible. Many types of new gauge interactions are possible. In addition to those leading to proton decay one can incorporate larger weak groups (involving $W_R^\pm$ or extra Z bosons[13]), technicolor (TC), extended technicolor (ETC), or horizontal interactions. Higgs mediated interactions are also possible[14] (e.g. some low mass scale models allow neutron oscillations $n \leftrightarrow \bar{n}$ mediated by Higgs exchange). Other options include the incorporation of supersymmetry (at low energies) or integer charge quarks.

It is therefore an experimental question not only to verify or disprove the basic ideas of grand unification but also to discriminate between models.

IV. Implications of Grand Unification

A. Predictions[15] for Coupling Constants and ρ.

The coupling constant predictions of grand unified theories depend on the details of the spontaneous symmetry breaking (SSB) pattern. In practice, clean predictions only emerge for theories involving just two mass scales, M_X and M_W, with a desert (a region with no gauge, Higgs, or fermion thresholds) in between. That is, if

$$G \underset{M_X}{\to} G_1 \underset{M_W}{\to} SU_3^c \times U_1^{EM}, \tag{4.1}$$

then the low energy running coupling constants g_s, g, and g' all approach g_G up to computable normalization factors as $Q^2 \to M_X^2$. One can therefore use the observed values of either α/α_s or $\sin^2\theta_W$ to predict M_X. Alternately, M_X can be eliminated yielding a relation between α/α_s and $\sin^2\theta_W$. This relation is just a consistency condition that the three couplings all meet at the same point. In order to evaluate this relation (i.e. predict $\sin^2\theta_W$) one must know the intermediate energy group G_1 and the G_1 assignments of all of the light (mass $\lesssim M_W$) fermions and Higgs bosons (in order to determine the renormalization group equations (RGE) for the running couplings). One must also know the G_1 assignments of an entire family (light and heavy) of fermions (in order to evaluate the normalizations). An explicit knowledge of G is not required, however.

For the case that $G_1 = G_s = SU_3^c \times SU_2 \times U_1$ the two independent ratios of coupling constants approach the asymptotic ratios[2]

$$\sin^2\theta_W = \frac{e^2}{g^2} \rightarrow \frac{\Sigma t_3^2}{\Sigma q^2}$$

$$\frac{\alpha}{\alpha_s} = \frac{e^2}{g_s^2} \rightarrow \frac{\Sigma t_{c_3}^2}{\Sigma q^2} \tag{4.2}$$

for $Q^2 \rightarrow M_X^2$, where t_{c3}, t_3, and q are the third component of color charge, third component of weak isospin, and electric charge of a fermion. The sums extend over an entire family of fermions. For F ordinary families, both asymptotic ratios are 3/8 (other weak interaction fermion assignments can lead to very different ratios[15].) In this case, $g_3 = g_s$, $g_2 = g$, and $g_1 = \sqrt{5/3}\,g'$ all approach g_G for large Q^2, as shown in Figure 2.

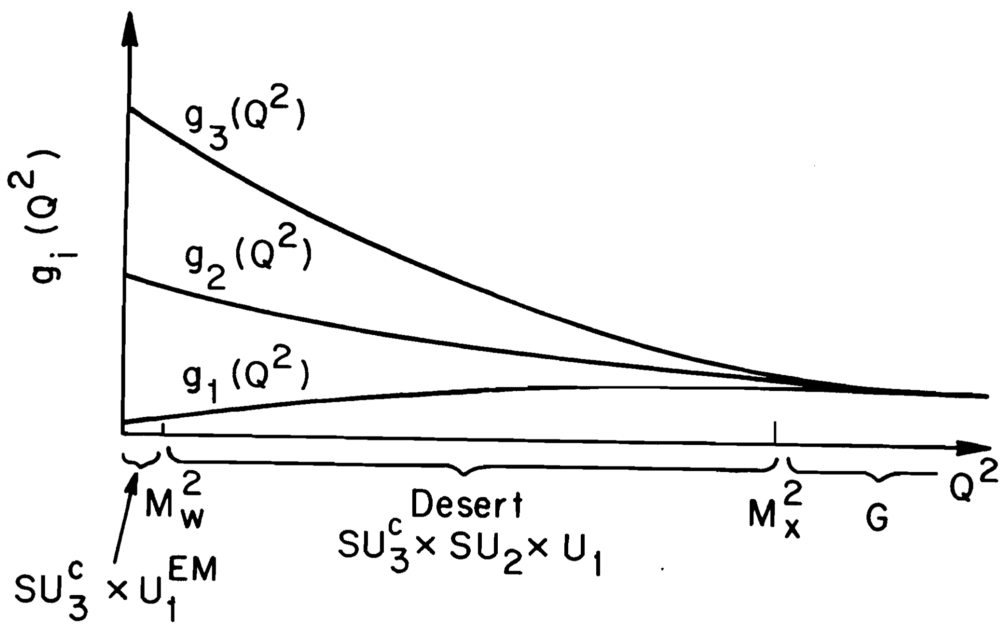

Figure 2. The behavior of the running coupling in 3-2-1 desert models. The apparent symmetry in each region is indicated.

For $M_W^2 < Q^2 < M_X^2$, the running coupling constant ratios are[16] (to lowest order in the RGE and treating thresholds as step functions)

$$\frac{\alpha(Q^2)}{\alpha_s(Q^2)} = \frac{3}{8}\left[1 - \frac{\alpha(Q^2)}{2\pi}\ (11 + \frac{n_H}{6})\ \ell n\ \frac{M_X^2}{Q^2}\right]$$

$$\sin^2\theta_W(Q^2) = \frac{3}{8}\left[1 - \frac{\alpha(Q^2)}{4\pi}\ \frac{(110-n_H)}{9}\ \ell n\ \frac{M_X^2}{Q^2}\right] \tag{4.3}$$

These lowest order equations are independent of F but depend weakly on the number n_H of light Higgs doublets.[14]

In principle one can predict M_X and $\sin^2\theta_W$ from (4.3), using the observed ratio of α/α_s at low energy as input. As emphasized by Marciano[17] and Goldman and Ross[18] and others[4], however, several refinements must be incorporated. These include (a) using the Q^2 dependence of α^{-1}:

$$\alpha^{-1}(M_W^2) \simeq \alpha^{-1}(0) - \frac{1}{3\pi}\sum_f q_F^2\ \ell n\ \frac{M_W^2}{m_f^2} \simeq 128.5\ , \tag{4.4}$$

where the sum extends over fermions with mass $m_f \leq M_W$. (b) Including threshold effects at M_W and M_X (the threshold effects largely cancel[4]). (c) Using two loop RGE (consistency then requires that one determine α_s using two loop QCD expressions and that radiative corrections be applied to neutral current data used to extract $\sin^2\theta_W$). There are now several analyses[4], all of which agree to within 1% in $\ell n\ M_X$ and 50% in M_X.

Predictions of 3-2-1 desert theories

The results for these theories (i.e. theories in which an arbitrary G breaks directly to $SU_3^c \times SU_2 \times U_1$ at M_X with a desert between M_W and M_X) with F light ordinary fermion families and n_H light Higgs doublets are[17,18,4]

$$M_X(\text{GeV}) = 2.4 \times 10^{14} \times (1.5)^{\pm 1} \times \left[\frac{\Lambda_{\overline{MS}}}{0.16\ \text{GeV}}\right] \times \left[\frac{1}{1.5}\right]^{n_H-1} \times (1.2)^{F-3} \tag{4.5}$$

$$\sin^2\hat{\theta}_W(M_W) = 0.2138 \pm 0.0025 + 0.006\ \ell n\left[\frac{0.16\ \text{GeV}}{\Lambda_{\overline{MS}}}\right] + 0.004\ (n_H-1) - 0.0004(F-3) \tag{4.6}$$

The stated errors in (4.5) and (4.6) are due to uncertainties in higher order terms, thresholds, heavy Higgs bosons[19], and the top quark mass ($15 < m_t < 50$ GeV is assumed). For the value[3] $\Lambda_{\overline{MS}} = 0.16\ ^{+0.100}_{-0.080}$ GeV for the QCD parameter, the log in (4.6) is $0\ ^{-0.0029}_{+0.0042}$. Two recent detailed analyses[17,20] yield the very weak F dependence in (4.5-4.6), although earlier studies[18,21] suggested a stronger F dependence.

Neutral Current Data

The experimental value of $\sin^2\theta_W$ obtained from deep inelastic scattering data and the SLAC polarized eD experiment (without radiative corrections) is[22]

$$\sin^2\theta_W = 0.229 \pm 0.009 \ (\pm 0.005), \tag{4.7}$$

where the second error is theoretical.[23]

Marciano and Sirlin[24] have recently computed the radiative corrections to $R_\nu = \sigma(\nu N \to \nu X)/\sigma\ (\nu N \to \mu^- X)$ and to the eD asymmetry. They find that radiative corrections reduce the value of $\sin^2\theta_W$ obtained from R_ν by 0.011 (of which 0.001 is from the neutral current cross section and 0.010 from the charged current). Similarly, $\sin^2\theta_W$ obtained from the eD asymmetry is reduced by 0.011. Similar results are obtained[25] by two groups who have calculated the leading log contributions to the radiative corrections to all orders. Marciano and Sirlin obtain[24]

$$\sin^2\hat{\theta}_W(M_W)\Big|_{\mathrm{exp}} = 0.215 \pm 0.012 \quad , \tag{4.8}$$

which corresponds to $M_W = (83.0 \pm 2.4)$ GeV and $M_Z = (93.8 \pm 2.0)$ GeV. (4.8) is in remarkable agreement with the theoretical prediction (4.6). This gives strong encouragement that the class of 3-2-1 desert theories may have something to do with the real world!

Some Other Models

By way of contrast, let us consider models with a left-right symmetric subgroup:

$$G \xrightarrow[M_X]{} SU_3^c \times SU_{2L} \times SU_{2R} \times U_1' \xrightarrow[M_{W_R}]{} G_s \xrightarrow[M_W]{} SU_3^c \times U_1^{EM} \tag{4.9}$$

If the V + A boson W_R has a mass $M_{W_R} \lesssim 10\ M_W$ and if one requires $M_X < m_P$ (the Planck mass) then the analogue of (4.3) implies[26]

$$\sin^2\theta_W > 0.29$$
$$\Lambda < 2.5 \text{ MeV} \tag{4.10}$$

which is clearly unacceptable. (One possible loophole, at least for the bad prediction for $\sin^2\theta_W$, is that if $M_{W_R} \sim M_{W_L}$ then the extra Z boson will modify the structure of the neutral current interaction.[27,28] It is possible to fit the neutral current data with a large $\sin^2\theta_W$ by adjusting 3 or 4 additional parameters in this case, but the success of the standard model would then be an accident. The V + A charged hadronic currents may also be problematic for non-leptonic hyperon decays and the K_L-K_S mass difference). In order to obtain acceptable values for $\sin^2\theta_W$ and Λ for the SSB pattern in (4.9), one requires[29] $M_{W_R} > 10^9$ Gev, which is much too large to be phenomenologically relevant.

Similar difficulties occur for models such as SU_4^4 if they break to the chiral color group[11] $SU_{3L}^c \times SU_{3R}^c \times SU_{2L} \times SU_{2R} \times U_1'$ in the region between M_X and M_W. For $M_X \sim 10^6$ GeV (suggested by α_s) one has the unacceptable value[4] $\sin^2\theta_W \simeq 0.30$.

The ρ Parameter

The parameter $\rho \equiv M_W^2/M_Z^2 \cos^2\theta_W$ measures the overall strength of the neutral current. In the standard model ρ can deviate from unity due to Higgs triplets ϕ_1 and heavy fermions. One has

$$\rho^{TH} = 1 + 4 \frac{\langle\phi_1\rangle^2}{\langle\phi_{\frac{1}{2}}\rangle^2} + \text{heavy fermion terms,} \tag{4.11}$$

where $\langle\phi_{\frac{1}{2}}\rangle$ is the ordinary doublet. The experimental value[22] $\rho = 0.992 \pm 0.017$ (± 0.011) is increased by radiative corrections[24,30] to

$$\rho_{exp} = 1.002 \pm 0.017 \ (\pm 0.011) \tag{4.12}$$

Most grand unified theories include Higgs triplets, but fortunately many of the 3-2-1 desert theories predict[31-33] the extremely small value

$$\frac{\langle\phi_1\rangle^2}{\langle\phi_{\frac{1}{2}}\rangle^2} \simeq \frac{M_W^2}{M_X^2} \sim 10^{-24} \tag{4.13}$$

In this case one can use (4.12) to limit the masses of heavy SU_2 doublet fermions with light partners.[34] The result is[15,22] $m_t < 335$ GeV and $m_{L^-} < 580$ GeV for a heavy lepton with a massless partner, both at 90% c.l.

B. Baryon Number Violation

In this section I describe some aspects of baryon number violation in the SU_5 model, in more general 3-2-1 desert models, and in models with low mass scales. More complete discussions may be found in Ref. (35-37).

The SU_5 Model (general Higgs structure).

Including mixing, the m^{th} fermion family is

$$5^*: \begin{pmatrix} \nu_m & \\ & (d^c A_R^{d\dagger})_m \\ e_m^- & \end{pmatrix}_L \qquad 10: \begin{pmatrix} (A_L^{e^+} e^+)_m & (A_L^u u)_m & (u^c A_R^{u\dagger})_m \\ & d_m & \end{pmatrix}_L , \tag{4.14}$$

where, for example, e_m^-, $m=1,\ldots, F$ is the m^{th} mass eigenstate charged lepton. In (4.14) A_L^u is the F x F generalized Kobayashi-Maskawa matrix, which is in principle known from charged-current weak interactions. Baryon number violation, however, involves transitions between adjacent columns in (4.14). All predictions for proton decay depend on the totally arbitrary and unknown F x F mixing matrices A_R^d, $A_L^{e^+}$, and A_R^u. These unknown mixing effects turn out to be the largest uncertainty in proton decay in the standard models. It is not even a priori obvious that the dominant amplitude is for $p \to e^+\pi^o$ rather than $p \to \tau^+\pi^o$, for example. Fortunately, things simplify enormously in some specific models. For example, the minimal SU_5 model with all fermion masses generated by Higgs 5's leads to $A_L^{e^+} = A_R^d = I$ and $A_R^{u*} = A_L^u K^*$, where K is a diagonal matrix of (CP-violating) phases that are irrelevant to proton decay. For the two family version of this model, the effective Lagrangian is[31,38] (suppressing color indices)

$$L_{SU_5} = \frac{g_5^2}{2M_X^2} \{ (\bar{u}_L^c \gamma^\mu u_L) [\bar{e}_R^+ \gamma_\mu d_R + \bar{\mu}_R^+ \gamma_\mu s_R$$

$$+\{(1+c^2)\bar{e}_L^+ + sc\ \bar{\mu}_L^+\} \gamma_\mu d_L + \{(1+s^2)\bar{\mu}_L^+ + sc\ \bar{e}_L^+\}\gamma_\mu s_L]$$

$$-[\bar{u}_L^c \gamma^\mu (c\, d_L + s\, s_L)][\bar{\nu}_{eR}^c \gamma_\mu d_R + \bar{\nu}_{\mu R}^c \gamma_\mu s_R]\} , \tag{4.15}$$

where $c = \cos\theta_c$, $s = \sin\theta_c$, and θ_c is the Cabibbo angle. Most of the specific results presented below are based on (4.15). More generally, Wilczek and Zee[39] have introduced the Kinship Hypothesis, which states that baryon-number-violating interactions mainly involve transition within the families (u, d, ν_e, e) or (c, s, ν_μ, μ) except for small (of order θ_c) mixing effects, as in the minimal SU_5 model. I will generally assume that the Kinship Hypothesis is true.

The Proton Lifetime

The naive estimate (3.2) for τ_p with (4.5) yields $\tau_p \simeq 2 \times 10^{29}$yr for $\Lambda_{\overline{MS}} \simeq 0.16$ GeV ($M_X \simeq 2.4 \times 10^{14}$ GeV) and[18,4] $\alpha_5 = g_5^2/4\pi \simeq 0.022$. As we will now see, more detailed calculations yield similar results.

These more detailed analyses (18, 21, 40-48) encounter the three classes of diagrams shown in Figure 3

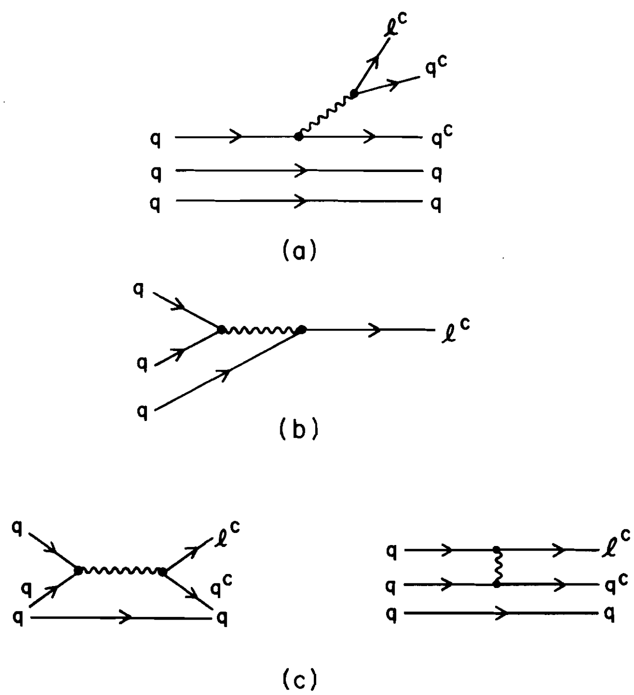

Figure 3. (a) Quark decay. (b) Three-quark fusion. (c) Two-quark fusion.

The quark decay diagram of 3a is suppressed by phase space.[40] The 3-quark fusion diagram of 3b, in which the initial quarks emit gluons or mesons, is usually assumed to be small. (It has recently been argued[47], however, that this diagram may dominate for decays in nuclei, but the theoretical uncertainties are large). A contrary position has been taken by Berezinsky, Ioffe, and Kogan[48], who have argued using QCD sum rules, PCAC, and pole approximations that the 3-quark diagram may dominate in free nucleon decays and that $\tau(p\to e^+\pi^0) \sim 10^{29}$yr for $M_X = 2.4 \times 10^{14}$ GeV. If they are right, then the simple SU_5 model is in serious trouble.

A number of authors have estimated[4] τ_p and τ_n assuming that the two-quark fusion diagrams dominate. The results are shown in Table 1. The spread of values is mainly due to uncertainties in computing the hadronic matrix elements (e.g. some calculations use SU_6 with a phenomenological wave function $\psi(0)$ for two quarks to overlap; others use bag model wave functions) and must be considered a genuine theoretical uncertainty. There are also smaller differences due to the treatments of the second family, mixings, and phase space (e.g. inclusive or exclusive sums on hadronic states, effective quark masses, etc.) The results have been normalized to common anomalous dimensions,[49] $\psi(0)$, and M_X (2.4×10^{14} GeV).

	$10^{29}\, a_p$	$10^{29}\, a_n$	τ_p (10^{29} yr)	τ_n (10^{29} yr)
JY[40]	3.7	4.3	1.2	1.4
EGNR[21]	4.8	---	1.6	---
GR[18]	2.4	3.7	6.8	1.2
GLOPR[41]	35 ρ_p	---	12 ρ_p	---
R[42]	(8.5-36)ρ_p	(7.7-29)ρ_n	(2.8-8.7)ρ_p	(2.6-9.7)ρ_n
D[43]	38 ρ_p	50 ρ_n	13 ρ_p	17 ρ_n
G[44]	41 ρ_p	48 ρ_n	13 ρ_p	16 ρ_n
DGS[45]	2 ρ_p	---	0.7 ρ_p	---
A[46]	(11-33)ρ_p	(10-29)ρ_n	(3.8-11)ρ_p	(3.3-9.7)ρ_n

Table 1. Lifetime estimates. The first three (last six) estimates are inclusive (exclusive). The last four calculations utilize MIT bag wave functions.

In Table 1, a_p and a_n are the coefficients in the approximate expressions

$$\tau_{p,n}(\text{yr}) = a_{p,n}\, M_X^4 \quad , \tag{4.16}$$

and $\rho_{p,n}$ are the fractions of two body final states in nucleon decay. From these results

$$\tau_p(\text{yr}) \simeq (0.8\text{-}13) \times 10^{29} \left[\frac{M_X}{2.4 \times 10^{14}\ \text{GeV}}\right]^4 \tag{4.17}$$

I will use the range of values in (4.17) as an estimate of the theoretical uncertainties (due to the hadronic physics) in τ_p (M_X). Combining (4.17) with (4.5) one has

$$\tau_p(\text{yr}) \simeq 3.2 \times 10^{29\pm1.3} \left[\frac{\Lambda_{\overline{MS}}}{0.16\ \text{GeV}}\right]^4 , \tag{4.18}$$

where the error is due to τ_p (M_X) and M_X $(\Lambda_{\overline{MS}})$.

For[4]

$$\Lambda_{\overline{MS}} = 0.160 \begin{matrix} +0.100 \\ -0.080 \end{matrix} \ \text{GeV} \tag{4.19}$$

one has finally

$$\tau_p(\text{yr}) = 3.2 \times 10^{29^{+2.1}_{-2.5}} , \tag{4.20}$$

where the error includes the uncertainty in $\Lambda_{\overline{MS}}$. Furthermore[35] the bound neutron lifetime is predicted to be $0.8 < \tau_n/\tau_p < 1.5$.

We see that the small values of $\Lambda_{\overline{MS}}$ that are now becoming accepted lead to the prediction of a very "short" proton lifetime. For example, $\Lambda_{\overline{MS}} = 200$ MeV implies τ_p in the range $(3.8 \times 10^{28} - 1.5 \times 10^{31})$yr, while $\Lambda_{\overline{MS}} = 100$ MeV yields $(2.4 \times 10^{27} - 9.6 \times 10^{29})$yr. The relation (4.18) between τ_p and $\Lambda_{\overline{MS}}$ is shown in Figure 4 along with the present experimental limit $\tau_p > 2 \times 10^{30}$ yr.

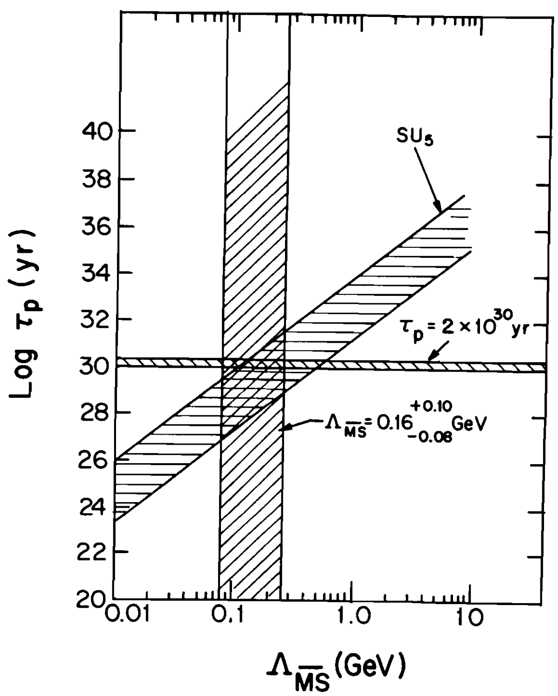

Figure 4. The SU_5 prediction for τ_p as a function of $\Lambda_{\overline{MS}}$.

Clearly, it should be easy to verify or disprove the prediction (4.20) of the minimal SU_5 model with small $\Lambda_{\overline{MS}}$ in forthcoming experiments.

Most other 3-2-1 desert models give predictions similar to (4.17), (4.18), and (4.20), although the exact coefficients a_p and a_n may be different. For example, the proton lifetime is predicted to be somewhat shorter in the SO_{10} model (because of the X' and Y' bosons), but this effect is smaller than the hadronic uncertainties in (4.17).

However, there are several theoretical loopholes that could allow nucleon lifetimes longer than (4.20). These include (a) modifying the light and/or heavy Higgs and fermion representations of the standard model (which affect M_X). These could probably not change τ_p by more than $\simeq 10^{\pm 2}$ without also destroying the successful prediction of $\sin^2\theta_W$ (an exception is supersymmetric models in which τ_p can be as large as 10^{45} yr.) (b) τ_p could be increased by violations of the Kinship Hypothesis. Even if the proton coupled predominantly to heavy channels it is unlikely that τ_p would be increased by much more than $\sin^{-2}\theta_c \simeq 20$, however[51] (unless the proton is made absolutely stable by imposing an extra symmetry - see below). (c) Models with three or more mass scales (such as (4.9)) can give arbitrarily long proton lifetimes. In such models the value of $\sin^2\theta_W$ is an input (to determine the extra mass scale), so the success of (4.6) would be an accident.

Branching Ratios in SU_5

A number of estimates (41-46, 48, 52-57) of the proton and neutron branching ratios have been made.[35] Although these differ in the hadronic matrix elements, mixings, and phase space treatments used, the most important differences are due to pionic recoil suppression effects[56] ($v_\pi/c \simeq 0.96$) and the method of projecting the spin of the recoiling q^c onto meson wave functions. The results of many of these estimates are given in Tables 2 and 3. Each column is labeled according to the value used for the ratio k/M of momentum to mass of the q^c: NR (nonrelativistic) assumes k/M = 0; R (relativistic) takes M = 0; and REC (intermediate recoil) takes k/M $\simeq$ 3/4. The spread in the three calculations of Kane and Karl[53] (KK) gives a reasonable estimate of the uncertainties, except there may be additional recoil suppressions of the pionic modes.

Mode	M[52]	GYOPR[41]	D[43]	G[44]	DGS[45]	KK[53]		
	REC	NR	R	R	R	NR	REC	R
$e^+\pi^0$	31	37	9	13	31	36	40	38
$e^+\rho^0$	15	2	21	20	21	2	7	11
$e^+\eta$	11	7	3	.1	5	7	1.5	0
$e^+\omega$	18	18	56	46	19	21	25	26
$\nu_e^c\pi^+$	12	15	3	5	11	14	16	15
$\nu_e^c\rho^+$	6	1	8	7	8	1.0	2.6	4
μ^+K^0	1	19	--	7	.5	18	8	5
$\nu_\mu^c K^+$	2	0	--	.5	--	0	.2	.6

Table 2. Predictions for branching ratios for proton decay into the major two body modes in the SU_5 model.

Mode	M[52] REC	GYOPR[41] NR	D[43] R	G[44] R	KK[53] NR	KK[53] REC	KK[53] R
$\nu_e^c\pi^o$	5	8	2	3	8	7	7
$\nu_e^c\rho^o$	3	.5	5	4	.6	1.2	1.8
$\nu_e^c\eta$	2	1.5	1	--	1.5	--	--
$\nu_e^c\omega$	3	3.5	14	10	5	5	5
$e^+\pi^-$	54	74	23	32	79	72	68
$e^+\rho^-$	27	4	55	48	6	12	19
$\nu_\mu^c K^o$	0	10	--	2	1.1	3	0.6

Table 3. Predictions for neutron decay branching ratios in SU_5.

Other Models

The baryon number violating interactions in a general model can be described by an effective Lagrangian[58-60]

$$L = CM^{4-d}\ O, \tag{4.21}$$

where O is an effective operator of dimension d, constructed from the light fields of the theory, M is the (superheavy) unification scale, and C is a dimensionless constant that is typically of order e^{n-2} for n external fields in O. This is analogous to the effective four-fermion operator $L_W = G_F\ JJ^\dagger/\sqrt{2}$ of the weak interactions, for which d = 6, n = 4, and $G_F/\sqrt{2} = g^2/8\ M_W^{\ 2}$. For $M >> M_W$, it suffices to consider $SU_3 \times SU_2 \times U_1$ invariant operators in (4.21). $SU_2 \times U_1$ breaking effects (suppressed by powers of M_W/M) can be incorporated by including the Higgs field (which can be replaced by its vacuum expectation value) in O.

3-2-1 Desert Theories

The lowest dimension baryon-number violating, color singlet, Lorentz scalar operators are four-fermion operators with[4] d = 6. Only the d = 6 operators are relevant for 3-2-1 desert theories because of the extremely large value of $M \gtrsim 10^{14}$ GeV. (d>6 interactions are suppressed by $(M_W/M)^{d-6}$). Weinberg[59] and Wilczek and Zee[58] have shown that, up to family indices, there are only six such d = 6 operators, and that (suppressing Lorentz, family, and G_s indices) all are of the form $qqq\ell$. These operators can be generated by gauge or Higgs exchange diagrams as in Figure 5a.

It is remarkable that these six operators all imply the following selection rules, which must be respected in all 3-2-1 desert theories[58,59,52,35]:

(a) $\Delta B = \Delta L$. For example $p \to e^+\pi^o$ is allowed, but $p \to e^-\pi^+\pi^+$ is forbidden;

(b) $\Delta S/\Delta B = -1$ or 0. Thus, $p \to \nu^c K^+$ but $n \not\to e^+K^-$;

(c) the $\Delta S = 0$ operators transform as strong isospin doublets, implying relations such as

$$\Gamma(p \to e^+\pi^0) = \frac{1}{2}\Gamma(n \to e^+\pi^-) \tag{4.22}$$

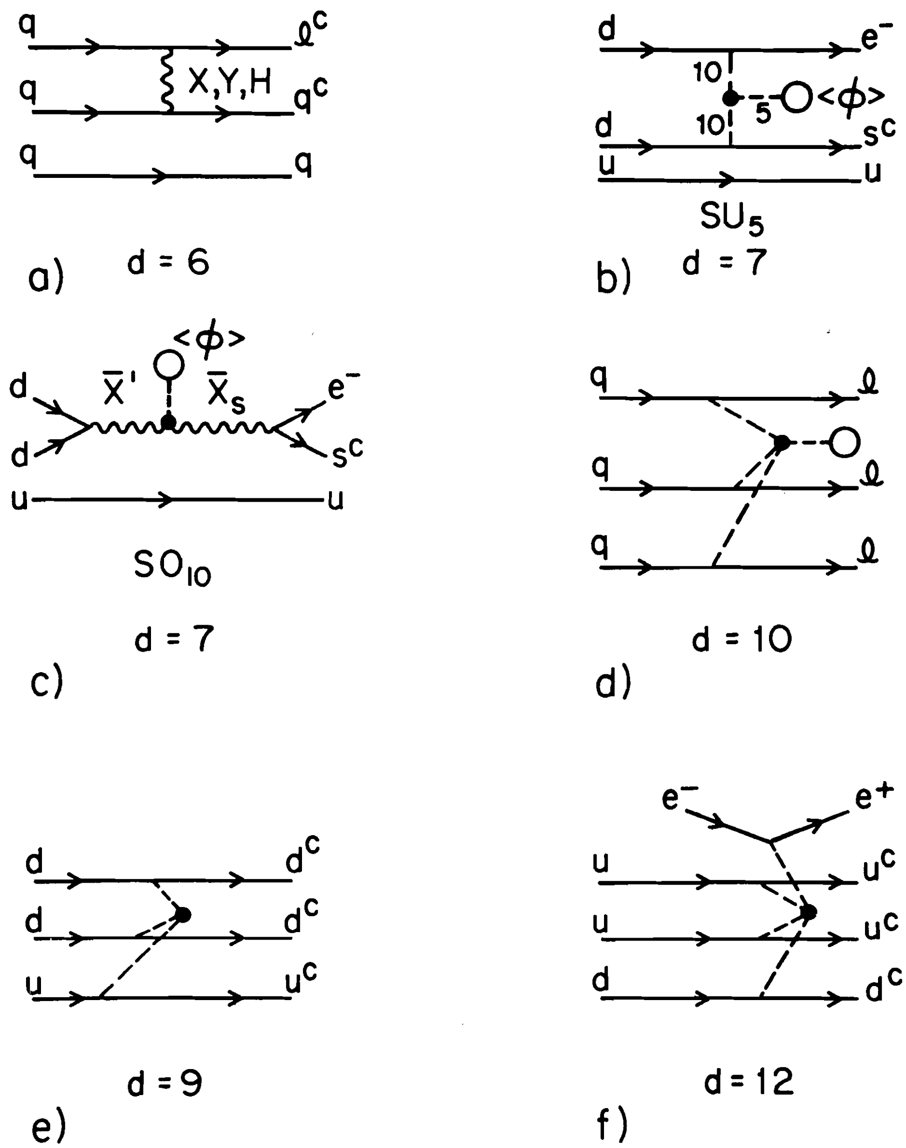

Figure 5. Typical diagrams leading to effective operators of various dimensions.

For operators involving (u, d, ℓ^-, ν_ℓ) only, where $\ell = e$ or μ, there are only four operators. For $\ell = e$ these are

$$O_1 = \bar{d}^c_L\, u_R\, [\bar{u}^c_R\, e^-_L - \bar{d}^c_R\, \nu_L]$$
$$O_2 = \bar{d}^c_R\, u_L\, [\bar{u}^c_L\, e^-_R]$$
$$O_3 = \bar{d}^c_R\, u_L\, [\bar{u}^c_R\, e^-_L - \bar{d}^c_R\, \nu_L]$$
$$O_4 = \bar{d}^c_L\, u_R\, \bar{u}^c_L\, e^-_R \quad , \tag{4.23}$$

with similar operators for $\ell = \mu$. Cabibbo mixing, which can modify the relative coefficients of the e^- and ν terms in O_1 and O_3 has been neglected (this should not be important unless there are large violations of the Kinship Hypothesis). It should be observed that the e^- terms in O_1 and O_2 are mapped into each other under space reflections, as are those in O_3 and O_4.

Hence, the most general effective Lagrangian for decays involving (u, d, e^-, ν_e) only in 3-2-1 desert theories is of the form[58,59]

$$L = \sum_{i=1}^{4} C_i\, O_i + \text{H.C.}, \tag{4.24}$$

where of course the C_i depend on the model.

Gauge Boson Mediated Processes

If the underlying mechanism is gauge boson exchange, then it can be shown[58,59] that $C_3 = C_4 = 0$, so that

$$L = C_1\, O_1 + C_2\, O_2 + \text{H.C.} \tag{4.24a}$$

Hence, only the ratio $r \equiv C_2/C_1$ can distinguish between models in this class for (u, d, e^-, ν_e) decays. (The hadronic uncertainties in τ_p are too large to effectively discriminate between models). For example, those who are skilled at Fierz transformations can immediately see from (4.15) that, for $\sin\theta_c = 0$, $r = 2$ in the SU_5 model (anomalous dimension effects modify[58] r to 2.1). Similarly, $r = 2\, M^2_{X'}/(M^2_X + M^2_{X'})$ for SO_{10}, which ranges from 0 to 2 as $M_{X'}/M_X$ goes from 0 to ∞.

Because the e^- parts of O_1 and O_2 are mapped onto each other under parity, the relative branching ratios for $e^+\pi^o/e^+\rho^o/e^+\eta/e^+\omega$ and $\nu^c\pi^+/\nu^c\rho^+$ are independent of r. That is, measurements of the specific hadronic final state in $\Delta S = 0$ decays will not distinguish between gauge-mediated models. On the other hand, the ratio of positrons to anti-neutrinos depends on r and is useful for distinguishing between models. For example,

$$\frac{\Gamma(O^{16}\to e^+X)}{\Gamma(O^{16}\to \nu^c X)} = 1 + r^2 \tag{4.25}$$

Other predictions are detailed in (35).

A useful signature of gauge mediated decays is that prompt muons emitted in $\Delta S = 0$ decays or $\Delta S = 1$ decays should have (different) universal[59] polarizations that depend only on parameters analogous to r. In the SU_5 model in (4.15), for example,

$$\Delta S = 0: \quad P(\mu^+) = -1$$

$$\Delta S = 1: \quad P(\mu^+) = \frac{1-(1+s^2)^2}{1+(1+s^2)^2} \simeq -0.05. \tag{4.26}$$

Quantities such as the muon polarizations or the ratios μ^+/e^+ and $(\Delta S=0)/(\Delta S=1)$ are dependent not only on the model but also the Higgs and mixing structure. Hence, their measurement would be a useful probe of these aspects of grand unified theories.[61]

Higgs Mediated Decays in 3-2-1 Desert Theories.

Proton decay can also be mediated by the exchange of Higgs bosons, which can generate all four operators in (4.23). The relative branching ratios $e^+\pi^o/e^+\rho^o/e^+\eta/e^+\omega$ and $\nu^c\pi^+/\nu^c\rho^+$ depend on the ratio $\lambda \equiv (C_4^2 + C_3^2)/(C_1^2 + C_2^2)$ (the ρ and η modes are enhanced[62,63] for $\lambda > 0$), so in principle it is possible to distinguish between gauge and Higgs mediated theories by careful measurements of these branching ratios. In practice this is difficult because of the hadronic uncertainties and also because λ can assume any value ≥ 0 in scalar mediated theories.

Another signal of the presence of C_3 and C_4 is that the prompt muon polarizations would no longer be universal.

A much more dramatic and obvious signal of Higgs mediated decays would be an enhancement of the branching ratios for muons and strange particles. This would occur in most models because of the tendency for Higgs particles to couple preferentially to heavier fermions. In fact, Golowich[62] has shown that in the minimal SU_5 model with a single Higgs 5 the branching ratio for $p \to \mu^+K^o$ would be essentially 100%. This would be a dramatic signal indeed!

Nuclear Effects

All foreseeable experiments involve nuclei rather than free nucleons. Nucleon decays within a nucleus are complicated by Fermi motion and also by the possibility that produced pions may be absorbed or elastically or charge-exchange scattered before getting out of the nucleus. For example, Sparrow has estimated[64] that only about half of the $p \to e^+\pi^o$ decays in water will lead to a back to back e^+ and π^o (and 40% of these are from the hydrogen atoms). Similarly, the secondary μ^+ rate should be reduced by $\simeq 2.5$ in water.

There may also be desirable effects. Dover et al.[65] and Arafune and Miyamura[66] have considered the process $p \to e^+ +$ (virtual π^o). The virtual π^o can be absorbed by another nucleon, producing an N, N* or Δ, as in Figure 6. The nucleon lifetime in nuclei may be shortened by anywhere from 5% to 50%, depending on short range correlations.

In addition, Fernández de Labastida and Ynduráin[47] have suggested that the three-quark fusion diagram of Figure 3b (with gluons radiated from the quarks absorbed by other nucleons) may dominate in nuclei (although the uncertainties are large).

All three of these mechanisms (interaction of a real meson, of a virtual

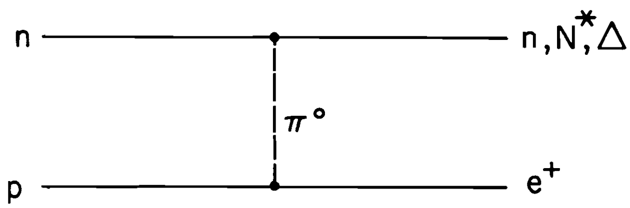

Figure 6. The absorption of a virtual pion in a nucleus.

meson, and 3-quark fusion) would generate recoiling nuclear fragments and possibly associated pions. The last two would lead to a continuous momentum spectrum (from 0 to m_p) for the emitted lepton, with values larger than $m_p/2$ favored.

Finally, $\Delta B = 2$ processes associated with $d > 6$ operators may be observable in nuclei.

Models with Low Mass Scales[35,12]

Operators with $d > 6$ could be important in models with mass scales $M << 10^{14}$ GeV. These operators may be distinguished experimentally from the $d = 6$ operators relevant to 3-2-1 desert theories by their very different selection rules (e.g. the $d = 7$ operators conserve $B + L$ rather than $B - L$). A number of low dimension operators are listed in Table 4, along with typical processes that they initiate, their selection rules, and the mass scales for which they would lead to nuclear or nucleon lifetimes in the observable 10^{30}-10^{33} yr range. (Unlike the 3-2-1 desert theories, there is no independent reason to expect the mass scales to actually be in this range. Nor, in general, is there a

d	0	Process	ΔB, ΔL, ΔS	M (GeV)
6	$qqq\ell$	$p \to e^+\pi^o$	$\Delta B = \Delta L = -1$ $\Delta S = 0,1$	$4 \times 10^{14} - 2 \times 10^{15}$
7	$qqq\ell^c\phi$	$n \to e^-K^+$	$\Delta B = -\Delta L = -1$ $\Delta S = 1$	$2 \times 10^{10} - 10^{11}$
7	$qqq\ell^cD$	$n \to e^-\pi^+$	$\Delta B = -\Delta L = -1$	$4 \times 10^9 - 2 \times 10^{10}$
10	$qqq\ell^c\ell^c\ell^c\phi$	$n \to \nu\nu e^-\pi^+$	$\Delta B = -1/3\ \Delta L = -1$	$(3 - 7) \times 10^4$
11	$qqq\ell\ell\ell\phi^2$	$p \to e^+\nu^c\nu^c$	$\Delta B = +1/3\ \Delta L = -1$	$(2 - 4) \times 10^4$
9	$qqqqqq$	$n \leftrightarrow \bar{n}$	$\Delta B = -2$	$4 \times 10^5 - 10^6$
		$pn \to \pi^+\pi^o$	$\Delta L = 0$	
		$nn \to 2\pi^o$		
12	$qqqqqq\ell\ell$	$pp \to e^+e^+$	$\Delta B = \Delta L = -2$	>500
		$H \leftrightarrow \bar{H}$		

Table 4. Baryon number violating operators of dimension d. q, ℓ, φ, and D represent quarks, leptons, Higgs field, and derivatives, respectively.

prediction of $\sin^2\theta_W$.) Many of the models which lead to these operators have Higgs mediated interactions. Some typical diagrams are shown in Figure 5.

The d = 9 six quark operator leads to the interesting processes of neutron-antineutron oscillations[67] and to dinucleon annihilation in nuclei. The operator can be rewritten as an effective dineutron operator $\delta m\ n^T n$, where $\delta m \sim e^4\ m_p{}^6/M^5$ (see 5e) is the neutron-antineutron transition mass. nn annihilations lead to a nuclear lifetime[4] $\Gamma_{nuc} \sim a(\delta m)^2/m_p$. For $\tau_{nuc} > 10^{30}$ yr and $a \sim 10^{-2}$ (for wave-function overlap effects) one requires $\delta m < 10^{-20}$ eV, leading to a free neutron-antineutron oscillation time $\tau_{n\bar{n}} = 1/\delta m > 10^5$ s. However, a number of recent more detailed estimates[66,68-71] of Γ_{nuc} find more stringent lower limits on $\tau_{n\bar{n}}$, ranging from 3×10^6 s to 5×10^7 s.

Two classes of baryon-number-violating models do not fit easily into the classification in Table 4. For example, there have been a number of models[72,73,4] in which the proton is made absolutely stable by imposing a globally conserved quantum number. This quantum number corresponds to baryon number for the ordinary fermions but not for a new class of "weird" quarks and leptons. Such theories therefore allow[74] a baryon asymmetry for the universe and predict the possibility of observing baryon number violation at accelerator energies[15] through such reactions as $e^+e^- \to Q\bar{Q} \to B\ell\ell\bar{E}^o\pi$'s, where Q is a new heavy (m ~ 50-100 GeV) quark and E^o is a stable massive (m ~ 1-10 GeV) neutrino.

In the integer-charge-quark versions of the Pati-Salam models[11,4] (e.g. $SU_4{}^4$) there is a $M_X \sim 10^4 - 10^6$ GeV leptoquark boson that can lead to quark decays such as $q \to \pi\nu$ with $\tau_q \sim 10^{-6} - 10^{-9}$s by mixing with the W. Proton decays into $3\nu\ \pi^+$, $3\nu\pi^+\pi^+\pi^-$, $\nu\nu e^-\ \pi^+\pi^+$, etc., with $\tau_p \sim 10^{29} - 10^{34}$ yr can then occur via the simultaneous decay of all three quarks.

Summary

The nucleon lifetime predictions for 3-2-1 desert theories are summarized in (4.17) - (4.20). If nucleon decay is observed then it will be of enormous interest to extract as much information about the decays as possible in order to discriminate between models. I have listed some of the measurable quantities in Table 5 along with what is actually tested by each type of measurement.

Quantity Measured	Aspect of Theory Probed
ΔB, ΔL, ΔS, ΔI selection rules	M, dimension of operator (general class of theory)
Lepton momentum spectrum Nuclear fragments	Nuclear effects
$\pi/\rho/\eta/\omega$; $P(\mu^+)$ universality; $B(\mu^+K^o)$	Gauge vs. Higgs mediation
e^+/ν^c ; $P(\mu^+)$	Gauge group and SSB
e^+/μ^+ ; $(\Delta S=0)/(\Delta S=1)$; $P(\mu^+)$	Mixings, Higgs, group, SSB

Table 5. Some observables in nucleon decay, and the aspects of grand unified models that they probe.

C. Cosmological Implications

An important laboratory for testing the ideas of grand unification is the universe itself. The baryon number violating interactions which are so weak at present may have been very important in the first 10^{-35} sec after the big bang, when the temperature T would have been comparable to M_X. Relics of that first instant, such as the net baryon or lepton number density of the universe or superheavy magnetic monopoles, may still be observable today.

The Baryon Asymmetry[75,76]

The observed ratio of baryon number to entropy in our part of the universe is[77]

$$\frac{k(n_B-n_{\bar{B}})}{s} \simeq \frac{kn_B}{s} \simeq \frac{1}{7}\frac{n_B}{n_\gamma} \simeq 10^{-10.8 \pm 1} \quad , \tag{4.27}$$

where n_B, $n_{\bar{B}}$, and n_γ are the number densities of baryons, antibaryons, and photons. If baryon number were absolutely conserved by all interactions, then the net baryon number would have to be postulated as an asymmetric initial condition on the big bang. (Alternately, there could be a large scale separation of baryons and antibaryons in the present universe. This view runs into severe difficulties, however. See 78, 79, 4 and Section V). One of the most exciting implications of grand unification is that baryon number may not be conserved; hence, it is possible that the observed baryon asymmetry was generated dynamically during the first instant after the big bang.

Three ingredients are needed to generate a net baryon number density $n_B-n_{\bar{B}}$ dynamically:[4] (a) B violating interactions (these alone are sufficient to erase or at least dilute an arbitrary baryon number that may have been present initially). (b) Non-equilibrium of the $\Delta B \neq 0$ interactions in the early universe (otherwise $n_B = n_{\bar{B}}$ by CPT and equilibrium). (c) CP and C violation (in order to distinguish between baryons and anti-baryons).

There are a number of possible mechanisms for generating a baryon asymmetry if conditions (a) - (c) are satisfied. The most common scenario is the following:[4] Soon after the big bang the $\Delta B \neq 0$ interactions involving the superheavy gauge bosons (which probably always remain in equilibrium) erase any initial baryon asymmetry. If the expansion and cooling rate of the universe is sufficiently rapid the longer lived superheavy colored Higgs bosons (such as the H^α in (3.4)) may drop out of equilibrium. This will occur if $T < M_H$ at the time of the H decays, which requires $M_H \gtrsim 10^{12} - 10^{13}$ GeV. The H and $\bar{H}$ decays can then generate a net baryon number.

For example, if r and 1-r are the relative branching ratios for $H \to q\ell$ and $H \to q^c q^c$, and $\bar{r}$ and $1-\bar{r}$ are those for $\bar{H} \to q^c \ell^c$ and qq, then the H and $\bar{H}$ decays will generate[80,4] a baryon number

$$\frac{kn_B}{s} \simeq 10^{-2}(r-\bar{r}) \quad , \tag{4.28}$$

where the 10^{-2} is roughly the ratio of Higgs particles to entropy at the time of decay and $(r-\bar{r})/2$ is the average baryon number produced per decay (much more detailed studies of the astrophysical parts of the calculation are described in (81, 82).

$r-\bar{r}$ is a CP violating quantity that must be generated by interferences, such as between the tree and loop diagrams in Figure 7.

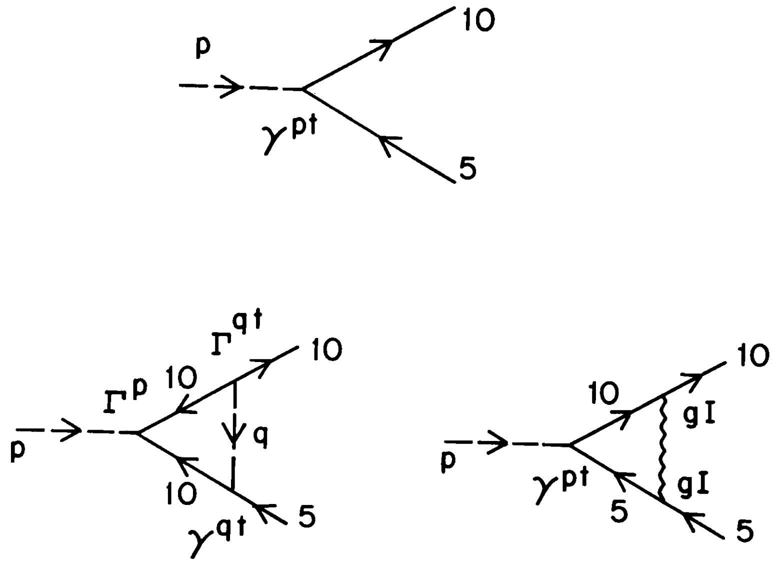

Figure 7. Contributions to $H_p \to q\ell$ in the non-minimal SU_5 model (p labels the Higgs multiplet). $r-\bar{r}$ can be generated by the interference of the first two diagrams.

It is difficult to estimate $r-\bar{r}$ (or even its sign) precisely, but its order of magnitude places useful constraints on model building.[82-87] For example, $r-\bar{r}$ vanishes at the one loop level of Figure 7 in the minimal SU_5 model with a single Higgs 5, because there are not enough independent phases in the Yukawa couplings. In this case, $r-\bar{r}$ is generated at the three loop level and is of order $\gamma^6 \sim (G_F\bar{m}_F^2)^3$, where $\gamma \sim g\, m_F/M_W$ is a Yukawa coupling and m_F is a fermion mass. In the standard 3 family model[86] $r-\bar{r} \sim 10^{-16}-10^{-20}$, which is much smaller than the required $10^{-8}-10^{-10}$. One way to generate an adequate baryon asymmetry in SU_5 is to add more Higgs multiplets [84,85] (all of the elements of which can be superheavy[14]). A nonzero $r-\bar{r}$ can then be generated at one loop level because of the independent Yukawa coupling phases. In this model there are too many unknown Yukawa couplings to compute $r-\bar{r}$ exactly, but one expects $r-\bar{r}$ of order $\gamma^2 \sim G_F\bar{m}_F^2$, which yields[84,85] the reasonable value $\sim 10^{-6}-10^{-10}$. Another possibility[88] is to assume that there are additional fermion families with masses $\gtrsim M_W$. Then $r-\bar{r} \sim \gamma^6$ can be sufficiently large.

There have been a number of recent studies of the baryon asymmetry in the SO_{10} model.[89-92] The difficulty with SO_{10} and similar models is that if the left-right symmetric weak subgroup is preserved down to a mass scale M_{W_R} (as in (4.9)) then charge conjugation C is also a good symmetry down to M_{W_R}. Hence, $r-\bar{r}$

will be suppressed at least[93] by powers of M_{W_R}/M_X. For example, if the SU_{2R} breaking is manifested by a heavy Majorana mass for the right-handed neutrino $N_R \leftrightarrow \nu_L^c$ then there is a cancellation[92] of order $(m_{N_R}^2 - m_{\nu_L}^2)/M_X^2$ between diagrams in Fig. 7 involving intermediate N_R and ν_L. In general, an adequate baryon asymmetry in SO_{10}-type models probably requires $M_{W_R} \gtrsim 10^{12}$ GeV (a possible loophole is mentioned in 93). As we have seen, the low energy coupling constants also suggest a large M_{W_R}.

In conclusion, it is difficult to predict kn_B/s precisely, because its magnitude and sign generally depend on too many unknown or poorly known quantities. It can, however, be used as a qualitative constraint on model building, and on the subsequent history of the universe. For example, non-adiabatic events, such as the supercooling associated with a Coleman-Weinberg type[94] $SU_2 \times U_1$ phase transition, could dilute kn_B/s unacceptably.[95] Other scenarios and possibilities, such as the evaporation of primordial black holes[4,96], the decay of heavy Majorana neutrinos[97], models involving low mass scales[96,98], processes involving out-of-equilibrium light particles[99], particle creation in an expanding universe[100], an anisotropic early universe[101], and TCP violation[102] are discussed in the literature.

The Lepton Asymmetry

The lepton asymmetry of the universe $k|n_L|/s$ could be large if there is a large excess of neutrinos over antineutrinos (or vice versa) in the universe. Observational limits (from the deceleration of the universe) only give the very weak limit[103]

$$\frac{|n_L|}{n_\gamma} \sim \frac{|n_\nu - n_{\bar{\nu}}|}{n_\gamma} < 8 \times 10^4 \ , \tag{4.29}$$

while a value $|n_L| \sim n_\gamma$ would have profound effects on nucleosynthesis[104] and would probably prevent the restoration of $SU_2 \times U_1$ in the early universe.[105]

It is usually argued[4] that such large values of $|n_L|$ will not occur in grand unification, and that typically $|n_L| \simeq n_B$. However, Harvey and Kolb[103] have recently found some examples of models in which $|n_L| \gg n_B$ if there are (a) large initial asymmetries in some quantum number, and (b) approximately conserved global quantum numbers.

Clearly, any brilliant ideas for probing the neutrino black-body radiation would be of great importance.

Superheavy Magnetic Monopoles[4]

Gauge theories in which a simple group G is spontaneously broken at a mass scale M_X to a subgroup $G_1 \times U_1$ containing an explicit U_1 factor necessarily possess 't Hooft-Polyakov magnetic monopoles[4] of mass $m \sim M_X/g^2$. (These are topologically stable classical configurations of the gauge and Higgs fields of the theories). All grand unified theories possess such monopole solutions since U_1^{EM} is unbroken. In particular, $M_X \simeq 10^{14}$GeV in 3-2-1 desert theories, so that $m \sim 10^{16}$ GeV.

These monopoles may have been produced prolifically in the early universe, especially at the time of the phase transition below which the grand unified symmetry was broken. Early estimates[106-108] based on causality arguments found an initial monopole density $n_m/n_B \simeq 10^{-3}$-10^{+4} if the transition was second order (this method of calculation has been questioned[109], however). m-$\bar{m}$ annihilations[106] could reduce a large initial density only to $n_m/n_B \simeq 1$, while observational limits[106,110-113] are many orders of magnitude more stringent than this (Table 6). This is the cosmological monopole problem.

Method	Density Limit (n_m/n_B)	Flux Limit ($m^{-2}yr^{-1}$)
Deceleration of universe[106]	10^{-13}	10^{-1}
Energy in galactic B fields[112,113]	10^{-20} - 10^{-25}	10^{-8} - 10^{-13}
Meteorites[111]	10^{-27} (if $m < 5 \times 10^{14}$ GeV)	---
Quark searches, deep mines[111]	---	300

Table 6. Upper limits on the monopole number density. The corresponding flux limits assume $n_B/n_\gamma = 10^{-10}$ and a monopole velocity $\beta \sim 10^{-2}$ from acceleration in galactic B fields. Ordinary monopole searches would not be sensitive to such massive, slow monopoles.

There have been many mechanisms proposed for suppressing monopole production, mainly by modifying the thermal history of the universe. These include assuming a strongly first order phase transition with extreme supercooling[114-116] (such a transition would lead to an exponentially expanding phase which could also solve the flatness and horizon problems of cosmology.[115] However, it is difficult to understand how such a phase could be "gracefully terminated" while still giving an adequate monopole suppression[116]), assuming that electric charge conservation was violated[117] for $M_X > T > T_c \simeq 1$ TeV, assuming that m-$\bar{m}$ pairs were confined by strings at the time of the $SU_2 \times U_1$ transition[118,119], and assuming that the initial temperature $T_{-\infty}$ of the universe as $t \to -\infty$ was too low to produce monopoles.[120] The theoretical predictions for n_m/n_B are summarized in Table 7.

It can be seen that the theoretical predictions for n_m/n_B range from 0 to values that are clearly ruled out. Unfortunately, these models of the early universe make few if any testable predictions. Experimental searches[101,111,121] for relic monopoles would be of great importance.

Model	n_m/n_B
Early annihilations[106]	< 1
Second order transition	
Causal estimates[106-108]	10^{-3} - 10^{+4}
Thermal estimate [109]	10^{10} exp $[-36\ M_\Phi/M_X]$
First order transition[114-116]	<<< 10^{-3}
U_1^{EM} broken above[117] 1 TeV	0
Confinement[118-119] at $T \sim M_W$	0
$T_{-\infty}$ too low to produce monopoles[120]	0

Table 7. Theoretical predictions for the monopole density.

D. Neutrino Masses and L Violation

The Standard Model

Fermion mass terms are always of the form $m\bar{\chi}_R\psi_L$ + H.C., where ψ_L (χ_R) are two-component left (right)-handed fields. The special case in which

$$\chi_R = \psi_R^c \equiv C\ \bar{\psi}_L^{\ T} \tag{4.30}$$

(i.e. χ_R is essentially the CP or CPT conjugate of ψ_L) yields a Majorana mass term[4]

$$m\bar{\psi}_R^c\ \psi_L = m\psi_L^T\ C\psi_L \tag{4.31}$$

(4.31) can be visualized as an interaction in which two ψ_L fields are annihilated. Hence, a Majorana mass term necessarily violates particle number by two units. The case in which $\chi_R \neq \psi_R^c$ is called a Dirac mass term. It does not violate particle number.

The neutrino masses are zero (and lepton number L is conserved) in the minimal $SU_2 \times U_1$ model. However, if one introduces a Higgs triplet then the Majorana mass term

$$a\ \bar{\nu}_R^c\ \nu_L = a\ \nu_L^T\ C\ \nu_L \tag{4.32}$$

becomes possible for the SU_2 doublet neutrino $\nu_L \underset{CP}{\leftrightarrow} \nu_R^c$. (4.32) transforms as $\Delta L = -2$, $\Delta I_W = 1$, where I_W is the weak isospin.

If one extends the model by introducing SU_2 singlets $\nu_R \underset{CP}{\leftrightarrow} \nu_L^c$ then two additional mass terms are possible. The Dirac mass term

$$m_D\bar{\nu}_R\nu_L = m_D\bar{\nu}_R^c\nu_L^c \tag{4.33}$$

satisfies $\Delta L = 0$, $\Delta I_W = ½$ and can be generated by the ordinary Higgs doublet. The Majorana term

$$S\bar{\nu}_R\nu_L^c = S\nu_L^{cT}\ C\nu_L^c \tag{4.34}$$

transforms as $\Delta L = +2$, $\Delta I_W = 0$, and can be obtained from a bare mass term or

Higgs singlet. In general, (4.32) - (4.34) can all be present, yielding a mass term (for one family)

$$(\bar{\nu}_R^c \; \bar{\nu}_R)\begin{pmatrix} a & m_D/2 \\ m_D/2 & S \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_L^c \end{pmatrix} + \text{H.C.} \tag{4.35}$$

This 2 x 2 matrix can be diagonalized to yield[122,4] two 2-component Majorana (i.e. self-conjugate under charge conjugation) mass eigenstates η_1 and η_2 with masses m_1 and m_2. In the special case of a pure Dirac mass term ($a = S = 0$) η_1 and η_2 turn out to be degenerate; η_1 and η_2 can then be combined to form a four-component Dirac field.

For F families one can write a 2F x 2F mass matrix of the form (4.35), with ν_L (ν_L^c) representing the F doublet (singlet) neutrinos and a, m_D, and S interpreted as F x F matrices. There are 2 F Majorana mass eigenstates in this case.

Grand Unified Theories[4]

The SU_5 model is very similar to $SU_2 \times U_1$ in that the neutrino masses are zero (and the quantum number B-L is conserved) unless (a) an extra Higgs 15 (which contains an SU_2 triplet) is added to the model and/or (b) additional fermions such as SU_5 singlets $\nu_L^c \underset{CP}{\leftrightarrow} \nu_R$ are added.

In the SO_{10} model each family is placed in a 16 dimensional representation that contains an SU_5 singlet ν_L^c. Hence, neutrino mass terms of the form (4.35) will in general be present.

For example, Higgs 10's will yield Dirac masses[4] $m_D = M^u$, where M^u is the F x F mass matrix for the charge 2/3 quarks. If these were the only neutrino mass terms then the predicted neutrino masses would be much larger than the experimental limits. One elegant solution to this problem[123] is to assume that the singlet neutrinos acquire a superheavy Majorana mass term S. S could be generated by the SU_5 singlet component of a Higgs 126 (model I), in which case one expects $S \sim M_X \sim 10^{14}$ GeV. If an elementary 126 is not introduced, S can still be generated by two-loop diagrams if a B-L violating Higgs cubic vertex is allowed[124] (model II). In this case, $S \gtrsim 10^5$ GeV. Assuming that $a = 0$ (small nonzero values will typically be induced by radiative corrections[4], but this should not alter the qualitative conclusions) one then has (for one family) the mass eigenstates

$$\eta_1 \sim \nu_L \; , \; \eta_2 \sim \nu_L^c \tag{4.36}$$

with masses

$$\begin{aligned} m_1 &\sim \frac{m_u^2}{S} \sim 10^{-9} \text{ eV } (<1 \text{ eV}) \\ m_2 &\sim S \sim 10^{14} \text{ GeV } (>10^5 \text{ GeV}) \end{aligned} \tag{4.37}$$

where $m_u \simeq 10$ MeV has been assumed. The numbers in parentheses refer to model II in which S is radiatively induced. Hence, one has a natural explanation of why the ordinary doublet neutrino $\nu_L \sim \eta_1$ is so much lighter than other fermions. The singlet neutrino $\nu_L^c \sim \eta_2$ is superheavy.

There have been a number of studies[125] of the generalizations of this model to F families, with a = 0, S superheavy, and m_D of the order of typical Dirac masses. Various assumptions were made concerning the detailed forms of m_D and S. The results are that for most m_D and S:

i) There are F light Majorana neutrinos, which are mixtures of the doublets ν_{eL}, $\nu_{\mu L}$,... and F superheavy mixtures of the singlets ν^c_{eL}, $\nu^c_{\mu L}$,... .

ii) The mass scale for the electron neutrino is probably too low (i.e. << 1 eV) to be experimentally interesting unless there are intermediate mass scales $S \gtrsim 10^5$ GeV (c.f. 4.37).

iii) In many cases there is a mass hierarchy

$$m_{\nu_e}/m_{\nu_\mu}/m_{\nu_\tau} = m_u{}^n/m_c{}^n/m_t{}^n \quad , \tag{4.38}$$

where, for example, n = 2 or 1 for models I and II above. If one succeeds in generating m_{ν_e} in the 10 eV range, then m_{ν_μ} and m_{ν_τ} are unacceptably large.

iv) There is little $\nu_e - \nu_\mu$ mixing.

It therefore appears that this specific SO_{10} model is not realistic if m_{ν_e} is really in the 10 eV range (although (iii) and (iv) can be evaded for some extreme values of the a priori unknown matrix S).

One can consider more complicated models (e.g. the E_6 model, which contains five 2-component neutral fields per family. An SU_5 model with extra fermions is considered in (126)). If one assumes that all $SU_2 \times U_1$ invariant mass terms are superheavy (the survival hypothesis[4]) then one expects in general that properties (i) and (ii) (but not (iii) and (iv)) will continue to hold. Of course, one can also consider the possibility that Majorana mass terms like S are zero (if forbidden by a symmetry like B-L) or are comparable to Dirac masses.

In summary, most grand unified theories (other than the minimal SU_5) predict neutrino masses at some level. A wide class of these models (those with superheavy $SU_2 \times U_1$ invariant Majorana mass terms) predict that the doublet neutrinos will be Majorana particles, typically with masses in the 10^{-9}eV to 10^2eV range. Experimental guidance in the field is desperately needed. The implications of several types of experimental observations are listed in Table 8.

E. The b Quark Mass

Many grand unified theories with minimal Higgs representations (e.g. SU_5 (SO_{10}) models in which only 5's (10's) contribute to fermion masses) predict relations between the charged lepton and charge -1/3 quark mass matrices[2,4] (which are generated by the same Yukawa couplings). For many of these theories one has the predictions[2]

$$m_b = m_\tau, \; m_s = m_\mu, \; m_d = m_e \quad , \tag{4.39}$$

for the running quark masses, evaluated at M_X. To compare with experiment the quark masses must be renormalized down to low energies.[31,131] The results of Nanopoulos and Ross[131] for m_b evaluated at the relevant scale $\mu \sim m_\Upsilon \sim 10$ GeV are

Observation	Possible Model
ν mass	M, D, C
Radiative decays[127]	
$\nu_2 \to \nu_1\gamma$	M, D, C
ν oscillations	
1st class: $\nu_e \leftrightarrow \nu_\mu \leftrightarrow \nu_\tau$	M, D, C
2nd class[122]: $\nu_L \leftrightarrow \nu_L^c$ (doublet) (singlet)	C
Neutrinoless double β decay[128], $e^- A(Z) \to \mu^+ A(Z-2)$, etc.[129]	M, C (or new Higgs or V + A)
ν magnetic moment[130]	D

Table 8. The implications of several possible observations. M refers to models in which the dominant mass terms are Majorana (i.e., $m_D \simeq 0$). D refers to Dirac masses ($a \simeq S \simeq 0$). C refers to models with comparable Majorana and Dirac mass terms.

shown in Figure 8 and can be represented by the approximate formula

$$m_b(\text{GeV}) \simeq 5.1 + 0.5 \, \ell n \left[\frac{\Lambda_{\overline{MS}}}{0.16 \text{ GeV}} \right] + F-3 \tag{4.40}$$

Phenomenological determinations of m_b include

$$m_b^{exp} = \begin{cases} (4.8 - 5.0) \text{ GeV, ref. } 132 \\ 4.65 \pm 0.05 \text{ GeV, ref. } 133 \end{cases} \tag{4.41}$$

There is some uncertainty (possibly[134] as much as 30%) in the precise connection between the theoretical value of m_b and the experimental value m_b^{exp}. Hence, I have somewhat arbitrarily chosen an experimental range (4.5 - 5.3) GeV to display in Fig. 8. It can be seen that the prediction for F = 3 and small (80-260 MeV) values for $\Lambda_{\overline{MS}}$ is in excellent agreement with the data, while the predictions for four families or the older larger values for $\Lambda_{\overline{MS}}$ tend to be too high.

The predictions in Figure 8 ignore the effects of Yukawa couplings in the RGE. If m_t or the masses of a fourth family are sufficiently large ($m \gtrsim 100$ GeV) then the large Yukawa couplings become important.[135,136] At tree level they increase the predictions[135] for m_b, but the potentially important two loop effects have not been calculated.[136]

The predicted value of m_s at the relevant scale $\mu \sim 1$ GeV is shown in Figure 8. The prediction is high compared to the typical phenomenological estimates[4] (150-300 MeV), but in this case the theoretical uncertainties are very large because one is in the strong coupling regime. A more serious difficulty with (4.39) is the renormalization independent prediction

$$\frac{m_d}{m_s} = \frac{m_e}{m_\mu} \simeq \frac{1}{200} \quad , \tag{4.42}$$

which differs by an order of magnitude from the current algebra determinations[4] $m_d/m_s \simeq 1/20$.

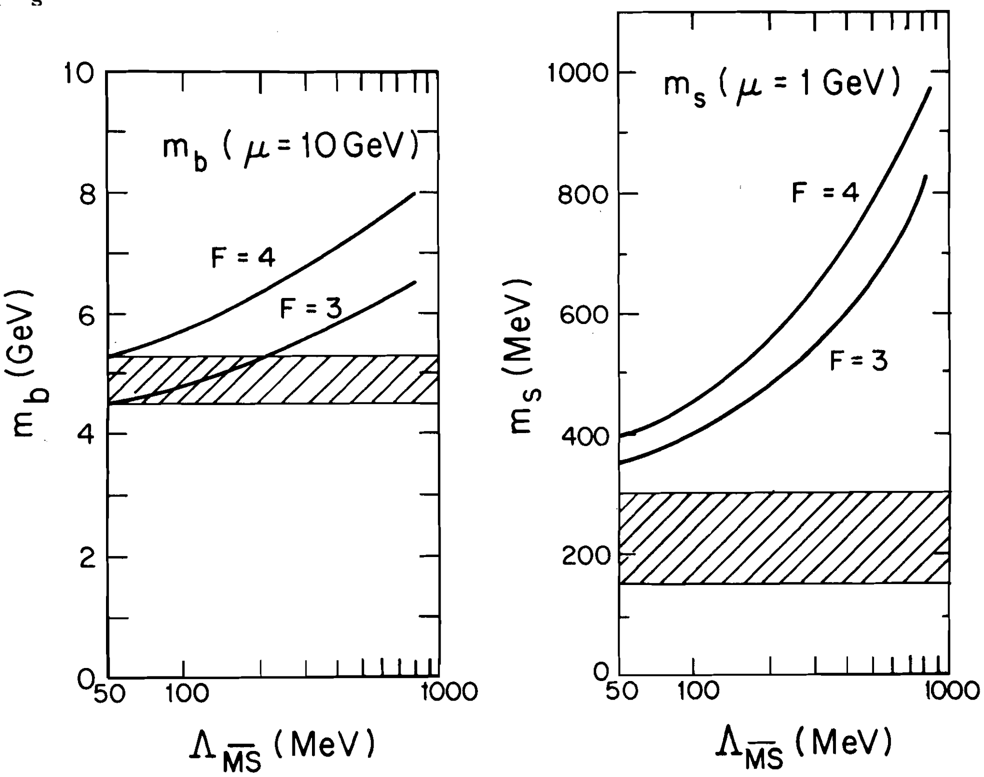

Figure 8. (a) The predictions[131] for m_b for 3 and 4 families, as a function of $\Lambda_{\overline{MS}}$. The phenomenological range 4.5-5.3 GeV is also shown. (b) the predicted value of m_s as a function of $\Lambda_{\overline{MS}}$ along with the phenomenological range 150-300 MeV.

(4.42) can be "fixed up" by adding additional Higgs representations which violate (4.39), but then in general the successful prediction of m_b will also be lost. The failure of (4.42) is one of the most serious problems for minimal SU_5 and related models. Some possible resolutions are referred to below.

V. Theoretical Issues and Problems

In this section I mention some of the outstanding theoretical problems in grand unification and briefly describe some recent developments. A more complete discussion of the less recent issues may be found in Ref. 4.

Fermions

Grand unified theories have been largely unsuccessful in predicting the fermion masses and mixing angles or in explaining the repetition of fermion families. In most of the simple models the fermion spectrum depends on arbitrary

Yukawa couplings, with the only improvement over the standard model being the predictions for m_b (along with the unsuccessful predictions for m_d/m_s). A few very specific models[137-139] yield $m_t \sim m_c m_\tau/m_\mu \sim 20 \pm 2$ GeV, but these generally require extra assumptions, and some models have other difficulties.[4]

It is of course possible that the fermion spectrum really does depend on many arbitrary parameters, but it would be very appealing if at least some of the masses could be predicted or explained. There have been several promising approaches.

One possibility is that effective (non-renormalizable) interactions coupling two fermions to more than one Higgs, such as

$$\psi^T C \psi H^n \Phi^m \tag{5.1}$$

may be generated by diagrams involving internal superheavy particles. (These could be tree diagrams mixing light and superheavy fermions[138], loop diagrams[124,140-149,4], or could even be due to whatever larger theory incorporates quantum gravity[150,151]). The coefficients would be suppressed by inverse powers of the superheavy mass, but this could be compensated for when Φ is replaced by its (superlarge) VEV. Such terms could transform differently from the elementary mass terms $\psi^T C \psi H$. For example, the product HH contains a piece transforming as a 15 of SU_5 and can therefore generate a Majorana mass for ν_L. (The models discussed in IV-E involving superheavy mass terms lead to such a term. See also (151)). Alternately, the term Φ H contains a 45 of SU_5. If the coefficient is sufficiently small such an effective term could resolve the problem with m_d/m_s without significantly affecting m_b. Another possible role of such effective mass terms would be to try to explain the hierarchy $(m_d, m_u, m_e) \ll (m_s, m_c, m_\mu) \ll (m_b, m_t, m_\tau)$. If extra constraints (e.g. due to the embedding of SU_5 in a larger gauge theory) are placed on the theory so that the tree level masses of the first or first two generations are zero, then they may acquire radiative masses from loop diagrams that are naturally smaller than the third family masses by powers of α. These ideas are very attractive. Unfortunately, nobody has succeeded in finding a compelling specific model out of the infinite range of possibilities.

Another interesting approach involves infrared quasi-stable fixed points in the RGE for the running masses.[152,135-136] The idea is that the Yukawa couplings $\Upsilon \sim g\, m_f/M_W$ associated with large mass fermions[153] ($m_f \gtrsim M_W$) are sufficiently large that they must be included in the RGE. The equations then often possess fixed points, which means that the running fermion masses at low energies tend to definite values, independent of their initial value at M_X (for a finite range of initial values). For example, the top quark mass, if sufficiently large, will tend to the value[136] $m_t \sim 240$, which is probably too large phenomenologically[154]. More relevant is the prediction[135,136] that the masses of a possible fourth fermion family (U,D,E,ν_E) are likely to be in the vicinity of $m_U \sim 222$ Gev, $m_D \sim 215$ GeV, $m_E \sim 60$ GeV.

It is perhaps worth summarizing some of the consequences of a massive fourth fermion family. On the positive side it could explain the cosmological baryon asymmetry[88], it has little effect on the predictions[17,20] for M_X and $\sin^2\theta_W$ and it could possibly be of relevance to the hierarchy problem.[155] On the other hand, a fourth family leads to too large a value for m_b in the simple models[31,131,135-136] (although the higher order diagrams could conceivably modify this[135,136]), and if the associated neutrino is massless it creates difficulties for the usual nucleosynthesis scenario[156] (these arguments are affected by neutrino masses[156] or large values[104] for $|n_L| = |n_\nu - n_{\bar{\nu}}|$, however).

Other possible approaches to the fermion mass problem include dynamical symmetry breaking[157], horizontal symmetries[4], and composite fermion models.[158] None of these approaches have led to a compelling model as of yet.

Low Energy Supersymmetry

There has been much recent interest[159-166] in combining grand unification with supersymmetry[167] (SS). In such theories there are new fermion partners (in the GeV-TeV range) of the standard model gauge bosons (gluinos, winos, binos) and the Higgs bosons (Higginos), as well as scalar partners of the ordinary fermions.

The major motivation for such models is that unbroken chiral symmetries might keep the Higginos massless when the gauge symmetry is broken. If the SS is good down to low energies, the Higgs particles will also be kept massless, therefore explaining the hierarchy problem (another motivation is the strong CP problem - see below).

For example, Georgi and Dimopoulos[159] have constructed a supersymmetric SU_5 model. Their model does not solve the hierarchy problem, however, because a ratio of Higgs parameters must be fine-tuned in the breaking of $SU_5 \to G_S$. The $SU_2 \times U_1$ and SS must then be explicitly broken by adding $\simeq 1$ TeV mass terms to the theory. Both of these steps are artificial, but at least the supersymmetry ensures that the fine-tuning will not be upset by radiative corrections. Several authors[160-162] have discussed the more elegant possibility that $SU_2 \times U_1$ and the supersymmetry are broken dynamically by technicolor-like forces[160,161], instantons[162], etc.

The major consequences of the low energy supersymmetry are (a) the existence of supersymmetric partners of the ordinary particles and of the Goldstino (Goldstone fermion) that occurs if the SS is broken spontaneously or dynamically. (b) These new particles modify the RGE for M_X and $\sin^2\theta_W$. For $n_H = 1$, M_X is increased[159,163] to $\simeq 10^{18}$ GeV so that $\tau_p \sim M_X^4 \simeq 10^{45}$ yr would be unobservable.[168] The prediction for $\sin^2\theta_W$ is unaffected, at least to lowest order. (There is some controversy[159,165] over whether the m_b prediction is changed). Observable proton lifetimes could occur for $n_H > 1$ (because of the effects of the extra Higginos on the RGE[165,166]) or if there happened to be colored Higgs bosons with the right mass to mediate observable decays.

CP Violation

CP violation can be either hard (i.e. explicit) or spontaneous[169]. Theories with hard breaking have little trouble generating an adequate baryon asymmetry, but in such theories it is very difficult to understand the small value of the strong CP parameter $\theta \lesssim 10^{-8}$-10^{-10} obtained from limits on the neutron electric dipole moment d_n. Some theories with spontaneous breaking can explain the small value of θ, but they have great difficulty in generating the baryon asymmetry and generally lead to an unacceptable domain structure of the universe (see below). These matters are thoroughly discussed in Ref. (4) and will not be repeated (See 170-171, however).

One possible solution to the strong CP problem is to add extra Higgs fields to the theory in such a way that one has a Peccei-Quinn[172] global symmetry U_1^{PQ} (i.e. the U_1^{PQ} current has a QCD anomaly). When non-perturbative instanton effects are taken into account the vacuum of the theory is expected[172,173] to have $\theta \simeq 0$ and the axion[173], the pseudo-Goldstone boson associated with the breaking of U_1^{PQ}, will have a mass $m_A \sim m_f\, f_\pi/\langle\phi\rangle$ and coupling to nucleons $g_{ANN} \sim f_\pi g_{\pi NN}/\langle\phi\rangle$ ($\langle\phi\rangle$ is the VEV which breaks U_1^{PQ}). If ϕ is a normal $SU_2 \times U_1$ doublet with $\langle\phi\rangle \sim 200$ GeV one has a normal or visible axion, with mass $M_A = 0($ 100 KeV) and $g_{ANN} = 0\ (10^{-3}\ g_{\pi NN})$. On the other hand, if ϕ is an $SU_2 \times U_1$ singlet then $\langle\phi\rangle >> M_W$ is possible.[174] In this case one can have a very light, very weakly coupled particle, referred to as an invisible axion. In fact, astrophysical arguments[175] require $m_A < 10^{-2}$ eV ($\langle\phi\rangle > 10^9$ GeV) in the invisible axion case.

Both visible and invisible axions can be incorporated into grand unified theories. Following the original suggestion of an invisible axion by Kim[174] in a non-unified model and its re-emergence in supersymmetric theories[176], an SU_5 model was constructed by Wise et al.[177] in which $\langle\phi\rangle$ is the grand unified scale M_X (ϕ is a complex 24 in this model), so that $M_A < 10^{-7}$ eV.

Such a model solves the strong CP problem and the axion is essentially unobservable, but the U_1^{PQ} symmetry must be imposed on the Lagrangian by hand. Georgi et al.[178] have therefore constructed a rather complicated SU_9 model in which the U_1^{PQ} emerges automatically (like B-L in SU_5). Models requiring an additional discrete symmetry have been constructed by Kim[179], and Barbieri et al[180] have suggested that U_1^{PQ} may be tied to a horizontal symmetry.

Cosmological Issues

The baryon and lepton number of the universe and the cosmological monopole problem were already discussed in Section IVC.

Many grand unified theories impose a discrete symmetry D on the Lagrangian in addition to the gauge symmetry. Examples include models with spontaneous CP violation (i.e. L is invariant under CP), models such as SU_4^4 involving semi-simple groups (in which a discrete symmetry that interchanges the factors is imposed in order to have a single gauge coupling constant), and the $\Phi \to -\Phi$ symmetry that is often imposed to restrict the couplings in the SU_5 model.[4] A very serious problem with all such models is that if D was unbroken in the early universe, then as the temperature dropped below the temperature T_c at which D was broken, causally separated domains would have formed in which the vacuum differed by D transformations.[181-184] For example, if CP is broken by the VEV of a Higgs field , then one could have $\langle\phi\rangle = v$ in one domain and $\langle\phi\rangle = v^*$ in another. (This would be one mechanism for forming separated matter and antimatter domains.[79,4])

The subsequent evolution of these domains is a complicated dynamical problem, but it is hard to avoid the conclusion that at least one domain wall would still be present within our present horizon ($R \simeq 10^{28}$ cm). The energy per unit area $\sqrt{\lambda}v^3$ (λ is a quartic Higgs coupling constant) in the domain wall would lead to unacceptable anisotropies and deceleration rates in the present universe unless $\sqrt{\lambda}\, v^3 < 10^{-5}$ GeV.[3] However, one expects the much larger values $\sqrt{\lambda}v^3 \sim 10^6$ GeV^3 (10^{42} GeV^3) for symmetries broken at the time of the $SU_2 \times U_1$ (grand unified) phase transitions. Hence, theories with discrete symmetries are in very serious difficulty.

Possible resolutions of the domain wall problem are to assume that D is softly broken by mass terms or dynamically broken before the completion of the phase transition,[185] to assume that D was never restored at high temperature[4], or to postulate that the domain walls were somehow destroyed.[182,184,186] For example, a subsequent strongly first order phase transition could have led to an exponential expansion of the sizes of individual domains.[186]

There has also been some discussion of topologically stable vortices in the early universe, mainly concerning the possibility that they could have been the seeds for galaxy formation.[181-184,187]

Other cosmological issues include other aspects of galaxy formation[188], phase transition[116], and $\bar{p}$ cosmic rays.[189]

Other Problems

Grand unified theories have several serious problems in addition to those already listed. These include the large number of parameters still present (23 in the minimal SU_5 model) and the fact that gravity is still not included.[190]

All gauge theories suffer from naturalness problems - that is, the existence of arbitrary parameters that are experimentally unnaturally small. One example is the strong CP parameter[4] $\theta \lesssim 10^{-8} - 10^{-10}$ of QCD. Another is the famous gauge hierarchy problem. The tiny ratio $(M_W^2/M_X^2) \simeq 10^{-24}$ does not occur naturally, but must be put into the theory by adjusting or fine tuning parameters to this precision[4,191-194]. Dynamical symmetry breaking[157] and low energy supersymmetries [159-166] were largely motivated by the desire to solve the hierarchy problem, but no compelling models have been constructed. (See also (155).) Perhaps the worst naturalness problem is the effective cosmological constant[195] (vacuum energy density) that is generated by all spontaneously or dynamically broken field theories. The cosmological constant generated by $SU_2 \times U_1$ (grand unified) symmetry breaking is $\simeq 50$ ($\simeq 100$) orders of magnitude larger than the observational limits. These must be cancelled to this incredible precision by the primordial cosmological constant (constant in the Lagrangian).

VI. Conclusions

Grand unified theories have many attractive features, including the underlying similarity of all of the observed interactions, the lack of a fundamental distinction between quarks, antiquarks, leptons, and antileptons, the explanation of charge quantization, and the dynamical explanation of the matter-antimatter asymmetry of the universe.

There are many possible grand unified theories, and it is an experimental problem not only to test the basic ideas but to distinguish between models. Observational implications include: (a) coupling constant predictions (the 3-2-1 desert theories are remarkably successful in their predictions of $\sin^2\theta_W$ and ρ); (b) baryon number violation (3-2-1 desert theories predict, for small values of $\Lambda_{\overline{MS}}$, a proton lifetime very close to the present limits); (c) cosmological implications for the baryon and lepton numbers of the universe, magnetic monopoles, domains, galaxy formation, etc.; (d) neutrino masses (most theories predict Majorana masses in the $10^{-9} - 10^{+2}$ eV range); (e) m_b (some of the simple models predict $m_b \sim 5$ GeV for small $\Lambda_{\overline{MS}}$ and F = 3); and (f) new low energy interactions. Various theories predict extended weak groups, dynamical symmetry breaking, horizontal interactions, new light Higgs particles[14], visible or invisible axions, CP violating effects, and various other classes of new scalars and fermions.

However, existing grand unified theories suffer from a number of theoretical problems, such as the lack of an explanation for the fermion spectrum and families, naturalness problems, and the absence of quantum gravity. As attractive as grand unified theories may be, they are clearly not the complete story.

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Discussion

T. Yanagida, Tohoku University: We have a strong constraint on neutrino masses in the minimum SO(10) model in order to explain the cosmological baryon number. The value of baryon asymmetry depends on the charge-conjugation symmetry in SO(10) and the neutrino masses are also related to the C-C-symmetry by so-called seesaw mechanism.

P. Langacker: Yes, that was one of the SO(10) models that I referred to.

R. E. Marshak, Virginia Polyt. Inst.: The speaker had very little time to cover a vast subject. Consequently, he could not point out some important distinguishing features between GUT models. I refer to features like zero or finite mass of the neutrino, B-L a generator of the group or not, the possibility of placing all quarks and leptons of a single generation in one representation or not and the possibility of an intermediate mass scale or not. It is along these lines that I could distinguish between "minimal" SU(5) and "minimal" SO(10) with its differing physical consequences of proton decay or, neutron oscillations, one or two (Majorana) neutrinos per generation, etc.

P. Langacker: As I tried to emphasize, there are a great variety of grand unified theories including the type that you are referring to. I think it is very important to probe these ideas (and other) in every possible way.