A TOUR OF PERTURBATIVE QCD

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1. Introduction.

Over the last four years an army of literally hundreds of particle physicists all over the world worked out numerous predictions of the Perturbative QCD. Many of these predictions have already been compared with the high energy data, others still await this confrontation. Still many others are to be worked out in the years to come.

Has QCD been tested in its perturbative regime to the extent that we can with justification begin to believe that we perhaps deal here with the correct theory of the Strong Interactions?

I will try to address this question at the end of my talk. Meanwhile I would like to invite you for a short Tour of Perturbative QCD. On our Tour we shall visit the following points of interest: i) Scale parameter Λ and the argument of α_{QCD}, ii) Deep-inelastic scattering, iii) Semi-inclusive processes, iv) $p_\perp$ distributions, double logs and Sudakov formfactors, v) Quarkonia, vi) Jets, and vii) Other theoretical news.

During our sight-seeing we shall put the emphasis on the results and confrontations with the experimental data rather than on technical details. The latter can be found in the by now numerous guides (A1-A20). At each point of interest we shall discuss in some detail various aspects of a given subject. At the end of our Tour we shall collect the most important results of our investigations with the hope to obtain a Grand View of the Present Status of Perturbative QCD. Only then shall we attempt to address the question posed at the beginning of this Introduction.

Before the departure for our tour it is perhaps useful to list three main aspects which we shall discuss along the way. These are: a) Extraction of the QCD scale parameter Λ from various processes, b) Higher Order Corrections to various processes, c) Double logs, Sudakov formfactors and generally soft gluon effects.

2. Scale Parameter Λ and the Argument of α_{QCD}.

In Perturbative QCD we deal with the expansions in powers of the effective strong interaction coupling constant $\alpha_{QCD} \equiv \alpha$, which depends on a large scale Q^2, relevant to a given process. To set the scene consider a physical quantity $P(Q^2)$ for which we have the following perturbative expansion in α

$$P(Q^2)=[\alpha_i(\eta Q^2)]^N[1+r_i^{(1)}(\eta)\ \frac{\alpha_i(\eta Q^2)}{\pi}+r_i^{(2)}(\eta)\ \frac{\alpha_i^2(\eta Q^2)}{\pi^2}+O(\alpha_i^3)] \tag{2.1}$$

where N is a power which depends on the process considered, and the coefficients $r_i^{(1)}(\eta)$, $r_i^{(2)}(\eta)$, etc. (also process dependent) are calculable in perturbative QCD. Now the point is (B1,B2) that both α_i and the coefficients r_i in Eq.(2.1) depend on the renormalization scheme used to calculate $P(Q^2)$. They also depend on the choice of the argument of α through the parameter η (B3). The renormalization scheme dependence of α_i and r_i is signaled in (2.1) by the index i, which may denote the following popular schemes discussed widely in the literature: i = MS (B4), $\overline{\text{MS}}$ (B2), MOM (B5,B6) among others (B7). To each scheme there is attached a scale Λ_i (e.g. $\Lambda_{\overline{MS}}$ or Λ_{MOM}) which is related to α_i by the following relation (B8) (numerical values below correspond to four effective flavors)

$$\alpha_i(\eta Q^2) = \frac{1.51}{\ln(\frac{\eta Q^2}{\Lambda_i^2})}\left\{1-0.74\ \frac{\ln\ln\frac{\eta Q^2}{\Lambda_i^2}}{\ln\frac{\eta Q^2}{\Lambda_i^2}}+O\left(\frac{1}{\ln^2\frac{Q^2}{\Lambda_i^2}}\right)\right] \tag{2.2}$$

where the first and the second term correspond to the well known one loop (B9) and two loop (B10) coefficients in the perturbative expansion for the renormalization group β function respectively (B11). The various scales Λ_i are related to each other. We have for instance (B12)

$$\Lambda_{MOM} = 2.16\ \Lambda_{\overline{MS}} \tag{2.3}$$

which corresponds to

$$\alpha_{MOM} = \alpha_{\overline{MS}}\left[1+3.21\ \frac{\alpha_{\overline{MS}}}{\pi}+O(\alpha_{\overline{MS}}^2)\right] \tag{2.4}$$

and

$$r_{\overline{MS}}^{(1)} = r_{MOM}^{(1)} + 3.21\cdot N \tag{2.5}$$

for a fixed value of η. Furthermore the η dependence of $r_i^{(1)}(\eta)$ for a fixed scheme i is given by

$$r_i^{(1)}(\eta) = r_i^{(1)}(1) + N \cdot 2.08 \ln \eta \quad . \tag{2.6}$$

Eqs. (2.3) and (2.4) express the important fact that the QCD scale Λ and the effective coupling constant α are not physical quantities and, dependently on the scheme considered, take different numerical values.

The point is however that the scheme dependence of the coefficients $r_i^{(1)}$, $r_i^{(2)}$, etc. is such that if r_i's and α_i are inserted into equation (2.1) then a scheme independent answer for $P(Q^2)$ is obtained (note, there is no index i on the l.h.s. of Eq. (2.1)). Strictly speaking the last statement is only exact if $P(Q^2)$ is calculated to all orders of perturbation theory. If the perturbative expansion is truncated as in Eq. (2.1) there is a left-over dependence of $P(Q^2)$ on the scheme considered which is related to the fact that different renormalization schemes give different estimates of higher order corrections ($O(\alpha^3)$ and higher) not included in the analysis. However, if we consider a class of schemes for which perturbative expansions are behaving well (i.e. higher orders are small) the dependence of $P(Q^2)$ on i will be weak even if only two first terms in the expansion are kept.

This turns out to be indeed the case (A2) for instance for deep-inelastic structure functions and the photon structure functions at not too large values of x, and also for total e^+e^- annihilation cross-sections. For these quantities as long as $\Lambda_{\overline{MS}} \leq 250$ MeV the $\overline{MS}$ and MOM schemes lead to the same predictions within a few %. For various quarkonia decays (see Section 6) where next to leading order corrections are sometimes large, the situation is somewhat worse and $\overline{MS}$ and MOM schemes are compatible with each other only within 10-15% (A2) indicating that higher order corrections not included in the analysis are not neglegible.

Now a few words about the parameter η in Eq. (2.1) which distinguishes between various choices for the argument of α. We may recall the discussions of 1979-80 of whether the α as extracted at PETRA from jet cross-sections had been "measured" at $Q^2 = 900$ GeV^2 or maybe at $Q^2 = 150$ GeV^2? At that time only the leading contributions to the three jet cross-sections had been fully known (this corresponds in Eq. (2.1) to N = 1 and $r_i = 0$) and the value of the extracted scale parameter Λ was very sensitive to the assumed relevant value of Q^2 (B13). Inclusion of next to leading order corrections to jet cross-sections (see Section 7) removed this sensitivity to a large extent (completely if all orders in α were taken into account) since each change of η in $\alpha_i(\eta Q^2)$ was compensated by the corresponding change in the parameter $r_i(\eta)$ (see Eq. (2.6)).

In summary the discussion of this section shows that it is essentially irrelevant which scheme for Λ and which η are used as long as

i) at least next to leading order corrections are taken into account (for the compensation mentioned above to occur)

and

ii) the next to leading order corrections in the renormalization schemes and for the choices of η considered are not too large (say smaller than 40% of the leading term).

For many quantities encountered in Perturbative QCD the $\overline{MS}$ scheme, the MOM scheme of ref.(B6) and any scheme with Λ_i satisfying $\Lambda_{\overline{MS}} \leqslant \Lambda_i \leqslant \Lambda_{MOM}$ fulfils the criterium ii), whereas the MS scheme does not. There are of course sometimes differences of a few to 10% in predictions obtained in $\overline{MS}$ and MOM schemes. I personally think that within our present understanding of higher order corrections in QCD we have to live with these uncertainties. But I am aware of the fact that some of my colleagues have different points of view. In particular Celmaster and Sivers (B3) argue that the MOM scheme is the best scheme. On the other hand Stevenson (B14) suggests a method for finding the best parameter η for a given renormalization scheme in such a way that the resulting answer is independent of η and i . At closer look Stevenson's procedure is similar to the scheme (see Section 3 and (B15)) in which all higher order corrections are absorbed into the scale Λ except for the two-loop contribution to the β function (see (2.2)).

One should also mention the suggestion of Pennington and Ross (B16) that, in the time-like processes, $|\alpha(Q^2)|$ in place of $\alpha(|Q^2|)$ should be used as an expansion parameter. The hope is that this will lead to a faster convergence of perturbative expansions. Although this idea is certainly an interesting one, I think more work is needed before it can be accepted as a working procedure. Similar comments apply to the interesting papers of ref. B17, where attempts are made to estimate $O(\alpha^2)$ corrections to deep-inelastic scattering on the basis of the presently known $O(\alpha)$ corrections to the structure functions and the three-loop contributions to the β function (B11).

Irrespective of personal views there are quantities in Perturbative QCD for which the requirement ii) is not satisfied. These are for instance the cases of the deep-inelastic structure functions for $x \to 1$, and of the massive muon production for $\mu^2 \ll p_\perp^2 \ll Q^2$. In these kinematical limits the next to leading (and higher) order corrections are large and a resummation of these corrections to all orders of perturbation theory has to be made. We shall encounter examples of it on our tour.

The discussion of this section shows how important the next to leading and higher order calculations are. Without them a meaningful extraction of the parameter Λ from the data and a meaningful comparison of values of Λ "measured" in various processes are not possible (B1).

In order to simplify the presentation, I have used throughout my talk the $\overline{MS}$ scheme, which seems to be the favorite scheme of the experimentalists.

3. Deep Inelastic Scattering.

We shall now visit the Deep Inelastic Scattering. This process has dominated many photon-lepton Conferences in the past and as we have seen this morning it is also an important part of the present conference (C1).

The main issues involved here are the logarithmic scaling violations in $F_2(x,Q^2)$ and $F_3(x,Q^2)$, higher twist effects, and the longitudinal structure functions. We shall discuss all these issues one by one.

3.1. Distinct regions of x.

In the discussion of QCD effects in deep-inelastic scattering it is useful
(A13) to distinguish four regions of x :

i) $0 \leq x \leq x_1(Q^2)$, the Regge region which is rather poorly understood at present,

ii) $x_1(Q^2) \leq x \leq x_2(Q^2)$, the region where the most action takes place. In this region the leading twist contributions are believed to dominate. Furthermore in this region the full cancellation of the virtual and soft real gluon emissions takes place and consequently there is only a single large (collinear) logarithm per loop in the Feynman Diagrams. These large collinear logarithms $[\alpha(Q^2) \log Q^2]^K$ have to be summed up to all orders in α which can be easily accomplished by the renormalization group techniques. The resulting formulae are known as moment equations (3.1) or evolution equations (3.2) which can be systematically expanded in powers of the effective coupling constant $\alpha(Q^2)$.

iii) $x_2(Q^2) \leq x \leq x_3(Q^2)$ with $1-x$ small but not too small. Since x in this region is rather close to the kinematical boundary the real emission of soft gluons is restricted and is therefore unable to fully compensate the reducing effects of virtual contributions. This results in large corrections of the type $\alpha(Q^2)\ln^2 \frac{1}{1-x}$ and $\alpha(Q^2)\ln \frac{1}{1-x} \ln\ln \frac{1}{1-x}$ which must be summed up to all orders of α. Since there are now two logarithms per loop standard renormalization group arguments do not apply and other techniques have to be used to sum the large corrections. Furthermore in this region higher twist contributions cannot be neglected any longer, at least for not sufficiently large values of Q^2.

iv) Finally for $x > x_3(Q^2)$ with $\log \frac{1}{1-x_3} \approx O\left(\frac{\log Q^2/\Lambda^2}{\log\log Q^2/\Lambda^2}\right)$ the higher twist contributions $O(1/Q^2)$ become dominant because the leading twist contributions are suppressed by Sudakov-like effects. The asymptotic behaviour of structure functions in this region in both Q^2 and x can however be studied by the renormalization group techniques.

The border lines between various regions are a bit fuzzy. A rough estimate on the basis of the formulae quoted below is that for $Q^2 = 20\ \mathrm{GeV}^2$ $x_1 \approx 0.02$, $x_2 \approx 0.7$ and $x_3 \approx 0.95$. With increasing Q^2, x_1 decreases while x_2 and x_3 increase.

We shall now in more detail discuss the regions ii) and iii) and briefly comment on the region iv).

3.2 Standard Approach.

3.2.1. Leading Twist (Theory).

Neglecting higher twist contributions the QCD predictions for the deep-inelastic structure functions F_2 and F_3 in the region ii) are usually expressed either in the form of the moments

$$M_n^{NS}(Q^2) = \int_0^1 dx\ x^{n-2}\ F^{NS}(x,Q^2) = M_n^{NS}\ (Q_0^2)\left[\frac{\alpha(Q^2)}{\alpha(Q_0^2)}\right]^{d_n^{NS}} [1 + R_n^{NS}\ \frac{(\alpha(Q^2) - \alpha(Q_0^2))}{\pi} + O(\alpha^2)] \tag{3.1}$$

or in the form of the evolution equations

$$Q^2\ \frac{\partial}{\partial Q^2}\ F^{NS}(x,Q^2) = \int_x^1 dz\ F^{NS}(\frac{x}{z},\ Q^2)\left[\frac{\alpha(Q^2)}{\pi}\ P^{(1)}(z) + \frac{\alpha^2(Q^2)}{\pi^2}\ P^{(2)}(z) + O(\alpha^3)\right]_+ \tag{3.2}$$

These equations are for F_3 and the non-singlet contributions to F_2 which dominate in the full region ii) except for $x < 0.3$ where singlet contributions become important. The moment equations and evolution equations (C2, C3, C4) for the singlet contributions are more complicated than Eqs. (3.1) and (3.2). They can be found in (C4, C5). The input moments $M_n^{NS}(Q_0^2)$ or the boundary condition $F(x, Q_0^2)$ to Eq.(3.2) have to be extracted from experiment at some not too small value of $Q^2 = Q_0^2$, say 5 - 10 GeV^2. The powers d_n (one-loop anomalous dimensions) and the coefficients R_n^{NS} are on the other hand calculable in QCD. The functions $P^{(1)}(x)$ and $P^{(2)}(x)$ contain the same information as d_n^{NS} and R_n^{NS} respectively.

Some technicalities should be mentioned here. The evaluation of the coefficients R_n^{NS} and the corresponding coefficients in the singlet sector $(R_n^{\pm})$ involves the calculations of the one-loop quark and gluon Wilson coefficient functions (B2) and of two-loop anomalous dimensions (C6). Furthermore these two calculations have to be done in the same renormalization scheme in order that the physical predictions for the structure functions are obtained. In the non-singlet sector there is a full agreement between calculations performed by various authors. In particular the recent calculations (C7) of the two-loop non-singlet anomalous dimensions agree with earlier calculations of Floratos, Ross and Sachrajda (C6). In the singlet sector there is a discrepancy between refs. (C8) and (C9) in the result for the element $\gamma_{GG}^{(1)}$ of the two-loop anomalous dimension matrix. Both calculations have been done in the $\overline{MS}$ scheme. When the results of both groups are transformed (C10) into the dimensional reduction scheme (DRS) (D11) which is frequently used in the supersymmetric calculations, the elements of the two-loop anomalous dimension matrix as calculated by Furmanski and Petronzio (C9) satisfy a so-called quark-lepton symmetry relation (C3): $\gamma_{GG}^{(1)} + \gamma_{qG}^{(1)} = \gamma_{qq}^{(1)} + \gamma_{Gq}^{(1)}$, whereas the results of Floratos et al. (C8) do not. As argued recently by Floratos (C12) this favors the result of ref. C9 . In any case the discrepancy just mentioned has essentially no phenomenological consequences, since the dominant contribution to $R_n^{\pm}$ and R_n^{NS} comes (at least in the $\overline{MS}$ scheme) from the coefficients functions calculated in (B2).

Finally it should be said that for not too large values of x or n (say $x < 0.7$, $n < 6$) the next-to-leading order corrections to F_2 and F_3 in $\overline{MS}$ and MOM schemes are not large (5 - 20%) and consequently it is believed that perturbative calculations can be trusted.

3.2.2. Phenomenological Results.

On the phenomenological side as has been shown in many analyses (C1,C5, C13,C14), the formulae like (3.1) and (3.2) agree very well (C1) with the existing deep-inelastic data (ep, μp, νN, $\bar{\nu}$N, etc.) even in the absence of higher twist contributions. Furthermore it has been found that the inclusion of next-to-leading order corrections (i.e. R_n^{NS}, $R_n^{\pm}$) in the phenomenological analyses improves the agreement of the theory with data. This is most clearly seen by utilizing the so-called Λ_n scheme (B1,B2,C15). The idea (B1) is to absorb all higher order corrections into the parameter Λ i.e., to put the formula (3.1) into the form of a leading order expression. The resulting scale Λ_n which replaces Λ of the L.O. expression becomes now n dependent with the n dependence predicted (B2,C15) by QCD:

$$\Lambda_n \approx \Lambda_{\overline{MS}} \quad \exp\left[\frac{R_n^{NS}}{d_n^{NS}}\right] \underset{\substack{\text{large}\\ n}}{\approx} \quad c\sqrt{n} \qquad\qquad c = \text{constant} \tag{3.3}$$

This n dependence is (as emphasized by Para and Sachrajda (C15)) renormalization prescription independent and agrees very well with the data (C16,C17). But what about values of $\Lambda_{\overline{MS}}$?

3.2.3. Values of $\Lambda_{\overline{MS}}$.

This morning some of us (C18) have been encouraged by our experimental colleagues that the value of 10 MeV for $\Lambda_{\overline{MS}}$ is not necessarily favored by the data. But as discussed by Dress (C1) the values of $\Lambda_{\overline{MS}}$ as extracted by various groups are smaller than two years ago. At this moment I am opening a table of values for $\Lambda_{\overline{MS}}$, which we shall collect on our tour (see table II at the end of this talk). The ones extracted from deep-inelastic scattering have been borrowed from Drees (C1), who finds

$$< \Lambda_{\overline{MS}} > \quad = \quad 157\ ^{+55}_{-40}\ \text{MeV}. \tag{3.4}$$

At first sight it would appear that this value is smaller than what one would naively expect for Λ_{QCD}. But it should be remarked that the corresponding value in the MOM scheme (apparently a more "physical" scheme than $\overline{MS}$) is $\Lambda_{MOM} \approx$ 350 MeV, a value which some of us guessed four years ago.

Since the details of various phenomenological analyses have been already presented by other speakers (C1), I will only make a few remarks.

First there exists one analysis of the CDHS data (C.17) in which much larger than (3.4) values of $\Lambda_{\overline{MS}}$ (450 ± 50 MeV) have been found. It is important to clarify this discrepancy. Second, it has been suggested by Roy (C19) that the low values of Λ found in high Q^2 μN experiments could be due to the presence of a hard intrinsic charm component in the nucleon. The corresponding threshold effect would cause an increase of $F_2(x,Q^2)$ at large x, and Q^2. Taking this effect into account and assuming the size of the intrinsic charm to be 2%, Roy finds that the net QCD scaling violations correspond to $\Lambda \approx$ 300-400 MeV. However as discussed at this conference there is essentially no evidence for such a charm component in the data. Consequently it is not clear how seriously one should take Roy's result. Finally there are a few remarks on higher twists but these will be made at the end of this section.

3.3 Large x behaviour.

For large x or large n the coefficients R_n in Eq.(3.1) behave like $(\ln n)^2$. When $\alpha(Q^2)(\ln n)^2$ or equivalently $\alpha(Q^2)\ln^2\frac{1}{1-x}$ are a substantial fraction of 1, one enters the region iii) of Section 3.1, perturbation theory breaks down and resummation of terms $\alpha(Q^2)^k(\ln^p\frac{1}{1-x})$ or $\alpha(Q^2)^k\ln^p n$ to all orders of perturbation theory has to be done. Such a resummation has been demonstrated by various authors (A3,A7,A13,C20-C23) in the so-called double leading log approximation (DLLA) in which the dominant terms in each order in α, i.e. $\alpha(Q^2)^k\ln^{2k}\frac{1}{1-x}$, are taken into account, but the terms $\alpha(Q^2)^k\ln^p\frac{1}{1-x}$ with $k<p<2k$ are neglected. One obtains for instance for the nucleon structure functions (A13):

$$F_2(x,Q^2) \sim (1-x)^3 \exp\left[-\frac{16}{33-2n_f} f(x,Q^2)\right] \tag{3.5}$$

where

$$f(x,Q^2) = \ln Q^2/\Lambda^2 \ \ln\ln Q^2/\Lambda^2 - \ln\frac{Q^2(1-x)}{\Lambda^2}\ \ln\ln\frac{Q^2}{\Lambda^2}(1-x) - \ln\frac{1}{1-x}\ \ln\ln\frac{1}{1-x}$$

The moment version of Eq.(3.5) is given by Eq.(3.7) of ref.(C.22). For $\alpha_s(Q^2)\log n \ll 1$ but $\alpha_s(Q^2)(\log n)^2 \approx O(1)$ the result (3.5) or its moment version correspond to the exponentiation of the dominant term in the parameter R_n i.e. $(\ln n)^2$ term. For $\ln\frac{1}{1-x} > O(\frac{\ln Q^2}{\ln\ln Q^2})$ or $n \geq \frac{Q^2}{Q_0^2}$, where $Q_0^2 \approx 1\ \text{GeV}^2$ the exponential in (3.5) behaves like a Sudakov formfactor and the structure function as given by (3.5) is strongly suppressed. Consequently the leading twist contributions on which Eq.(3.5) is based cease to be important and higher twist contributions take over. This is the region iv) of Section 3.1. In this region one can show that (C24)

$$F_2(x,Q^2) \sim \frac{G_M(Q^2)}{Q^2}\sum_N C_N\left[\frac{\ln Q^2}{\ln Q^2(1-x)}\right]^{-2A_N} \tag{3.6}$$

where $G_M(Q^2)$ is the magnetic formfactor of the nucleon, A_N are anomalous dimensions of three fermion operators and C_N are calculable coefficients which have not been calculated so far.

The result like (3.5) is valid only for large x and it would be interesting to have a formalism which would interpolate between the intermediate x region (ii)) and the large x region (iii)). It has been suggested (C20,C22,C23) that so-called improved evolution equations (compare with (3.2))

$$Q^2 \frac{\partial F^{NS}(x,Q^2)}{\partial Q^2} = \frac{1}{\pi}\int_x^1 dz\, F^{NS}(\frac{x}{z},Q^2)\left[\alpha(Q^2\frac{(1-z)}{z})\,P^{(1)}(z)\right]_+ \tag{3.7}$$

which correspond to rescaling of the argument of α from Q^2 to $Q^2(1-z)/z$ may do this job. The change of the argument of α is an effect of taking properly the kinematical constraints on the $k_\perp$ of the emitted gluons (A3,A7). Thus it is argued that by a simple rescaling of the argument of α (in the leading order expressions) one can resume the most important corrections to all orders of perturbation theory. Indeed as has been shown in refs. (C20,C22,C23) an equation like (3.7) is for $x \to 1$ equivalent to (3.5) to the DLLA accuracy. One can question, however, the quality of Eq.(3.7) in interpolating between intermediate and large x regions. Such an interpolation should include correctly the terms like $\alpha(Q^2)^k \ln^p \frac{1}{1-x}$ $(k < p < 2k)$ which have not been taken into account in Eqs.(3.5) and (3.7). We thus need a systematic approach in which we can calculate the corrections to Eqs.(3.5) and (3.7). There is a hope that the methods developed recently by Collins and Soper (see Section 5) may serve this purpose.

Furthermore there is the question raised by some of my colleagues (C25) whether an equation like (3.7) is consistent with the factorization of mass singularities. The point is that such a factorization is only true at fixed Q^2 and not $W^2 = Q^2(1-x)/x$.

How relevant is all this for the deep-inelastic scattering phenomenology and for the data fitting? Personally I do not think it is of great relevance. It has been shown for instance in ref. (C14) that the numerical differences between equations like (3.7) and the explicit calculations of the next-to-leading order corrections (Eq.(3.1)) become important only for $x > 0.7$ where the structure functions are small, data are poor, and the uncertainties due to higher twist contributions are non-negligible.

On the theoretical side, however, the studies of refs. (A3,A7,A13,C20-C25) are very important and this for two reasons. First they show that the large x behaviour of the twist two contributions to the deep-inelastic structure functions and in particular the large higher order corrections can be brought under control. Second the resummation methods discussed in the above papers turn out to be useful for such processes in which (Sudakov-like) effects are experimentally better "visible" than in the deep-inelastic scattering. We shall come to this in Section 5.

3.4 Longitudinal Structure Functions.

In the parton model with spin ½ quarks and in the absence of target mass corrections and higher twist contributions, the longitudinal structure function is zero (C26). In QCD and in the leading twist approximation one finds

$$\frac{M_n^{(L)}(Q^2)}{M_n^{(2)}(Q^2)} = \frac{4}{3}\,\frac{\alpha(Q^2)}{\pi(n+1)}\left[1 + B_n\,\frac{\alpha(Q^2)}{\pi} + O(\alpha^2)\right] \tag{3.8}$$

where $M_n^{(L)}$ and $M_n^{(2)}$ are the moments of the non-singlet longitudinal (F_L) and F_2 structure functions respectively. It has been known already for some time (C27) that the first term in (3.8) cannot account for the albeit poor data. Especially for large n or x values the leading order prediction of Eq.(3.7) lies systematically below the data. For small x, where singlet contributions (not shown in (3.8)) dominate, the agreement of the theory with data is quite good. The disagreement between theoretical predictions and the intermediate and large x data for F_L might not be a problem for QCD, however, and could be due to our neglect of higher twist contributions, target mass effects, non-perturbative effects etc., which are present in QCD but are difficult to calculate (C28).

Irrespective of this it is of importance to check how large the next-to-leading order corrections are to the ratio in (3.8) (i.e. the coefficients B_n). Such calculation is also necessary if we want to use in Eq.(3.8) the same value for the scale parameter Λ (e.g. $\Lambda_{\overline{MS}}$) as the one obtained from the phenomenology of F_2 and F_3 structure functions. The coefficients B_n have been recently calculated by Duke, Kimel and Sowell (C29). They find that B_n's vary slowly with n and in the $\overline{MS}$ scheme change from 4.3 to 7.5 when n is varied from 2 to 10. The corrections are therefore non-negligible and have the right sign. However due to the decrease of $\alpha_{\overline{MS}}$ as compared to the leading order α_{LO} (C30), the net effect of the next-to-leading order corrections to the ratio (3.8) is very small (C31). Consequently there is still room in the data for diquarks and higher twists effects (C28). So let us say a few words about the latter.

3.5 Higher Twists

At low values of Q^2 one has to worry, in addition to logarithmic scaling violations, about power-like scaling violations. In perturbative QCD they are represented by target mass effects, heavy quark mass effects and by contributions of operators of higher twist. Here we shall only discuss the latter. In the presence of higher twist contributions, Eq.(3.1) generalizes to

$$M_n^{NS}(Q^2) = \sum_{\substack{t=2\\ \text{even}}} \frac{A_n^{(t)}}{[Q^2]^{t-2}}\,[\alpha(Q^2)]^{d_n^{(t)}}\,[1 + R_n^{(t)}\,\frac{\alpha(Q^2)}{\pi} + \ldots] \tag{3.9}$$

where the sum runs over various twist (t) contributions: leading twist ($t = 2$), twist four ($t = 4$) and so on. $A_n^{(t)}$ are incalculable hadronic matrix elements of spin n, twist t operators. $d_n^{(t)}$ and $R_n^{(t)}$ are calculable numbers, e.g. $d_n^{(2)} = d_n^{NS}$ and $R_n^{(2)} = R_n^{NS}$. It should be remarked that there are many operators of a given twist > 2 contributing to Eq.(3.9) so this equation is in reality more complicated than we have shown. Consequently there are many unknown non-perturbative parameters $A_n^{(t)}$ $(t>2)$ which have to be extracted from the data. This makes the phenomenology of higher twist contributions very complicated.

Since the parameters $A_n^{(t)}$ are incalculable at present (see, however, below) one can study phenomenologically the effects of higher twist contributions in deep inelastic scattering by using "QCD motivated" parametrizations of the terms $t > 2$ in Eq.(3.9). Unfortunately there is no full agreement between phenomenologists on the importance of higher twist contributions in the scaling violation analyses. Some physicists find that they are (C32) or could be (C33) important. Others find (C34) that they are negligible except in the $x \to 1$ region. One should also mention here the analysis of the CDHS group (C35). They find that for $Q^2 > 10\ \mathrm{GeV}^2$ and $x < 0.7$ the higher twist contributions are probably negligible but are important at lower Q^2 i.e. in the SLAC region.

Clearly in order to settle the issue a systematic attack of the higher twist problem is very desirable. Some progress in this direction has already been made:

a) the anomalous dimensions of some of the twist four (t=4) operators have been calculated (C36);

b) the coefficient functions of certain classes of higher twist operators have been analyzed in ref. (C37) and in particular in ref. (C38);

c) diagrammatic approaches to higher twist contributions have been suggested in refs. (C39) and (C40). These papers also address the question of higher twist contributions to the semi-inclusive processes;

d) one should also mention the explicit calculations (C41) of certain higher twist contributions to semi-inclusive processes, and finally

e) there exist very interesting calculations of the matrix elements $A_n^{(2)}$ (C42) and $A_n^{(4)}$ (C38) in the framework of the MIT bag model. Neglecting logarithmic scaling violations one finds (C38) for $n = 2$ moment

$$M_{n=2}^{ep-en}(Q^2) = M_{n=2}^{(t=2)}\Big|_{BAG} - \frac{(60\ \mathrm{MeV})^2}{Q^2} \underset{Q^2=5}{\approx} 0.1 - 7\cdot 10^{-4} . \tag{3.10}$$

Thus in the MIT bag model the twist four contribution to the $n = 2$ moment is smaller than 1%. For higher n the higher twist contributions are expected to be more important. The calculation is in progress. Note also the negative sign in Eq.(3.10). If this feature would continue to higher n then the inclusion of

"MIT bag motivated" higher twist contributions into the analysis of deep-inelastic data would increase the scale parameter Λ rather than decrease it, as assumed in many phenomenological analyses.

One may of course ask how much the bag model has to do with QCD? In spite of this the analyses of refs. (C38) and (C42) are very interesting and undoubtedly one should pursue in this direction with the hope of gaining a better understanding of higher twist contributions.

4. Semi-Inclusive Processes.

In 1977-78 it was shown by a group of physicists (D1 - D3) that Perturbative QCD calculations could be extended to other processes such as semi-inclusive processes, discussed here and in Section 5, and jet cross-sections, discussed in Section 7.

The main issues involved in semi-inclusive processes are higher order QCD corrections for the integrated (over $p_\perp$) cross-sections, the resulting corrections to parton model relations connecting various processes, and the study of transverse momentum ($p_\perp$) distributions. We shall discuss all these issues here and in the following Section.

4.1. Basic Structure.

There are six (including deep-inelastic scattering) inclusive and semi-inclusive processes for which a huge amount of data is now available and which therefore deserve particular attention. These are:

$$\begin{array}{lll} \text{i)}\ eh \to eX & \text{ii)}\ e^+e^- \to hX & \text{iii)}\ h_1h_2 \to \mu^+\mu^- X \\ \text{iv)}\ eh_1 \to eh_2 X & \text{v)}\ e^+e^- \to h_1h_2X & \text{vi)}\ h_1h_2 \to h_3X \end{array} \tag{4.1}$$

In perturbative QCD the formulae for the processes listed in Eq.(4.1) have the following general structure:

$$\sigma_h = \sum_i f_i^h(x,Q^2) \otimes \sigma_i(x,\alpha(Q^2)) \tag{4.2}$$

for the processes i) and ii),

$$\sigma_{h_1h_2} = \sum_{ij} f_i^{h_1}(x_1,Q^2) \otimes \sigma_{ij}(x_1,x_2,\alpha(Q^2)) \otimes f_j^{h_2}(x_2,Q^2) \tag{4.3}$$

for the processes iii) - v) and

$$\sigma_{h_1h_2}^{h_3} = \sum_{ijk} f_i^{h_1}(x_1,p_\perp^2) \otimes f_j^{h_2}(x_2,p_\perp^2) \otimes \sigma_{ij}^k(x_{1,2,3},\alpha(p_\perp^2)) \otimes f_k^{h_3}(x_3,p_\perp^2) \tag{4.4}$$

for the process vi)

In the above equations $f_i^h(x,Q^2)$ stand either for the parton distributions (quark, antiquark, gluon) which measure the probability for finding a parton of type i in a hadron h with the momentum fraction x , or they stand for the fragmentation functions, which measure the probability for a parton of type i to decay into a hadron h carrying the fraction x of the parton momentum. σ_{ij} are the relevant parton cross-section and the summation in Eqs.(4.2) and (4.3) is over quarks, antiquarks and gluons. The $\otimes$ denotes symbolically a convolution, an example of which can be found in Eq.(3.2).

Strictly speaking the separation of the (physical) cross-sections σ_h , $\sigma_{h_1h_2}$ and $\sigma_{h_1h_2}^{h_3}$ into parton distributions, parton cross-sections and fragmentation functions is not unique beyond the leading order (D4) and depends on the definition of parton distributions and fragmentation functions. This is analogous to the arbitrariness in the definition of the effective coupling constant encountered in Section 2. Two definitions have been discussed in the literature:

Definition A. (D4)

The parton distributions and fragmentation functions are defined by the space-like (S) and time-like (T) cut vertices (D3) respectively which are normalized at Q^2 and defined by the $\overline{MS}$ scheme. The moment version of the evolution equations for such defined densities is (in the non-singlet sector) as follows:

$$\begin{bmatrix} \langle q^{NS}(Q^2)\rangle_n \\ \langle D^{NS}(Q^2)\rangle_n \end{bmatrix} = \begin{bmatrix} \langle q^{NS}(Q_0^2)\rangle_n \\ \langle D^{NS}(Q_0^2)\rangle_n \end{bmatrix} \left[\frac{\alpha(Q^2)}{\alpha(Q_0^2)}\right]^{d_n^{NS}} \left[1+\begin{bmatrix} Z_{nS}^{NS} \\ Z_{nT}^{NS} \end{bmatrix} \frac{(\alpha(Q^2)-\alpha(Q_0^2))}{\pi}\right] \tag{4.5}$$

where

$$\langle q^{NS}(Q^2)\rangle_n \equiv \int_0^1 dx \quad x^{n-1}\, q^{NS}\,(x,Q^2) \tag{4.6}$$

and similarly for the fragmentation functions $D^{NS}(z,Q^2)$. Only two-loop anomalous dimensions and two-loop β function contributions are included into the coefficients Z_{nS}^{NS} and Z_{nT}^{NS} . The remaining higher order corrections which come from one-loop QCD corrections to the relevant parton subprocesses are included into the short distance functions σ_{ij} and σ_i which we have called parton cross-sections. We shall discuss these cross-sections below. Z_{nS}^{NS} and Z_{nT}^{NS} differ from each other due to the difference between two-loop anomalous dimensions of space-like cut vertices (D3) (relevant for deep-inelastic scattering and parton distributions in general, see Section 3) and the two-loop anomalous dimensions of time-like cut vertices (relevant for fragmentation functions) which have been calculated last year in ref. (D5). This difference corresponds to the breakdown of the so-called Gribov-Lipatov relation (D6). However, both Z_{nS}^{NS} and Z_{nT}^{NS} are very small (~ 0.5) for $n < 10$ and grow only like $\ell n\, n$ for large n. Consequently evolution equations defined by (4.5) are essentially the same as the leading order equations and furthermore they are almost the same for $q(x,Q^2)$ and $D(x,Q^2)$.

Definition B. (D7)

All higher order corrections to deep-inelastic scattering and to $e^+e^- \to hx$ are absorbed into the definition of parton distributions and fragmentation functions respectively. The evolution equations now take the form (4.5) with Z_{nS}^{NS} and Z_{nT}^{NS} replaced by

$$[R_n^{NS}]_{S,T} = [Z_n^{NS}]_{S,T} + [B_n^{NS}]_{S,T} \tag{4.7}$$

where $(B_n^{NS})_S$ and $(B_n^{NS})_T$ come from the one-loop corrections to the deep-inelastic scattering and the one-loop corrections to $e^+e^- \to hx$ respectively. Since $B_{nS,T}^{NS}$ are large for large n (see below), the new evolution equations differ substantially at large n (large x) from the corresponding leading order equations. Furthermore, as we shall discuss below, B_{nS}^{NS} and B_{nT}^{NS} differ considerably from each other which is mainly due to the continuation of Q^2 from space-like to time-like region. Consequently the evolution equations for parton distributions and parton fragmentation functions, given by the definition B, differ at "low" values of $Q^2 < 100\ \mathrm{GeV}^2$ where the next-to-leading order corrections (in particular for fragmentation functions) are not negligible.

4.2. Summary of Higher Order Corrections to Semi-Inclusive Processes.

We begin this summary by discussing the processes i) - v) in Eq.(4.1). Let us denote the moments of the cross-sections σ_h and σ_i of Eq.(4.2) generally by

$$\sigma_n(Q^2) = \int_0^1 dx\, x^{n-2}\, \sigma(x,Q^2) \tag{4.8}$$

and the double moments of the cross-sections $\sigma_{h_1h_2}$ and σ_{ij} of Eq.(4.3) by

$$\sigma_{nm}(Q^2) = \int_0^1 dx_1 \int_0^1 dx_2\, x_1^{n-2}\, x_2^{m-2}\, \sigma(x_1,x_2,Q^2) \tag{4.9}$$

where x, x_1 and x_2 are the relevant scaling variables. Then the moments of the cross-sections for the processes i)- v) of Eq.(4.1) can be written (neglecting obvious overall factors) as $4\pi\alpha_{EM}/3Q^2$ in (4.12) as follows:

$$\sigma_n^{DIS}(Q^2) = \langle q(Q^2)\rangle_n \; [1 + B_{nS} \; \frac{\alpha(Q^2)}{\pi} + O(\alpha^2)] \tag{4.10}$$

$$\sigma_n^{e^+e^- \to hX} = \langle D(Q^2)\rangle_n \; [1 + B_{nT} \; \frac{\alpha(Q^2)}{\pi} + O(\alpha^2)] \tag{4.11}$$

$$\sigma_{nm}^{DY}(Q^2) = \langle q(Q^2)\rangle_n \; \langle \bar{q}(Q^2)\rangle_m \; [1 + B_{nm}^{DY} \; \frac{\alpha(Q^2)}{\pi} + O(\alpha^2)] \tag{4.12}$$

$$\sigma_{nm}^{eh_1 \to h_2 eX}(Q^2) = \langle q^{h_1}(Q^2)\rangle_n \; \langle D^{h_2}(Q^2)\rangle_m \; [1 + B_{nm}^{eh} \; \frac{\alpha(Q^2)}{\pi} + O(\alpha^2)] \tag{4.13}$$

$$\sigma_{nm}^{e^+e^- \to h_1 h_2 X}(Q^2) = \langle D^{h_1}(Q^2)\rangle_n \; \langle D^{h_2}(Q^2)\rangle_m \; [1 + B_{nm}^{e^+e^-} \; \frac{\alpha(Q^2)}{\pi} + O(\alpha^2)] \tag{4.14}$$

We have used here the definition A of parton densities and we have expanded the parton cross-sections in powers of α keeping only the next-to-leading terms. Furthermore to make the formulae more transparent we have not summed over quarks and antiquarks. The parton cross-sections for these additional subprocesses are exactly the same to this order as the parton cross-sections shown in (4.10)-(4.14). The references where the explicit calculations of the parameters B_n and B_{nm} can be found are listed in (D8-D13) and in Table III at the end of this talk.

We have found (D14) that for large n and m the results for various coefficients B_n and B_{nm} in (4.10)-(4.14) can be summarized by the following simple formulae

$$\begin{bmatrix} B_{nS} \\ B_{nT} \end{bmatrix} \Rightarrow F_n^{(1)} \; + \; \begin{bmatrix} 0 \\ 1 \end{bmatrix} \frac{2}{3}\pi^2 \tag{4.15}$$

and

$$\begin{bmatrix} B_{nm}^{DY} \\ B_{nm}^{eh} \\ B_{nm}^{e^+e^-} \end{bmatrix} \Rightarrow F_{nm}^{(2)} \; + \; \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} \frac{2}{3}\pi^2 \tag{4.16}$$

where the universal functions $F_n^{(1)}$ and $F_{nm}^{(2)}$ are given by ($\overline{MS}$ scheme)

$$F_n^{(1)} = \frac{2}{3}\,[\log n]^2 + 1.77\,[\log n] - 2.2 - \frac{\pi^2}{9} \tag{4.17}$$

and

$$F_n^{(2)} = \frac{2}{3}([\log n]^2 + [\log m]^2 + 2\log m \log n) + 1.54\ ([\log n] + [\log m]) - 4.4 - \frac{4}{9}\pi^2 \ . \qquad (4.18)$$

Thus,

1) For large n and m there is a universality in the n and m dependence of B_n and B_{nm} at the level of $(\log n)^2$, log m log n, log n and constant terms, which is broken only by a process dependent number ([$\cdots$]) of the 2/3 π^2 terms.

2) We have found (D14) a simple counting rule for the number of "2/3 π^2" terms one has to add to the universal function for a given process. This "2/3 π^2 counting rule" reads as follows: count the number of target hadrons and/or hadrons detected in the final state, which are on the other side of a large momentum, say Q^2. Using this rule, one immediately reproduces Eqs.(4.15) and (4.16).

3) For low n the universal behaviour of B_{nS} and B_{nT} is no longer satisfied; $B_{nT} - B_{nS}$ is smaller than $\frac{2}{3}\pi^2$. However, the universality of B_{nm}^{eh}, B_{nm}^{DY} and $B_{nm}^{e^+e^-}$ is (except for $\frac{2}{3}\pi^2$ terms) satisfied within 5% if n, m $\geqslant$ 6 .

4) The next-to-leading order corrections to all processes ii) - v) are much larger than the ones found in deep-inelastic scattering and they are large at all values of x or n due to the π^2 terms which come mainly from the continuation of Q^2 from the space-like to the time-like region. Slightly smaller corrections for all processes i) - v) are found if the MOM scheme is used.

5) In order to study QCD corrections to parton model relations which connect various processes it is useful to use the definition B of parton distributions and parton fragmentation functions. In this case B_n^{eh}, $B_{nm}^{e^+e^-}$ and B_{nm}^{DY} are replaced by

$$\begin{bmatrix} \tilde{B}_{nm}^{DY} \\ \tilde{B}_{nm}^{eh} \\ \tilde{B}_{nm}^{e^+e^-} \end{bmatrix} = \begin{bmatrix} B_{nm}^{DY} \\ B_{nm}^{eh} \\ B_{nm}^{e^+e^-} \end{bmatrix} - \begin{bmatrix} B_{nS} \\ B_{nS} \\ B_{nT} \end{bmatrix} - \begin{bmatrix} B_{nS} \\ B_{nT} \\ B_{nT} \end{bmatrix} \qquad (4.19)$$

and Eq. (4.16) by

$$\begin{bmatrix} \tilde{B}_{nm}^{DY} \\ \tilde{B}_{nm}^{eh} \\ \tilde{B}_{nm}^{e^+e^-} \end{bmatrix} \rightarrow \tilde{F}_{nm}^{(2)} + \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} \frac{2}{3}\pi^2 \qquad (4.20)$$

where

$$\tilde{F}^{(2)}_{nm} = \frac{4}{3}\log m \cdot \log n - 0.23\,[\log n + \log m] - \frac{2}{9}\pi^2 \tag{4.21}$$

$\tilde{F}^{(2)}_{nm}$ are renormalization scheme independent.

Since now the $\frac{2}{3}\pi^2$ terms have been absorbed into the definition of the fragmentation functions the parton model relations between $eh\to eX$, $e^+e^-\to hX$ and $eh\to ehX$ as well as between $e^+e^-\to hX$ and $e^+e^-\to h_1h_2X$ are mainly violated by the $(\log n)\cdot(\log m)$ terms which introduce non-factorization in n and m, absent in the leading order and in the parton model. The parton model relation between $eh\to eX$ and $h_1h_2\to\mu^+\mu^-X$ is also violated by the [2] $\frac{2}{3}\pi^2$ term.

Let us summarize five main findings which have important implications for future research and for confrontation with the data.

A) The next-to-leading order corrections to the processes ii) - v) are large for all n and m due to the terms $(\log n)^2$, $(\log m)^2$, $(\log n)\cdot(\log m)$ and the π^2 terms. The resummation of these corrections to all orders of perturbation theory is necessary. The $(\log n)^2$, $(\log m)^2$, etc. terms can be summed as in the deep-inelastic scattering (see Sec. 3.3). It has also been suggested (D15) that the π^2 terms can be summed to all orders in $\alpha(Q^2)$ by using the asymptotic formula for the elastic quark form factor (D16). I think that this method of summing the π^2 terms related to the continuation from space-like to time-like Q^2 is quite reasonable. It should be, however, kept in mind that not all π^2 terms are summed by this method; e.g., the π^2 terms of Eqs. (4.17), (4.18) and (4.21). Consequently the π^2 terms require further study.

B) Scaling violations in massive muon production and in all time-like processes are expected to be <u>larger</u> (for a given $\Lambda_{\overline{MS}}$) than in deep-inelastic scattering (D17). In particular we find the prediction

$$\Lambda_{eff}^{e^+e^-\to hX} \approx (2-3)\ \Lambda_{eff}^{DIS} \tag{4.22}$$

where the effective scales Λ are to be extracted by means of leading order expressions from $e^+e^-\to hX$ and deep-inelastic scattering data. Although scaling violations in $e^+e^-\to hX$ have been seen both at PETRA and PEP, it is not yet clear whether Eq. (4.22) is in agreement with data.

C) One expects the cross-section for the massive muon production to be renormalized by roughly <u>factor 2</u> relative to the standard Drell-Yan formula, i.e.

$$\left[\frac{d\sigma}{dM^2}\right]^{hh\to\mu^+\mu^-X} \approx K\left[\frac{d\sigma}{dM^2}\right]^{DY} \tag{4.23}$$

with $K \approx 2$. This is confirmed by the data $(pN, \pi N)$ (D18). Is this a great success of QCD or just an accident?

D) One expects the non-factorization in n and m in the processes iii) - v). In particular in the semi-inclusive deep-inelastic scattering $eh_1 \to eh_2 X$ this corresponds to non-factorization in x and z variables. It is not clear whether the presence of such non-factorization in the data has been firmly established (A18).

E) One should also mention the large next-to-leading order corrections to $h_1 h_2 \to h_3$ (large $p_\perp$) X, which have been found by Ellis, Furman, Haber and Hinchliffe (D.19). If $\alpha(\hat{s})$ ($\hat{s}$ is the c.m.s. energy of the parton subprocess, say qq) is the expansion parameter, the next-to-leading order corrections are roughly by a factor 1 - 2 larger than the Born cross-section. These large corrections have been confirmed by Furmanski and Slominski (D20). They point out however (see also ref. B3) that the corrections are small in the whole range of $p_\perp$ if $\alpha(\hat{s}/7.4)$ instead of $\alpha(\hat{s})$ is used as the expansion parameter. Thus in the end it might be that the process in question is not outside our control. For the cross-section at 90^0 one can find, using the results of ref.(D20), the following simple formula

$$E \frac{d\sigma}{d^3p} \underset{90^0}{\simeq} \frac{1}{p_\perp^4} \alpha^5_{\overline{MS}} \left(\frac{4p_\perp^2}{7.4\ \Lambda^2_{\overline{MS}}} \right) \simeq \frac{1}{p_\perp^4} \cdot \frac{1}{p_\perp^{6\Lambda_{\overline{MS}}+0.7}} \tag{4.24}$$

which is valid for $2 \leqslant p_\perp \leqslant 10$ GeV and $0.1 \leqslant \Lambda_{\overline{MS}} \leqslant 0.5$ GeV. The first factor is the nominal power related to the parton subprocess. The second is the effect of the leading and next-to the leading QCD corrections. For $\Lambda_{\overline{MS}} \approx 0.2 - 0.3$ GeV the effective power is 6 - 6.5 which is not so far from the experimentally measured value ~ 8. The remaining $p_\perp$ dependence seen in the data can presumably be explained by the intrinsic $k_\perp$ effects. The calculations of refs. (D19) and (D20) involve only the QCD corrections to the qq subprocess. Before a detailed phenomenological analysis can be done, the QCD corrections to other subprocesses, such as Gq, $q\bar{q}$ and GG have to be computed.

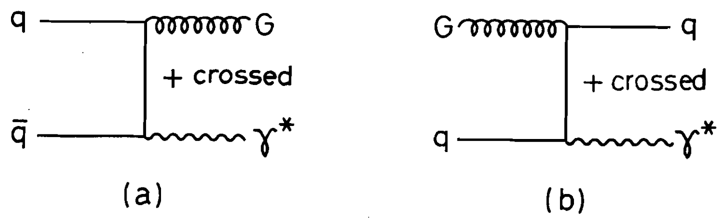
There have been a few more calculations of higher order QCD corrections to semi-inclusive processes. They are listed in Table III (see the end of this talk).

5. $p_\perp$ Effects.

5.1. Preliminaries.

Among the most spectacular QCD effects are $p_\perp$ effects which are caused by gluon bremsstrahlung. These have been most extensively studied in the massive muon pair production and in e^+e^- annihilation. We shall concentrate here mainly on the massive muon pair production.

In the standard Drell-Yan model and in the absence of the primordial $k_\perp$ of annihilating quarks and antiquarks the transverse momentum $p_\perp$ of the muon pair is zero. In QCD $p_\perp$ is no longer zero and its perturbative component receives the dominant $O(\alpha)$ contribution from the diagrams of Fig. 1. The distributions resulting from these diagrams have been calculated by various authors (E1). The result is compared with the data (E2) in Fig. 2. It is clear that the diagrams of



(Fig. 1)

Fig. 1 cannot reproduce the data. In particular the shape at small and intermediate $p_\perp$ is wrong. Furthermore at low $p_\perp$ the predicted distribution behaves as $1/p_\perp^2$ contrary to the data which is rather flat. For large $p_\perp^2 \sim O(Q^2)$ the situation is much better but the theoretical prediction lies somewhat lower than the measured distribution, especially in πN scattering.

A little thinking convinces us however that there is as yet no need to worry or to panic, and this for four reasons.

<u>First</u> there is something positive in Fig. 2. The data show large $p_\perp$ effects in accordance with theoretical expectations. Furthermore the predicted increase of $\langle p_\perp \rangle$ with s and Q^2 is confirmed by the data (E2).

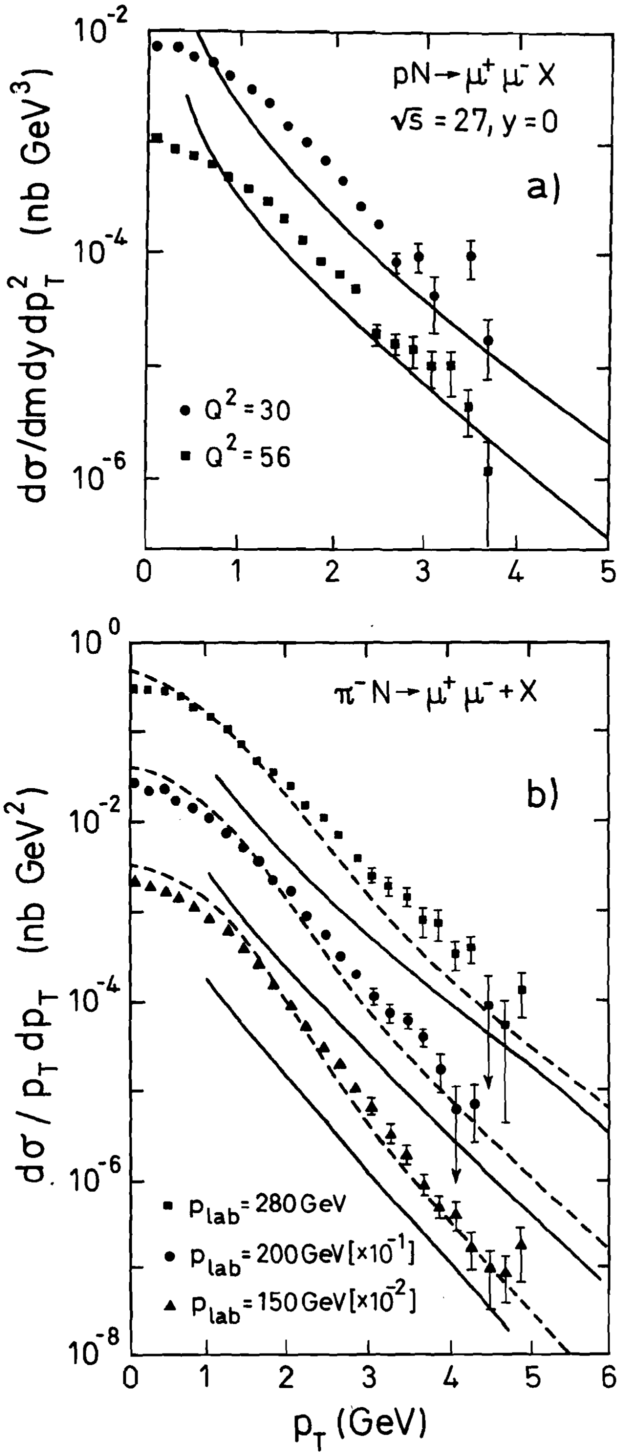
<u>Second</u> the theoretical predictions shown in Fig. 2 are based on the expression

$$\frac{d\sigma}{dQ^2 dp_\perp^2} \sim \frac{4\pi\alpha_{EM}^2}{9SQ^2} \int dx_1 dx_2\, q(x_1,Q^2)\, q(x_2,Q^2)\, \sigma(x_1,x_2,\tau,\frac{Q^2}{p_\perp^2}) \tag{5.1}$$

which only applies for $p_\perp^2 \approx O(Q^2)$. If $p_\perp^2$ is $O(Q^2)$ at order α_s^k there is only a k-fold logarithmic divergence due to mass (collinear) singularities,while the infrared (soft) divergences cancel between real and virtual gluon emissions. The mass singularities can be factored out and the left-over large logarithms $\alpha_s^k \log^k Q^2$ can be resummed to give Q^2 dependent parton distributions. For $p_\perp^2 \ll Q^2$ but $p_\perp^2 \gg \mu^2$ the situation is more complicated. Now at each order in perturbation theory the dominant corrections to the standard Drell-Yan process are of the form $\alpha_s^k \ln^{2k}(Q^2/p_\perp^2)$ arising from the emission of k gluons which are both soft <u>and</u> collinear. If $Q^2 \gg p_\perp^2$ the perturbation theory breaks down and these large logarithms have to be summed to all orders of perturbation theory. This region of $p_\perp$ is analogous to the region iii) (large x) of Section 3. We shall deal with it below. In any case we should not be surprised that a formula like (5.1) disagrees with data for $p_\perp^2 \ll Q^2$.

<u>Third</u> at low $p_\perp$ one may expect non-perturbative effects related to the intrinsic (primordial) $\langle k_\perp \rangle$ to be of importance. The usual procedure (E3) is to convolute the perturbative result of Fig. 2 with the primordial distribution chosen to have the form

$$f(k_\perp^2) \sim \exp[-\frac{k_\perp^2}{2\langle k_\perp^2 \rangle}] \tag{5.2}$$



(Fig 2.) Calculations to $O(\alpha)$ in QCD (solid lines) of the $p_\perp$ distributions of muon pairs are compared to data in a) pN interactions and b) πN interactions. The figures are from ref. (E4). The dashed line in Fig. 2a corresponds to the $O(\alpha)$ result convoluted with the primordial $k_\perp$ according to the procedure of ref. (E3).

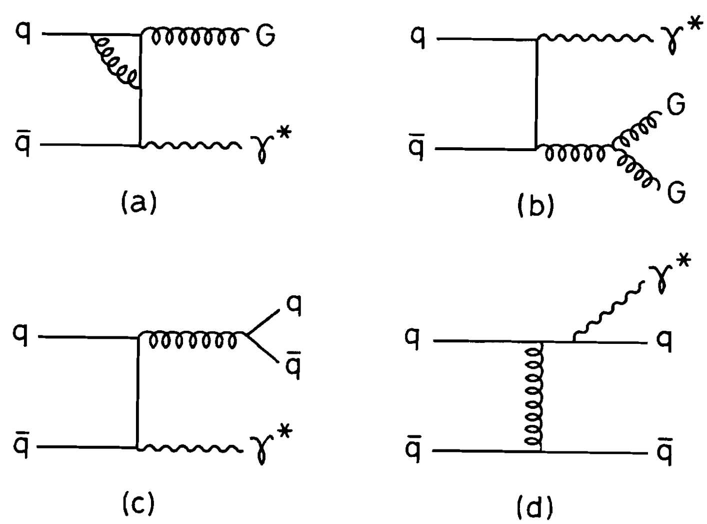
Choosing $\langle k_\perp^2 \rangle \sim 1\ \mathrm{GeV}^2$ one obtains a better agreement with the data (see the dashed line in Fig. 2b). This procedure is however very ad hoc and the agreement reached with the data cannot be regarded as a success of the theory. Nevertheless the primordial $k_\perp$ effects should somehow be taken into account. How they are really important can only be answered once the perturbative part of the $p_\perp$ distributions at relatively low $p_\perp (O(1\ \mathrm{GeV}))$ is correctly taken into account. We shall come to this in Section 5.3.

Fourth one may ask whether the next-to-leading order QCD corrections which are here $O(\alpha_s^2)$ could modify substantially the leading order result of Fig. 2 ?

We shall now address some of the above questions.

5.2. $p_\perp^2 \sim O(Q^2)$ region.

As can be seen in Fig. 2 , the $p_\perp$ distributions resulting from the diagrams of Fig. 1 are consistent with the pN data at $p_\perp \approx Q \approx 4$ GeV but are substantially below the πN data. Even after the inclusion of the intrinsic $k_\perp$ (in the amount sufficient to fit the low $p_\perp$ data) the πN data lie by a factor $K' \approx 2.4$ (E4) above the theoretical predictions. Now in pN scattering the Compton subprocess (Fig. 1b) dominates for $p_\perp^2 \sim O(Q^2)$, whereas in πN scattering the annihilation subprocess (Fig. 1a) is more important. One could then expect (E4) that, if QCD is going to agree with the πN data for $p_\perp^2 \sim Q^2$, the higher order corrections to the annihilation subprocess should be substantial. Indeed it has been found recently by Ellis, Martinelli and Petronzio (E5) and also by Perlt(E6) that the $O(\alpha^2)$ QCD corrections to the "non-singlet cross-sections" (e.g. $(\pi^+ - \pi^-)p$ or $(\bar{p} - p)p$) are substantial. The calculation involves the subprocesses $q\bar{q} \to q\bar{q}\gamma^*$ and $qq \to qq\gamma^*$ (E7) in addition to the virtual and real gluon radiation corrections to the lowest order subprocess $q\bar{q} \to G\gamma^*$. Typical diagrams are shown in Fig. 3 . For $\Lambda \approx 400$ MeV the ratio



(Fig. 3)

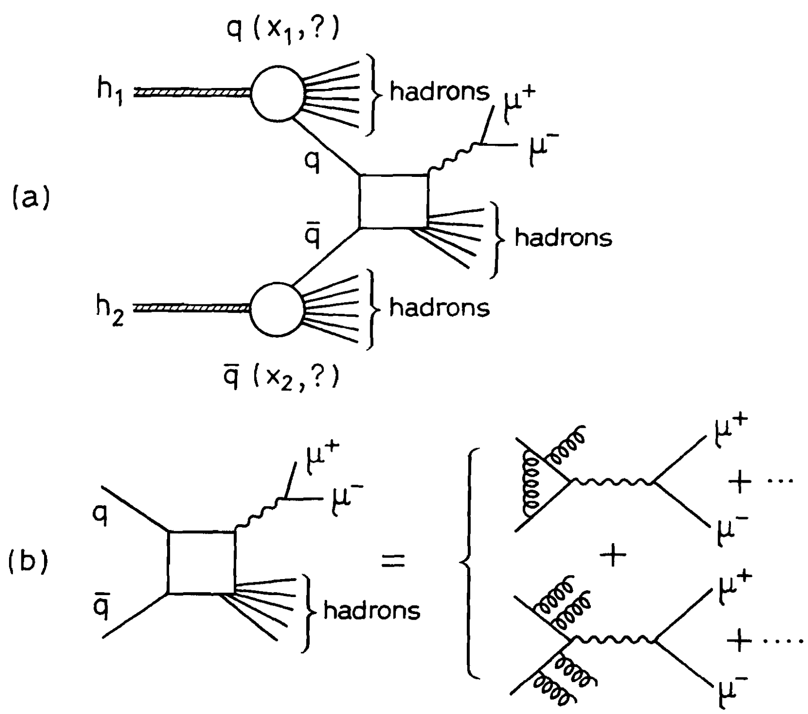
$$K'(p_\perp) = \frac{\frac{1}{p_\perp}\frac{d\sigma}{dQdp_\perp}(O(\alpha_s)+O(\alpha_s^2))}{\frac{1}{p_\perp}\frac{d\sigma}{dQdp_\perp}(O(\alpha_s))} \tag{5.3}$$

changes from 2.3 to 1.5 when $p_\perp$ is changed from 2.5 to 5 GeV at $Q = 6.5$ GeV. Thus as in the case of the integrated (over $p_\perp$) cross-sections (see Section 4) one finds a correction factor of order 2 and furthermore, as there, it appears that this factor is welcomed by the data. There remains of course the worry that since the corrections are so large, the perturbative calculations of $p_\perp$ distributions at present values of Q^2 cannot be trusted. In view of the fact that these large corrections are welcomed by the data it is important to investigate whether the resumation of the most important higher order corrections to all orders in α could be done.

But what about the small and intermediate $p_\perp$? Can we do better, than it is shown in Fig. 2, within the perturbation theory without introducing a large and ad hoc intrinsic $k_\perp$ for partons ?

5.3. Small and Intermediate $p_\perp$ (E8 - E19).

The massive muon pair production cross-section as given in (5.1) is schematically illustrated in Fig. 4a where the circles stand for parton distributions, and the square denotes the parton cross-section. The later cross-section which we denote by $d\sigma/dp_\perp^2$ can be calculated in perturbation theory as shown in Fig. 4b.



(Fig. 4)

The questions which we want to ask now are:

a) What is the scale which enters the parton distributions when $p_\perp^2 << Q^2$?

b) What happens to $d\sigma/dp_\perp^2$ when $p_\perp^2 << Q^2$?

c) Can we use perturbation theory to evaluate $d\sigma/dp_\perp^2$ at $p_\perp = 0$?

We shall now answer all these questions one by one.

5.3.1. The scales in the parton distributions.

In the discussion of $p_\perp$ distributions in the massive muon pair production it is useful to distinguish three regions in $p_\perp$. In each of these regions the parton distributions which enter formulae like (5.1) are to be evaluated at different scales. One finds (E16-E19):

Region	Scale in $q(x,?)$
$p_\perp^2 \sim Q^2$	Q^2
$Q^2 >> p_\perp^2 > (p_\perp^2)_0$	$p_\perp^2$
$p_\perp^2 \leqslant (p_\perp^2)_0$	$(p_\perp^2)_0$

where

$$(p_\perp^2)_0 \equiv [\Lambda^2]^{\frac{1}{1+c}} [Q^2]^{\frac{1}{1+c}} \qquad c = \frac{16}{33-2f} \quad . \tag{5.4}$$

Here f is the number of flavors. For $Q^2 = 100$ GeV^2 and $\Lambda = 500$ MeV, $(p_\perp^2)_0 \approx 2.5$ GeV^2. The important point is that even for $p_\perp^2 \approx 0$ the parton distributions are to be evaluated at $(p_\perp^2)_0$, which for the example quoted above and in general, is large enough for the perturbative calculations to make sense. We shall discuss it in more detail in Sect. 5.3.3.

5.3.2. Parton Cross-Sections for $p_\perp^2 << Q^2$.

In Section 5.2 we have discussed the parton cross-sections in the region $p_\perp^2 \approx Q^2$. We shall now present what happens to $d\sigma/dp_\perp^2$ in the two remaining regions listed above.

As we have mentioned in Section 5.1 when $\mu^2 << p_\perp^2 << Q^2$ double logarithms $\alpha^k \log^{2k} \frac{Q^2}{p_\perp^2}$ appear in the parton cross-sections and perturbation theory breaks down. These double logarithms can be interpreted as a result of an incomplete cancellation between soft virtual and real gluon emissions. If we want to have any reliable QCD predictions for the massive muon pair production in this region we have to sum all these large corrections to all orders of perturbation theory.

Quite generally one can write

$$\frac{1}{\sigma_0} \frac{d\sigma}{dp_\perp^2} \approx \frac{\alpha}{p_\perp^2} \log \frac{Q^2}{p_\perp^2} \Big\{A + \alpha(B_1 \log^2 \frac{Q^2}{p_\perp^2} + B_2 \log \frac{Q^2}{p_\perp^2} + B_3)$$

$$+ \alpha^2 (C_1 \log^4 \frac{Q^2}{p_\perp^2} + C_2 \log^3 \frac{Q^2}{p_\perp^2} + \ldots)$$

$$+ \ldots \Big\} \qquad (5.5)$$

where $\sigma_0 = 4\pi\alpha^2_{EM}/9Q^2$. Then in the so-called double leading logarithmic approximation (DLLA) in which only the dominant terms in each order in α, i.e. $\alpha^k \log^{2k} \frac{Q^2}{p_\perp^2}$, are summed, one obtains (E8-E11)

$$\frac{1}{\sigma_0} \frac{d\sigma}{dp_\perp^2} \sim \frac{\alpha}{p_\perp^2} \log \frac{Q^2}{p_\perp^2} \exp \left[-\frac{2\alpha}{3\pi} \log^2 \frac{Q^2}{p_\perp^2} \right] . \qquad (5.6)$$

This formula is only valid for a fixed α . For a running α the argument of the exponential in (5.6) is somewhat more complicated (E9, E16). The exponential itself can be regarded as an effective quark formfactor. It gives the probability for the massive (Q^2) muon pair production in $q\bar{q}$ annihilation without emission of gluons having transverse momenta $k_\perp$ greater than $p_\perp$ (the transverse momentum of the muon pair). When $p_\perp^2 << Q^2$ this probability is very small. Indeed for $p_\perp^2 = 0$ the cross-section is predicted to be zero (Fig. 5).

If there is a strong cancellation between the leading logarithms for small $p_\perp^2$, it is quite probable that the subleading logarithms neglected so far could have an important contribution in this $p_\perp$ region, and could fill the dip of Fig. 5. This indeed seems to be the case as first discussed by Parisi and Petronzio (E11) and recently in more detail by other authors (E17-E18).

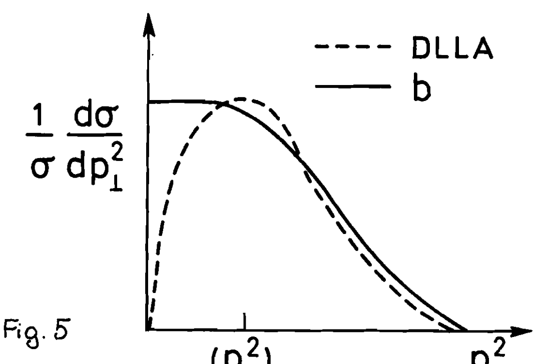


Fig. 5

Let us first recall that in DLLA the dominant contribution comes from multi-gluon emissions with one gluon having $k_\perp^2 \approx p_\perp^2$ and the remaining gluons having $(k_\perp^i)^2 << p_\perp^2$. It turns out that for small $p_\perp^2$ the most important contributions come from multi-gluon emissions with two or more gluons having $k_{\perp i}^2 >> p_\perp^2$ which add vectorially to give a small $p_\perp^2$ of the muon pair. As discussed in detail in refs.(E17) and (E18) the contributions in question are suppressed in each order of α by at least $\log^3 \frac{Q^2}{p_\perp^2}$ as compared to the contributions which enter DLLA. However after all orders are summed these subleading logarithms dominate over the DLLA contributions and fill the dip at small $p_\perp$. As pointed out first by Parisi and Petronzio (E11), and recently discussed by various authors (E15-E19), the subleading logarithms in question can be "pulled out" from the series (5.5) by using the exact transverse momentum conservation. This is usually most easily done by working in the impact parameter space b instead of $k_\perp$ space (E10-E15, E17-E19).

Defining $\tilde{\sigma}(b,Q^2)$ by

$$\frac{1}{\sigma_0}\frac{d\sigma}{dp_\perp^2} \sim \int d^2b \exp[-i\vec{b}\cdot\vec{p}_\perp]\,\tilde{\sigma}(b,Q^2) \tag{5.7}$$

one obtains

$$\tilde{\sigma}(b,Q^2) = \sum_i e_i^2\, q(x_1, \frac{1}{b^2})\bar{q}(x_2, \frac{1}{b^2})\, e^{-\hat{S}_0(b^2,Q^2)} \tag{5.8}$$

where (for fixed α)

$$\hat{S}_0(b^2,Q^2) = \frac{2\alpha}{3\pi}\log^2(Q^2b^2). \tag{5.9}$$

We observe that Eqs.(5.7)-(5.9) correspond to the exponentiation of the leading logarithms in the b-space rather than in the $k_\perp$ space as it is done in the DLLA (see (5.6)). Thus exponentiating in the b-space one sums effectively some sub-leading logarithms in the $k_\perp$ space (those related to the exact transverse momentum conservation) in addition to the leading logarithms which enter DLLA. Numerically it turns out (E17,E18) that for $p_\perp^2 > (p_\perp^2)_0$ the DLLA and the "b-space method" lead to very similar results, whereas for $p_\perp^2 < (p_\perp^2)_0$ the two give very different predictions. Instead of a dip predicted by DLLA, the b-space approach gives (E16-E19) (see Fig. 5)

$$\frac{1}{\sigma_0}\frac{d\sigma}{dp_\perp^2} \xrightarrow[p_\perp^2/Q^2\to 0]{} \frac{A}{\Lambda^2}\,[\log\frac{Q^2}{\Lambda^2}]^{\frac{1}{2}}\ [\frac{Q^2}{\Lambda^2}]^{-0.6} \tag{5.10}$$

where the power 0.6 corresponds to 4 flavors.

5.3.3. Small $p_\perp$ Region.

We are now in a position to answer the question c) of whether we can use perturbation theory to evaluate $d\sigma/dp_\perp^2$ at $p_\perp \approx 0$. As remarked already, for $p_\perp^2 \ll (p_\perp^2)_0$ the dominant contribution to $d\sigma/dp_\perp^2$ comes from multi-gluon emissions with $(k_\perp^i)_{gluons} \approx (p_\perp^2)_0 \gg p_\perp^2$. Consequently even for very small $p_\perp$ the argument of α and of the parton distributions will be $(p_\perp^2)_0$ and not $p_\perp^2$. But since $(p_\perp^2)_0$ (see (5.4)) is for sufficiently large Q^2 much larger than Λ^2, the perturbation theory can be safely applied. It should be stressed, however, that at not too large values of Q^2 one should expect the non-perturbative effects (intrinsic $k_\perp$) to be important. It is, however, interesting to observe (E11) that due to the exponential in (5.8) the large b region (where the non-perturbative effects are most important) is more and more suppressed as Q^2 increases. Consequently at very large Q^2 the Fourier transform in (5.7) will be dominated by the small b region (short distances), sensitivity to the intrinsic $k_\perp$ will be lost, and the $p_\perp$ distributions in the massive muon production will be fully predicted within perturbation theory in the whole region of $p_\perp$!! This is of course only a dream at presently available energies, but at Isabelle and Tevatron energies this dream could be partially realized. So let us see what our experimental friends can observe at such high energies.

5.3.4. Large Q^2 Predictions

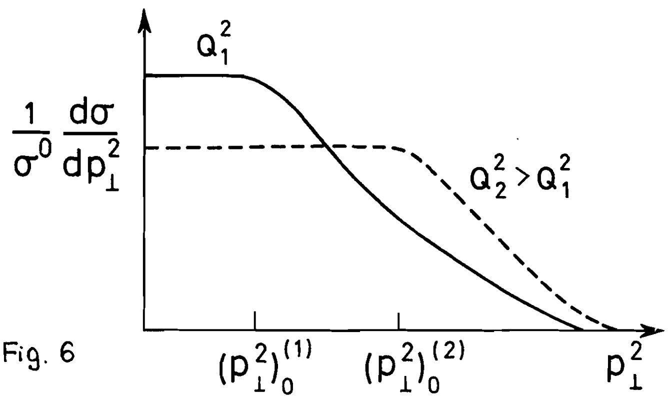


Fig. 6

They should observe (see Fig. 6) a plateau in $p_\perp^2$, extending from $p_\perp^2 \approx 0$ to $p_\perp^2 = (p_\perp^2)_0 \approx \Lambda^{1.2} Q^{0.8}$ (E16), and a decreasing with $p_\perp^2$ (at fixed Q^2) cross-section for $p_\perp^2 > (p_\perp^2)_0$. With increasing Q^2 the length of the plateau should increase as $\Lambda^{1.2}Q^{0.8}$ and its height should decrease as $Q^{-1.2}$. Consequently for values of $p_\perp^2$ somewhat larger than $(p_\perp^2)_0$ an increase of the cross-section with Q^2 is expected. Note that the height and the length of the plateau depend sensitively (as a power) on the scale parameter Λ . Thus in principle the massive muon pair production offers us a possibility for a precise determination of the scale Λ by using very high energy machines. Note that this is opposite to many other experiments (e.g. deep-inelastic scattering) in which the sensitivity to Λ is lost at such high (say $Q^2 \geqslant 500$ GeV^2) energies. However, before the scale Λ which enters the formulae above can be identified with $\Lambda_{\overline{MS}}$ more theoretical work is needed. Some progress in this direction has already been made by Collins and Soper (E19).

5.3.5. Systematic Approach

We have seen above that the inclusion of certain sub-leading logarithms (in the $k_\perp$ space) modified substantially the DLLA result. The obvious question (E18) is then whether other subleading logarithms not taken as yet into account could have an important effect on everything that we just said. In order to answer this question a systematic approach is needed. Such an approach has recently been proposed by Collins and Soper. We shall only present here their final formula which applies in the whole range of $p_\perp^2$. It reads as follows (E20):

$$\frac{Q^2 \; d\sigma}{dx_1 dx_2 d^2p_\perp} = \Sigma + Y \tag{5.11}$$

where Σ dominates for $p_\perp^2 << Q^2$ and it combines with Y for $p_\perp^2 \approx Q^2$ to reproduce the standard perturbative results of Section 5.2. Σ is given essentially by a formula like (5.7) with

$$\hat{S} = \int_{1/b}^{\sqrt{x_1 x_2 Q^2}} \frac{d\mu}{\mu} \left[\log \frac{x_1 x_2 Q^2}{\mu^2} \bar{\gamma}_K(g(\mu)) - 2\bar{K}(b,g(\tfrac{1}{b}))\right] \tag{5.12}$$

replacing $\hat{S}_0$ of Eq.(5.9). The functions $\bar{\gamma}_K$ and $\bar{K}$ are fully calculable in perturbation theory and the claim is that once the one-loop and two-loop contributions to $\bar{\gamma}_K$ and one-loop result for $\bar{K}$ are inserted into (5.12) the remaining corrections are small. The results (see (5.7)) discussed in the previous sections correspond to setting $\bar{K}$ to zero and taking only one loop contributions to $\bar{\gamma}_K$ into account. $\bar{K}$ is already known but the two-loop $\bar{\gamma}_K$ has still to be calculated. Once it is known the scale Λ in Eq.(5.10) can be related to $\Lambda_{\overline{MS}}$.

It should be remarked that a complete proof of the Collins-Soper formula (5.11, 5.12) is still missing, but there exists a proof (E21) of an analogous formula relevant for Energy-Energy correlations in $e^+e^- \to h_1 h_2 X$.

In summary it seems that a lot of progress has been made towards the understanding of $p_\perp$ distributions in the massive muon production. However it is crucial to check whether the predictions discussed here are not spoiled by diseases found by Doria, Frenkel and Taylor (see Section 8) in the integrated over $p_\perp$ cross-sections at the two-loop and higher twist level.

Finally we should mention that the phenomenological application of formulae like (5.7)-(5.9) has been made recently by Chiappetta and Greco (E22), who find a good agreement of the theory with πN and pN data after the inclusion of the intrinsic $\langle k_\perp^2 \rangle_{int} \approx 0.4$ GeV^2. Thus the inclusion of multiple-gluon effects into the phenomenological analysis improves the agreement of the theory with the low $p_\perp$ data without the need for a large O(1 GeV) intrinsic $k_\perp$.

5.4. Miscellaneous Remarks

Another place where the physics discussed above can be studied is energy-energy correlations (E23), $d\Sigma/d\cos\theta$, which can be measured in e^+e^-annihilation. The important variable is now θ , the angle between the momenta of two particles a and b detected in the final state (E24) ($e^+e^- \to a + b +$ anything). For $60^0 < \theta < 120^0$ the standard perturbation theory can be used. One finds (E25) that a good description of the data can only be obtained after the inclusion of a substantial non-perturbative (fragmentation) contribution. The non-perturbative component decreases with the increasing energy (like 1/W) but even at the highest energy it corresponds to roughly 40% of the full result. The non-perturbative effects are expected (E23) to be much smaller (decreasing like $1/W^2$) in the asymmetry defined by

$$A(\theta) = \frac{d\Sigma}{d\cos\theta}(\pi-\theta) - \frac{d\Sigma}{d\cos\theta}(\theta) . \tag{5.13}$$

Indeed, the analysis of the PLUTO group (E26) shows that the perturbative result is in good agreement with the data at 30 GeV. It should be mentioned that these results are based on the leading order ($O(\alpha)$) of perturbation theory. The $O(\alpha^2)$ corrections are now being computed (E27).

For small θ the physics is essentially identical to the one encountered at low $p_\perp$ in the massive muon pair production. Detailed discussions can be found in (E12), (E17), (E18) and (E21). The first comparisons with the data are encouraging.

There are other quantities where multiple gluon emissions play an important role. These are the total transverse jet momentum distributions (E10, E15, E26) and acolinearity distributions in deep-inelastic scattering (E13).

6. Heavy Quarkonia Decays.

The next set of quantities which we shall encounter on our tour are the leptonic, photonic and hadronic decay widths of heavy quarkonia and their hyperfine splittings. It is believed (A17,F1) that in QCD all these quantities can be written in a factorized form as follows:

$$\Gamma = |\phi(0)|^2 \quad C(\alpha_s(m^2)) \tag{6.1}$$

where $\phi(0)$ is the wave function at the origin of the $Q\bar{Q}$ system and $C(\alpha_s(m^2))$ is a short distance function which can be calculated in Perturbative QCD and which has an expansion in α_s of the type given in Eq.(2.1). Furthermore m is the mass of the constituent quark. The function $C(\alpha_s(m))$ is obtained by evaluating the amplitude for annihilation of a quark and an antiquark into gluons, and in higher orders into gluons, quarks and antiquarks. The wave function $\phi(0)$, which contains long distance effects (binding effects) cannot be calculated by perturbative methods, but can be obtained from a potential model. It is sometimes useful to take appropriate ratios of various partial widths and eliminate $|\phi(0)|^2$ from the analysis. The resulting quantities are then fully calculable in Perturbative QCD and consequently good for QCD tests.

To our best knowledge there is no rigorous proof of the factorization in Eq. (6.1) but there exist arguments (A17, F1) that at least the leading and the next-to-leading order QCD corrections to various ratios considered below are independent of the binding energy and can be meaningfully calculated in perturbation theory.

After these general remarks we can now have a closer look at the outcome of various calculations.

6.1. Large Corrections to P-state Decays.

Barbieri, Caffo, Gatto and Remiddi (F2) have calculated the one loop QCD corrections to the annihilation widths into hadrons and into two photons of the P-wave quarkonium states. In order to confront the results of these calculations with the existing data we consider (F2) the following ratios (h≡hadrons)

$$R_1 \equiv \frac{\Gamma(0^{++}\to h)}{\Gamma(2^{++}\to h)}\ \frac{\Gamma(2^{++}\to\gamma\gamma)}{\Gamma(0^{++}\to\gamma\gamma)} = 1 + 6.5\ \frac{\alpha(4m^2)}{\pi} = 1.6 \pm 0.2 \qquad (6.2)$$

$$R_2 \equiv \frac{4}{15}\ \frac{\Gamma(0^{++}\to h)}{\Gamma(2^{++}\to h)} = 1 + 12.\ \frac{\alpha(4m^2)}{\pi} = 2.15 \pm 0.35 \qquad (6.3)$$

where the numerical values of the coefficients of α/π are for the charmonium family. For the bottomium family the corresponding coefficients are 4.0 and 9.5. It should be emphasized that the coefficients in question are true physical predictions of QCD and do not depend on the renormalization scheme used to calculate Γ's. Thus the sign and the size of the corrections can be directly confronted with the data. As shown in Eqs. (6.2) and (6.3), for the full range of values of Λ considered (0.1 GeV < Λ < 0.5 GeV), the corrections to the leading order predictions (i.e. 1) are sizeable. It is then interesting to note (F2) that the present data (F3), which give $R_1 \geqslant 1.45$ and $R_2 = 2.1 \pm 0.5$, require these large corrections. The sign and the size of the corrections agree with the data!! This enthusiasm is tempered, however, by the fact that the next-to-leading order corrections to R_1 and R_2 are large and it is not clear whether we should trust perturbation theory. But the situation could have been even worse. We could have found very small corrections or corrections with a negative sign in which case there would not be much hope for the agreement of the theory with data (unless non-perturbative effects were very large). I therefore think that at least on a qualitative level QCD predictions for R_1 and R_2 for the charmonium family have survived the confrontation with experiment.

For more quantitative tests it is important to make a similar comparison for the bottomium family. The corresponding predictions are $R_1 = 1.26 \pm 0.05$ and $R_2 = 1.63 \pm 0.13$ i.e., corrections are smaller than in the charmonium case and the perturbative expansions are expected to behave better.

6.2. Leptonic Widths of Ψ and Υ, Hadronic Widths of Paraquarkonia and Hyperfine Splittings.

The large corrections to R_1 and R_2 discussed above are by no means the only large corrections encountered in the quarkonia physics. It has been known for a long time that the one-loop corrections to the leptonic width of the S-state orthoquarkonia (1^{--}) were substantial. One has (F4)

$$\Gamma^V\left(\begin{smallmatrix}\Upsilon\\ \Psi\end{smallmatrix} \to \mu^+\mu^-\right) = \frac{4\pi e^2\alpha^2_{EM}}{m^2}\ |\phi(0)|^2\ \left[1 - 5.3\ \frac{\alpha(m^2)}{\pi}\right] \qquad (6.4)$$

where e and m are the charge and the mass of the constituent quark respectively. Furthermore α_{EM} is the electromagnetic coupling constant. The one-loop corrections here have an opposite sign to those found in the P state decays and they reduce the Born term prediction by roughly a factor 1.8 ± 0.3 for Ψ and a factor 1.5 ± 0.2 for Υ when $0.1 < \Lambda < 0.3$ GeV.

Similarly the one-loop corrections to the hadronic widths of paraquarkonia have been found to be substantial. In the $\overline{MS}$ scheme one has (F5)

$$\Gamma^{PS}\left(\begin{matrix}\eta_c\\ \eta_b\end{matrix} \to h\right) = \frac{8\pi}{3}\,\frac{\alpha^2_{\overline{MS}}(m^2)}{m^2}\,|\phi(0)|^2\,\left[1 + \begin{pmatrix}5.27\\ 4.85\end{pmatrix}\frac{\alpha_{\overline{MS}}(m^2)}{\pi}\right] \tag{6.5}$$

It is then interesting to observe that the recently calculated (F6) one-loop QCD corrections to hyperfine splittings turn out to be very small (2 - 7%):

$$\Delta E\begin{pmatrix}\Psi-\eta_c\\ \Upsilon-\eta_b\end{pmatrix} = \frac{32\pi}{9}\,\frac{\alpha_{\overline{MS}}(m^2)}{m^2}\,|\phi(0)|^2\,\left[1 + \begin{pmatrix}0.77\\ 0.14\end{pmatrix}\frac{\alpha_{\overline{MS}}(m^2)}{\pi}\right] \tag{6.6}$$

The important question is then whether the predictions (6.4) and (6.5) (large corrections) and the prediction (6.6) (small corrections) can be made simultaneously consistent with the data, and this for reasonable values of $\alpha_{\overline{MS}}$. In order to answer this question one can either take various ratios of quantities in (6.4)-(6.6) in which case the dependence on $|\phi(0)|^2/m^2$ is eliminated, or use a potential model from which $|\phi(0)|^2$ can be obtained. Using the potential model of ref.(F7) one finds (see Table I) that the existing data for the charmonium family are well represented by Eqs.(6.4),(6.5) and (6.6) with $\Lambda_{\overline{MS}} = 200 \pm 100$ MeV (F8).

Table I

Quantity	Theory ($\Lambda_{\overline{MS}} = 200 \pm 100$)	Experiment	
$\Gamma(\Psi \to \mu^+\mu^-)$	$4.7 \pm .6$	$4.8 \pm .6$	keV
$\Gamma(\eta_c \to h)$	22 ± 10	$20\,^{+16}_{-11}$	MeV
$\Delta E(\Psi-\eta_c)$	84 ± 21	119 ± 9	MeV
$\Gamma(\Upsilon \to \mu^+\mu^-)$	0.98 ± 0.06	$1.16 \pm .17$	keV
$\Gamma(\eta_b \to h)$	7.1 ± 2.1	?	MeV
$\Delta E(\Upsilon-\eta_b)$	$39. \pm 5$	?	MeV

This is consistent with the analysis of the authors of ref.(F6) who, using various ratios of quantities in (6.4),(6.5) and (6.6) and employing the method of ref. (B15) find $\Lambda_{\overline{MS}} = 160 \pm 90$ MeV. Remembering various theoretical uncertainties such as non-perturbative effects, relativistic corrections and higher order corrections which have not been taken into account, the analysis presented above can only be regarded as semi-quantitative. For this reason also the precise determination of the scale parameter $\Lambda_{\overline{MS}}$ cannot be made from Ψ spectroscopy. For the Υ family the situation is expected to be better as we shall now discuss.

6.3 $\Lambda_{\overline{MS}}$ from Hadronic Width of Orthoquarkonia.

One of the most interesting and at the same time difficult calculations in Perturbative QCD in the last year has been done by Lepage and McKenzie (F9) who evaluated the long awaited α^4 corrections to the hadronic widths of Ψ and Υ. The calculation involves the transitions $Q\bar{Q} \to 4G$, $2Gq\bar{q}$ in addition to the virtual corrections to the 3G decay mode. Since the perturbative expansion for the hadronic widths of S-wave orthoquarkonia begins with α^3 these quantities are very useful for a precise determination of the scale parameter Λ. Choosing the $\overline{MS}$ scheme Lepage and McKenzie find (F9)

$$\Gamma_h\left({\Psi \atop \Upsilon} \to \text{hadrons}\right) = \frac{160}{81}\,(\pi^2 - 9)\,\alpha^3_{\overline{MS}}(M^2)\,\frac{|\phi(0)|^2}{M^2}\,\left[1 + \begin{pmatrix}4.9\\3.8\end{pmatrix}\frac{\alpha_{\overline{MS}}(M^2)}{\pi}\right] \tag{6.7}$$

which when combined with (6.4) gives

$$\frac{\Gamma_h}{\Gamma^V_{\mu^+\mu^-}} = \frac{10}{81}\,\frac{(\pi^2-9)}{\pi\, e^2}\,\frac{\alpha^3_{\overline{MS}}(M^2)}{\alpha^2_{EM}}\,\left[1 + \begin{pmatrix}10.2\\9.1\end{pmatrix}\frac{\alpha_{\overline{MS}}(M^2)}{\pi}\right] \tag{6.8a}$$

$$= \frac{10}{81}\,\frac{(\pi^2-9)}{\pi\, e^2}\,\frac{\alpha^3_{\overline{MS}}(m^2)}{\alpha^2_{EM}}\,\left[1 + \begin{pmatrix}0.80\\0.45\end{pmatrix}\frac{\alpha_{\overline{MS}}(m^2)}{\pi}\right]. \tag{6.8b}$$

In obtaining (6.8b) we have used $M = 2m$, where M is the quarkonium mass. Note that if the argument of α is chosen to be m the corrections to the ratio above are very small. Comparing Eq.(6.8a,b) with the data for the Υ family (F10), Lepage and McKenzie find $\Lambda_{\overline{MS}} = 121^{+44}_{-31}$ MeV and $\Lambda_{\overline{MS}} = 100^{+34}_{-25}$ MeV from (6.8a) and (6.8b) respectively. The difference between these two values together with the quoted errors which come from the data show how accurately one can at present determine Λ from Υ decays. Smaller values of $\Lambda_{\overline{MS}}$ are obtained for the charmonium but the analysis is much less reliable. As shown in Table I the QCD prediction for $\Gamma_{\mu^+\mu^-}(\Upsilon)$ when combined with the potential model of ref.(F7) agrees quite well with the data. Also in ref.(F9) predictions for $\Gamma(\Psi, \Upsilon \to \gamma + \text{hadrons})$ can be found.

6.4. Summary and Outlook.

i) It seems that the QCD predictions for various decay rates of quarkonia agree well with experiment. An exception are the E1 Transitions (F11) where the discrepancy between theory and experiment amounts to a factor 2 to 4, but at the moment it is not clear whether this is the problem of QCD or of the existing potential models. Furthermore only leading order prediction for the E1 transitions are known and the higher order QCD corrections could turn out to be as large as in the case of the P-state decays.

ii) Radiative corrections to various decay rates turn out to be small and large at right places. One can, however, worry a bit that in the cases where they are large, perturbative calculations cannot be fully trusted.

iii) Because of large corrections the charmonium family can offer us only qualitative tests. More quantitative confrontation can be made for the bottomium family once $\Delta E(\Upsilon-\eta_b)$ and $\Gamma(\eta_b \to h)$ are measured in addition to the already known $\Gamma(\Upsilon \to \mu^+\mu^-)$ and $\Gamma(\Upsilon \to h)$. Good data for all these four quantities could allow us precise determination of Λ . Finally as discussed at length in ref. (F12), the toponium family, if not too heavy (F13), would be a very good place for QCD tests and for the determination of the $Q\bar{Q}$ potential. For a 40-50 GeV heavy toponium a large portion of the $Q\bar{Q}$ potential is expected to be Culombic, in which case the confrontation with the QCD potential, which is known up to the two-loop level (F14) will be possible.

iv) The values of $\Lambda_{\overline{MS}}$ extracted from various decay rates are compatible with each other, although the values extracted from the hadronic and leptonic widths are somewhat lower than the one extracted from hyperfine splittings (F8). Furthermore the values of $\Lambda_{\overline{MS}}$ are compatible with the ones found in deep-inelastic scattering analysis.

v) Finally a better understanding of factorization (6.1), of the non-perturbative effects and of the relativistic effects is clearly needed. The latter effects are probably quite important for the charmonium family but are estimated (F12) to be relatively small for bottomium and toponium families.

Further aspects of the Quarkonia physics can be found in the talk of Shifman (F11).

We have not discussed here the jet studies in quarkonia physics. Among the recent papers on this subject is the study of gluon jets in heavy paraquarkonium decay (F15).

7. Jets and e^+e^- Annihilation.

One of the most popular topics in Perturbative QCD is jets. These have been studied most extensively in e^+e^- annihilation. There have already been many talks on jets at this conference (G1) and consequently we shall concentrate here only on the highlights of jet physics.

The procedure for all jet calculations consists essentially of three steps. In Step 1, one calculates the diagrams of the type given in Fig. 7 : a) zeroes order in α , b) $O(\alpha)$, c)- e) $O(\alpha^2)$ and generally f) $O(\alpha^n)$.

In Step 2 one integrates the results of the first step over various variables. Subsequently so-called infrared safe quantities (G2) can be constructed. The most popular among these are: i) Sterman-Weinberg cross-sections (G2) and ii) average jet measurements such as thrust (G3), acoplanarity (G4), Fox-Wolfram parameters (G5) and energy-energy correlations (G6). One is also interested in iii) $p_\perp$ distributions of the hadrons in the final state and in iv) the average hadronic multiplicity. Finally in Step 3 hadronization effects and generally non-perturbative effects have to be taken into account before a given quantity can be compared with the experimental data.

The first two steps are believed to be well understood in Perturbative QCD, whereas the last step which involves long distance phenomena can only be handled by invoking hadronization models (G7). There is also a belief (partly justified for some quantities) that the calculations of the first two steps can be done

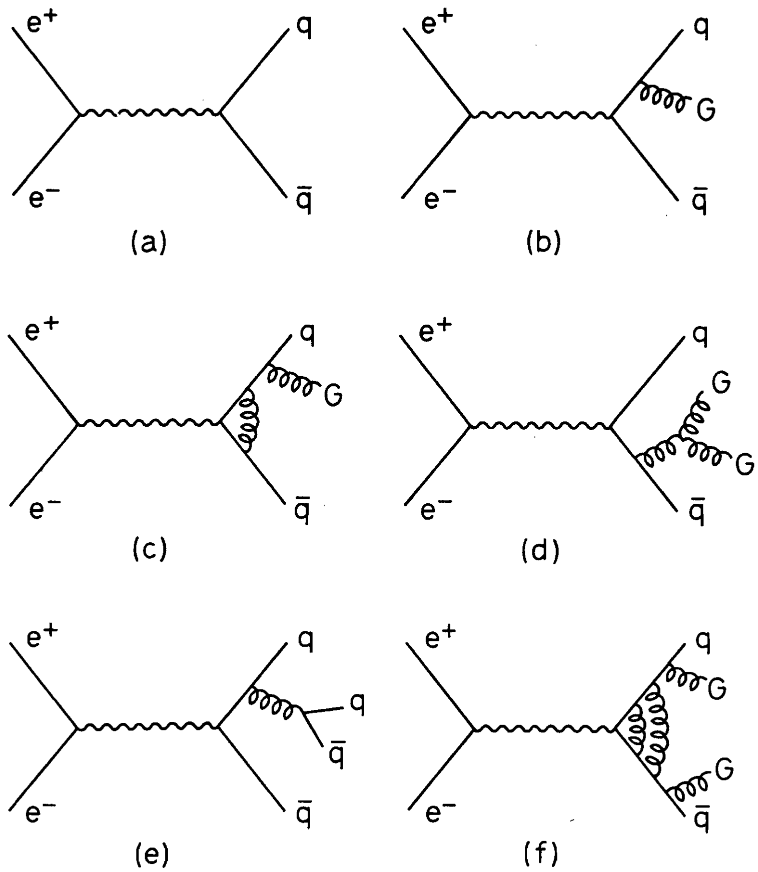


Fig. 7

independently of the last step. Furthermore it is hoped that for some quantities and at sufficiently high energies the results of the first two steps will be insensitive to the hadronization effects of step 3 . Such quantities would then be suitable for "clean" QCD tests. Unfortunately it does not seem that anybody so far succeeded in finding a quantity which would be completely clean at the accessible energies. Consequently quantitative tests and confrontations of QCD with the available experimental data should be considered with some caution. On the other hand qualitative tests at present energies have much firmer basis. In fact it appears that on the qualitative level all QCD predictions for jet cross-sections are in accord with the experimental findings.

In the following we shall make a biased and probably incomplete list of the most interesting confrontations of QCD with the experimental data in e^+e^- annihilation. Subsequently we shall discuss a hot topic of this symposium: α^2 corrections to the jet cross-sections.

7.1. Highlights of Jets and of e^+e^- Annihilation.

1. Observation of a two jet structure at SPEAR between 3.0 and 7.4 GeV, which subsequently has been confirmed by groups at Doris.
2. Studies of the angular distribution of the jet axis with respect to the beam

direction revealed $1+\cos^2\theta$ distribution supporting spin ½ for the quarks (scalar quarks would give $1-\cos^2\theta$).

3. The measured value of $R^{e^+e^-}$ agrees very well with the QCD prediction. The higher order corrections turn out to be small (G8).
4. Deviations from two jet structure have been observed by several groups at Doris. In particular the analysis by the PLUTO group (G9) suggests the decay $T\to 3G$. Furthermore the analysis of Koller and Krasemann (G10), the analysis a'la Ellis-Karliner (G11) by the TASSO group (G12) and the recent paper by Koller, Sander, Walsh and Zerwas (G13) support the spin 1 for the gluon.
5. Three jet events in the nonresonant region at 30 GeV have been observed by various groups (G14). Extensive analyses of various distributions support the belief that the three jet events come from hard gluon bremsstrahlung (G15). In particular one observes broadening of $p_\perp$ distributions with increasing energy. Furthermore some differences between the quark and gluon fragmentation have been observed in accordance with QCD expectations (G16).

On the theoretical side there have been many developments in the jets physics. Some of them are listed below.

6. Jet calculus (G17).
7. α^2 corrections to the jet cross-sections and to the event shapes which we shall discuss in Sect. 7.2.
8. Generalizations of the Sterman-Weinberg formula. The present status is summarized very nicely in Sect. 6 of ref. (A13).
9. Energy-energy correlations discussed in Sect. 5.4.
10. Average hadronic multiplicities to be discussed by Mueller in his talk at this symposium.

7.2. Higher Order Calculations and the Parameter Λ.

In Sections 3 and 6 we have discussed the values of the scale parameter Λ as extracted from the deep-inelastic scattering and quarkonia decays respectively. It is important to check whether the jet cross-sections give the same value for Λ. One year ago various experimental groups at DESY found the values of Λ using the leading order ($O(\alpha)$) jet cross-sections. For reasons discussed in Section 2 these values cannot be meaningfully compared with the values of $\Lambda_{\overline{MS}}$ found in Sections 3 and 6. For such a comparison to make sense next-to the leading order corrections ($O(\alpha^2)$) to the jet cross-sections have to be calculated. Such a calculation involves the Born diagrams contributing to four jet cross-sections $e^+e^-\to q\bar{q}\,GG$ and $e^+e^-\to q\bar{q}\,q\bar{q}$ and also the virtual (loop) corrections to the three jet process $e^+e^-\to q\bar{q}G$.

The four jet cross-sections have been first calculated in ref.(G18) and the result of this calculation has been subsequently confirmed by two other groups (G19,G20). The first full calculation of order α^2 (i.e. including virtual corrections) has been done by Ellis, Ross and Terrano (ERT) (G20) and subsequently by two other groups (G21-G23). The answers for the matrix elements (see step 1 above) obtained by all groups agree with each other, whereas there seemed to exist (at least before this symposium) some disagreements in subsequent steps of various analyses.

The authors of ref.(G23) found large and positive corrections to the thrust distribution (G24). This result has been confirmed in refs.(G25 - G29) where the matrix elements of ERT (G20) have been used. On the other hand Fabricius, Kramer, Schierholz and Schmitt (FKSS) (G21) found small and negative corrections to the thrust distribution. The latter authors have also calculated (G22) the generalized 3-jet Sterman-Weinberg cross-section, and found that for certain values of the cut-off parameters ε and δ the corrections were small. A similar result has been obtained by Sharpe (G27), who used the matrix elements of ERT.

In view of all these results, we want to ask now the following questions:

i) How large are the $O(\alpha^2)$ QCD corrections to the thrust distribution as defined by Farhi (G3) in 1976?

ii) Which distributions (thrust, Sterman-Weinberg cross-sections, etc.) are useful for QCD tests?

iii) How large is $\Lambda_{\overline{MS}}$ as extracted from the jet cross-sections?

Here are the answers to all these questions.

i) In order to answer the first question I organized two short meetings (G30) with the physicists, who were directly involved in the calculations mentioned above. It has been concluded that only the authors of refs.(G24 - G29) calculated the thrust distribution as defined in (G3). The authors of ref.(G21) calculated a different distribution (call it $d\sigma/dT'$) and therefore there is no wonder that they obtained a result which differs from (G23 - G29). The discussion of the difference is somewhat too technical to be presented here. In summary then, the $O(\alpha^2)$ QCD corrections to the thrust distribution are large and positive (G31). One has for instance:

$$\frac{1}{\sigma}\frac{d\sigma}{dT}\bigg|_{T=.85} = 4.8\ \alpha_{\overline{MS}}(Q^2)\ \left[1+16.\frac{\alpha_{\overline{MS}}}{\pi}\right] \tag{7.1}$$

and

$$\frac{1}{\sigma}\frac{d\sigma}{dT}\bigg|_{T=.70} = 0.2\ \alpha_{\overline{MS}}(Q^2)\ \left[1+24.\frac{\alpha_{\overline{MS}}}{\pi}\right] \tag{7.2}$$

which for $\alpha_{\overline{MS}}$ (1200 GeV^2) $\approx$ .13 corresponds to a 60% and 100% correction respectively. For $0.75 < T < 0.90$ the corrections are roughly 60%.

ii) What about other distributions? FKSS have found small ($\sim 20\%$) corrections to the 3-jet Sterman-Weinberg cross-sections for $\varepsilon = 0.2$ and $\delta = 45^0$, but huge corrections (a factor $1/3 \pm 1$) for $\varepsilon = 0.1$ and $\delta = 30^0$. This agrees with the calculations of Sharpe (G27) who finds that the $O(\alpha^2)$ corrections to Sterman-Weinberg 3-jet cross-sections are less than 25% for $\varepsilon > 0.05$ and $36^0 < \delta < 60^0$, but that they are larger for other values of ε and δ.

As emphasized by Clavelli and Wyler (G26,G32), for small T the $O(\alpha^2)$ corrections to $d\sigma/dT$ must be large because the phase-space for the $O(\alpha)q\bar{q}G$ contribution ends at $T = 2/3$ whereas for the $O(\alpha^2)$ $q\bar{q}GG$ contribution it extends to

$T = \frac{1}{\sqrt{3}} \approx 0.58$. This is clearly seen in Eqs.(7.1) and (7.2). This is unfortunate because in the range of T far from $T=1$ the non-perturbative effects are expected to be smallest and consequently one would believe that this is the best region for QCD tests. Clavelli and Wyler suggest therefore to seek variables which have the same kinematic boundaries in all orders of α. One possibility (G26) is to divide events in two with respect to the plane normal to the thrust axis and use the invariant mass M_H of the heavier jet as a variable. In all orders in α one has $M_H^2/S \leqslant 1/3$. Using the matrix elements of ERT, Clavelli and Wyler find that the $O(\alpha^2)$ corrections to the distribution in question are at most 40% in the whole range of M_H.

In summary it seems that thrust distributions for $0.75 < T < 0.90$, Sterman-Weinberg cross-sections for $\varepsilon > 0.05$ and $36^0 < \delta < 60^0$, and M_H distributions of ref.(G26) can be meaningfully compared with the data. The corrections to the thrust distribution are somewhat large but I do not think that they are large enough to prevent the determination of $\Lambda_{\overline{MS}}$ from these distributions (G33). I think it is important to find out which of the three distributions mentioned above is least sensitive to non-perturbative effects. Talking to various people during this symposium I get the impression that different opinions exist on this issue.

iii) But what about the values of $\Lambda_{\overline{MS}}$? Unfortunately at the moment of this writing there is no full agreement on this value. The authors of refs.(G23, G25) and in particular Ali (G29) find $\Lambda_{\overline{MS}} \approx 100 \pm 50$ MeV from the thrust distribution (G34). A similar result has been obtained in ref.(G26) by studying the M_H distribution. On the other hand FKSS find $\Lambda_{\overline{MS}} \approx 480$ MeV by comparing the 3-jet Sterman-Weinberg cross-sections with data. Because of large experimental errors the value $\Lambda_{\overline{MS}} \approx 300$ MeV could also fit the latter cross-sections. The discrepancies in the values of $\Lambda_{\overline{MS}}$ just mentioned do not look so bad if one talks about $\alpha_{\overline{MS}}$ instead of $\Lambda_{\overline{MS}}$. $\Lambda_{\overline{MS}} = 100 \pm 50$ MeV and say $\Lambda_{\overline{MS}} \approx 400 \pm 100$ MeV correspond to $\alpha_{\overline{MS}} = 0.125 \pm 0.01$ and $\alpha_{\overline{MS}} \approx 0.16 \pm 0.01$ at $Q^2 \approx 1200\ \mathrm{GeV}^2$ respectively. This amounts to a 30% discrepancy. Nevertheless it is important to clarify why the values of $\Lambda_{\overline{MS}}$ or $\alpha_{\overline{MS}}$ extracted from various distributions are so different.

8. Theoretical News

During the last two years there have been several important theoretical results in perturbative QCD, which we have not discussed in this review. For completeness, however, we shall make a (probably incomplete) list of these achievements.

8.1. Wee Parton Cancellation.

In connection with the semi-inclusive processes in which there are two hadrons in the initial state (e.g. $pp \to \mu^+\mu^- X$) or two detected hadrons in the final state (e.g. $e^+e^- \to h_1 h_2 x$) there is an important question of whether the soft gluons' exchanges between the colliding (or detected) hadrons are cancelled by the real gluon emissions. Explicit calculations of order α have shown that this is indeed the case. However, an all order proof was missing for some time. Recently such a proof for $e^+e^- \to h_1 h_2 x$ has been demonstrated by Collins and Sterman (H1). An analogous proof for the massive muon production is however still missing.

8.2. Doria-Frenkel-Taylor disease.

We should mention a very important finding of Doria, Frenkel and Taylor (H2). These authors made a study of the infrared behaviour of the inclusive process $qq \to$ virtual photon + anything ($O(\alpha^2)$). Their study has been repeated by Di'Lieto, Gendron, Halliday and Sachrajda (H3) who, although finding some errors in the intermediate steps of the calculations of ref.(H2), confirmed the main result of Doria et al: for the process in question the Bloch-Nordsieck cancellation of infrared divergences fails. The left infrared divergence is $O(m^2/Q^2)$ and will undoubtedly complicate the study of higher twist contributions to massive muon production. Generally a similar feature is expected for processes with two hadrons in the initial state (H2 - H4). The above results raise the following important question: does the Bloch-Nordsieck mechanism work for leading twist contributions to the processes in question in order α^k with $k > 2$?

It is possible to cancel the infrared singularities mentioned above by forming a coherent state of soft gluons (I4), but the final answer must clearly depend on how this state is formed and consequently the predictive power of the theory is lost.

8.3. Exclusive Processes

During the last two years the QCD predictions for the hadronic formfactors and the elastic scattering at large angles have been worked out by various people, in particular by Farrar and Jackson (H5), Brodsky and Lepage (H6), Efremov and Radyushkin (H7), Parisi (H8), Duncan and Mueller (H9) and Landshoff and Pritchard (H10). Further references and the discussion of the results of these papers can be found in the talk by Mueller.

I will just mention here that in ref.(H11) the next to the leading order corrections (i.e. $O(\alpha^2)$) to the hard scattering amplitude relevant for the pion formfactor have been calculated. The corrections turn out to be substantial. However in order to obtain the full $O(\alpha^2)$ corrections to the pion formfactor, the next to the leading order corrections to the parton distribution amplitudes have still to be computed.

8.4. Two-photon Processes

Photon structure functions F_2^{γ}, F_L^{γ}, etc., which can be measured in $e^+e^- \to e^+e^-$ + hadrons have attracted the attention of many people, whose names can be found in the talk of Bardeen. As opposed to the hadronic structure functions, F_i^{γ} can be fully calculated if Q^2 is large enough. For $Q^2 \approx 5 - 10$ GeV^2 only the $x > 0.4$ region can be fully predicted in Perturbative QCD since the small x region receives an important vector dominance contribution. The recent PLUTO data (H12) seem to agree for $x > 0.4$ with the shape and normalization of F_2^{γ} as predicted by QCD. The value of $\Lambda_{\overline{MS}}$ turns out to be 200 ± 100 MeV (H13).

For completeness we have listed in Table III the references to higher order calculations which are relevant to two-photon physics (H14-H17).

8.5. Average Hadronic Multiplicities

Very interesting predictions have been made for the energy dependence of the average hadronic multiplicities. Details can be found in the talk by Mueller and in (H18).

9. Grand View of Perturbative QCD.

We are approaching the end of our tour. Let us enumerate the successes and spectacular results of Perturbative QCD as well as the problems which have to be solved in the future.

9.1. Seven Wonders of Perturbative QCD.

A) Roughly ten years ago theorists began a search for a theory of strong interactions which would explain approximate Bjorken scaling and the ratio $R^{e^+e^-}$. QCD turned out to be such a theory. Moreover it predicted calculable logarithmic deviations from the exact Bjorken scaling and the deviations from the parton model prediction for $R^{e^+e^-}$. The high statistics experiments performed over the last four years have shown deviations from the free parton model predictions in accordance with QCD. Furthermore it has been found by theorists that higher order corrections to deep-inelastic structure functions and to $R^{e^+e^-}$ were rather small already at presently available energies implying that perturbative calculations for these quantities can be trusted.

It has also been found that there are other quantities for which the leading order QCD predictions agree well with the data, and for which higher order corrections turned out to be small. The list of these quantities includes photon structure functions at intermediate x values and hyperfine splittings among others.

B) It is then important to notice that the same theory which gives small higher order corrections to the quantities mentioned above gives large corrections to almost all semi-inclusive processes. In particular one finds large renormalization of the Drell-Yan cross-section by roughly factor 2, scaling violations in fragmentation functions which are predicted to be larger than those for parton distributions, and substantial non-factorization effects, in particular in semi-inclusive deep-inelastic scattering. Large QCD corrections are furthermore found in $p_\perp$ distributions in the massive muon production and in the leptonic, hadronic and photonic widths of quarkonia. Some of these corrections can be made smaller by suitably redefining the expansion parameter α, but others which are renormalization prescription independent are the true predictions of the theory. It is then interesting to observe that essentially all of the large corrections which came out of various theoretical calculations are required by the data. This is in particular the case in the Drell-Yan cross-sections and various widths of quarkonia.

C) There are other spectacular predictions of the Perturbative QCD which have been confirmed by the data. We should mention first of all the three jet events which have been found in agreement with QCD on the qualitative and, to some extent also, quantitative level.

D) Also various large $p_\perp$ effects, as the ones found in the massive muon pair production, e^+e^- annihilation, and deep-inelastic scattering, and which are believed to be the consequence of hard gluon bremsstrahlung, belong to the spectacular predictions of QCD, which have been confirmed by the data.

E) Other spectacular results are related to multiple soft gluon emissions and consequently to the infrared structure of the theory. These are in particular $p_\perp$ distributions in the Drell-Yan process at small $p_\perp$ values, where one expects a decrease of the cross-section with the increasing energy, and the average multiplicities for which a fast increase with the energy is predicted. Although the present data seem to indicate the expected increase of average multiplicities, more work has to be done on the theoretical side before a meaningful quantitative confrontation with the data is possible. Other spectacular confrontations of QCD with the data are expected in the Exclusive processes and the Two-photon Processes.

F) It should also be emphasized that essentially for all quantities for which the relevant calculations have been done the higher order corrections improve the agreement of the theory with data. In particular this is the case of the deep-inelastic structure functions (F_2, F_3), $p_\perp$ distributions in the massive muon production at intermediate and large $p_\perp$ values and also quantities mentioned under B).

G) Finally as shown in Table II there is a remarkable (with few exceptions, see below) agreement between the values of $\Lambda_{\overline{MS}}$ extracted from various experiments. Roughly the present "world $\Lambda_{\overline{MS}}$ " turns out to be

$$\Lambda_{\overline{MS}} = 160 \begin{array}{l} + 100 \\ - \ \ 80 \end{array} \text{MeV}$$

In spite of the Seven Wonders which we have encountered on our trip it is obvious that there still remain many problems which have to be solved before we can be completely satisfied with ourselves and with QCD. Let us list some of them.

9.2. Seven Problems to be Solved in Perturbative QCD.

$\tilde{A}$) The study of higher twist effects in deep-inelastic scattering and in other processes should be continued. Also better understanding of the on set of hadronization and of non-perturbative effects is clearly needed. Some progress in this direction has been made by Shifman, Vainshtein and Zakharov (I1)

$\tilde{B}$) Better understanding of the origin of large higher order corrections found in various processes and in particular the study of their resumation is desirable.

$\tilde{C}$) Further study of Sudakov-like effects is clearly needed. In particular one needs a systematic method for calculating corrections to double leading logarithmic approximation. Important progress in this direction has been made by Collins and Soper (E19).

Table II: Values of $\Lambda_{\overline{MS}}$ and $\alpha_{\overline{MS}}$

Quantity	$\Lambda_{\overline{MS}}$ (MeV)	$\alpha_{\overline{MS}}$ ($Q^2 = 30$ GeV^2)
Structure functions:		
CDHS	210^{+80}_{-70}	0.18 ± 0.02
CDHS (moments)	250 ± 80	0.19 ± 0.02
EMC, H_2	145^{+150}_{-90}	0.165 ± 0.035
EMC, Fe	170^{+155}_{-105}	0.17 ± 0.04
BCDMS	85^{+96}_{-78}	0.15 ± 0.03
BEBC	140^{+95}_{-35}	0.165 ± 0.02
GGM	150^{+150}_{-110}	0.165 ± 0.035
Charm	240 ± 120	0.190 ± 0.01
Ref. C17	450 ± 50	0.23 ± 0.01
$\Gamma(\Psi \to \mu^+\mu^-)$ $\Gamma(\eta_c \to h)$	200 ± 100	0.175 ± 0.025
$\Delta E(\Psi - \eta_c)$	350 ± 50	0.21 ± 0.01
$\Gamma(\Upsilon \to h)$	120 ± 45	0.16 ± 0.01
Sterman-Weinberg 3 jet cross-section (G22)	480 (± 200)	0.24
Thrust (G29)	110^{+70}_{-50}	0.155 ± 0.020
Photon Structure Functions	200 ± 100	0.175 ± 0.025

$\tilde{D}$) One would like to have some "Clean QCD Tests" in which problems $\tilde{A}$ and $\tilde{B}$ are avoided. In particular a clean experimental test of the non-abelian structure of QCD (I2) and of asymptotic freedom would be very important.

$\tilde{E}$) Further exploration of two photon physics, of exclusive processes and of various spin effects (I3) is certainly of interest.

$\tilde{F}$) There are still various rigorous proofs to be demonstrated. Some have been listed in Section 8.

$\tilde{G}$) Finally it would be good to clarify the origin of the difference between the large values of $\Lambda_{\overline{MS}} \approx 400 - 500$ MeV as extracted by Duke, Owens and Roberts

(C17) from the deep-inelastic scattering data and by FKSS (G22) from the Sterman-Weinberg 3 jet cross-sections, and the small values of $\Lambda_{\overline{MS}} \approx 160^{+100}_{-80}$ MeV found by the rest of the world.

Table III: Higher Order Calculations

Quantity	Ref.	Quantity	Ref.
Two-Loop β function	B10	Three-Loop β function	B11
Two-Loop Space-Like Anomalous Dimensions	C6 C7 C8 C9	Two-Loop Time-Like Anomalous Dimensions	D5
Deep-inelastic Scattering F_2, F_3	B2 C8	Deep-inelastic Scattering F_L	C29
Massive Muon Production ($q\bar{q}$,qG)	D8	Massive Muon Production (qq)	D9
$e^+e^- \to hx$	D10	$e^+e^- \to h_1h_2x$	D11
eh $\to$ ehx	D12 D13	hh $\to$ hx	D19 D20
hh $\to$ jet x	D21	Polarized Deep-Inelastic Scattering	C43
$p_\perp$ distributions in Drell-Yan	E5 E6 E7	Large $p_\perp$ direct photons	D22
Four Jets	G18 G19 G20	Full α^2 corrections to Jets	G20 G21 G23
$R^{e^+e^-}$	G8	Three Loop Corrections to Hadronic Multiplicities	H18
Real Photon Structure Functions F_2^γ	H14	Virtual Photon Structure Functions	H15
Paraquarkonia 0^{-+}	F5	Orthoquarkonia 0^{++}, 2^{++}	F2
Orthoquarkonia 1^{++}	F2	Hadronic Width 1^{--}, $1^{--} \to \gamma x$	F9
Leptonic Widths 1^{--}	F4	Hyperfine Splittings	F6
F_3^γ	H16	Direct photons in e^+e^- Collisions	H17
Large $p_\perp$ Direct Leptons	D23	Pion Formfactor	H11

We can now address the question posed at the beginning of the Introduction. I personally believe that in view of the many successful QCD predictions discussed above there is a very good possibility that Quantum Chromodynamics is the correct theory of the Strong Interactions. However in view of the remaining problems which have to be solved both in the short and the long distance domain, we still have to work very hard before we can be sure that this is indeed the case. In other words: "Although substantial progress has been done (see Table III) it seems we shall still have a lot of fun in the years to come."

I would like to thank many of my friends for numerous informative discussions, and NORDITA and colleagues of NORDITA/NBI for warm hospitality.

Footnotes and References

+ Permanent address.

A1. Peterman, A., Phys. Rep. 53C (1979) 157.

A2. Buras, A.J., Rev. Mod. Phys. 52 (1980) 199.

A3. Dokshitzer, Y.L., D.I. Dyakonow and S.I. Troyan, Phys. Rep. 58C (1980) 269.

A4. Ellis, J. and C.T. Sachrajda, CERN-preprint TH-2782 (1979) Cargèse Summer Institute 1979.

A5. Ellis, J., Proceedings of the 1979 International Symposium on Lepton and Photon Interactions at High Energies, Fermilab, 1979 ed. T.B.W. Kirk and H.D.I. Abarbanel.

A6. Lectures by S.D. Ellis, R.D. Field, C.H. Llewellyn-Smith and A.J. Buras in "Quantum Flavordynamics, Quantum Chromodynamics and Unified Theories" ed. K.T. Mahanthappa and J. Randa, Plenum Press (1980).

A7. Brodsky, S.J. and G.P. Lepage, SLAC-Pub-2447 (1979), SLAC-Pub-2762 (1981).

A8. Konishi, K., Nordita 81/25 (1981).

A9. Reya, E., Phys. Rep. 69C (1981) 197.

A10. Llewellyn-Smith, C.H., Proceedings of the 1980 International Conf. on High Energy Physics, Madison (1980).

A11. Buras, A.J., "Seven Topics in Perturbative QCD", Fermilab-Conf-80/79-THY, Physica Scripta 23, (1981) 863.

A12. Talks by J. Lefrancois, F. Sciulli and B.H. Wiik in the Proceedings of the 1980 International Conf. on High Energy Physics, Madison (1980).

A13. Mueller, A.H., Phys. Rep. 73C (1981) 237.

A14. Ali, A., DESY 81-016 (1981).

A15. Sachrajda, C.T., Lectures presented in Les Houches Summerschool, 1981.

A16. Bjorken, J.D. SLAC-Pub-2372.

A17. Remiddi, E., Lectures given at the International School of Physics E. Fermi, Varenna 1980.

A18. Schmitz, N., Max-Planck Institute preprints, MPI-PAE/Exp.El. 88 and 89 and talk given at this conference.

A19. Schellekens, A.N.J.J., "Perturbative QCD and Lepton Pair Production", Ph.D. Thesis, (1981).

A20. Söding, P. and G. Wolf, DESY 81-013 (1981); H. Meyer, University of Wuppertal preprint (1981).

B1. Bace, M., Phys. Lett. 78B (1978) 132.

B2. Bardeen, W.A., A.J. Buras, D.W. Duke and T. Muta, Phys. Rev. D18 (1978) 3998.

B3. Celmaster W. and D. Sivers, Phys. Rev. D23 (1981) 227.

B4. t'Hooft, G., Nucl Phys. B61 (1973) 455.

B5. Barbieri, R., L. Caneschi, G. Curci and E. D'Emilio, Phys. Lett. 81B (1979) 207.

B6. Celmaster, W. and R.J. Gonsalves, Phys. Rev. Lett. 42 (1979) 1435 and Phys. Rev. D20 (1979) 1420.

B7. Schellekens, A.N., Nuovo Cimento, 24 (1979) 513; M.R. Pennington and G.G. Ross, Phys. Lett. 86B (1979) 331; L.F. Abbott, Phys. Rev. Lett. 44 (1980) 1569; C. Rozenzweig and Monsay, Phys. Rev. D23 (1981) 1217; L. Caneschi, Phys. Lett. 98B (1981) 295; E. Braaten and J.P. Leveille, DOE-ER/00881-178; A. Dhar and V. Gupta, Phys. Lett. 101B (1981) 432; R. Coquereaux, Ann. Phys. (N.Y.) 125 (1980) 401; Phys. Rev. D23 (1981) 1365.

B8. The relation between Λ_i and α_i as given in (2.2) is not the only possible one, but the one in (2.2) defines $\Lambda_{\overline{MS}}$ and Λ_{MOM} as used in refs. (B2) and (B6). See also A.J. Buras, E.G. Floratos, D.A. Ross and C.T. Sachrajda, Nucl. Phys. B131 (1977) 308.

B9. Politzer, H.D., Phys. Rev. Lett. 30 (1973) 1346; D.J. Gross and F. Wilczek, Phys. Rev. Lett. 30 (1973) 1323.

B10. Caswell, W., Phys. Rev. Lett. 33 (1974) 244; D.R.T. Jones, Nucl. Phys. B75 (1974) 531.

B11. Three loop contributions to the β functions have been calculated by O.V. Tarasov, A.A. Vladimirov and A.Y. Zharkov, Phys. Lett. 93B (1980) 429. They are, in contrast to the one-and two-loop contributions, renormalization prescription dependent and cannot be used without various two-loop Wilson coefficient functions, which are not known at present.

B12. See ref. (B6) and E. Braaten and J.P. Leveille ref. (B7). The numerical value in (2.3) is for the Landau Gauge.

B13. See B.H. Wiik (A12) and P. Söding and G. Wolf (A20).

B14. Stevenson, P., Phys. Lett. B100 (1981) 61, Phys. Rev. D23 (1981) 2916 and Univ. of Wisconsin preprint DOE-ER/00881-213 (1981). For critical discussions of this work see J. Kubo and S. Sakakibara, Univ. of Dortmund preprint, DO-TH 81/07 and M.R. Pennington, Univ. of Durham preprint 1981.

B15. Grunberg, G., Phys. Lett. 95B (1980) 70.

B16. Pennington, M.R. and G.G. Ross, Phys. Lett. 102B (1981) 167. See also A.I. Sanda, Phys. Rev. Lett. 42 (1979) 1658.

B17. Moshe, M., Phys. Rev. Lett. 43 (1979) 1851; A. Blumenfeld and M. Moshe, Technion-PH-81-29 (1981).

C1. See the talks by G. Smadja, J. Wotschack, J.Drees, H. Montgomery, M. Strovink and E. Fisk in these proceedings.

C2. Altarelli, G. and G. Parisi, Nucl. Phys. B126 (1977) 298.

C3. See ref. (A3).

C4. See refs. (A2) and (C7).

C5. See also A. Gonzales-Arroyo, C. Lopez and F.J. Yndurain, Nucl. Phys. B153 (1979) 161; Nucl Phys. B159 (1979) 512; Nucl Phys. B166 (1980) 429.

C6. Floratos, E.G., D.A. Ross and C.T. Sachrajda, Nucl. Phys. B129 (1977) 66; ibid. B139 (1978) 545 E.

C7. Curci, G., W. Furmanski and R. Petronzio, Nucl. Phys. B175 (1980) 27; E.G. Floratos, R. Lacaze and C. Kounnas, Phys. Lett, 98B, (1981) 89 , 285.

C8. Floratos, E.G., D.A. Ross and C.T. Sachrajda, Nucl. Phys. B152 (1979) 493.

C9. Furmanski, W. and R. Petronzio, Phys. Lett. 97B (1980) 437.

C10. Antoniadis, I. and E.G. Floratos, LPTENS 81/1 (1981).

C11. Siegel, W., Phys. Lett. 84B (1979) 193; D.M. Capper, D.R.T. Jones and P. Van Nieuwenhuizen, Nucl. Phys. B167, (1980) 479.

C12. Floratos, E.G., LPTENS 81/5 (1981).

C13. See for instance D.W. Duke and R.G. Roberts, Nucl. Phys. B166 (1980) 243; ref. C5; M.R. Pennington and G.G. Ross, Nucl. Phys. B179 (1981) 324. Further references can be found in A.J. Buras, Proceedings of the Madison Conference (1980).

C14. Barnett, R.M., D. Schlatter and L. Trentadue, Phys. Rev. Lett. 46 (1981) 1659.

C15. Para, A. and C.T. Sachrajda, Phys. Lett. 86B (1979) 331.

C16. Duke, D.W. and R.G. Roberts, ref. C13; H.L. Anderson et al., Fermilab-Pub-79/30-EXP.

C17. Duke, D.W. J.F. Owens and R.G. Roberts, CERN preprint 1981.

C18. Such a low value of $\Lambda_{\overline{MS}}$ would lead in most Grand Unification models to the proton's life time $O(10^{25}$ years) as compared to the experimental lower limit $\sim 10^{30}$ years.

C19. Roy, D.P., Phys. Rev. Lett. 47 (1981) 213. The idea of the hard intrinsic charm in the nucleon is due to S.J. Brodsky, C. Peterson and N. Sakai, Phys. Rev. D23 (1981) 2745.

C20. Curci, G. and M. Greco, Phys. Lett. 92B, (1980) 175.

C21. Parisi, G., Phys. Lett. 90B (1980) 295.

C22. Amati, D., A. Bassetto, M. Ciafaloni, G. Marchesini and G. Veneziano, Nucl. Phys. B173 (1980) 429.

C23. Ciafaloni, M., Phys. Lett. 95B (1980) 113; M. Ciafaloni and G. Curci, S.N.S. 1/1981.

C24. Parisi, G., Phys. Lett. 84B (1979) 225; A. Duncan and A.H. Mueller, Phys. Lett. 90B (1980) 159.

C25. Baulieu, L., E.G. Floratos and C. Kounnas, private communication.

C26. Callan, C.G. and D.J. Gross, Phys. Rev. Lett. 22 (1969) 156.

C27. De Rujula, A., H. Georgi and H.D. Politzer, Ann. of Phys. 103 (1977) 315; A.J. Buras, E.G. Floratos, D.A. Ross and C.T. Sachrajda, Nucl. Phys. B131 (1977) 308.

C28. Abbott, L.F., E.L. Berger, R. Blankenbecler and G. Kane, Phys. Lett. 88B (1979) 157; W.B. Atwood, SLAC-Pub-2428 (1979); A. Gonzales-Arroyo, C. Lopez and F.J. Yndurain, Phys. Lett. 98B,(1981) 215; E.M. Haacke, Phys. Lett. 97B (1980) 427.

C29. Duke, D.W., J.D. Kimel and G.A. Sowell, Phys. Rev. to be published.

C30. Buras, A.J., Fermilab-Pub-80/43-THY (May,1980).

C31. This is now being checked by S. Coulson and R. Ecclestone.

C32. Donnachie, A. and P.V. Landshoff, Phys. Lett. 95B (1980) 437.

C33. Abbott, L.F., W.B.Atwood and R.M. Barnett, Phys. Rev. D22 (1980) 582.

C34. Duke, D.W. and R.G. Roberts, Phys. Lett 94B (1980) 417; M.R. Pennington and G.G. Ross, ref. C13.

C35. Eisele, F., Invited talk at the IV Symposium on Elementary Particle Physics, Kazimierz , May 24-31,1981.

C36. Gottlieb, S., Nucl. Phys. B139 (1978) 125; M. Okawa, Nucl. Phys. B172 (1980) 481, B187 (1981) 71.

C37. Luttrell, S.P., S. Wada and B.R. Webber, Nucl. Phys. B188 (1981) 219.

C38. Jaffe, R.L. and M. Soldate, MIT preprints CTP 931 and CTP 937.

C39. Politzer, H.D., Nucl. Phys. B172 (1980) 349.

C40. Furmanski, W., Jagellonian University preprint, TPJU-12/81.

C41. Farrar, G.R. and D.R. Jackson, Phys. Rev. Lett. 35 (1975), 1416; E.L. Berger and S.J. Brodsky, Phys. Rev. Lett. 42 (1979) 940; E.L. Berger, Phys. Lett. 89B (1980) 241; E.L. Berger, T. Gottschalk and D. Sivers, Phys. Rev. D23 (1981) 99; E.L. Berger and S.J. Brodsky, SLAC-Pub-2749, ANL-HEP-PR-81-14.

C42. R.L. Jaffe and G.G. Ross, Phys. Lett. 93B (1980) 313.

C43. Kodaira, I., S. Matsuda, K. Sasaki and T. Uematsu, Nucl. Phys. B159, (1979) 99; I. Kodaira, K. Sasaki, S. Matsuta, T. Muta and T. Uematsu, Phys. Rev. D20 (1979) 627; J. Kodaira, Nucl. Phys. B165 (1980) 129.

D1. Dokshitzer, Y.L., D.I. Dyakonow and S.I. Troyan, ref. A3; G. Sterman and S. Weinberg, Phys. Rev. Lett. 39 (1977) 1436; H.D. Politzer, Phys. Lett. 70B (1977), 430, Nucl. Phys. B129 (1977) 301; C.T. Sachrajda, Phys. Lett. 73B

(1978) 185, Phys. Lett. 76B (1978) 100; W. Furmanski, Phys. Lett. 77B (1978) 312; Y. Kazama and Y.P. Yao, Phys. Rev. Lett. 41(1978) 611, Phys. Rev. D19 (1979) 3111, 3121; W.B. Frazer and J.F. Gunion, Phys. Rev. D19 (1979) 2447; C.H. Llewllyn-Smith, Acta Phys. Austriaca Suppl. XIX (1978) 331; A.V. Radyushkin, Phys. Lett. 69B (1977) 245.

D2. Amati, D., R. Petronzio and G. Veneziano, Nucl. Phys. B140 (1978) 54, B146 (1978) 29; R.K. Ellis, H. Georgi, M. Machacek, H.D. Politzer and G.G. Ross, Phys. Lett. 78B (1978) 281, Nucl. Phys. B152 (1979) 285; C. Libby and G. Sterman, Phys. Lett. 78B (1978) 618, Phys. Rev. D18 (1978) 3252, 4737; G. Sterman, Phys. Rev. D19 (1979) 3135, Phys. Rev. D17 (1978) 2773, 2789.

D3. Mueller, A.H., Phys. Rev. D18 (1978) 3705; S. Gupta and A.H. Mueller, Phys. Rev. D20 (1979) 118.

D4. Baulieu, L. and C. Kounnas, Nucl. Phys. B141 (1978) 423; I. Kodaira and T. Uematsu, Nucl. Phys. B141 (1978) 497.

D5. Curci, G., W. Furmanski and R. Petronzio, Nucl. Phys. B175 (1980) 27; W. Furmanski and R. Petronzio, Phys. Lett. 97B (1980) 437; J. Kalinowski, K. Konishi and T.R. Taylor, Nucl Phys. B181 (1981) 221; J. Kalinowski, K. Konishi, P.N. Scherbach and T.R. Taylor, Nucl. Phys. B181 (1981) 253; E.G. Floratos, R. Lacaze and C. Kounnas, Phys. Lett. 98B (1981) 89, 285; H. Okada, T. Munehisa, K. Kudoh and K. Kitani, TIT/HEP-64/1980 and Phys. Lett. 102B (1981) 49.

D6. Gribov, V. and L. Lipatov, Sov. J. Phys. 15 (1972) 675.

D7. Altarelli, G., R.K. Ellis and G. Martinelli, Nucl. Phys. B143 (1978) 521.

D8. Kubar-Andre, J. and F.E. Paige, Phys. Rev. D19 (1979) 221; Altarelli, G., R.K. Ellis and G. Martinelli, Nucl. Phys. B157 (1979) 461; K. Harada, T. Kaneko and N. Sakai, Nucl. Phys. B155 (1978) 169; B. Humpert and W.L. Van Neerven, Phys. Lett. 84B (1979) 327, Phys. Lett. 85B (1979) 293.

D9. Contogoutis, A.P. and J. Kripfganz, Phys. Rev. D20 (1979) 2295; A.N. Schellekens and W.L. Van Neerven, Phys. Rev. D21 (1980) 2619, D22 (1980) 1623.

D10. Altarelli, G., R.K. Ellis, G. Martinelli and S.Y. Pi, Nucl. Phys. B160 (1979) 301; J. Ambjørn and N. Sakai, Nordita preprint 80/14; G. Curci, W. Furmanski and R. Petronzio, Nucl. Phys. B175 (1980) 27; R. Baier and K. Fey, Z. Phys. C2 (1979) 339; E. Floratos, C. Kounnas and R.Lacaze, LPTENS 81/3 (1981).

D11. Altarelli,G. et al., Nucl. Phys. B160 (1979) 301.

D12. Sakai, N., Phys. Lett. 85B (1979) 67; G. Altarelli et al. Nucl. Phys. B160 (1979) 301; J. Ambjørn and N. Sakai, ref. D10; R. Baier and K. Fey, Z. Phys. C2 (1979) 339.

D13. Sheiman, J., I. Hinchliffe and H.E. Haber, LBL preprint 11577 (1980).

D14. Buras, A.J. and N. Sakai, unpublished. See Sect. 4.5 of ref. (All).

D15. Parisi, G., Phys. Lett. 90B (1980) 295; G. Curci and M. Greco, Phys. Lett. 92B (1980) 175.

D16. Korthals-Altes, C.P. and E. de Rafael, Nucl. Phys. B106 (1976) 237, B125 (1977) 275; Phys. Lett. 62B (1976) 320; R. Coquereaux and E. de Rafael, Phys. Lett. 69B (1977) 181; Y.L. Dokshitzer, Soviet Physics, JETP 46 (1977) 461; J.M. Cornwall and G. Tiktopoulos, Phys. Rev. D13 (1976) 3370; A. H. Mueller, Phys. Rev. D20 (1979) 2037.

D17. For a detailed discussion see Floratos et al., ref. (D10).

D18. For a detailed discussion and references see J. Kubar et al., Nucl. Phys. B175 (1980) 251; see also M. Chaichian, M. Hayashi and M. Roos, HU-TFT-81-10 (1981).

D19. Ellis, R.K., M.A. Furman, H.E. Haber and I. Hinchliffe, Nucl Phys. B173 (1980) 397.

D20. Furmanski, W. and W. Stominski, Jagellonian University preprint, TPJU-11/81 (1981).

D21. Furman, M.A. CU-TP-182 (1980).

D22. Aurenche, P., J. Lindfors, Nucl. Phys. B168 (1980) 296; A.P. Contogouris, L. Marleau and S. Papadopoulos, Phys. Rev. D22 (1980) 1629.

D23. Aurenche, P. and J. Lindfors, Nucl. Phys. B185 (1981) 274.

E1. Politzer, H.D., Nucl.Phys. B129 (1977) 301; G. Altarelli, G. Parisi and R. Petronzio, Phys. Lett. 76B (1978) 351, 76B (1978) 356; H. Fritzsch and P. Minkowski, Phys. Lett. 73B (1978) 80; C. Michael and T. Weiler, contribution to the XIIIth Recontre de Moriond, Les Arcs, France (1978); K. Kajantie and R. Raitio, Nucl. Phys. B139 (1978) 72; K. Kajantie, J. Lindfors and R. Raitio, Phys. Lett. 74B (1978) 384; J. Cleymans and M. Kuroda, Phys. Lett. 80B (1979) 385; J.C. Collins, Phys. Rev. Lett. 42 (1979) 291; K. Kajantie and J. Lindfors, Nucl. Phys. B146 (1978) 465; F. Halzen and D. Scott, Phys. Rev. D18 (1978) 3378, D19 (1979) 216; E.L. Berger, ANL-HEP-PR-78-12, ANL-HEP-CP-80-24.

E2. Stroynowski, R., Phys. Rep. 71C (1981) 1.

E3. See for instance G. Altarelli et al. ref.(E1).

E4. Halzen, F. and D.M. Scott, DAMTP 81/12 and DOE-ER/00881-205 (March 1981).

E5. Ellis, R.K., G. Martinelli and R. Petronzio, CERN preprint, TH-3079 CERN (May 1981).

E6. Perlt, H., Karl-Marx-Univ. preprint,(1981).

E7. Chaichian, M., M. Hayashi and T. Honkarante, Nucl. Phys. B175 (1980) 493.

E8. Dokshitzer, Yu.L., D.I. Dyakonov and S.I. Troyan, Phys. Rep. 58 (1980) 269; C.L. Basham, L.S. Brown, S.D. Ellis and S.T. Love, Phys. Lett. 85B (1979) 297; G.C. Fox and S. Wolfram, Caltech Rep. No. CALT 68-723 (1979), unpublished; A.V. Smilga, Nucl. Phys. B161 (1979) 449; W. Marquardt and F. Steiner, Phys. Lett. 93B (1980) 480; C.Y. Lo and J.D. Sullivan, Phys. Lett. 86B (1979) 327; J.G. McKitterick, Univ. of Illinois preprint ILL-(TH)-80-22 (1980).

E9. Ellis, S.D. and W.J. Stirling, Phys. Rev. D23 (1981) 214.

E10. Curci, G., M. Greco and Y. Srivastava, Phys. Rev. Lett. 43 (1978) 834, Nucl. Phys. B159 (1979) 451.

E11. Parisi, G. and R. Petronzio, Nucl. Phys. B154 (1979) 427.

E12. Baier, R. and K. Fey, Z. Phys. C2 (1979) 339.

E13. Cleymans, J. and M. Kuroda, Phys. Rev. Lett. 45 (1980) 1923, Univ. of Bielefeld preprint, BI-TP 80/17.

E14. Craigie, N.S. and H.F. Jones, Nucl. Phys. B172 (1980) 59; H.F. Jones and J. Wyndham, Nucl. Phys. B176 (1980) 466.

E15. Curci, G. and M. Greco, Phys. Lett. 79B (1978) 406, 92B (1980) 175, Frascati preprint LNF-81/7 (1981); M. Greco, LNF-81/10 (March 1981).

E16. Mueller, A.H., Sect. 12 of ref. (A13).

E17. Rakow P.E.L. and B.R. Webber, Nucl. Phys. B187 (1981) 254.

E18. Ellis, S.D., N. Fleishon and W.J. Stirling, Seattle University preprint, RLO-1388-852.

E19. Collins, J.C. and D.E. Soper, Univ. of Oregon preprint OITS, 155 (1981) and talk presented at Mariond Workshop of Lepton Pair Production, Les Arcs, France, 1981.

E20. Here $x_1 = q^+/P_1^+$; $x_2 = q^-/P_2^-$ where the light-cone coordinates are used and q and P_i are the momenta of the virtual photon and of the incoming hadrons respectively.

E21. Collins, J.C. and D.E. Soper, "Parton Distributions and Decay Functions" OITS-166 (1981).

E22. Chiappetta, P. and M. Greco, Centre de Physique Théorique preprint, CPT.81/P.1307. June 1981).

E23. Basham, C.L., L.S. Brown, S.D. Ellis and S.T. Love, Phys. Rev. Lett. 41 (1978) 1585, Phys. Rev. D19 (1979) 2018.

E24. θ is defined in such a way that $\theta = 0$ corresponds to the back to back configuration.

E25. Brown, L.S. and S.D. Ellis, University of Washington preprint RLO-1388-850.

E26. Berger, Ch. et al. Phys. Lett. 99B (1981) 292.

E27. Ellis, S.D. and W.J. Stirling in preparation.

F1. Appelquist , T. and H.D. Politzer, Phys. Rev. D12 (1975) 1404; A. Duncan and A.H. Mueller, Phys. Lett. 93B (1980) 119; M.E. Peskin, Nucl. Phys. B156 (1979) 365; G. Bhanot and M.E. Peskin, Nucl. Phys. B156 (1979) 391; I. Muzinich and F. Paige, Phys. Rev. D1 (1980) 1151; S. Gupta, SLAC-Pub- (1981).

F2. Barbieri, R., M. Caffo, R. Gatto and E. Remiddi, Phys. Lett. 95B (1980) 93 and CERN preprint 3071 (1981). See also R. Barbieri, M. Caffo and E. Remiddi, Nucl. Phys. B162 (1980) 220.

F3. See for instance M. Oreglia, SLAC-Pub - 2529 (1980).

F4. Karplus, R. and A. Klein, Phys. Rev. 87 (1952) 848; R. Barbieri, R. Gatto, R. Kögerler and Z. Kunszt, Phys. Lett. 57B (1975) 455.

F5. Barbieri, R., G. Curci, E. d'Emilio and E. Remiddi, Nucl. Phys. B154 (1979) 535; P. Pascual and R. Tarrach, Univ. of Barcelona preprint 1980; K. Hagiwara, C.B. Kim and T. Yoshino, Nucl. Phys. B177 (1981) 461.

F6. Buchmüller, W., Y.J. Ng, S.H.H. Tye, CLNS 81/497 Fermilab-Pub-81/46 THY. The leading logarithm has also been calculated by R. Barbieri, R. Gatto and E. Remiddi, CERN preprint (1981).

F7. Buchmüller, W., G.Grunberg and S.H.H. Tye, Phys. Rev. Lett. 45 (1980)103, 45 (1980) 587E .

F8. For $\Delta E(\Psi-\eta_c)$ a better agreement with the data could be obtained using $\Lambda_{\overline{MS}} = 350 \pm 50$ MeV.

F9. Lepage, G.P. and P.B. McKenzie, CLNS/81-498.

F10. Alm, M.S. et al., J.J. Mueller et al., Phys. Rev. Lett. 46 (1981) 1181.

F11. See the talk by M.A. Shifman in these proceedings.

F12. Buchmüller, W. and S.H.H. Tye, Phys. Rev. D24 (1981) 132.

F13. For toponium of mass of order 60-80 GeV, the weak decays become very important which makes the QCD tests difficult.

F14. Fischler, W. Nucl. Phys. B129 (1977) 157; A. Billoire, Phys. Lett. 92B (1980) 343.

F15. Kamal, A.N. J. Kodaira, T. Muta, SLAC-Pub-2725 (April 1981).

G1. See the talks by R. Felst, W. Braunschweig and D. Haidt in these proceedings.

G2. Sterman, G. and S. Weinberg. Phys. Rev. Lett. 39 (1977) 1436; A. De Rujula, J. Ellis, E.G. Floratos and M.K. Gaillard, Nucl. Phys. B138 (1978) 387.

G3. Farhi, E., Phys. Rev. Lett. 39 (1977) 1587.

G4. De Rujula, A. et al., ref.(G2).

G5. Fox, G.C. and S. Wolfram, Phys. Rev. Lett. 41 (1978) 1581, Nucl. Phys. B149 (1979) 413, Phys. Lett. 82B (1979) 134.

G6. Basham, C.L., L.S. Brown, S.D. Ellis and S.T. Love, Phys. Rev. D17 (1978) 2298, Phys. Rev. Lett. 41 (1978) 1585, Phys. Rev. D19 (1979) 2018.

G7. Field, R.D. and R.P. Feynman, Nucl. Phys. B136 (1978) 1; B. Andersson, G. Gustafson, C. Peterson, Z. Physik C1 (1979) 105, C3 (1980) 22; B. Andersson, G. Gustafson, T. Sjöstrand, Z. Physik C6 (1980) 235 and Phys. Lett. 94B (1980) 211; P. Hoyer, P. Osland, H.G. Sander, T.F. Walsh and P. M. Zerwas, Nucl. Phys. B161 (1979) 349; A. Ali, E. Pietarinen, G. Kramer and J. Willrodt, Phys. Lett. 93B (1980) 155 , A. Ali, E. Pietarinen and J. Willrodt, DESY Report T-01/1980.

G8. Dine M. and J. Sapirstein, Phys. Rev. Lett. 43 (1979) 668; K.G. Chetyrkin, A.L. Kataev and F.V. Tkachov, Phys. Lett. 85B (1979) 277; W. Celemaster and R.J. Gonsalves, Phys. Rev. Lett. 44 (1980) 560.

G9. PLUTO Group, Phys. Lett. 82B (1979) 449.

G10. Koller,K. and H. Krasemann, Phys. Lett. 88B (1979) 119.

G11. Ellis, J. and I. Karliner, Nucl. Phys. B148 (1979) 141.

G12. See B.H. Wiik ref. (A12).

G13. Koller, K., H.G. Sander, T.F. Walsh and P.M. Zerwas, DESY preprint 1981.

G14. Brandelik, R. et al., Phys. Lett. 86B (1979) 243; D.P. Barber et al., Phys. Rev. Lett. 43 (1979) 830; Ch. Berger et al., Phys. Lett. 86B (1979) 418; W. Bartel et.al., DESY-report 79/80 (1979) and Phys. Lett. 91B (1980) 142.

G15. Ellis, J., M.K. Gaillard and G.G. Ross, Nucl.Phys. B111 (1976) 253.

G16. Bartel, W. et al., JADE-collaboration DESY, 81/009 (1981).

G17. Konishi,K., A. Ukawa and G. Veneziano, Phys. Lett. 80B (1979) 259, and ref. (A8).

G18. Ali, A., J.G. Körner, Z. Kunszt, J. Willrodt, G. Kramer, G. Schierholz and E. Pietarinen, Phys. Lett. 82B (1979) 285, and Nucl. Phys. B167 (1980) 454.

G19. Gaemers, K.J.F. and J.A.M. Vermaseren, Z. Physik C7 (1980) 81.

G20. Ellis, R.K. D.A. Ross and A.E. Terrano, PRL 45 (1980) 1226 and Nucl. Phys. B178 (1981) 421.

G21. K. Fabricius, I. Schmitt, G. Schierholz and G. Kramer, Phys. Lett. 97B (1980) 431.

G22. Fabricius, K., G. Kramer, G. Schierholz and I. Schmitt, DESY 81/35 (1981); G. Schierholz, DESY 81/42 (1981).

G23. Vermaseren, J.A.M., K.J.F. Gaemers and S.J. Oldham, CERN-preprint 3002.

G24. Also large and positive corrections to C distributions have been found (G20).

G25. Kunszt, Z. Phys. Lett. 99B (1981) 429 and CERN preprint, TH-3141-CERN (August 1981).

G26. Clavelli, L. and D. Wyler, Bonn-University, BONN-HE-81-3 (July 1981).

G27. Sharpe, S, Berkeley preprint, LBL-13018 (July 1981).

G28. Ellis, R.K. and D.A. Ross, CERN preprint, TH-3131-CERN (1981).

G29. Ali, A., "the QCD effective coupling constant in e^+e^- annihilation", DESY preprint (August 1981).

G30. Among the participants of these meetings, which took place during the symposium, were A. Ali, L. Clavelli, R.K. Ellis, K.J.F. Gaemers, C. Kramer, Z. Kunszt and G. Schierholz. Similar conclusions have been reached by T. Gottschalk (see a very recent and a very interesting Caltech preprint).

G31. As conjectured by D.A.Ross, Nucl. Phys. B188 (1981) 109 a part of these large corrections could be summed to all orders.

G32. See also T. Chandramohan and L. Clavelli, Phys. Lett. 94B (1980) 409 and BONN-HE-80-22 (Nov. 1980).

G33. See also the comment by A. Ali after this talk.

G34. A. Ali (and H.Newman) used also the oblateness as obtained by Mark-J. Collab.

H1. Collins, J. and G. Sterman, Stony Brook Preprint ITP-SB-80-63 (1980).

H2. Doria, R., J. Frenkel and J.C. Taylor, Nucl. Phys. B168 (1980) 93.

H3. Di'Lieto, C., S. Gendron, I. G. Halliday and C.T. Sachrajda, Nucl. Phys. B183 (1981) 223.

H4. Andrasi, A., M. Day, R. Doria, J. Frenkel and J.C. Taylor, Nucl. Phys. B182 (1981) 104.

H5. Farrar, G. and D. Jackson, Phys. Rev. Lett. 43 (1979) 246.

H6. Brodsky, S.J. and G.P. Lepage, ref. (A7) and references therein,

H7. Efremov, A.V. and A.V. Radyushkin, Phys. Lett. 94B (1980) 245.

H8. Parisi, G., Phys. Lett. 84B (1979) 225).

H9. Duncan, A. and A. Mueller, Phys. Lett. 90B (1980) 159, Phys. Rev. D21 (1980) 1636.

H10. Landshoff P.V. and D.J. Pritchard, Z. Phys. C6 (1980) 69.

H11. Field, R.D., R. Gupta, S. Otto and L. Chang, Univ. of Florida preprint, 80-21 (1980); F.M. Dittes and A.V. Radyushkin, JINR-E2-80-688 (1980).

H12. See the talk by P. Wedemeyer at this symposium.

H13. Buras, A. J. and D.W. Duke, unpublished.

H14. Bardeen, W.A. and A.J. Buras, Phys. Rev. D20 (1979)166; D.W. Duke and J.F. Owens, Phys. Rev. D22 (1980) 2280.

H15. Uematsu, T. and T.F. Walsh, Fermilab preprint, September 1981.

H16. Sasaki, K., SLAC-Pub-2712 (March 1981).

H17. Sasaki, K., SLAC-Pub-2707 (March 1981).

H18. Mueller, A.H., Columbia preprint, CU-TP-197 (1981).

I1. Shifman, M.A., A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B147 (1979) 385, 448, 519.

I2. See for instance, J.G. Körner, G. Schierholz and J. Willrodt, Nucl. Phys. B185 (1981) 365; C.S. Lam, G. Marleau, P.S. Yeung and B.A. Li (McGill Univ. preprint, 1981).

I3. Olsen, H.A., P. Osland, I. Øverbø, Phys. Lett. 97B (1980) 286, Nucl. Phys. B171 (1980) 209; C.S. Lam and Wu-ki Tung, Phys. Rev. D21 (1980) 2712; P.W. Johnson and Wu-ki Tung, Phys. Rev. Lett. 45 (1980) 1382 and IIT preprint , 1981; F. Baldracchini, N.S. Craigie, V. Roberto and M. Socolovsky, Trieste preprint 1981.

I4. C. Nelson, Fermilab preprint 1981.

Discussion

R. L. Jaffe, MIT: I would just like to comment that the $x \to 1$ behavior of higher twist effects depends upon longitudinal quark and gluon distributions in the infinite momentum frame which are conceptually independent from twist two distributions. Therefore, the $x \to 1$ behavior of higher twist effects remains at present unknown.

A. J. Buras: Thank you for this comment. I fully agree with you.

G. Wolf, DESY: A question to the $O(\alpha_s^2)$ corrections to the 3 jet production in e^+e^-. Using the x_1 variable of FKSS, it seems that one can define a kinematical region in which the order α_s^2 contribution is reasonably small. Is that also true when one works with thrust?

A. J. Buras: Yes, there is a region let's say from 0.75 to 0.95 where the corrections to thrust distributions are of order 60 %, for α_s about 0.13, when one works in the $\overline{MS}$ scheme.

G. Wolf: Yes, but that's much bigger than in the other case.

A. J. Buras: Yes, but I do not think that these substantial corrections prevent determination of $\Lambda_{\overline{MS}}$ from the data on thrust. Furthermore the distributions discussed by FKSS have small corrections in a rather limited range of S. The phenomenology of these distributions has still to be done. The question is that what this group should really do is take various ε and δ and, for each ε and δ, extract α and see whether they exactly get the same α, and see how useful their distributions are. I think Ali wants also to make a comment.

G. Kramer, Univ. of Hamburg: In the meantime, I can tell you that we varied ε and δ quite a lot and haven't found anything.

A. J. Buras: o.k. very good.

A. Ali, DESY: Well, the matter of corrections to jet distributions actually is a matter of taste and temperament. And you can reduce the corrections also in thrust distributions if you subtract the genuine four jet events from your total data sample. You can do this for example by looking at the acoplanarity distribution which is a finite and well calculable quantity in QCD, and is a well-defined quantity also in experiment. I know that there are definite events seen experimentally which lie above the three jet prediction, in other words the Born term $q\bar{q}G$ - on top of that -. So the experimental data provides you naturally with a handle on the four jet events and you can subtract that out both from the theory and experiment and I have done this and the resulting corrections to the thrust distribution to the genuine three jet processes is only 30-35 %.

B. Stella, Univ. of Rome: Can you guess how large will be the next order corrections, α_s^3 ?

A. J. Buras: I would not like to guess, but extrapolating, they could be of order of 30 %. But this is only a hope.