EXCLUSIVE PROCESSES IN QCD

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I. Introduction

In the last few years we have achieved an enormous increase in our ability to calculate high energy processes from perturbative QCD.[1] Originally it was thought that only sufficiently inclusive processes were amenable to perturbative techniques. We realize now, however, that almost every high energy process has its Q^2 or momentum transfer dependence determined to a large extent by perturbative QCD. In the end I think that the inclusive processes will furnish more precise tests of QCD, but our understanding of exclusive processes adds a depth of knowledge of short distance phenomena which is very satisfying.

In this talk I shall concentrate on purely exclusive processes[2] although I shall in a few instances deal with inclusive processes near a kinematic limit where their inclusiveness may be in question. In Sec. II form factors and exclusive decays of heavy quarkonium states will be discussed. In Sec. III elastic wide angle elastic scattering will be considered with emphasis placed on the energy dependence for a fixed angle. The $x \to 1$ limit of structure functions is discussed in Sec. IV. This is a limit which matches on, in a rather complicated way, with transition form factors. In Sec. V the idea of intrinsic charm is considered, mostly from a conceptual viewpoint as to its definition and possible existence. In Sec. VI there is a brief discussion of calculations of matrix elements which occur in deeply inelastic scattering by use of a bag model. In Sec. VII wee parton cancellations and Sudakov corrections for μ-pair production are considered. Sec. VIII concerns soft particle production and the multiplicity of hadrons in a jet.

II. Elastic Form Factors

One of the striking theoretical developments in QCD in the past few years has been the realization that the asymptotic behavior of elastic form factors is determined by perturbative QCD.[3-9, 1] For particles which communicate with a quark-anti-quark pair, mesons, this result is now firmly established. For baryons the result is likely true as a consequence of a Sudakov suppression of unwanted terms. We begin with a discussion of the asymptotic behavior of the pion form factor.

A. Meson Form Factors

An obvious question occurs. How can one calculate the asymptotic behavior of the pion form factor when one cannot give any reasonable description of a pion as a Nambu-Goldstone boson in QCD? The answer to this question is that factorization allows one to separate a Q^2 dependent part of the form factor which is calculable in perturbation theory from a Q^2 independent part which is not directly calculable but which is related, at least partially, to the pion weak decay constant. Thus, we begin by considering the transition form factor for a q-$\bar{q}$ pair of momentum p to go into a q-$\bar{q}$ pair of momentum p' under the influence of the electromagnetic current. Such a process is illustrated in Fig. 1. We shall take a frame where $p_\mu = (p_0, 0, 0, p)$ and $p_\mu' = (p_0', 0, 0, -p)$. We suppose p^2, p'^2, $p \cdot k$, $p' \cdot k'$, k^2 and k'^2 remain fixed as $Q^2 = 2p \cdot p'$ becomes large. We shall obtain

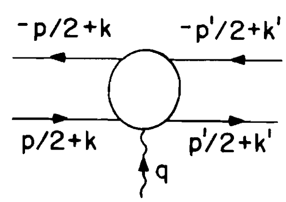

Fig. 1
A q-$\bar{q}$ transition form factor.

the pion form factor by letting $p^2 \to m_\pi^2$ and $p'^2 \to m_\pi^2$ at the end of the calculation. Call $x = |\vec{k}|/|\vec{p}|$ and $x' = |\vec{k}'|/|\vec{p}'|$. Then after a suitable projection of spinor indices we consider a scalar transition form factor $T(p, k; p', k')$.

Order by order in perturbation theory one proves for large Q^2 that

$$T(p, k; p', k') = \int_{-1}^{1} dx\, dx'\, v(p, k, x)\, T(x, Q^2, x')\, v(p', k', x') \ . \tag{1}$$

Corrections to (1) are of order (m/Q). $v(p, k, x)$ depends on the details of near mass shell QCD so we do not expect to be able to reliably calculate such an object. $T(x, Q^2/\mu^2, x')$, on the other hand, depends only on far off shell behavior and is calculable by the renormalization group. We expect (1) to be true in general and an evaluation of $T(x, Q^2, x')$ to be possible within perturbative QCD.

Using the renormalization group one finds

$$T(x, Q^2/\mu^2, x') \underset{Q^2 \to \infty}{\to} \sum_{N, N' = 0, 2, 4, \cdots} C_N(x)\, T_{NN'}(Q^2/\mu^2, g^2)\, C_{N'}(x') \tag{2}$$

with

$$T_{NN'}(Q^2/\mu^2, g^2) = (\ln Q^2/\mu^2)^{-\gamma_N - \gamma_{N'}}\, T_{NN'}(1, g^2(Q^2)) \ . \tag{3}$$

In general one cannot evaluate

$$\int_{-1}^{1} dx\, v(p, k, x)\, C_N(x) \ , \tag{4}$$

however, for $N = 0$ and $p^2 \to m_\pi^2$, (4) is proportional to the weak decay constant f_π. Thus one may write, taking $p^2, p'^2 \to m_\pi^2$,

$$F_\pi(Q^2) = 16\pi f_\pi^2 \frac{\alpha(Q^2)}{Q^2} \left[1 + \sum_{N=2,4,\cdots} c_N (\ln Q^2/\mu^2)^{-\gamma_N} \right]^2 \tag{5}$$

where the γ_N are positive and increasing with N.

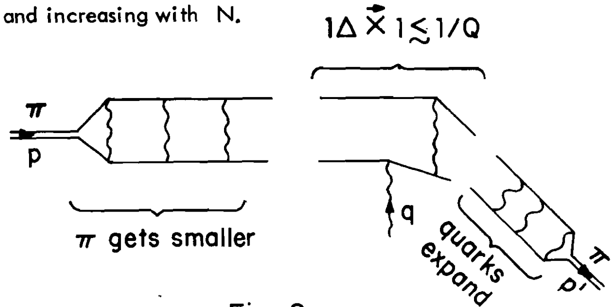

Fig. 2

A space-time description of the pion form factor.

Now that we have seen that the asymptotic behavior of the form factor of the pion (or of any other meson communicating with a q-$\bar{q}$ system) is calculable, let me take a moment to describe, in terms of a space-time picture, why such form factors are calculable. In Fig. 2 I have illustrated what occurs. Suppose the virtual photon acts at $x_\mu = 0$. Then at a time $t_1 \approx -p/\mu^2$, with $\mu \approx 350$ MeV, the wave function of the pion starts to become small. That is, at this time the valence q-$\bar{q}$ pair begins to shrink in transverse size and any gluons present start to be absorbed. By the time the photon acts, the q-$\bar{q}$ pair is within a size $|\Delta \vec{x}| \lesssim 1/Q$. The virtual photon turns one of the quarks around. Then, within a time $\Delta t \lesssim 1/Q$ the q (or $\bar{q}$) which has been struck interacts with the spectator $\bar{q}$ (or q) turning it around also. At this point the q-$\bar{q}$ pair starts to expand to a normal pion size, which process is completed at a time $t_2 \approx p/\mu^2$. The process of collapse and expansion of the q-$\bar{q}$ pair (and the accompanying gluons) is calculable in perturbative QCD since short distances are involved. The non-calculable part is the wave function of the pion before the collapse and after the final expansion of the q-$\bar{q}$ pair in the final state. There is no teleology involved here, of course. The wave function of the pion is simply fluctuating, in the interaction picture, as it normally does. These short distance fluctuations are calculable in QCD.

B. Baryon Form Factors

In order to emphasize that the calculation of baryon form factors is not on so sound a footing as it is for mesons, let me outline two possible procedures for dealing with baryon form factors.[8]

Procedure 1. Assume factorization of the 3-quark transition form factor illustrated in Fig. 3. Thus one supposes one has an analog of Eq. 1. Next, apply the renormalization group just as has been outlined for the meson case. This procedure gives a well-defined answer for the asymptotic behavior of $G_M(Q^2)$ for the nucleon. And

$$G_M(Q^2) \approx \frac{32\pi^2}{9} \frac{\alpha^2(Q^2)}{Q^4} \sum_{n,m} b_{nm} (\ln Q^2/\mu^2)^{-A_n-A_m} \tag{6}$$

results.[5] Unfortunately, here none of the b_{nm} can be related directly to any well measured process.

Procedure 2. Begin calculating in perturbation theory for the 3-quark transition form factor. If the quark has a nonzero mass, one finds an answer which disagrees with Procedure 1. In fact one finds a series of Sudakov logarithms in this case.[8]

What should one do? Very possibly, even probably, Sudakov effects suppress the bad logarithms in Procedure 2 and make Procedure 1 the correct prescription. Hopefully, one will be able to prove that this is the case in the near future.

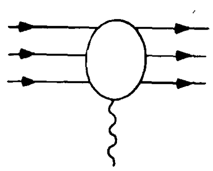

Fig. 3

A three quark form factor.

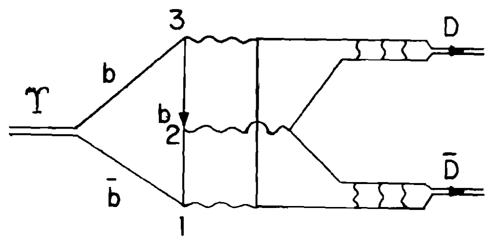

Fig. 4

$\Upsilon \to D + \bar{D}$

C. Υ-Decays

Exclusive decays of the Υ are formally very similar to form factors.[10] The following processes have been considered. (i) $\Upsilon \to K\bar{K}$ and $\Upsilon \to D\bar{D}$,[11] (ii) $\Upsilon \to$ proton-anti-proton,[12] (iii) $\Upsilon \to \gamma + \pi + \pi$.[13] Let me describe very briefly Chao's calculation of $\Upsilon \to D\bar{D}$. Consider the graph shown in Fig. 4 where $\Upsilon \to 3g \to D\bar{D}$ is illustrated. Although the Υ is not an especially small object the annihilation takes place within a short distance. That is, the points 1, 2, 3 of Fig. 4 are within a spatial size $|\Delta\vec{x}| \lesssim 1/M_\Upsilon$. The decay $\Upsilon \to D\bar{D}$ then consists of (i) a collapse of the $b-\bar{b}$ in the upsilon to a point, (ii) the transition of $b-\bar{b}$ into a pair of oppositely moving q-$\bar{q}$ pairs, and finally (iii) the fitting of these quark-anti-quark pairs into the wave functions of the D and $\bar{D}$. (i) is exactly the same as for the process $\Upsilon \to e^+e^-$ and so has been measured. (ii) is calculable in QCD and has been calculated by Chao. (iii) requires the knowledge of the D's wave function. This is model dependent, just as the c_N in Eq. 5 are model dependent, and becomes the major uncertainty in the calculation. Chao finds

$$\frac{\Gamma_{\Upsilon \to K\bar{K}}}{\Gamma_{\Upsilon \to \text{all hadrons}}} \approx 10^{-6} \left(\frac{f_K}{f_\pi}\right)^4$$

and

$$\Gamma_{\Upsilon \to D\bar{D}} \approx (50\text{-}100) \left(\frac{f_D}{f_K}\right)^4 \Gamma_{\Upsilon \to K\bar{K}} .$$

III. Wide Angle Elastic Scattering

Although I do not think it likely that wide angle elastic scattering will become a good quantitative testing ground for QCD, it is an interesting theoretical question to what extent energy and angular dependences are predictable for these processes. Let me begin my discussion by giving the contrasting power counting rules suggested by Brodsky, Farrar[14] and by Landshoff.[15] For simplicity the discussion will be phrased in terms of $\pi-\pi$ elastic scattering.

In the Brodsky, Farrar picture elastic scattering occurs much as does the hadron scattering part of an elastic electron hadron interaction. Refer to the graph shown in Fig. 5. According to Brodsky and Farrar, before the scattering the valence quark-anti-quark pair (3, 4) in the pion comes within a transverse distance $|\Delta\underline{x}| \lesssim 1/\sqrt{s}$ as does the pair (1, 2). The Fock states of the two pions, in the center-of-mass system, are supposed to consist only of their q-$\bar{q}$ valence pairs. Upon colliding, two interactions between the pions are necessary in order to turn both q and $\bar{q}$ around. A simple counting of variables then indicates the necessity of a further hard interaction in order that the outgoing q-$\bar{q}$ systems have low masses. Dimensional counting for these various hard scatterings, three of them, gives $\frac{d\sigma}{dt} \underset{\substack{s\to\infty \\ \theta \text{ fixed}}}{\to} \frac{1}{s^6} f(\theta)$.

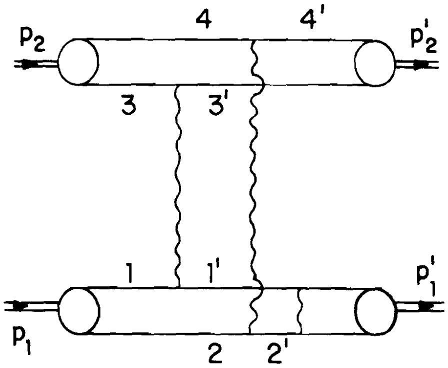

Fig. 5

A hard scattering contribution to wide angle elastic scattering.

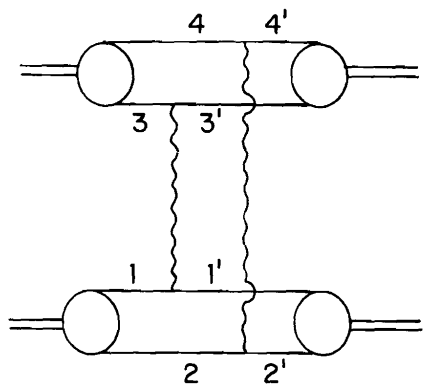

Fig. 6

The Landshoff graph.

In the Landshoff picture, see Fig. 6, the pions also consist only of their q-$\bar{q}$ valence pairs. However, now one views the q and $\bar{q}$ in a particular pion as not being close together in transverse coordinate space. (In momentum space this means that the q and $\bar{q}$ are not to be very far off mass shell.) There are supposed to be two scatterings of almost identical angle which turn the q-$\bar{q}$ pairs around, leaving them in a low mass system. That is, $\theta(1, 1') \approx \theta(2, 2') \approx \theta(3, 3') \approx \theta(4, 4')$. A power counting for such scatterings leads to

$$\frac{d\sigma}{dt} \underset{\substack{s\to\infty \\ \theta \text{ fixed}}}{\to} \frac{1}{s^5} \tilde{f}(\theta).$$

Thus, the Landshoff type of scattering would seem to dominate wide angle elastic scattering. However, there is an apparent danger in the situation envisioned by Landshoff. The valence q-$\bar{q}$ in the pion are not close together, and hence do not form a very local color neutral system. Thus it is unlikely that collinear and soft gluons will be absent. However, gluons must be absent if the scattering is to be purely elastic. In general the price one pays for picking out a particular piece of the wave function not having collinear and soft gluons is a Sudakov factor. (In the Brodsky, Farrar type of scattering the q and $\bar{q}$ are close together and so form a local color singlet. A color singlet does not need to have a gluon cloud and so one expects no Sudakov suppression in that case.)

In fact there are Sudakov corrections to the Landshoff graphs. After taking into account leading double logarithmic effects one finds[1]

$$\frac{d\sigma}{dt} = \frac{1}{\Delta^2}\left|\int_{-1}^{1} dx_1\, dx_2\, dx_1'\, dx_2' \sum_i \phi_\pi(x_1, \Xi_i \Delta)\, \phi_\pi(x_2, \Xi_i \Delta)\, \phi_\pi(x_1', \Xi_i \Delta)\right.$$
$$\left.\phi_\pi(x_2', \Xi_i \Delta)\, e^{-2c\left[\ln \Delta(\ln\ln \Delta - \ln\ln \Xi_i \Delta) - \ln 1/\Xi_i\right]} H_i(p, p', x, x')\right|^2 \qquad (7)$$

where Ξ_i is a combination of x_1, x_2, x_1', x_2' and θ. H_i is a particular "hard" part and ϕ_π is a renormalization group evolved wave function of the pion. (A judicious use of the techniques of the type developed by Collins and Soper[16] might allow one to show that (7) is the dominant term in an asymptotic series for $d\sigma/dt$.) Numerically it turns out that one is somewhat closer to Brodsky, Farrar than to Landshoff since $\frac{d\sigma}{dt} \underset{\Delta\to\infty}{\to} \frac{1}{\Delta^5}\, \Delta^{-4c \ln \frac{4c+1}{4c}}$ as far as power dependences are concerned. $c = \frac{4c_F}{11 - \frac{2}{3} n_f}$.

IV. $x \to 1$ Limit of Structure Functions

There are at least five distinct regions, in x, for a deeply inelastic structure function. These regions are: (i) $x \to 0$; (ii) x finite, away from 0 or 1; (iii) $1 - x < 1$ but $\ln \frac{1}{1-x}/\ln Q^2 < \lambda$; (iv) $\ln \frac{1}{1-x}/\ln Q^2 > \lambda$ but $Q^2(1-x)$ large and λ to be specified later; (v) $Q^2(1-x)$ fixed as Q^2 grows. Region (i) is the Regge region for which no systematic discussion exists in the QCD context. Region (ii) is the normal region where one may directly apply renormalization group improved perturbation theory in order to confront data with QCD. Region (v) is the region of elastic and transition form factors which we have previously discussed. We shall now summarize the status of our understanding of regions (iii) and (iv).

Suppose for fixed x away from zero or one we let Q^2 become large and keep only the dominant twist (dominant power in Q^2) contributions to νW_2 for, say, the proton. If we now let x go near 1 there are contributions like $\alpha(Q^2) \ln^2 1/1-x$ which vitiate a standard renormalization group calculation when $\alpha(Q^2)\ln^2 1/1-x$ is of order 1. (In fact a careful analysis reveals terms more complicated in structure than simple powers of $\alpha(Q^2) \ln^2 1/1-x$.) So far we have no systematic treatment of all the leading and subleading logarithms in $1 - x$. Such a systematic analysis should be possible. Brodsky, Lepage[17] and Amati, Bassetto, Ciafaloni, Marchesini, Veneziano[18] have suggested that one may sum the leading $1 - x$ singular terms simply by a judicious modification of the argument of the running coupling in a renormalization group or Altarelli-Parisi equation. This conjecture has been verified in an Abelian theory by an explicit calculation of the leading Sudakov effects in the presence of a running coupling.[1] (In an axial gauge the non-Abelian Sudakov case follows directly from the Abelian calculation.) One finds

$$\nu W_2 = c(1-x)^3 \exp\left\{-\frac{4c_F}{11-\frac{2}{3}n_f}\left[\ln Q^2 \ln\ln Q^2 - \ln Q^2(1-x)\ln\ln Q^2(1-x) - \ln\frac{1}{1-x}\ln\ln\frac{1}{1-x}\right]\right\}. \qquad (8)$$

However, until one has a method of systematically improving Eq. 8, this formula should be taken with some caution. In particular the scale to be used in (8) will be set by subleading effects. This is our description of region (iii).

In addition to the leading twist terms, those terms in νW_2 which have a factor of $(1-x)^3$ near $x = 1$ but which have no inverse power of Q^2, there are in general non-leading twist terms. In particular we should expect $(1-x)^2/Q^2$ terms coming from twist-4 operators, $1-x/(Q^2)^2$ terms coming from twist-6 operators and $1/(Q^2)^3$ terms coming from twist-8 operators. The twist-4 and twist-6 terms also have Sudakov factors although the twist-8 contribution has no Sudakov suppression. The twist-2 and twist-8 contributions are comparable when $1/(Q^2)^3$ is of the same size as the expression on the right hand side of Eq. 8. This happens when $\ln 1/1-x = \lambda \ln Q^2$ with λ satisfying $3(1-\lambda) = 4c_F/11 - \frac{2}{3} n_f \left[\ln 1/1-\lambda + \lambda \ln 1-\lambda/\lambda\right]$. Thus, for $\ln\frac{1}{1-x}/\ln Q^2 > \lambda$ the twist-8 contribution dominates the twist-2 contribution. (Whether or not there is a region either like $\lambda_i < \ln\frac{1}{1-x}/\ln Q^2 < \lambda$ or like $\lambda < \ln\frac{1}{1-x}/\ln Q^2 < \lambda_i$ in which twist-4 or twist-6 terms dominate νW_2 is a question which can be answered only when the Sudakov calculation has been done for those terms in the Wilson expansion. In the following we suppose, for simplicity, that twist-4 and twist-6 terms never dominate νW_2.)

So long as $Q^2(1-x)$ remains large, but $\ln\frac{1}{1-x}/\ln Q^2 > \lambda$, we are in region (iv). In this region we can write[6,8]

$$\nu W_2 \approx \frac{G_M(Q^2)}{Q^2}\,\alpha(Q^2)\sum_{N=0}^{\infty} C_N\left[\frac{\ln Q^2}{\ln Q^2(1-x)}\right]^{-2A_N} \tag{9}$$

where the A_N are anomalous dimensions of a particular 3-nucleon operator and C_N is a pure, calculable, N-dependent number.

V. Intrinsic Charm

We have heard earlier in this conference a number of discussions on the phenomenological predictions of the intrinsic charm proposal of Brodsky, Hoyer, Peterson and Sakai[19] (BHPS). Rather than attempting to evaluate the successes or failures of this hypothesis, I shall here try to explain the idea of an intrinsic heavy quark component in a low mass hadron and to show the conceptual difficulties which arise when one imagines that this is a long lived component.

To begin let us recall the Witten,[20] Georgi-Politzer[21] discussion of heavy quark, say charm, contributions to deeply inelastic structure functions. One may write

$$\int_0^1 \nu W_2(x,Q^2)\,x^n dx = \sum_{i=1}^{3}(p\,|\,O_n^{(i)}\,|\,p)\;E_n^{(i)}(Q^2) + (p\,|\,O_n^{(c)}\,|\,p)\;E_n^{(c)}(Q^2) \tag{10}$$

where the $O_n^{(i)}$ are local operators corresponding to ordinary quarks and gluons, and $O_n^{(c)}$ corresponds to a charmed quark operator $\bar{c}(x)\,\gamma_{\mu_1}\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n}c(x)$. Charmed quark effects show up in three places. (i) There are charm effects in $E_n^{(i)}(Q^2)$; (ii) $(p\,|O_n^{(i)}|\,p)$ may depend on charmed quarks; and (iii) the second term on the right hand side of Eq. (10) depends explicitly on charmed quark effects. The Symanzik,[23] Appelquist-Carazzone[24] theorem tells one that (ii) and (iii) give effects of order μ^2/m_c^2 where $\mu \approx 300$ MeV. (i) can give terms of order 1 which are calculable at large Q^2. It is the calculation of these order 1 terms that Witten and Georgi-Politzer discussed. Such contributions correspond to a small value of x in νW_2. Intrinsic charm, a large x component, must be a part of the μ^2/m_c^2 terms that Witten and Georgi-Politzer are unable to handle systematically.

Write the state of the proton as

$$|\,\psi_p\rangle = \sqrt{1-\alpha^2}\;|\,\psi_p\rangle^{\text{normal}} + \alpha\,|\,\psi_p\rangle^{c\bar{c}} \tag{11}$$

where $|\,\psi_p\rangle^{\text{normal}}$ is that part of the proton state which can be given only in terms of light quark and gluon Fock space states in the proton's rest frame. $|\,\psi_p\rangle^{c\bar{c}}$ is that part of the proton state which includes $c\bar{c}$ pairs in its Fock space description with renormalization done on a light mass scale. α^2 is of order μ^2/m_c^2. Bag model calculations of Donoghue and Golowich[24] find $\alpha^2 \approx 0.01$-0.02. BHPS argue that such an intrinsic charm contribution is sufficient to account for the large x_F charm production observed at the ISR. Bertsch, Brodsky, Goldhaber and Gunion[25] suggest that diffractive charm production may dominate ordinary diffractive dissociation in hadronic collisions with large nuclei. Roy[26] has then noted that the intrinsic charm component of BHPS should give a significant contribution to the large x part of νW_2, partially compensating the scaling violations due to ordinary quarks. The status of these phenomenological considerations can be found elsewhere in the proceedings of this conference.

In Fig. 7 I have illustrated the interaction picture evolution of quanta in a proton bound state. Solid lines are quarks, wiggly lines are gluons and heavy solid lines are charmed quarks. The time scale for emission and absorption of a gluon, in an infinite momentum frame of the proton, is $\tau_0 \approx p/\mu^2$. There are also $c\bar{c}$ fluctuations, illustrated in Fig. 7a, on a time scale $\tau_1 \approx p/m_c^2$. If k^2, in Fig. 7a, is greater than m_c^2 then these contributions are calculable by the techniques of Witten and Georgi-Politzer. If $k^2 < m_c^2$, such contributions are part of the μ^2/m_c^2 contribution which is not calculable by perturbative QCD. However, such a $c\bar{c}$ component is not the intrinsic charm component of BHPS since the heavy quarks do not in general carry a particularly large part of the momentum of the proton. Also, such a short lived fluctuation cannot contribute to diffractive dissociation since diffractive effects should average over times of order p/μ^2. We suspect it is these short lived $c\bar{c}$ fluctuations which Donoghue and Golowich have calculated in the bag model. Thus we believe that the bag calculation has little to do with the idea of intrinsic charm.

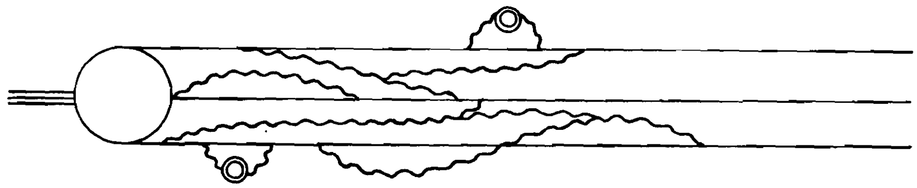

Fig. 7a A short lived $c\bar{c}$ fluctuation.

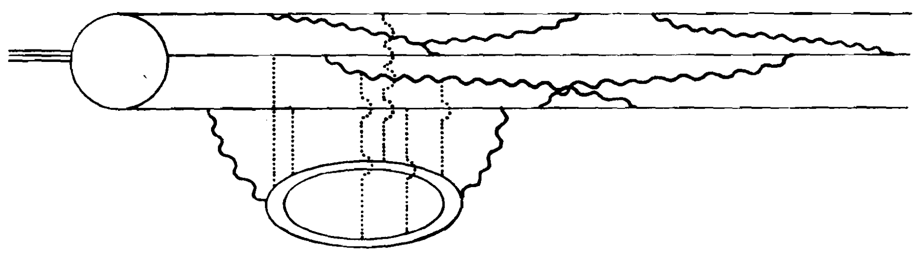

Fig. 7b A possible long lived intrinsic charm component.

The hypothesized intrinsic charm must correspond to a long lived $c\bar{c}$ pair as illustrated in Fig. 7b. Here we suppose that the $c\bar{c}$ pair lives a time $\tau_0 \approx p/\mu^2$. However, in order that this be the case it is necessary that the $c\bar{c}$ pair interact with the rest of the bound state on a time scale $\tau_1 = p/m_c^2$ so that the $c\bar{c}$ does not separate spatially from the rest of the proton. The c and $\bar{c}$ have a lower velocity then the light components of the proton. It is these rapid interactions which give most of the momentum p to the $c\bar{c}$ system in the infinite momentum frame. Unfortunately, there does not seem to be a QCD interaction strong enough to connect the $c\bar{c}$ pair with the rest of the proton with anything like a τ_1 scale of interaction. Thus we feel that it will be very hard to build a realistic model of an intrinsic, long lived, charm component of the proton.

As a theoretical prediction I think the idea of intrinsic charm is suspect. If it should happen that phenomenology forces us into the conclusion that a long lived intrinsic charm component of the proton exists, we would then have the task of trying to find an interaction strong enough to keep the $c\bar{c}$ system bound in the proton.

VI. Calculation of Matrix Elements

In many of the problems we have discussed up to this point it has been possible to calculate Q^2 dependences, but absolute predictions have been elusive. With respect to some of the apparent successes of QCD, in particular for deeply inelastic structure functions, it has even been suggested that the $1/Q^2$ corrections to the dominant term in QCD may be important. For this reason it is important to attempt, admittedly model dependent, calculations of matrix elements in QCD.

Jaffe and Ross[27] have calculated the matrix elements of the twist-2 non-singlet operators occurring in deeply inelastic scattering. The calculation was done using an MIT bag model. Combining these matrix elements, with a renormalization group calculation for the Wilson coefficients, a good agreement between theory and experiment was found. This agreement could be taken as indirect evidence that higher twist effects are small. It is much better, however, to have some direct evidence as to the size of higher twist terms.

Jaffe and Soldate[28] have begun an analysis and calculation of higher twist terms following the program discussed by Politzer.[29] So far they have calculated, completely, one higher twist non-singlet term. If one writes

$$\int_0^1 \nu W_2^{N.S.}(x, Q^2)\, dx = \text{Twist-2 part} - \frac{1}{Q^2} (p \,|\, O \,|\, p)\, E_2(Q^2) \quad , \tag{12}$$

they have evaluated $(p|O|p)$. $E_2(Q^2)$ is taken from the parton model. Normalizing $E_2 = 1$

$$\frac{1}{Q^2}(p|O|p) = \frac{\Delta^2}{Q^2} \tag{13}$$

with $\Delta \approx 60$ MeV is found. At $Q^2 = 5$ the higher twist contribution is $< 1\,\%$ of the lower twist term. It is important to have this calculation extended to other moments to be sure that the above calculated smallness of the twist-4 term is not accidental.

Calculations of this variety can be very important in letting us decide where perturbative QCD is most likely to give accurate predictions. It would be very nice to have some calculations of the type of matrix elements which appear in form factors and in Υ-decays.

VII. μ-Pair Production

μ-pair production is one of the oldest and most interesting processes for which a parton model and QCD analysis has met with some considerable success. The process is illustrated in Fig. 8. With $x_i = Q^2/2p_i \cdot q$ and $\underset{\sim}{q}$ the transverse momentum of the μ-pair in the center-of-mass of the colliding hadrons the parton model, with QCD modified parton distributions, predicts

$$\int \frac{d\sigma}{d^4q}\, d^2\underset{\sim}{q} = \frac{8\pi\alpha^2}{3N_c(Q^2)^2} \sum_F \ell_F^2 \left\{ x_1\, P^F(x_1, Q^2)\, x_2\, P^{\bar{F}}(x_2, Q^2) + 1 \leftrightarrow 2 \right\}. \tag{14}$$

$\alpha(Q^2)$ corrections to the above formula give the famous "κ" factor which brings theory and experiment into reasonable agreement.

At very high energies there are two distinct regions where $\underset{\sim}{q}^2/Q^2 \ll 1$.[1,30,31] (i) When $\underset{\sim}{q}^2 \lesssim (Q^2)^{c/1+c}$, with $c = 4c_F/11 - \frac{2}{3}n_F = 16/25$ for 4 flavors, one has

$$\frac{d\sigma}{d^4q} \approx \frac{8\pi\alpha^2}{3N_c(Q^2)^2}\left(\frac{Q^2}{\Lambda^{*2}}\right)^{-c\ln\frac{1+c}{c}} \sqrt{\frac{2c\ln Q^2}{\pi(1+c)^2}} \sum_F \ell_F^2 \left\{ \xi \frac{\partial}{\partial\xi} x_1 P^F(x_1,\xi)\, \zeta\frac{\partial}{\partial\zeta} x_2 P^{\bar{F}}(x_2,\zeta) + 1 \leftrightarrow 2 \right\}_{\xi=\zeta=(Q^2)^{\frac{c}{1+c}}} \tag{15}$$

(ii) For $\underset{\sim}{q}^2 \gtrsim (Q^2)^{\frac{c}{1+c}}$ one has

$$\frac{d\sigma}{d^4q} \approx \frac{8\pi\alpha^2}{3N_c(Q^2)^2} \sum_F \ell_F^2 \exp\left\{-c\left[\ln Q^2(\ln\ln Q^2 - \ln\ln \underset{\sim}{q}^2) - \ln Q^2/\underset{\sim}{q}^2\right]\right\}$$
$$\times\, \underset{\sim}{q}^2 \frac{\partial}{\partial \underset{\sim}{q}^2}\left\{ x_1 P^F(x_1, \underset{\sim}{q}^2)\, x_2 P^{\bar{F}}(x_2, \underset{\sim}{q}^2) + 1 \leftrightarrow 2 \right\}. \tag{16}$$

The general behavior of Eqs (15) and (16) is illustrated in Fig. 9.

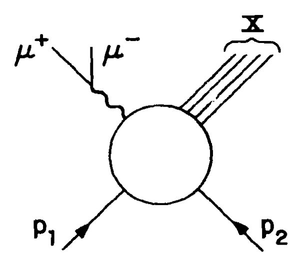

Fig. 8

μ-pair production.

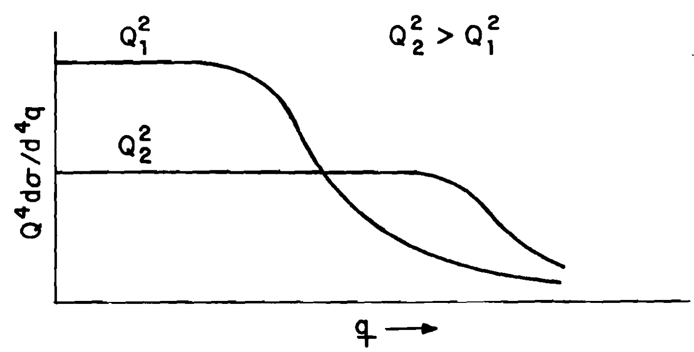

Fig. 9

$\underset{\sim}{q}^2$-distribution in μ-pair production.

Qualitatively, the striking effect shown in Fig. 9 is the decrease of $d\sigma/d\underline{q}^2$, at $\underline{q}^2=0$, as Q^2 increases. At the same time the $\underline{q}^2$ distribution of $d\sigma/d^4q$ broadens, the region of flatness in $\underline{q}^2$ increasing as $(Q^2)^{16/41}$. These effects should be very evident at ISABELLE. The absolute normalization of $d\sigma/d^4q$ at $\underline{q}^2=0$ is given once one knows Λ^*. Collins and Soper[16] have given a prescription for calculating Λ^* in terms of the QCD parameter Λ. This prescription involves the evaluation of two loop diagrams which have yet to be done.

Equations 15 and 16 are not yet at a rigorous level, although there is a prescription by Collins and Soper[16] for calculating corrections to these equations. In fact, as emphasized by Bodwin, Brodsky and Lepage (BBL),[32] even Eq. 14 is not a rigorous consequence of QCD. The problem is soft gluon exchanges which have been a stumbling block in attempts to prove factorization in processes where two or more hadrons of fixed momenta are involved. In case all the explicitly observed hadrons are in the final state, for example in $\gamma(Q^2)\to h(p_1)+h(p_2)+\cdots+h(p_n)+x$ with $\gamma(Q^2)$ a highly off-shell photon and $h(p_i)$ an observed hadron with momentum p_i, Collins and Sterman[33] were able to give convincing arguments that all soft gluon exchanges cancel order by order in perturbation theory. They were not able to prove an analogous cancellation for μ-pair production because of some troublesome $i\epsilon$'s. BBL then gave an explicit example in μ-pair production where soft-gluon exchanges do not cancel in a non-Abelian theory. At first glance this would seem to be a great setback for QCD phenomenology. Before showing why I think one can avoid the disease found by BBL, let me describe the example they consider.

BBL deal with the graphs shown in Fig. 10 where the gluon lines ℓ_1 and ℓ_2 are presumed to be soft. BBL estimate the effects of these graphs by making a Glauber approximation where the lines $k_1+\ell_2$, $k_1+\ell_1+\ell_2$, $p_2-k_2+\ell_1$, $p_2-k_2+\ell_1+\ell_2$ in Fig. 10a are put on-shell and where the lines $k_1+\ell_2$, $k_1-\ell_1$, $p_2-k_2-\ell_1$, $p_2-k_2+\ell_2$ in Fig. 10b are put on-shell. In an Abelian theory the contribution of the graph in Fig. 10a exactly cancels that of the graph in Fig. 10b. However, in the non-Abelian case the ratio of the contributions of the graph in Fig. 10a to that of Fig. 10b is

$$-\frac{\mathrm{tr}\{T^aT^bT^aT^b\}}{\mathrm{tr}\{T^aT^aT^bT^b\}} = -\frac{C_F(C_F-\frac{1}{2}C_A)}{C_F^2} = \frac{1}{N^2-1} \tag{17}$$

for color SU(N). Thus we believe, following BBL, that wee gluons will not cancel order by order in perturbation theory.

It turns out that there are Sudakov corrections to the graphs shown in Fig. 10.[34] A leading double logarithmic approximation shows that these Sudakov effects precisely remove the part of the wee gluon contribution which does not cancel between the two graphs in Fig. 10. Thus we may suspect, and hope, that the disease found by BBL is cured by Sudakov and that it is acceptable to apply standard QCD phenomenology to the Drell-Yan process. We may anticipate a rigorous verification of this conjecture in the next year or so.

VIII. Soft Particle Production and Multiplicity

When $x\to 0$ single particle inclusive annihilation cross sections in $\ell^+\ell^-$ collisions are no longer governed by a straightforward application of the renormalization group. This shows up, for example, in a series of the form

$$\gamma_n(g^2) = \frac{g^2}{n-1}\sum_{m=1}^{\infty} c_m\left(\frac{g^2}{(n-1)^2}\right)^{m-1} + \text{less singular terms} \tag{18}$$

for the anomalous dimension matrix. A number of groups[35-38] have suggested that a solution to the Altarelli-Parisi equations with a careful handling of kinematics, in order to treat the running coupling properly, might be a way to go beyond the renormalization group and obtain small x results. Such an approach has been successful in obtaining the Sudakov effects as $x\to 1$. Following this procedure one obtains an average multiplicity of produced hadrons growing like an exponential of $\sqrt{\ln Q^2}$. Such an increase comes from a square root branch point of $\gamma_n(g^2)$ in n after summing the series indicated in (18).

However, in contrast to the $x\to 1$ case, it now appears that the above procedure gives the correct form for γ_μ, but not the correct values of the parameters appearing in the square root.[39] The origin of this difficulty is that non-planar graphs are just as important as planar graphs. We may understand this in the following way. The average multiplicity is determined by

$$\bar{n}(Q^2)\sigma = \int 2E\,\frac{d\sigma}{d^3p}\,\frac{d^3p}{2E} \tag{19}$$

or

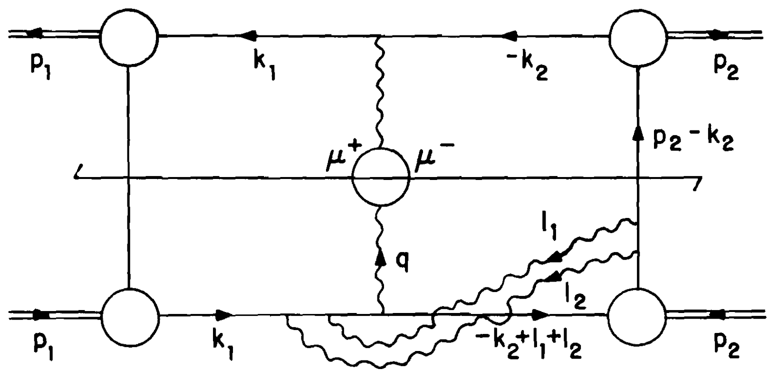

Fig. 10a

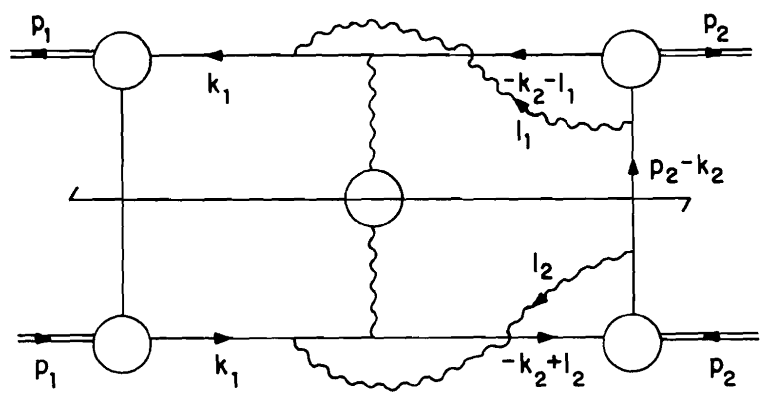

Fig. 10b

Soft gluon exchanges in μ-pair production.

$$\frac{4\bar{n}(Q^2)}{\pi(Q^2)} = \int_{2P/Q}^{1} 2E\,\frac{d\sigma}{d^3p}\,x\,dx \tag{20}$$

where $d\sigma/d^3p$ is the inclusive cross section for producing a particle of momentum p and mass P. p corresponds to a fraction, x, of the maximum possible momentum of a produced particle. In (20) factors of $1/x^2 \ln^n 1/x$ in $2E\,d\sigma/d^3p$ get changed into $\ln^{n+1} Q/P$ factors. Thus terms which are non leading in $2E\,d\sigma/d^3p$, as far as powers of $\ln Q^2$ are concerned, may have additional powers of $\ln 1/x$ and be part of a leading series in $\ln Q^2$ as far as $\bar{n}(Q^2)$ is concerned. This is exactly what happens for non-planar graphs.

Through order g^6 the singlet anomalous dimension for decay processes is consistent with[39]

$$\gamma_n = \frac{1}{4}\left[-(n-1)^2 + \sqrt{(n-1)^2 + \frac{8\alpha C_A}{\pi}}\,\right]. \tag{21}$$

(Planar graphs have a form identical to (21) except that the 4 and 8 are replaced by 2 and 4 in that equation.) We may now use factorization, which states

$$(Q^2)^2 \int 2E\,\frac{d\sigma}{d^3p}\,x^n\,dx \underset{Q^2\to\infty}{\longrightarrow} A_n E_n(Q^2) \tag{22}$$

and the renormalization group, which gives

$$E_n(Q^2) = e^{\int_{\mu^2}^{Q^2} \frac{d\lambda^2}{\lambda^2}\gamma_n\left[g^2(\lambda^2)\right]} \tag{23}$$

to find

$$\bar{n}(Q^2) \propto e^{\int_{\mu^2}^{Q^2} \frac{d\lambda^2}{\lambda^2}\gamma_1\left[g^2(\lambda^2)\right]}\,. \tag{24}$$

Although $\gamma_1(g^2)$ does not make sense order by order in perturbation theory, the form (21) does make sense evaluated at $n = 1$ so one gets $\gamma_1(g^2) = \sqrt{\alpha C_A/2\pi}$. Then one easily finds

$$\bar{n}(Q^2) \propto e^{\sqrt{\frac{2C_A}{\pi b}\ln Q^2}} \tag{25}$$

with $b = 33-2n_f/12\pi$. It should be possible to extend the calculation of the leading singularities in γ_μ to all orders in g^2.

The picture which emerges from the above three loop calculation is consistent with a branching process (see Fig. 11) but where $\theta(k_i, k_{i+1}) > \theta(k_{i+1}, k_{i+2})$ in the center of mass of the current initiating the process and in a light-like axial gauge. A planar set of graphs gives a branching process with the angular constraint

$$\frac{1-\cos\theta(k_i, k_{i+1})}{2} > \frac{k_{i+2}}{k_{i+1}}\,\frac{1-\cos\theta(k_{i+1}, k_{i+2})}{2}\,. \tag{26}$$

One can use the form (21) for γ_μ to solve for $\frac{d\sigma}{dx}$ at small x. One finds

$$\frac{d\sigma}{dx} \propto \frac{e^{\sqrt{\frac{2C_A}{\pi b}\ln Q^2}}}{\sqrt{\ln^{3/2} Q^2}}\; e^{-3\sqrt{\frac{8C_A}{\pi b}}\,\frac{\left(\frac{1}{4}\ln Q^2 - \ln\frac{1}{x}\right)^2}{\ln^{3/2} Q^2}} \tag{27}$$

so long as $\dfrac{\left(\frac{1}{4}\ln Q^2 \,\lambda\, \ln\frac{1}{x}\right)^2}{\ln^2 Q^2} << 1$.

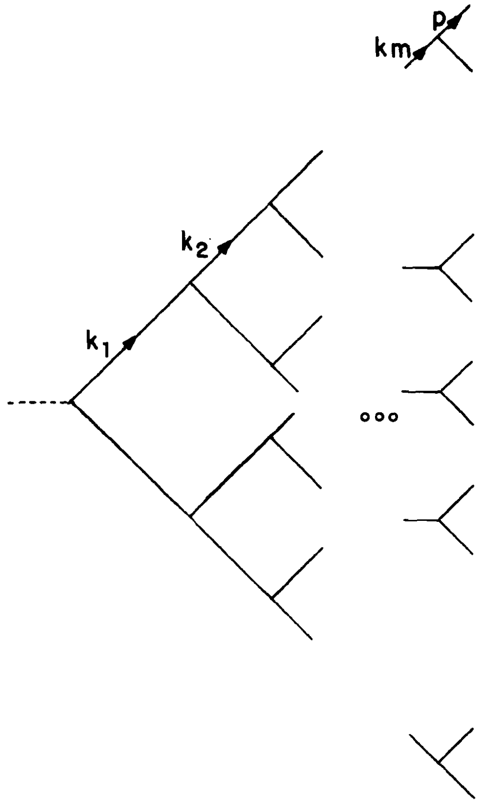

Fig. 11

Inclusive annihilation as a branching process.

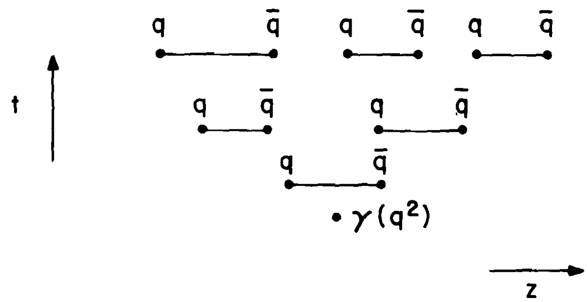

Fig. 12

A string-breaking picture of virtual photon decay.

If it turns out that these low order calculations are not misleading and that something like a branching process is correct, then the idea of preconfinement[40] becomes very important. If a branching process is a correct description of the evolution of a jet, then one may follow the evolution of quarks and gluons down to off shell masses on the order of a few GeV or perhaps even less. It is only at this late stage in jet evolution that non-perturbative effects become important. But, if non-perturbative effects only come in at a very late stage it must be true that an effective local color neutralization has taken place completely within perturbative QCD. This neutralization is preconfinement. It is an important problem to show that preconfinement occurs for a general non-Abelian color group.

The picture above is in striking contrast to the Bjorken[41] and Casher, Kogut, Susskind[42] picture of jet evolution. In Fig. 12 we have illustrated the evolution of a virtual photon into a q-$\bar{q}$ pair, the formation of a color string between the q and $\bar{q}$, and the subsequent breakings of that string as the q and $\bar{q}$ separate in space. In this picture non-perturbative effects come in at the early stages of jet evolution. In such a case we should no longer expect to be able to calculate the multiplicity without a deeper understanding of non-perturbative QCD.

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Footnote

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Discussion

L.M. Jones, Univ. of Illinois: Is there any reason why "diquark" wave function and form factor at large Q^2 can't be calculated in the same way for the pion? I realize there will be divergences because it is not a color singlet, but you might expect these to factor off and cancel out when physical processes are calculated.

A.H. Mueller: Yes, you might. However, there would be a Sudakov factor which would suppress the diquark form factor. This means that the diquark mass could never be small.