MULTIQUARK HADRONS

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"The report of my death was an exaggeration."

Mark Twain, Cablegram from London to a New York newspaper, 2 June 1897

In the past year reports of the death of a certain class of multiquark states, the narrow baryonia, have circulated widely.[1-4] There is little doubt that the reports are well-founded and that the fervor of the last few years on the part of both theorists and experimentalists concerning such states should be laid to rest. On the other hand, there are many other, albeit less spectacular, aspects of multiquark physics which have survived and some which are flourishing. It is my intention to describe a selection of these without attempting to be complete or scholarly. I have been saved from those responsibilities by the recent appearance of several good reviews both of theory[4,5,6] and experiment,[3,7,8] and by the fact that this is a lepton-photon symposium to which few papers on this topic have been contributed. To make things easier I will restrict my remarks almost entirely to the $Q^2\bar{Q}^2$ system, the major exception being some remarks at the end about $Q^6$ and the nucleon-nucleon force.

Among the observable manifestations of multiquark physics to survive the baryonium massacre are rather broad, overlapping, conventional resonances in baryon-antibaryon scattering which may still properly be called "baryonium." Also flourishing are the non-resonant enhancements, threshold effects and background phases seen in low energy scattering and perhaps associated with multiquark configurations. A new subject which has attracted some interest at this conference is the admixture of multiquark components, in particular of heavy (c or b) quark components, in the nucleon's "intrinsic" wavefunction. Finally, the application to the nucleon-nucleon system of dynamical methods developed in the study of multiquark hadrons, while still in its infancy, may lead to some basic understanding of the short range component of the nucleon-nucleon force. With these topics in mind the remainder of this talk is organized as follows:

1. An overview: multiquark hadrons as $N_C \to \infty$
2. The dynamics of light $Q^2\bar{Q}^2$ systems: the P-matrix and the meson-meson continuum.
3. What, if anything, are the scalar mesons?
4. How to fit data in the P-matrix scheme
5. Heavy quarks in the nucleon's wavefunction
6. $\gamma\gamma \to Q^2\bar{Q}^2$
7. Quarks, QCD, and the nucleon-nucleon force

## 1. Multiquark hadrons as $N_C \to \infty$

The limit of QCD as the number of colors ($N_C$) goes to infinity has proved to be a remarkably good guide to the phenomenology of ordinary mesons[9] and baryons.[10] I will review its predictions for (color singlet hadrons) of the form $Q^m\bar{Q}^m$ ($m \leq N_C$) in hopes it is as good a guide to their behavior.

The first, and perhaps most important, realization regarding the $N_C \to \infty$ behavior of $Q^m\bar{Q}^m$ is that there are (at least) two very different systems as $N_C \to \infty$ which are indistinguishable at finite $N_C$. The first is $Q^k\bar{Q}^k$ with k-fixed as $N_C \to \infty$, which I will call Type I. The second is $Q^{N_C-k}\bar{Q}^{N_C-k}$, again with k-fixed as $N_C \to \infty$, which I will call Type II. The appearance of two different classes of objects as $N_C \to \infty$ suggests (to me at least) that at $N_C = 3$ there may be two classes of $Q^2\bar{Q}^2$ systems with rather different properties.

The type I objects may be exemplified by $Q^2\bar{Q}^2$. This system was studied first by 't Hooft.[11] Its properties are reviewed by Coleman[9] and Witten.[10] To summarize:

1. In the limit $N_c \to \infty$, $Q^2\bar{Q}^2$ is unbound. It is merely the meson-meson continuum.
2. In order $1/N_c$, $Q^2\bar{Q}^2$ differs from the $(\bar{Q}Q)^1-(\bar{Q}Q)^1$ continuum by diagrams which mix in $(\bar{Q}Q)^A-(\bar{Q}Q)^A$ components [the superscripts denote singlet (1) and adjoint (A) representation of $SU(N_c)$].

Notice that there is no "zero width approximation" for these objects. They should be regarded as components of the meson-meson continuum with weak color exchange forces which couple confined (colored) channels to the continuum.

To derive these results define a color-singlet quark quadrilinear

$$D(x) \equiv \frac{1}{N_c}\,\bar{Q}Q\bar{Q}Q(x)\,, \tag{1.1}$$

normalized so

$$\langle 0|D(x)D(0)|0\rangle \sim O(1) \text{ as } N_c \to \infty \tag{1.2}$$

Any D(x) can be decomposed in terms of color singlet bilinears,

$$D(x) = \cos\theta\, B_{12}(x)B_{34}(x) + \sin\theta\, B_{14}(x)B_{32}(x) \tag{1.3}$$

where

$$B_{ij}(x) \equiv [\bar{Q}_i(x)Q_j(x)]^1$$

and the indices i,j, etc. refer to labels (such as spin and flavor) other than color. The Green's function of Eq. (1.2) contains three sorts of terms as shown in Figure 1: 1) disconnected terms such as $\langle B(x)B(0)\rangle\langle B(x)B(0)\rangle$ which are $O(1)$ as $N_c \to \infty$ (Figure 1a); 2) connected, "direct" terms such as $\langle B_{12}(x)B_{34}(x)B_{12}(0)B_{34}(0)\rangle_c$ which are $O(1/N_c^2)$ as $N_c \to \infty$ (Figure 1b); and 3) connected, "exchange" terms such as $\langle B_{12}(x)B_{34}(x)B_{14}(0)B_{32}(0)\rangle_c$ which are $O(1/N_c)$ as $N_c \to \infty$ (Figure 1c). The counting of powers of $N_c$ is shown in Figure 1 in the double line rotation of 't Hooft: to read off the power of $N_c$ associated with any graph, supply a factor of $1/N_c$ at each $Q^2\bar{Q}^2$ vertex, $1/\sqrt{N_c}$ at each $Q\bar{Q}g$ vertex, and $N_c$ for each loop. The first non-trivial interactions, those of Figure 1c, will mix $(\bar{Q}Q)^1-(\bar{Q}Q)^1$ with $(\bar{Q}Q)^A-(\bar{Q}Q)^A$ if regarded in any fixed basis (e.g. $(\bar{Q}_1Q_2)-(\bar{Q}_3Q_4)$).

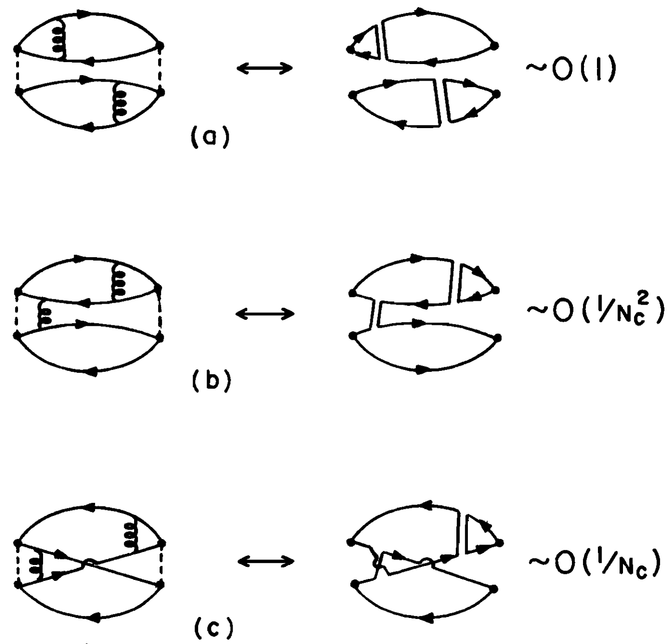


Fig. 1. Typical diagrams (in ordinary and double line notation) contributing to the three terms in $D(x)D(0)$. The $Q\bar{Q}Q\bar{Q}$ insertion is denoted •----• to allow color indices to be more easily followed.

Type II objects contain an infinite number of quarks and antiquarks as $N_c \to \infty$. My remarks about them are derived from Witten's Hartree model of baryons as $N_c \to \infty$.[10,12] One must distinguish "exotics", in which quarks and antiquarks share no common flavor, so that $Q\bar{Q}$ annihilation is forbidden, from "non-exotics" in which annihilation is possible. The properties of non-exotic $Q^{N_c-k}\bar{Q}^{N_c-k}$ as $N_c \to \infty$ are as follows:

(1) In the limit $N_c \to \infty$ the ground states and low-lying excitations of $Q^{N_c-k}\bar{Q}^{N_c-k}$ for different (finite) k are degenerate and mix strongly.
(2) The $Q^{N_c-k}\bar{Q}^{N_c-k}$ ground state is separated from baryon-antibaryon ($B\bar{B}$) threshold by a mass gap of $O(N_c)$.
(3) The decay of $Q^{N_c-k}\bar{Q}^{N_c-k}$ eigenstates into mesons is suppressed as $N_c \to \infty$.
(4) The width of excited $Q^{N_c-k}\bar{Q}^{N_c-k}$ states (above $B\bar{B}$ threshold) into $B\bar{B}$ + mesons is at least $O(\sqrt{N_c})$.
(5) "Funny" color states such as $[Q^{N_c-k}]^{\lambda}[\bar{Q}^{N_c-k}]^{\bar{\lambda}}$ where $\lambda$ is some special representation of $SU(N_c)$ are not a distinct class as $N \to \infty$. They are

degenerate with other $Q^{N_c-k}\bar{Q}^{N_c-k}$ states and mix strongly with them.

I conclude that non-exotic Type II states as $N_c \to \infty$ look something like "baryonium." They do couple strongly to $B\bar{B}$ with $\Gamma/M \sim O(1/\sqrt{N_c})$ and weakly to mesons. However, they do not have definite quark number, or an affinity for $B\bar{B}$ threshold, or show any sign of metastable (narrow) color configurations. In short, they are rather dull, broad (note meson widths are $O(1/N_c)$) overlapping resonances in the $B\bar{B}$ channel. If the lightest $Q^{N_c-k}\bar{Q}^{N_c-k}$ state is below $B\bar{B}$ threshold it can only decay into mesons and is narrower. Exotic Type II states share the properties listed above except for (2); instead they are degenerate with $E\bar{E}$ threshold, where $E(\bar{E})$ is an exotic baryon (antibaryon).

To derive these results in Witten's model one must first find the appropriate Hartree potentials for the $Q^{N_c-k}\bar{Q}^{N_c-k}$ system. In Witten's model as $N_c \to \infty$ the dynamics of a baryon is determined by the Hartree potential generated by the graph of Fig. 2a. The mass of the ground state baryon is $N_c\varepsilon_0$, where $\varepsilon_0$ is the lowest of the single-particle Hartree energies $\{\varepsilon_i\}$, which are independent of $N_c$ up to $O(1/N_c)$. So $B\bar{B}$ threshold occurs at $M_{th}=2N_c\varepsilon_0 + O(1)$. The Hartree potential for $Q^{N_c-k}\bar{Q}^{N_c-k}$ contains, in addition, contributions from the annihilation graph of Figure 2b. This system has Hartree energies $\{\eta_i\}$. The lowest states of $Q^{N_c-k}\bar{Q}^{N_c-k}$ therefore have masses $M(k) = 2(N_c-k)\eta_0 + O(1)$. Mixing between configurations with different k proceeds by the diagram of Fig. 2c which is O(1) as $N_c \to \infty$ ($1/\sqrt{N_c}$ at each vertex, but $O(N_c)$ initial quarks or antiquarks to emit the gluon). Because of the annihilation potential $\eta_0$ and $\varepsilon_0$ differ at O(1), which establishes point (2). The mass matrix for $Q^{N_c-k}\bar{Q}^{N_c-k}$ states contains a term $O(N_c)$ proportional to the identity plus O(1) diagonal and off-diagonal elements. This establishes point (1). The argument for point (3) is given in Ref. 10. Regarding point (5), all color configurations are solutions to the same Hartree equations and therefore differ in mass by O(1) unless a non-zero fraction of quarks and/or antiquarks are excited. Different color configurations mix at O(1) by the diagram of Fig. 2d. Finally, the width of $Q^{N_c-k}\bar{Q}^{N_c-k}$ into $B\bar{B}$+mesons can be established as follows: consider an excited $Q^{N_c-k}\bar{Q}^{N_c-k}$ state just above $B\bar{B}$ threshold. Since the $Q^{N_c-k}\bar{Q}^{N_c-k}$ system and the $B\bar{B}$ scattering state are solutions to the same Hartree equations with the same energy up to O(1) in $N_c$, all but a finite number of quarks and antiquarks must have unit overlap in the two states. $Q^{N_c-k}\bar{Q}^{N_c-k}$ therefore decays into $B\bar{B}$ by the diagrams of Fig. 2c and d which are O(1). The phase space for a state of mass $O(N_c)$ to decay into two others of $O(N_c)$ grows only like $\sqrt{N_c}$, so $\Gamma_{B\bar{B}} \sim O(\sqrt{N_c})$. More highly excited $Q^{N_c-k}\bar{Q}^{N_c-k}$ states may emit mesons (O(1)) as well as decay directly to $B\bar{B}$.

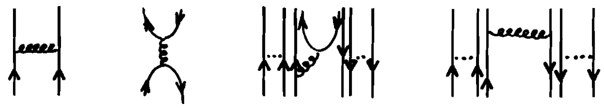


Figure 2: Diagrams relevant to the $N_c \to \infty$ behavior of $Q^{N_c-k}\bar{Q}^{N_c-k}$

What is one to conclude from all this about $N_c$=3? If $N_c \to \infty$ is a reliable guide I expect two classes of $Q^2\bar{Q}^2$ systems: Type I are predominantly continuum meson-meson states with relatively weak color-mixing forces; Type II are rather broad $B\bar{B}$ resonances weakly coupled to mesons, unless they are below $B\bar{B}$ threshold, in which case they may be narrower. There is considerable evidence for $B\bar{B}$ resonances resembling Type II $Q^2\bar{Q}^2$ states seen in $p\bar{p} \to \pi^+\pi^-$ and $\pi^0\pi^0$ and perhaps in the $p\bar{p}$ total cross section.[3,7] The Type I $Q^2\bar{Q}^2$ system will be seen to bear a haunting resemblance to the low mass $Q^2\bar{Q}^2$ "primitives" of the bag model.

## 2. The dynamics of light $Q^2\bar{Q}^2$ systems: the P-matrix and the meson-meson continuum

The color singlet $Q^2\bar{Q}^2$ system has two qualitatively different sorts of channels: $[(\bar{Q}Q)^{8}(\bar{Q}Q)^{8}]^{1}$ in which quark-antiquark pairs are coupled to the octet of color [we are now back in SU(3)], and $[(QQ)^{1}(QQ)^{1}]^{1}$ in which quark-antiquark pairs are color singlets. There are no strong confining forces between constituent $Q\bar{Q}$-pairs in the latter case so the system is free to dissociate into a meson pair. The $Q^2\bar{Q}^2$ system is different in this respect from either $Q\bar{Q}$ or $Q^3$ which can only couple to hadron scattering states by first creating a $Q\bar{Q}$-pair from the vacuum. In the absence of other barriers one suspects that $Q^2\bar{Q}^2$ will be more intimately coupled to open channels than ordinary mesons or baryons.[13] In model calculations of the spectra of $Q^2\bar{Q}^2$ "states" the coupling to open channels is usually ignored. In the bag model, for example, s-wave multiquark eigenstates are constructed by populating the lowest mode in a spherical cavity with quarks and antiquarks. For $Q^2\bar{Q}^2$ this confines color singlet as well as color octet channels, and generates an infinite tower of zero width "states." Two years ago, Low and I[14] pointed out that these "states" should not be interpreted as poles in the S-matrix since they have been obtained using unphysical boundary conditions. Instead, we suggested, they should be identified as poles in a quantity, the "P-matrix", which has poles at the energies of Hamiltonian eigenstates subject to confining boundary conditions in all channels. We called such objects "primitives."

The P-matrix is algebraically related to the S-matrix and can be extracted from scattering data. Its pole locations and residues can then be compared with the predictions of models. In the case of the s-wave scattering of spinless mesons with only one open channel the P-matrix is given by

$$P = k\cot(kb+\delta(k)), \quad (2.1)$$

where k is the center of mass momentum, $\delta(k)$ is the phase shift, and b is the meson-meson separation at which the confining boundary condition is applied. Comparison of theory and experiment takes place as follows: the theorist imposes confining boundary conditions at some convenient value of b and predicts an infinite tower of primitives with energies $\{E_n\}$. The experimentalist measures $\delta(k)$ and constructs P according to Eq. (2.1) using the theorist's value of b. The experimentalist's P will have an infinite tower of poles (because of the factor of kb in Eq. (2.1)) at energies $\{E'_n\}$. The test of the model is whether the $\{E_n\}$ and the $\{E'_n\}$ agree.

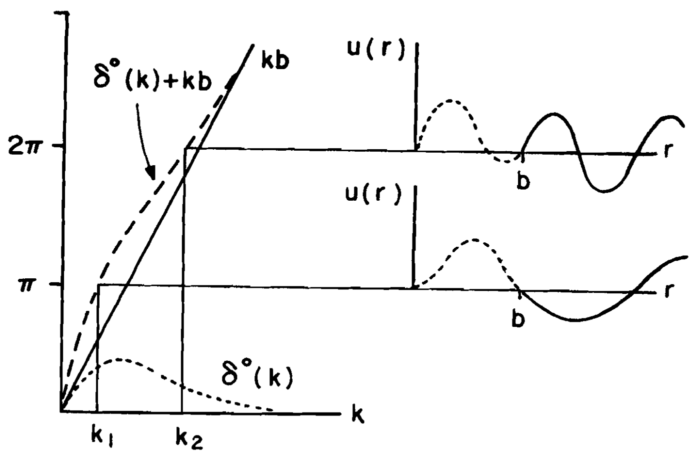


Figure 3.

In Figure 3 I attempt to illustrate why poles in P have something to do with confined quark model eigenstates. The dotted curve in Figure 3 shows a hypothetical s-wave phase shift $\delta^0(k)$. The dashed curve gives $\delta^0(k)+kb$. When $\delta^0(k)+kb$ equals to $n\pi$, P has a pole, and the radial wavefunction has a zero at r=b. At this

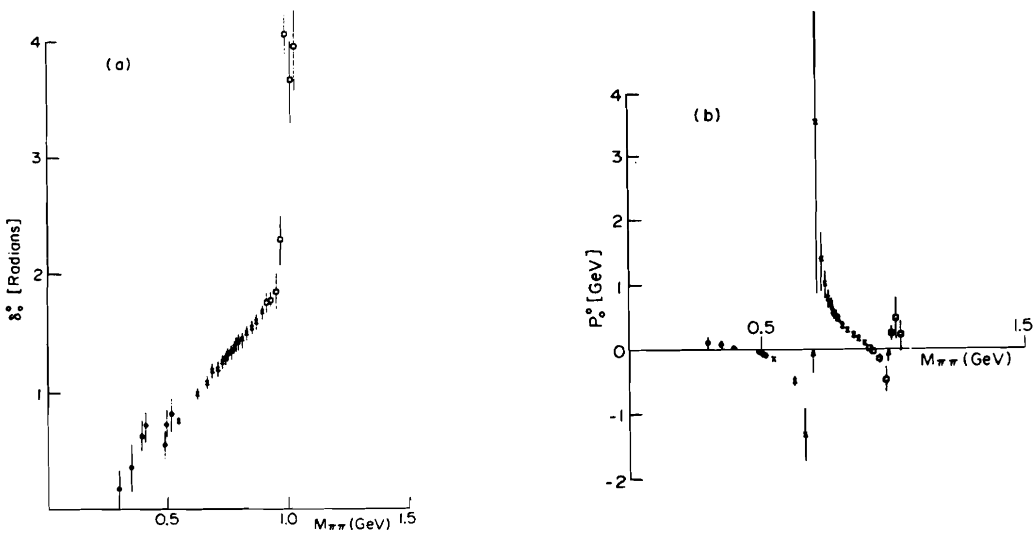


Fig. 4. a) Phase shift, and b) P-matrix for low energy s-wave I=0 $\pi\pi$-scattering.

energy it matches smoothly onto a uniformly confined quark wavefunction. Notice that even for $\delta(k)=0$ the P-matrix will have poles corresponding to the energies at which the non-interacting radial wavefunction, sin kr, has a node at r=b. Thus even in the absence of meson-meson interactions a theorist performing a calculation with universally confining boundary conditions must expect to find an infinite tower of $Q^2\bar{Q}^2$ primitives. Further discussion of the P-matrix formalism along with further motivation and examples can be found in Refs. 4, 6, and 14-16.

Figures 4 and 5 illustrate the application of the P-matrix formalism to $\pi\pi$ scattering. The measured $\pi\pi$ s-wave phase shifts in the I=0 and I=2 channels are shown in Figs. 4a and 5a. The corresponding P-matrix elements obtained from Eq. (2.1) with b chosen according to the prescription of Ref. 14 are shown in Figs. 4b and 5b. The poles which appear at ∿690 MeV in I=0 and ∿1040 MeV in I=2 correspond closely to the predicted energies of $Q^2\bar{Q}^2$ primitives in the bag model.[13] Clearly poles in the P-matrix do not necessarily correspond to resonances or even rapid variations in the S-matrix, and the predictions of quark models cannot be checked by a superficial perusal of phase shifts or effective mass plots. In the case of $\pi\pi$ scattering in the s-wave, the attraction seen in the I=0 channel and the repulsion seen in the I=2 channel are correlated with the color-magnetic quark-quark interactions which lower or raise the mass of the P-matrix pole respectively.

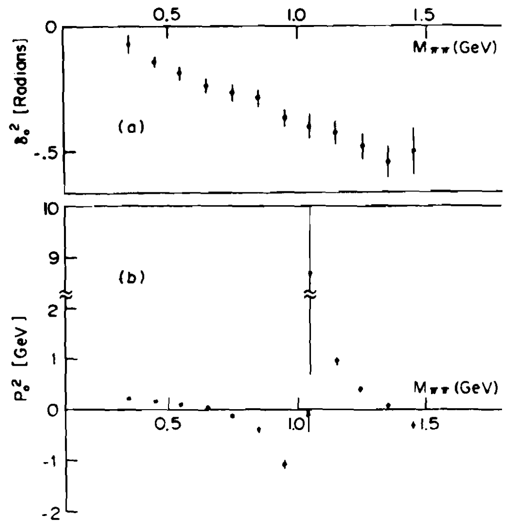


Fig. 5 a) Phase shift, and b) P-matrix for low energy s-wave I=2 $\pi\pi$-scattering

In order to reconstruct the P-matrix it is necessary to measure the entire S-matrix. This severly limits the channels in which the P-matrix methods can be used. In addition to the meson-meson s-wave, some s-wave meson-nucleon[17] and nucleon-nucleon[18] channels have been studied with some success.

3. What, if anything, are the scalar mesons?

The resonances and enhancements seen in the s-wave scattering of pseudoscalar mesons do not appear to be ordinary $Q\bar{Q}$-states. Numerous attempts to group them into a conventional SU(3)-flavor nonet have failed.[19-21] There are too many of them, and their masses, mixing angles, and decay couplings would be unnatural. Some years ago Johnson and I[22,13] suggested that the origin of the difficulty might be that the lightest of them are $Q^2\bar{Q}^2$ objects. This could account for several of the more troubling aspects of their behavior (e.g. the near degeneracy and affinity for $K\bar{K}$ threshold of the $\delta(960)$ and $S^*(990)$, and the lightness relative to other scalars of the $\varepsilon(700)$). One noticeable problem with the $Q^2\bar{Q}^2$ assignment was the absence of a light (∿1000 MeV) strange scalar meson to fill out the $Q^2\bar{Q}^2$ $0^{++}$ nonet. Later, Low and I[14] pointed out that if s-wave meson-meson scattering is analyzed in the P-matrix formalism the $Q^2\bar{Q}^2$ assignment looks more attractive. In particular the $K\pi$ s-wave has a P-matrix pole very near 1000 MeV as required. Furthermore the exotic $\pi^+\pi^+$ and $\pi^+K^+$ channels also show P-matrix poles at energies close to the values predicted by the bag model.[13]

Since the publication of Ref. 13 there has been further work on the quark content of the scalar mesons both from the P-matrix point of view and in the context of other dynamical models. I believe it is an appropriate time to review once again the status of the scalar mesons. I will first summarize what has been done using the P-matrix formalism and then discuss some other quantitative analyses. The reader will undoubtedly notice my prejudice against analyses which do not keep careful track of the distinction between effects seen in the data (S-matrix) and parameters of the quark model (P-matrix).

The input to any attempt to understand scalar mesons are the enhancements and resonances observed in s-wave meson-meson scattering. The $0^{++}$ "objects" of

**Table I**

**Resonances, enhancements, and other objects seen in the $0^{++}$ channel**

| I | Y | Traditional Name | Comments |
|---|---|---|---|
| 0 | 0 | $\epsilon(700)$ | Non-resonant, low mass $\pi\pi$ enhancement |
| | | $S^*(990)$ | Narrow, appears at $K\bar{K}$ threshold and couples more strongly to $K\bar{K}$ than $\pi\pi$. |
| | | $\epsilon'(1300)$ | Well-established resonance seen in $\pi\pi\rightarrow\pi\pi$ and $\pi\pi\rightarrow K\bar{K}$. |
| 1 | 0 | $\delta(960)$ | Narrow, appears just below $K\bar{K}$ threshold. Seen in $\eta\pi$ invariant mass and as threshold enhancement in $K\bar{K}$ |
| | | $\delta'(1300)$ | Possible resonance, poorly established. |
| $\frac{1}{2}$ | ±1 | $\kappa(1000)$ | Non-resonant, low mass $\pi K$ enchancement. |
| | | $\kappa(1400)$ | Well-established resonance seen in $\pi K\rightarrow\pi K$. |
| 2 | 0 | ------ | Slowly falling phase shift in $\pi\pi\rightarrow\pi\pi$ from threshold through $\sim$1500 MeV. |
| $\frac{3}{2}$ | ±1 | ------ | Slowly falling phase shift in $\pi K\rightarrow\pi K$ from threshold through $\sim$1500 MeV. |

interest are listed in Table I. The entries in the table correspond to effects observed directly in data rather than to states claimed on the basis of phenomenological analyses. References to the original literature may be found in the reviews of Montanet[3] and Protopopescu and Samios.[8] The results of P-matrix analyses of these channels, where available, are summarized in Table II. All are the results of one channel analyses except for the I=Y=0 system for which a two channel ($\pi\pi$ & $K\bar{K}$) analysis has been carried through by two groups (with consistent results).[23,24] The absence of measurements of $\pi\eta$ phase shifts makes it impossible to study the I=1, Y=0 channel with P-matrix methods. To extend the analysis of the I=1/2, Y=±1 channel to the region of the $\kappa(1400)$ requires $K\eta$ phase shifts which do not as yet exist. Since prominant resonances are inevitably associated with nearby P-matrix poles it is quite likely that the $\delta(960)$ and $\kappa(1400)$ would be seen in P-matrix analyses at masses not far from the physical states.

The problem of classification is to put the P-matrix poles of Table II (augmented by the $\kappa(1400)$ and $\delta(960)$) into correspondence with the primitives expected in a confined quark model. In Table III I list the lightest $0^{++}$ primitives expected in the bag model.[13] The masses of $Q\bar{Q}$ primitives are guesses since bag model calculations of orbitally excited light-quark configurations are not reliable. The possibility of mixing between $Q\bar{Q}$ and $Q^2\bar{Q}^2$ configurations is ignored in the table. A possible assignment of P-matrix poles to primitives is suggested in the final column of Table III. This assignment is based on the quantitative structure of the mass spectrum and the channel couplings in Table II and should be taken <u>cum grano salis</u>. It is uncertain, for example, whether to associate the I=Y=0 P-matrix pole at 1460 MeV with the lightest $s\bar{s}$ primitive or with a $Q^2\bar{Q}^2$ primitive in the next lightest multiplet. Any serious attempt to classify the effects seen in the low energy meson-meson s-wave will require better data (e.g. more channels extended to higher energies) and better theory (e.g. bag calculations which go beyond the symmetric cavity model and which allow for $Q\bar{Q}$-$Q^2\bar{Q}^2$ mixing). It is notable, however, that there appear to be enough P-matrix poles in the data to accomodate the multitude of primitives predicted at low energies in the model.

**Table II**

**Poles found in P-matrix analyses of s-wave Meson-Meson scattering**

| I | Y | Mass (MeV) Ref. | Residue (GeV$^3$) | Channel Coupling |
|---|---|---|---|---|
| 0 | 0 | 690 [24] | .064 | --- |
| | | 1030±30 [23,24] | $\sim$.10 | $\lvert\xi_{\pi\pi}/\xi_{K\bar{K}}\rvert\sim 1$ |
| | | 1260±50 [23,24] | $\sim$.30 | $\lvert\xi_{\pi\pi}/\xi_{K\bar{K}}\rvert\sim 4$ |
| | | 1420±20 [23,24] | $\sim$.20 | $\lvert\xi_{\pi\pi}/\xi_{K\bar{K}}\rvert\sim\frac{1}{4}$ |
| $\frac{1}{2}$ | ±1 | 960 [24] | .079 | --- |
| 2 | 0 | 1040 [24] | .21 | --- |
| $\frac{3}{2}$ | ±1 | 1190 [24] | .22 | --- |

**Table III**

**Classification of $0^{++}$ Primitives**

<u>LIGHTEST $Q^2\bar{Q}^2$ NONET</u>

| Quark Content | I | Y | Predicted Mass (MeV) | A Possible Assignment (Traditional name and/or P-matrix pole) | |
|---|---|---|---|---|---|
| $ud\bar{u}\bar{d}$ | 0 | 0 | 650 | $\epsilon(700)$ | P(690) |
| $(u\bar{u}+d\bar{d})s\bar{s}$ | 0 | 0 | 1100 | $S^*(990)$ | P(1000) |
| $u\bar{s}(u\bar{u}+d\bar{d})$... | $\frac{1}{2}$ | ±1 | 900 | $\kappa(1000)$ | P(960) |
| $ud\bar{s}\bar{s}$ | 1 | 0 | 1100 | $\delta(960)$ | insufficient data |

<u>LIGHTEST $Q\bar{Q}$ NONET</u>

| Quark Content | I | Y | Predicted Mass (GeV) | A Possible Assignment (Traditional name and/or P-matrix pole) | |
|---|---|---|---|---|---|
| $(u\bar{u}+d\bar{d})$ | 0 | 0 | $\sim$1200 | $\epsilon(1300)$ | P(1260) |
| $s\bar{s}$ | 0 | 0 | $\sim$1500 | ---- | ---- |
| $u\bar{s}$... | $\frac{1}{2}$ | ±1 | $\sim$1350 | $\kappa(1400)$ | insufficient data |
| $u\bar{d}$... | 1 | 0 | $\sim$1200 | $\delta'(1300)$ | insufficient data |

<u>LIGHTEST $Q^2\bar{Q}^2$ EXOTICS</u>

| Quark Content | I | Y | Predicted Mass (MeV) | A Possible Assignment (Traditional name and/or P-matrix pole) | |
|---|---|---|---|---|---|
| $uu\bar{d}\bar{d}$ | 2 | 0 | 1150 | --- | P(1040) |
| $uu\bar{s}\bar{d}$... | $\frac{3}{2}$ | ±1 | 1350 | --- | P(1250) |
| $uu\bar{s}\bar{s}$... | 1 | ±2 | 1550 | --- | insufficient data |

Immediately after the suggestion that some scalar mesons may be $Q^2\bar{Q}^2$ configurations rather than $Q\bar{Q}$, several attempts were made to infer their quark content from their couplings to better understood particles or to weak or electromagnetic probes. These effects have been inconclusive. Most, if not all, fail to distinguish between the parameters of the physical resonances or enhancements which are observed experimentally, and the parameters of the P-matrix poles which are predicted by the theory. For example the I=Y=0 bag primitive predicted at ∿1100 MeV with quark content $(u\bar{u}+d\bar{d})s\bar{s}$ is predicted to decouple from the $\pi\pi$ channel. On the other hand, the physical enhancement associated with this primitive may or may not couple to $\pi\pi$ depending on the structure of terms in the P-matrix which are non-singular in the vicinity of 1100 MeV. Roughly speaking the physical couplings of enhancements or resonances reflect P-matrix parameters qualitatively except in the vicinity of important thresholds. In the case at hand this caveat applies primarily to the $\delta(960)$ and S*(990) which are very close to $K\bar{K}$ threshold. Another shortcoming of some analyses is their use of Breit-Wigners to describe the propagation of virtual scalar mesons. This is suspect for the $\varepsilon(700)$ and $\kappa(1000)$ which are not even resonant and for the S*(990) and $\delta(960)$ whose shapes are strongly affected by the $K\bar{K}$ threshold.

Holmgren and Pennington[25] studied the couplings of the $\pi\pi$ s-wave and p-wave in the $\varepsilon(700)$ and $\rho(770)$ regions. They found $g^2_{\varepsilon\pi\pi}/g^2_{\rho\pi\pi} \sim 2.1 \pm 0.1$, $g^2_{\varepsilon N\bar{N}}/g^2_{\rho N\bar{N}} \sim 4\pm1$, and $g^2_{\varepsilon K\bar{K}}/g^2_{\rho K\bar{K}} \sim .90\pm.09$, all of which seems to support the interpretation of the $\varepsilon(700)$ as a $ud\bar{u}\bar{d}$ configuration. Aaron and Goldberg[26] studied the decay $\eta\to\pi^+\pi^-\pi^0$ mediated by the isospin violating Hamiltonian $H_{TAD}=1/2\ (m_u-m_d)(\bar{u}u-\bar{d}d)$. They reduce in the $\pi^0$, use PCAC to show the amplitude must have a linear zero at $E_{\pi^0}=0$, parametrize it linearly and then fix the remaining parameter by the value of $\langle\pi^+\pi^-|\bar{u}\gamma_5 u+\bar{d}\gamma_5 d|\eta\rangle$ at $E_{\pi^+} = E_{\pi^-} = \frac{m_\eta}{2}$ . They saturate this matrix element with the $\varepsilon(700)$ and get a good fit if it is mostly $Q^2\bar{Q}^2$ as opposed to zero if it is pure $Q\bar{Q}$. In a similar spirit, Golowich studied whether the $\varepsilon(700)$ could explain the $\Delta I = 1/2$ enhancement in $K\to2\pi$ decays. He used a P-matrix motivated picture to compute the effect as $K\to2\pi$ of a $Q\bar{Q}$ or $Q^2\bar{Q}^2$ object in the $\pi\pi$ $0^{++}$ channel at about 700 MeV. Unfortunately he found neither produces a significant enhancement. On balance the evidence seems to favor a $Q^2\bar{Q}^2$ assignment for the $\varepsilon(700)$.

The evidence concerning the $\delta$*(990) and $\delta(960)$ is more confusing. Greenhut and Intemann have studied these objects in a series of papers.[28-30] In Ref. 28 they argue for a $Q^2\bar{Q}^2$ assignment of the $\delta$-based branching ratio for $\eta\to\pi\gamma\gamma$ which they assume to be $\delta$-dominated. In Ref. 29 they compare the $\pi\pi$ and $K\bar{K}$ couplings of the $\varepsilon$ and S* which they find to agree qualitatively with the expectations of the $Q^2\bar{Q}^2$assignment. Finally in Ref. 30 they study $\eta'\to\pi\gamma\gamma$ in an isobar model assuming scalar dominance, and conclude in favor of the $\delta(960)$ being a $Q\bar{Q}$ configuration. There are problems with these analyses: In Ref. 30 all $Q^2\bar{Q}^2$ couplings are normalized by the physical width of the $\delta(960)$ which is probably badly distorted by the nearby $K\bar{K}$ threshold. The S*(990) is assumed to decouple from $\pi\pi$ if it is a $Q^2\bar{Q}^2$ object. In light of these it is difficult to judge how seriously to regard their conclusions. Braman and Massó in a recent letter[31] and a paper submitted to this conference[32] have used similar methods to try to establish a $Q\bar{Q}$ assignment for the $\delta(960)$. Their argument is simple. They assume $\eta'\to\pi\gamma\gamma$ is dominated by the virtual $\delta$ intermediate state: $\eta'\to\delta\pi\to\eta\pi\pi$ and from this conclude $\Gamma_{\eta'\to\eta\pi\pi} \propto g^2_{\eta'\delta\pi}g^2_{\eta\delta\pi} \propto [\Gamma_{\delta\to\eta\pi}]^2K$, where $K=g^2_{\eta'\delta\pi}/g^2_{\eta\delta\pi}$. If the $\delta$ contains non-strange quarks only (as in the $u\bar{d}$ assignment) then only the non-strange components of the $\eta$ and $\eta'$ participate. So $K=K_0$, where $K_0$ is the ratio of non-strange quark components in the $\eta'$ and $\eta$. If, on the other hand, the $\delta$ is a $u\bar{d}s\bar{s}$ object the situation is reversed and $K=K_S$, where $K_S$ is the analogous ratio for strange quarks in the $\eta'$ and $\eta$. Assuming the $\eta$ and $\eta'$ have no glueball component, $K_S = 1/K_0$. Braman and Massó take $\Gamma_{\delta\to\eta\pi} = 50\pm10$ MeV to get $K=.60\pm.25$ which they compare with a "measured" value of $K_0=.55\pm.06$ (from $\eta$ and $\eta'$ production experiments and mass formulae). This argument appears to favor the $Q\bar{Q}$ assignment of the $\delta(960)$ rather strongly. [Note the $Q^2\bar{Q}^2$ assignment "predicts" $K=1/K_0 \cong 1.8$]. It is, however, very sensitive to the value of $g_{\eta\delta\pi}$: notice that $\Gamma_{\eta'\to\eta\pi\pi}$ goes like $(g_{\eta\delta\pi})^4K$. A 30% increase in $g_{\eta\delta\pi}$ would increase K to 1.8 in agreement with the $Q^2\bar{Q}^2$ analysis. The connection between the apparent width of the $\delta(960)$ and the channel couplings of the intrinsic quark "state" is made subtle by the nearby $K\bar{K}$ threshold. The analysis of Ref. 31 ignores these subtleties and cannot be trusted at the 30% level.

Finally Achasov, Devyanin and Shestakov[33] have pointed out that the "width" of the $\delta(960)$ depends crucially on how one interprets the background underneath it. The conventional (and naive) treatment subtracts a smooth background and fits a narrow Breit-Wigner. As an alternative they include the "background" in their δ(960) and find a much broader object with a narrow cusp at $K\bar{K}$ threshold. Flatté[34] pointed out similar possibilities some years ago. Such exercises serve to emphasize the need for a dynamical interpretation of quark model "states."

My conclusions are that the scalar meson problem is still confused and will remain so until data are analysed with theoretical techniques which recognize the dynamical differences between the zero width objects catalogued in quark models and the strongly coupled effects seen by experiment. The P-matrix formalism can be applied to simple two-body scattering data, but no similar approach exists as yet for less highly constrained processes. What data have been analysed with P-matrix methods appear to support the quark model calculations of $Q\bar{Q}$ and $Q^2\bar{Q}^2$ primitives summarized in Table III.

While theorists are busy trying to develop the methods suggested above, experimenters might consider helping out by providing heretofore unmeasured meson-meson scattering amplitudes. Top on my list is $\pi\eta\to\pi\eta$ or $\pi\eta\to K\bar{K}$ in the δ-region and beyond. These amplitudes would help theorists sort out the confused structure of the very important I=1 $0^{++}$ channel. Other potentially interesting channels are $\pi\pi\to\eta\eta$ and $K\pi\to K\eta$.

## 4. Fitting data in a P-matrix formalism[36]

As I have described it, use of the P-matrix formalism appears to require complete knowledge of the S-matrix. This is unfortunate since data rarely are available in all potentially important channels. With incomplete data one cannot construct P and examine its poles. Several groups[24,35] have recently attempted a different and more traditional approach, namely fitting a P-matrix to the existing, perhaps incomplete data. One parametrizes the P-matrix, e.g. as a single pole or a sum of a few poles, perhaps being guided by some theoretical expectations, and fits the parameters to whatever data is available. The advantages are obvious: it is traditional, having been used with the K-matrix for years, and it allows one to make use of fragmentary data. The problem is that the P-matrix fits reported to me have turned out very poorly. Fortunately the problem has a simple solution. Furthermore, the solution provides some insight into the subtleties of multiquark physics. I will take some time to describe it, hoping to encourage people to fit data with the P-matrix.

First let me illustrate the problem. Consider the $\pi\pi$ I=2 phase shift shown in Fig. 5a. Direct construction of $P=k\cot(kb+\delta(k))$ yields a pole at $M_0\cong 1040$ MeV with residue $R_0\approx .22$ GeV$^3$. An obvious (but wrong!) ansatz to fit to this data is a P-matrix consisting of a single pole plus a constant

$$P_0(s) = C_0 + \frac{R_0}{s-M_0^2} \tag{4.1}$$

The constant is included in recognition that even in the simplest models (e.g. non-relativistic potential models) the P-matrix requires subtraction. Eq. (4.1) depends on three parameters which can be fit to the data. In Fig. 6 I show the results of such a fit (dashed line). $R_0$ and $M_0$ were fixed at the values quoted

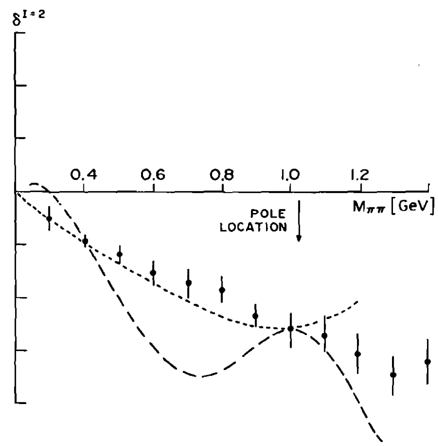


Fig. 6. P-matrix fits to I=2 s-wave $\pi\pi$ phase shift. Dashed curve is naive fit; dotted curve is correct fitting algorithm.

above. $C_0$ was (somewhat arbitrarily) chosen to fit $\delta(k)$ at $M_{\pi\pi}$ = 400 MeV. The fit is as bad as one could imagine. It is constrained to agree at $M_{\pi\pi}$ = 400 MeV and to have the correct magnitude and slope at $M_{\pi\pi}$ = 1040 MeV. It hardly touches another data point.

What went wrong? The answer is very simple: $\delta(k)=0$ does not correspond to $P=0$. Instead $\delta(k) = 0$ requires $P=P_0=k\cot kb$ or

$$P_0 = \frac{1}{b} + 2k^2 b \sum_{n=1}^{\infty} \frac{1}{k^2b^2-n^2\pi^2} \tag{4.2}$$

Trivial data require a P-matrix which is a sum of precisely spaced poles with specific residues. In comparison consider the K matrix, $K \equiv 1/k \tan\delta(k)$, for a single s-wave channel. Clearly $\delta=0$ implies $K=0$ and vica versa. Adding poles to the trivial K-matrix, $K_0\equiv 0$, therefore has a local effect on the phase in the vicinity of the added pole. Starting with $P=0$ is equivalent to starting with the hard-sphere phase $\delta(k)=-kb$ which is a highly non-trivial and improper "trivial" starting point. Looking back at Figure 6 it is clear that except for the wiggle introduced by the pole near 1050 MeV, the dashed line is trying to look like $\delta(k) \sim -kb$.

The way around this problem is now obvious: one should begin with a "trivial" P-matrix, $P_0=k\cot kb$, not with $P_0=0$. The poles in $P_0$ are to be interpreted as the $Q^2\bar{Q}^2$ primitives which are associated with the non-interacting meson-meson system. [Even if two mesons ($Q\bar{Q}$ states) do not interact at all, the universally confined $Q^2\bar{Q}^2$ system will possess an infinite tower of primitives]. To fit a P-matrix to the data one should move the poles in $P_0$ as required, interpreting them as $Q^2\bar{Q}^2$ primitives, and add poles to $P_0$ as required, interpreting them as $Q\bar{Q}$ primitives, since these are not present at all in $P_0$. To test this idea I return to the $\pi\pi$ I=2 data. Only $Q^2\bar{Q}^2$ primitives are expected. The algorithm outlined above suggests a fit of the form

$$P = C_0 + \frac{R_0}{s-M_0^2} + \left(k\cot kb - \frac{2\pi^2/b}{k^2b^2-\pi^2}\right) \tag{4.3}$$

I have removed the first pole from $P_0$ and let its location and residue be fit to the data. Once again it is a 3 parameter fit. The parameters are chosen as before. The fit is shown as the dotted curve in Fig. 6. In Figure 7 the same two fitting techniques are applied to the I=0 $\pi\pi$ s-wave in the $\varepsilon(700)$ region. Clearly the new algorithm provides much better fits. It should make it possible to extend P-matrix analyses to many systems for which only incomplete data are available. The generalizations of the new fitting algorithm to many channels and higher partial waves are easily constructed by setting S=1 in the algebraic equation relating the S and P-matrices to obtain $P_0$. The resulting expression for $P_0$ in terms of trigonometric functions can be expanded in terms of simple poles much like cotkb. Subtleties such as the scattering of particles of unequal mass will be discussed elsewhere.[37]

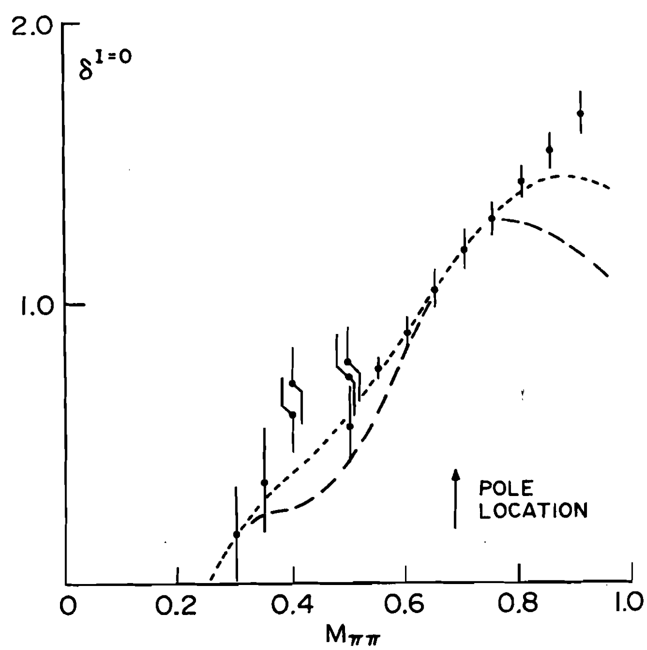

Fig.7. Same as Fig. 6 for I=0 s-wave $\pi\pi$ phase shift.

5. Heavy quark admixtures in the nucleon

A manifestation of multi-quark physics quite different from the ones I have been discussing has recently been suggested by Brodsky, Hoyer, Peterson, and Sakai (BHPS).[38] They have suggested that the nucleon may contain a non-negligible admixture of charm-anticharm quark pairs in its "intrinsic" wavefunction. These are not the "sea" quark pairs found at short distances (and low Bjorken-x) within the nucleon generated by the familiar QCD process of gluon bremsstrahlung and quark pair creation from an original "intrinsic" wavefunction without pairs. Instead these are supposed already to be present in the nucleon's wavefunction at large distances (or low mass scales). The motivation for the suggestion of BHPS is that a $c\bar{c}$ component in the nucleon with a hard $x_{Bj}$-distribution could account for the surprisingly large cross section for forward production of charmed hadrons in hadron-hadron scattering.

The argument of BHPS requires the $c\bar{c}$-quarks to have appreciable intensity at large $x_{Bj}$. They argue on the basis of energy denominators in the infinite momentum frame that the larger a quark's mass the harder its parton distribution. Another, perhaps equivalent, way to reach the same conclusion was suggested by de Rujula.[39] He uses the correspondence between x or $p_3$ in the infinite momentum frame and $p^+=1/\sqrt{2}\ (E+p_3)$ in the laboratory. It is reasonable to assume that in bound state configuration $Q^3c\bar{c}$ all quark momenta are comparable and $O(1/R)$. Since $m_c \gg m_u$, $m_d$, or $m_s$ it follows trivially that $p^+$ for a charmed quark will characteristically exceed $p^+$ for an up, down or strange quark.

Although the admixtures required by BHPS are only at the 1% level, their consequences are far reaching. Some of these were pointed out by them, others have been discussed in contributions to this conference by Gavai and Roy[40] and by Roy[41]. Gavai and Roy analyze dimuon production, $\mu N\to\mu\mu x$, assuming the intrinsic charm distribution of BHPS. The standard mechanism for dimuon production, consisting of the hard subprocess $\gamma g\to c\bar{c}$ followed by a hard fragmentation for $c(\bar{c})\to D(\bar{D})+x$, provides an adequate fit to existing dimuon data. There is no room for the additional contribution through direct muon scattering off a charmed quark. If, however, the charm fragmentation function is somewhat softer, then Gavai and Roy find that present data are compatible with the BHPS intrinsic charm content of the nucleon. Upon closer inspection, I find that there is essentially zero overlap between the region of $x_{Bj}$ covered by present $\mu N\to\mu\mu X$ experiments[42,43] ($x_{Bj}\lesssim 0.1$) and the region of $x_{Bj}$ in which BHPS require intrinsic $c\bar{c}$ quarks ($0.3\lesssim x_{Bj}\lesssim 0.8$). So the analysis of Gavai and Roy does not test BHPS directly, only its extrapolation to smaller $x_{Bj}$. The paper by Roy[41] points out that even a small intrinsic $c\bar{c}$ component in the nucleon can lead to significant scaling violation at large $x_{Bj}$. At $x\sim 0.5$ the c-quark distribution function is $\sim$10% of the ordinary quark's, according to BHPS. So, argues Roy, a 10% increase may be expected as charm threshold is crossed. Analyses of scaling violations which ignore this effect would substantially underestimate $\Lambda_{QCD}$.

The proposal of BHPS is interesting in itself as an explanation of the anomalously large production of charm in hadron-hadron scattering. It can and should, but so far has not, been tested directly in muon induced dimuon production as suggested by Roy and Gavai (and in fact, in neutrino induced dimuon production as suggested by Brodsky, Peterson, and Sakai[38]).

## 6. $\gamma\gamma\to Q^2\bar{Q}^2$

Studies of the $4\pi$ final state in $\gamma\gamma$ scattering[44] indicate an enhancement in the $\rho^0\rho^0$ channel near threshold. Li and Liu[45] and Achesov, Devyanin and Shestakov[46] have suggested that the enhancement is due to the production of a $J^{PC}=2^{++}$ $Q^2\bar{Q}^2$ state (or states). The suggestion is motivated by vector meson dominance argument: for almost real photons the hadronic contribution to $\gamma\gamma$ scatterings is assumed to proceed through a virtual $\rho^0\rho^0$ intermediate state. Near threshold one might expect to find the same sort of $Q^2\bar{Q}^2$ enhancements as appear, for example, in the $K\bar{K}$ system (the S* and $\delta$). The tables of $Q^2\bar{Q}^2$ primitives[13] provide $J^{PC}=2^{++}$ (and $0^{++}$) candidates in the right mass range predominantly coupled to vector mesons, to explain the data.

The study (both experimental and theoretical) of such processes is still in its infancy. Perhaps, as Refs. 45 and 46 suggest, low energy $\gamma\gamma$ scattering will be fertile ground for developing multiquark spectroscopy. I would nevertheless remind enthusiasts of the history of baryonium and suggest that identifying bumps in cross sections with "states" in catalogues is not sufficient. It is necessary to understand the dynamics which connect the zero-width primitives of the quark model with enhancements or resonances in $\gamma\gamma$ scattering.

## 7. Quarks, QCD, and the nucleon-nucleon force

Quarks and QCD are enjoying a vogue among nuclear physicists who would like to describe the short distance behavior of many nucleon systems in terms of quark and gluon degrees of freedom. [The longer range nucleon-nucleon interactions appear to be well-described both theoretically and phenomenologically in terms of one and two pion exchange]. Most particle theorists would agree, I think, that we understand too little about low energy, non-perturbative effects in QCD to attempt any description of the nuclear force in fundamental terms. On the other hand, ad hoc models of QCD bound state dynamics like the bag model or the non-relativistic quark model (NRQM) can be adapted to study the nuclear force. It would be interesting to know if the wealth of data on the nuclear force (including the nucleon-nucleon resonances discussed at this conference) are compatible with these popular models of quark-gluon dynamics. So far there is no broadly successful QCD motivated model for the nuclear force. I know of two approaches to the problem which have received attention in the last few years. First is the attempt to calculate the NN potential either from a bag model[47,48] or from a NRQM.[49,50] The second[18,51] is based on the P-matrix.[14,52]

The basic idea of the NN potential calculations is to study the six-quark system in a Born-Oppenheimer approximation. One identifies center of mass coordinates of two $Q^3$ subsystems and calculates the energy with the separation between the two $Q^3$ subsystems constrained to some value, $\rho$. This is then interpreted as the potential $V(\rho)$. De Tar[48] and Liberman[49] were the first to attempt such calculations using the bag model and NRQM respectively. They treated the problem as a 1 channel problem - the $Q^3$ subunits were required to have nucleon quantum numbers - and found substantial repulsion at short distances in $V(\rho)$. One of the important technical details of these calculations is the need to include p-wave in addition to s-wave quark states in one's basis in order to be able to describe the separation of a $Q^6$ system into two separated $Q^3$-systems. Later Harvey[50] performed calculations in the non-relativistic quark model in which he allowed configuration mixing and many channels (e.g., $\Delta\Delta$ or $\underset{\sim}{N}\cdot\underset{\sim}{N}$, where $\underset{\sim}{N}$ is a color octet nucleon). Harvey found that with these improvements the short range repulsion vanished. Recently Bender and Dosch[53] have compared bag, NRQM, and string model calculations and proposed a resolution to the discrepancy between Harvey's and the earlier work. They point out that the crucial factor determining whether a model leads to repulsion at short distances is the energy cost of exciting a quark from the s- to the p-wave, and not the presence of other channels. Too small an s-p splitting leads to an overestimate of the $s^4p^2$ admixture at short distances, which lowers the energy. In Ref. 50 a mismatch between the scale of the confining potential and the scale of the quark wavefunctions effectively reduces the s-p splitting. Bender and Dosch find repulsion at short distances in all models with realistic s-p splittings, though the repulsion is not as large as that found by De Tar and Liberman.

Regardless of their success or failure in producing a phenomenologically acceptable potential, I worry that the dynamical framework of these NN potential calculations may be inadequate to the problem. The output of a hypothetical successful calculation would be some multichannel potential $V(\rho)$ defined in the space of nucleon-nucleon (or more generally, baryon-baryon) scattering states. Its long range behavior presumably would somehow match onto conventional meson exchange models. The new physics would be at short distances. This, unfortunately, is exactly the place where I believe a two-body potential makes little sense. Confinement in QCD sets in at distances of order $1/\Lambda_{QCD} \sim R_p \sim 1$ fm. At distances say a factor of two less than this, one does not *a priori* expect color singlet or $Q^3$ correlation to dominate. It would appear to be necessary to treat the full six-quark system. It would appear miraculous if a two-body potential would provide a good description of this problem with many more degrees of freedom.

Calculations of the NN potential based on the NRQM have an additional problem which I feel is very serious both conceptually and practically, namely the presence of van der Waal's forces. As is well known, non-relativistic models with confining forces between quarks will, in general, generate long range, $1/r^p$, forces between color singlets. For a particularly nice demonstration and discussion of this see Ref. 54. The long range force corresponds to an iteration of the confining potential as shown in Fig. 8. The conceptual problem is that power law forces imply, on the basis of very general analyticity requirements, the presence of massless particles in the model[55]: the $1/r^p$ force requires a cut in the complex t-plane with branch point at t=0. This in turn corresponds to

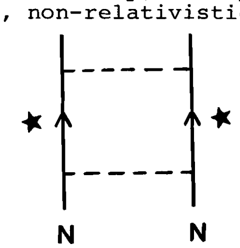


Fig.8. Iteration of the confining potential which gives rise to strong van der Waal's forces. The intermediate states labelled * are color-octet baryons.

a continuum with spectrum continuing down to mass zero. One can object that potential models, being non-relativistic, do not satisfy these analyticity requirements. Nevertheless I think one must regard the non-relativistic potential as the static limit of some relativistic exchange. Either that or action-at-a-distance! The t-channel in Figure 8 has glueball quantum numbers; a flavor and color singlet without quarks. The long range van der Waal's force can therefore be identified with a continuum of glueball states which can be exchanged in the t-channel. Although the direct limits on van der Waal's forces are not very strong, the existence of a continuum of strongly interacting glueballs beginning at mass zero is certainly excluded. It is unsatisfying to work in a model which is in such violent disagreement with experiment. In practice the lightest glueball mass is probably of the order of or somewhat greater than 1 GeV corresponding to a range of $\sim .2$ fm. So the van der Waal's forces in the NRQM represent an approximation whereby glueball exchange with range $\lesssim .2$ fm is replaced by a power law which is more important than any conventional exchanges at large distances. One solution would be to cut off the NRQM calculation of the potential at some intermediate distance and then match to some external meson exchange picture. The bag model does not suffer from this problem. Two nucleon bags separated by large distances interact only by the exchange of virtual meson bags. On the other hand, the problem of bag fission has yet to be solved. The P-matrix formalism avoids the problem of bag fission but at the price of again breaking the problem into two parts - an interior region where quark and gluon degrees of freedom are used and the exterior where meson and/or nucleon degrees of freedom are appropriate.

It turns out that a formalism identical to the P-matrix was developed and applied many years ago in nuclear physics by Feshbach and Lomon[52]. The analog to the P-matrix was called by them the f-matrix. They were interested in the NN force at intermediate and large distances. They parametrized the short distance part of the interaction by the logarithmic derivative of the wavefunction at a (small) separation b: $f(b)=\psi'/\psi|_b$. Among other things this allowed them to avoid having to make detailed assumptions about the NN dynamics at short distances which was (and is) little understood. Given f(b) one can integrate out through the region of conventional exchanges and calculate phase shifts. Feshbach and Lomon found that a constant or weakly energy dependent f(b) provided a good description of low energy NN scattering. The P-matrix picture of Ref. 14 turns this scheme on its head. f and P are identical. According to Ref. 14, quark model eigenstates are the poles in P (or f). In the NN-system the quark model primitives are all well above NN-threshold so at low energies it is reasonable to assume P (or f) depends only weakly on the energy. At higher energies P (or f) should show poles corresponding to quark model primitives.

This suggests a unified model of the nucleon-nucleon force in which quark model calculations at short distances are matched to meson exchange at large distances. Shatz and I[18] and Lomon and his collaborators[51] have analyzed low energy NN scattering in the $^3S_1$-$^3D_1$ and $^1S_0$ channels in this way and have found P-matrix poles in the experimental data in reasonable agreement with bag model predictions. These methods are still very crude and have not been applied to other partial waves where interesting data exist.

Before leaving the baryon-baryon system I would like to mention the most striking bag model prediction: the existence of a stable dihyperon primitive with mass (of the P-matrix pole) 2165 MeV[56]. The only evidence against such an object are two observed double-$\Lambda$ hyperfragments[57] (one arguable) which may perhaps be interpreted as evidence that two $\Lambda$'s do not bind strongly. The dedicated search carried out at Brookhaven several years ago did not have sufficient sensitivity. The mass quoted in Ref. 56 must be revised in light of the P-matrix formalism. Since the quark model calculation is performed subject to confining constraints (in channels like $\Lambda\Lambda$ which are not confined) the mass can only go down when physical boundary conditions are adopted. Thus Soldate[59] found that the P-matrix pole at 2165 MeV generates a bound state S-matrix pole roughly 100 to 130 MeV lower in mass[60]. This object should definitely be sought in more sensitive experiments.

## 8. Conclusions

Although the excitement of the past few years concerning narrow baryonium states has died, multiquark physics is alive and flourishing. One must, however, be careful when looking for its manifestations. They are often far from obvious. I have identified some areas where I feel multiquark effects may be important. In nearly every case, however, it is necessary to go beyond bump hunting or even conventional methods of analyzing scattering data in order to study multiquark configurations. Above all it is essential to recognize the difference between the zero width objects catalogued in naive quark models and the background phases, enhancements, and resonances seen in scattering experiments with physical boundary conditions.

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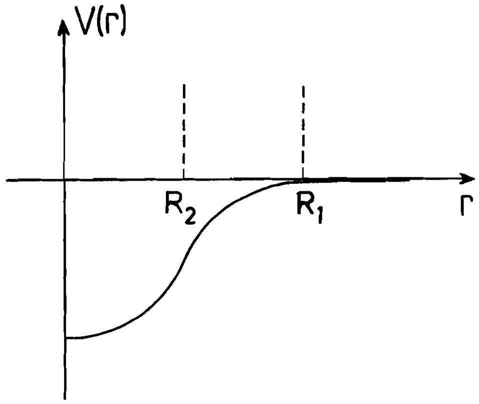
## Discussion

J.D. Sullivan, Univ. of Illinois: Is the transformation between the P-matrix and the S-matrix always known, and is the radius b that enters arbitrary or is it the bag model radius?

R.L. Jaffe: The transformation is known for two-body scattering. When the system cannot be described even effectively by two-body scattering, e.g. by an isobar model, then I don't know how to use the P-matrix formalism. Now the parameter b appears both in the theory and the analysis of experiment. When a theorist does a calculation he puts in confinement at some radius and that determines his parameter b. So he publishes a set of eigenenergies and he tells the world the radius at which he imposed the confining boundary condition. If he imposed the boundary condition at some other radius he would get different energies. Now if the experimentalist wants to test this theorist's predictions he first measures $\delta$ and then constructs the P-matrix with the theorist's value of b and compares the energies of the poles he finds with the energies the theorist predicts. b is chosen for the convenience of the theoretical calculation. If I chose b to be $\infty$ then I would effectively be calculating the S-matrix. If I choose b to be 5 fm then, in order to get a reasonable estimate of energies, I have to allow bags to fission because in the radius of 5 fm a $q^2\bar{q}^2$ state with the quantum numbers of two pions would like to be a pion here and a pion there. So I choose a value of b small enough so that I can do the calculation without having to consider bag fission: In principle b is arbitrary but in practice it is not.

M. Peskin, Cornell University: I have two questions: First, is it necessary to consider b as a parameter fixed independently of the mass of the state? At least in earlier MIT bag model calculations, the radius of the bag varied with the state. Secondly, if one is really free to adjust b arbitrarily, one could set $b = 1/10$ fm, in which case the P-matrix is computable by perturbation theory in QCD. Then the spectrum can be computed from perturbation theory. That can't be right! What is wrong with this prescription?

R. L. Jaffe: First, b is in fact chosen to be energy dependent. It could be energy independent, but we use the energy dependent value given by the bag virial theorem. It goes like the mass of the state to the 1/3rd power. As to your second question, b can be any radius outside the important region of interactions. Let's think about potential theory: For a potential like the one in the figure, a theorist could put a barrier at $R_1$ and calculate all the eigenstates. These determine the location of the P-matrix poles and the scattering wave at one of these poles consists of the solution to the Hamiltonian problem matched to a free scattering wave. With the barrier instead at $R_2$, one has to match not a free scattering wave but a wave interacting in that potential. You will get the same answer if you preserve the interactions outside of the point you build your barrier.




J. D. Jackson, Univ. of California, Berkeley: For those of the audience under fifty, I point out that the P-matrix formalism or its equivalent was discussed in the context of radar research during the Second World War and in the context of nuclear reaction theory in 1947 by E. P. Wigner and L. Eisenbud, Phys. Rev. 72 (1947) 29, and T. Teichmann and E. P. Wigner, Phys. Rev. 87 (1952) 123. Theorists wishing to learn the application of such techniques should consult these papers, the review by A. M. Lane and R. G. Thomas in Rev. Mod. Phys. 30 (1958) 257, or the book by R. G. Sachs, "Nuclear Theory".

R. L. Jaffe: To be precise $P = 1/R$. If you are really pedestrian like I, I suggest Viki Weisskopf's book with Blatt. Chapter X contains a very nice discussion. I should add that the R-matrix corresponds to the boundary condition $\Psi' = 0$, so the purpose of that boundary condition was to find the best impedance match. If you have found a pole in the R-matrix the scattering wave comes into the interaction region with zero slope and therefore matches onto a wave with large intensity inside. This is a familiar criterion for a resonance in the good old sense. Our boundary condition is exactly the opposite, for physical reasons: because it is confining, not transmitting.