THEORY OF HEAVY QUARK ANTIQUARK STATES

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Introduction

When I think of the modern theory of heavy quarkonium I recall the beginning of one of the stories in "Gesta Romanorum". It sounds like that. A noble man has a white cow which he liked very much because she was white and because she gave milk. The existing theory of heavy quark states is one of the most elegant branches of QCD ("white"), and, what may be more important, it provides with unique possibilities in investigation of other issues, such as physics of glueballs, properties of the non-perturbative QCD vacuum, and so on ("gives milk").

In my talk I will not try to summarize all theoretical knowledge on the subject - even a brief review of various competing approaches would take too much time. Instead, I will concentrate on a few results, which, I think, might stimulate further development. The main topics are:

(i) Masses and leptonic widths of $Q\bar{Q}$ levels from QCD sum rules;
(ii) Pre-Coulomb behaviour of heavy systems;
(iii) Multipole expansion in decays $\psi' \to J/\psi\pi\pi$, $\Upsilon' \to \Upsilon\pi\pi$, ...;
(iv) Exclusive hadronic decays.

Some other questions to be discussed in brief are: the spectrum of open charm and beauty states, effects due to weak interaction, radiative decays and the problem of glueballs. I would like to notice in advance that the non-relativistic potential model [1] which served as a basis for very many papers devoted to $Q\bar{Q}$ states is only occasionally mentioned. This is not because the model has become unpopular but, rather, because it seems to be well-known to the audience. There exists a lot of nice reviews [2] written by experts and exhaustive account of latest results is given by Martin at the Lisbon Conference [3].

I. SPECTRUM

Quarks and Their Masses

Existence of two heavy quarks, c and b, is established now for certain. Their electric charges are $+2/3$ and $-1/3$ and they form the charmonium and bottonium families, respectively. It is usually believed that there should exist the third heavy quark t, not yet found experimentally, with electric charge $+2/3$.

The most important characteristic of heavy quarks is their mass. Since they are permanently confined one cannot weigh an isolated quark just in the same way as one does it, say, with muons. Still, it is possible to introduce the notion of the so-called current quark - an object which is essentially deprived of its gluonic cloud. (To be more exact, only soft gluons are eliminated, hard gluons result in logarithmic effects which are readily accounted for.)

The current quark mass depends on a normalization point and enters all calculations which are based on fundamental chromodynamics. For c and b quarks it was determined [4,5] from QCD sum rules (see below):

$$m_c \simeq 1.35 \text{ GeV}, \qquad m_b \simeq 4.80 \text{ GeV}. \tag{1}$$

I quote here the numbers referring to an on-shell mass - a gauge invariant quantity well-defined in perturbation theory. Euclidean mass, also often mentioned in literature [4], is gauge dependent. Say, in the Landau gauge

$$m(p^2 = -m^2) = m(p^2 = m^2)\left\{1 - \frac{2\alpha_s \ln 2}{\pi} + \dots\right\},$$

and in other gauges the coefficient of the α_s term varies. For charmed quarks there exist at present many independent estimates of euclidean mass [4,6-8]. They all agree with each other and with eq. (1):

$$m_c \simeq 1.25 \text{ GeV} \quad (p^2 = -m_c^2\text{, Landau gauge}). \tag{2}$$

Unfortunately, the situation with b quarks is worse. The only relevant analysis which I am aware of [8] yields $m_b(\text{euclidean}) \simeq 4.26$ GeV, and this number is too small - in order to agree with eq. (1) it should be larger by approximately 150 MeV. The discrepancy is presumably due to the fact that certain Coulombic effects important in the Υ family were neglected in [8].

Those who work with constituent quarks usually get larger masses. This is natural, of course, since they include in their quarks some gluon clots as well.

It is worth noting that

$$M_{J/\psi} > 2m_c \tag{3}$$

but

$$M_\Upsilon < 2m_b\,; \quad 2m_b - M_\Upsilon \simeq 130 \text{ MeV} \quad [5]. \tag{4}$$

While eq. (3) is quite transparent, eq. (4) might seem surprising. Indeed, confinement effects tend to increase the mass of the resonance as compared to the doubled mass of the quark. What happened in the $b\bar{b}$ system? In the Υ family the Coulomb attraction becomes numerically important and overcompensates the positive mass shift due to confinement forces.

As for the hypothetical t quark, its absence in DESY implies that [9]

$$m_t > 18.5 \text{ GeV} . \tag{5}$$

Now, to understand the quarkonium spectrum we should know, apart from the quark masses, the nature of the binding forces. According to modern views quarks live in a complicated medium - non-perturbative QCD vacuum, densely populated by long-range fluctuations of gluon fields. These non-perturbative fluctuations lower the vacuum energy-density as compared to its perturbative value. If one injects a $Q\bar{Q}$ pair the quark color field somewhat freezes the fluctuations in the surrounding domain [10] which results in an effective attraction between Q and $\bar{Q}$.

It is important to realize that in real systems like J/ψ or Υ the attraction force can be described by no static potential. Indeed, the impact on the gluon medium can be reduced to a potential only if the medium has time to tune itself and follow the (slow) quark motion. In other words, the potentiality condition is

$$\omega_{\text{quark}} \ll \omega_{\text{glue}} \tag{6}$$

where ω denotes a characteristic frequency. In the charmonium and upsilonium families the characteristic frequencies are of order

$$\omega_{\text{quark}} \sim M_{\psi'} - M_{\psi} \sim M_{\Upsilon'} - M_{\Upsilon} \sim 0.6 \text{ GeV}, \tag{7}$$

and frequencies inherent to the gluonic medium are approximately the same. Validity of the multipole expansion (see below) implies even that $\omega_{\text{glue}} < \omega_{\text{quark}}$. If so, one can expect large deviations from the potential picture. The expectation is confirmed, in a sense, by recent analyses of various relativistic effects [11].

Gluon Condensate

Peculiar properties of the QCD vacuum responsible for the formation of spectrum are not yet understood completely. Still, some coarse characteristics are known. The net effect of long-range gluon fluctuations is measured, for instance, by the vacuum expectation value of gluon field squared

$$\langle \text{vac} | G^a_{\mu\nu} G^a_{\mu\nu} | \text{vac} \rangle \neq 0 . \tag{8}$$

On one hand, this parameter reduces in a straightforward way to the vacuum energy density [7]

$$\varepsilon_{\text{vac}} = \frac{1}{4} \langle \text{vac} | \Theta_{\mu\mu} | \text{vac} \rangle \simeq -\frac{9}{32} \langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^a_{\mu\nu} \rangle , \tag{9}$$

where $\Theta_{\mu\nu}$ is the energy-momentum tensor. I used the fact that $\Theta_{\mu\mu}$ is determined in QCD by the so-called triangle anomaly [12]:

$$\Theta_{\mu\mu} = \frac{\beta(\alpha_s)}{4\alpha_s} G^a_{\mu\nu}G^a_{\mu\nu} + \sum_q m_q\bar{q}q \simeq -\frac{9\alpha_s}{8\pi} G^a_{\mu\nu}G^a_{\mu\nu}.$$

On the other hand, the gluon condensate (8) plays a special role in heavy quarkonium physics. Why? The $Q\bar{Q}$ pair forming a quarkonium level is in a colorless state, and, hence, its coupling to the vacuum fields is of a dipole type,

$$H_{int} = -\frac{1}{2}g(t_1^a - t_2^a)\vec{r}\vec{E}^a \tag{10}$$

where $\vec{E}^a$ is the chromoelectric field and $t_{1,2}$ stand for the color SU(3) generators acting on the quark and antiquark indices, respectively. For transitions between colorless states the first order term in H_{int} vanishes, and the leading effect reduces to the second order term proportional to

$$\langle\vec{E}^a\vec{E}^a\rangle = -\frac{1}{4}\langle G^a_{\mu\nu}G^a_{\mu\nu}\rangle \quad ,$$

plus further iterations.

The vacuum expectation value (8) was introduced in ref. [13] where its magnitude was estimated from charmonium sum rules:

$$\langle vac|\frac{\alpha_s}{\pi} G^a_{\mu\nu}G^a_{\mu\nu}|vac\rangle \simeq 1.2\cdot 10^{-2}\ \mathrm{GeV}^2 \ . \tag{11}$$

Recent analyses [5,8,14,15] based on similar principles but involving more data indicate that it can actually be larger by a factor of 1.5 - 2.

Comparing eqs. (11) and (9) we see that ε_{vac} is negative. This is in full accordance with the fact that in a confining theory non-perturbative fluctuations should lower the vacuum energy density.

Pre-Coulomb Behaviour

In one particular case information coded in eq. (11) is sufficient to build a genuine and exhaustive theory of quarkonium levels. If the quark mass m is large enough the quarks are bound essentially by the Coulomb force at distances of order k_n^{-1} where k_n is given by the following relation:

$$k_n = \frac{m}{n}\frac{2}{3}\alpha_s(k_n) \tag{12}$$

(n is the principle quantum number). At large k_n the radius of the orbit is small compared to the characteristic wave length of the vacuum fluctuations, and hence

$$|rD_\mu G_{\alpha\beta}| \ll |G_{\alpha\beta}| \quad ; \tag{13}$$

all higher-order non-perturbative effects can be neglected. The life of quarks becomes simple: they form a Coulomb system which experience, however, the influence of an external constant field. This field is (a) weak, (b) random, (c) chromoelectric (corrections due to chromomagnetic fields are suppressed by two powers of $\alpha_s(k_n)$).

Being put in this way the problem has an elegant and exact solution (based on the operator product expansion) originally described by Voloshin and Leutwyler [16-18]. They managed to find an analytic answer for the level shifts, namely,

$$M_{nl} = 2m - \frac{k_n^2}{m} \{1 - \frac{m^2}{k_n^6} n^2 a_{nl} <\frac{\pi\alpha_s}{18} G^2>\} \tag{14}$$

where l is the orbital angular momentum and a_{nl} is a known coefficient function of order unity, say, $a_{10} \simeq 1.65$, $a_{20} \simeq 1.78$, etc. (m stands for the so-called on-shell mass, see discussion above). This formula can be, perhaps, useful in $t\bar{t}$ phenomenology; it is even more important theoretically since it provides a quantitative answer to the question at which quark masses the Coulomb-like picture sets in.

The expansion parameter in eq. (14) is, evidently,

$$\frac{m^2}{k_n^6} n^2 <\frac{\pi\alpha_s}{18} G^2>$$

and it becomes of order unity in the $b\bar{b}$ system. (I mean the ground state, $k_1(b\bar{b}) \simeq 0.96$ GeV if $\alpha_s(\text{GeV}) = 0.3$, i.e. $\Lambda_{QCD} = 100$ MeV). For lighter quarks the binding force has nothing to do with the Coulomb interaction, and the latter is completely negligible. On the contrary, heavier quarks form almost perfect Coulomb levels with very small deviations. The upsilon family lies somewhere in-between - here the Coulomb terms are competing with non-perturbative ones.

It is instructive to examine also the n-dependence which turns out to be very sharp. Already the first excited level in $b\bar{b}$ is completely non-Coulomb. For $n = 2$ the lower boundary of the Coulomb domain shifts to $m \gtrsim 20$ GeV. For such masses the number of excited levels below the continuum threshold is rather large [19]

$$n \simeq 2\left(\frac{m_t}{m_c}\right)^{1/2} \gtrsim 7-8 ,$$

and one can enjoy a rich spectrum of dynamical scenarios in one and the same quarkonium family.

Of much practical interest is the result for $\Gamma(1^3S_1 \to e^+e^-)$,

$$\Gamma = \Gamma_{Coul}|1 + A_z/A_\gamma|^2 \{1 + \frac{m^2}{k_1^6} 4.93 <\frac{\pi\alpha_s}{18} G^2>\} . \tag{15}$$

I quote here the expression derived by Voloshin, Leutwyler's one is somewhat different. Γ_{Coul} is purely Coulombic width,

$$\Gamma_{Coul} = 4\pi(\text{quark charge})^2 \frac{\alpha^2}{m^2} \frac{k_1^3}{\pi}\left(1 - \frac{16\alpha_s(m)}{3\pi}\right) , \tag{16}$$

and a correction factor $|1 + A_Z/A_\gamma|^2$ is due to the Z-boson contribution [20]. The curve for a reduced width,

$$\tilde{\Gamma} \equiv \Gamma(1^3S_1 \to e^+e^-)/(\text{quark charge})^2 |1 + A_Z/A_\gamma|^2 \tag{17}$$

is displayed in fig. 1. It is surprisingly flat in the domain $m \gtrsim 10$ GeV where the result is reliable. When m becomes less than 10 GeV, the Voloshin-Leutwyler theory fails, and other methods should be and were successfully applied for calculation of quarkonium parameters.

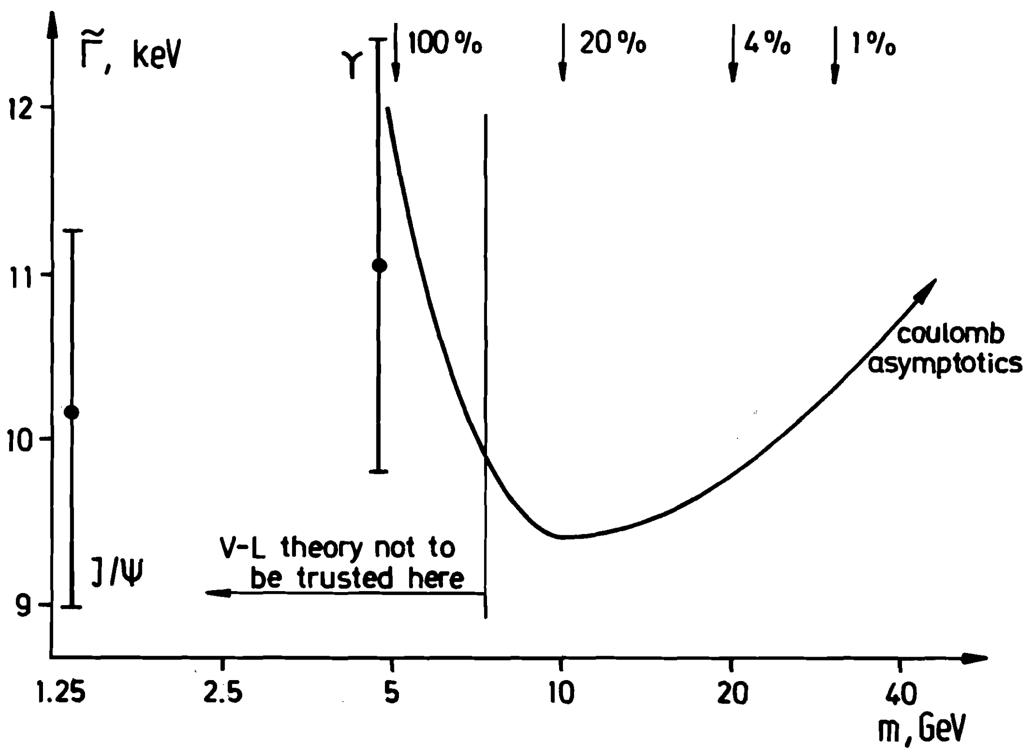

Fig. 1 Theoretical prediction for the reduced leptonic width, eq. (17), versus quark mass. The experimental points for J/ψ and Υ are drawn for comparison. The numbers in the upper part of the plot indicate the value of the G^2 correction.

QCD Sum Rules

The size of charmonium and upsilonium is too large to apply directly the technique described above. Still, there exists a roundabout way, a certain extrapolation procedure, allowing to extract accurate predictions for lowest-lying states with various quantum numbers in terms of fundamental quantities.

Consider, say, the vector channel in charmonium. The spectral density R_c is defined in a standard way

$$R_c \quad = \quad \sigma(e^+e^- \to \text{charm})/\sigma(e^+e^- \to \mu^+\mu^-)$$

where $\sigma(e^+e^- \to \text{charm})$ includes J/ψ, higher resonances, and charmed continuum. Everybody knows that single resonance structures are not resolved in present-day QCD and only smeared cross section is predicted. Thus, we are forced to make formally a step back as compared to the situation discussed above. Instead of a particular level we consider now weighted sums over many levels. However, if the weight function is steep enough, the sum may be practically saturated by the lowest-lying state, and we arrive at a (quasi) theory of such states.

In other words, everything depends on our ability to calculate integrals $\int R_c(s)f(s)ds$ with steep weight functions. Just like the world history is sometimes reduced to an enumeration of battles and kings, the history of QCD is expressed by a list of functions $f(s)$ discussed theoretically in this or that period of time.

Due to time limitation it is impossible to trace now ancient history [21-23], although all the stages were, of course, very important. At present, the approach to resonance physics based on QCD sum rules is already a well-developed method with a rather wide range of applications.

The basic theoretical steps are simple. We start with a two-point function with appropriate quantum numbers, for instance, to analyse R_c we choose

$$\Pi_{\mu\nu} \quad = \quad i\int dx\ e^{iqx}\langle 0|T\{\bar{c}\gamma_\mu c(x),\ \bar{c}\gamma_\nu c(0)\}|0\rangle\ . \qquad (18)$$

The simplest graph contributing to $\Pi_{\mu\nu}$ is depicted in fig. 2. Moreover, due to famous asymptotic freedom [24] it is the only one which survives in deep euclidean domain of Q^2. In one-to-one correspondence to asymptotic freedom there is a trivial smooth behaviour of $R_c(s)$ at large s .

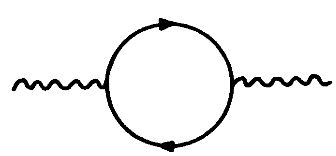

Fig. 2 The lowest-order diagram for the correlation function (18). Continuous lines depict heavy quarks and wavy lines currents

When we move from the euclidean domain towards the physical one interactions become important bringing in some additional mass scale. Respectively, the smooth curve for $R_c(s)$ becomes less smooth at smaller s, and there appear resonance structures.

The interaction which shows up first and turns out to be most important is depicted in fig. 3. The crosses on the gluon lines indicate that they are non-perturbative, and the diagram reduces to vacuum expectation value (8) times a known function of Q^2 [13, 6-8].

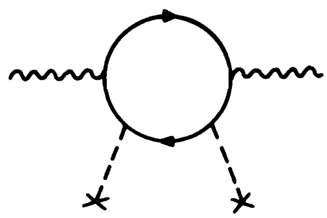

Fig. 3 Coupling of quarks to vacuum fields. Dashed lines depict gluons.

On the other hand, $\Pi_{\mu\nu}$ can be expressed in terms of R_c via the general dispersion relation. Thus, the resonance properties appear to be correlated with the fundamental vacuum parameters.

I skip a lot of details here which are essential but, perhaps, not so interesting to the audience. The list of technicalities is very large: moment technique, Borel transformation (relativistic and non-relativistic), vacuum expectation values of higher-dimension operators, and so on and so on. Those, who are interested should look through original papers [5-8]. Just to illustrate characteristic features of the method let me reproduce a plot from one of the first works [7] (fig. 4). It displays the ratio of the moments

$$r_n = \int \frac{R_c(s)}{s^{n+1}}\, ds \Big/ \int \frac{R_c(s)}{s^n}\, ds = \frac{n^2-1}{n^2+\frac{3}{2}n}\ \frac{1}{4m_c^2}\Big\{1 - \frac{(6n+14)n(n+1)(n+2)}{(2n+3)(2n+5)} \times \langle \tfrac{4}{9}\pi\alpha_s G^2\rangle (4m_c^2)^{-2} + \dots\Big\} \tag{19}$$

versus n. For large n all the contributions except for the J/ψ die away, and $r_n \to M_\psi^{-2}$. Unfortunately, in present-day theory it is impossible to tend n to ∞ since non-perturbative terms blow up. At $n = 5$-6, however, the term proportional to $\langle G^2\rangle$ is still controllable. On the other hand, at such n the J/ψ contribution exceeds 95 %. Thus, the J/ψ mass is fixed in terms of the quark mass to one percent accuracy.

Historically the problem was reversed: the quark mass and $\langle G^2\rangle$ were fitted to reproduce M_ψ. With these parameters in hand masses of other ground levels are unambiguously predicted. I would like to show you recent results for charmonium P levels obtained by Reinders et al. [8] (fig. 5). In all the cases there is a

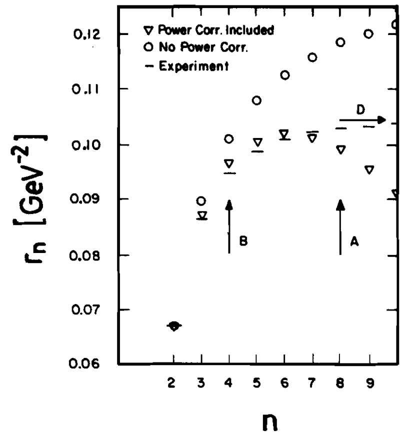

Fig. 4 The ratio of moments versus n. (For definitions see eq.(19).) Arrow A marks the 20 % level for the $\langle G^2\rangle$ correction. Arrow B separates the regions of small and large experimental uncertainties: to the right of this arrow the uncertainty is $\lesssim 1$ %. Arrow D shows the asymptotic value of r_n.

nice stability plateau (this fact is due to some technical improvements worked out in [8]). Its position is in excellent agreement with the experimental value of mass. The authors also predict the position of the elusive $^1P_1$ level:

$$M(1\,{}^1P_1) \quad = \quad 3.51 \pm 0.01 \text{ GeV} . \tag{20}$$

This seems to be the most accurate and reliable estimate existing today and, I believe, it will be confirmed after the discovery of the level.

The analogous analysis in the upsilon family is hampered by necessity of accounting for the Coulomb interaction. All difficulties were overcome, one by one, in works of Voloshin [25,5] who used a non-relativistic version of the Borel technique.

He considers first the 1^- channel and extracts the precise value of the b-quark mass, which I have already quoted. Then, he addresses a harder problem of 1S-1P splitting. The final result for M(1P) is [5]

$$M(1P, b\bar{b}) \quad = \quad 9.83 \pm 0.03 \text{ GeV} . \tag{21}$$

It is interesting to note that various potential models give here a spectrum of predictions ranging from 9.86 up to 9.94 (the most typical number is 9.90-9.92). Future experiments will reveal the degree of our theoretical understanding.

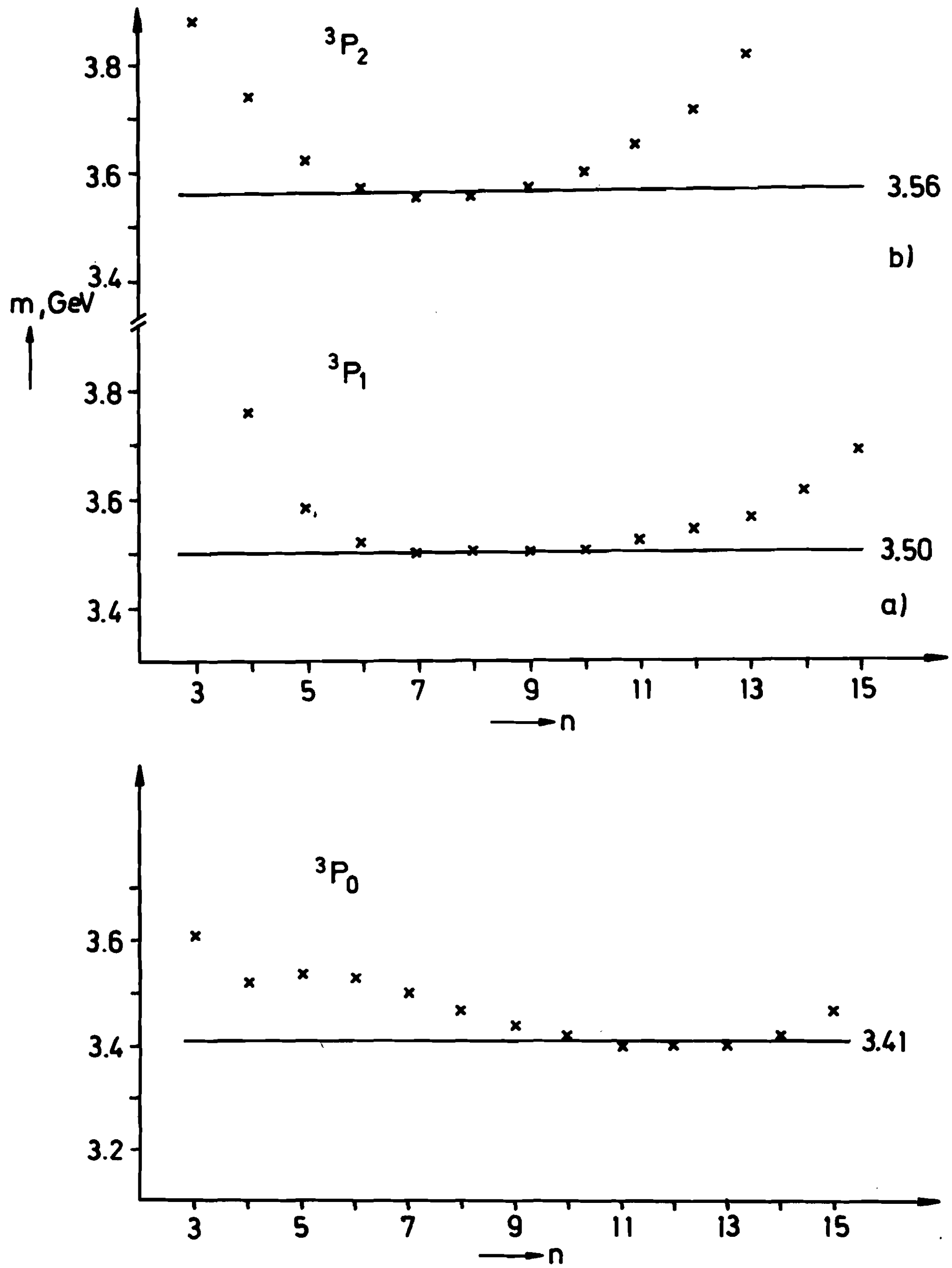

Fig. 5 Masses of charmonium P levels from QCD sum rules (borrowed from ref. 8)

Spin Effects

Spin dependence of forces acting between bound quarks is one of the oldest questions in quarkonium physics. Conventional one-gluon exchange gives rise to some spin effects summarized by the familiar Breit-Fermi Hamiltonian [26]. This is not the end of the story, however. Coupling of quarks to the vacuum gluon fields induces additional spin dependence, even at short distances. This non-perturbative contribution by no means can be reduced to any kind of a potential [27,17].

Perhaps, many remember yet the dramatic history of the η_c particle. It was found first at a wrong place, 2.83 GeV, while the QCD sum rules implied [6]

$$m_{\eta_c} = 3.00 \pm 0.03 \text{ GeV} .$$

The SLAC discovery of the 2.98 state was one of the major successes of the QCD-based ideas.

In a sense, the story repeats now in the $b\bar{b}$ family. There is no unanimous opinion among theorists on the problem of the $\Upsilon - \eta_b$ mass difference. Investigators of the potential models usually give [2] 60 to 80 MeV, although in some works a smaller splitting is predicted (20 to 40 MeV according to ref. 28). A rather subtle analysis is required in order to extract $M_\Upsilon - M_{\eta_b}$ from the sum rules. This was done [29] with the result

$$M_\Upsilon - M_{\eta_b} \simeq 30 \text{ MeV} . \tag{22}$$

The effects of gluon condensate here turn out to be almost negligible, $\lesssim 5$ MeV.

Later, literally the same method was used by Reinders et al. [8] who obtained, however, $\Delta M \simeq 60$ MeV. Perhaps, this number is overestimated because it does not account for the Coulomb corrections.

Recently it was proposed [17] to approach the problem from large-mass side. Sufficiently heavy quarkonium levels are Coulombic, and the leading non-perturbative correction can be included explicitly [17,30]. Say, the splitting in the ground state $(n=1)$ is [31]

$$M(1^3S_1) - M(1^1S_0) = \frac{32\pi}{9} \frac{\alpha_s(m)}{m^2} |\psi_1(0)|^2 + \langle \frac{\pi\alpha_s}{18} G^2 \rangle (4mk_1^2)^{-1} \frac{688}{153} , \tag{23}$$

where $\psi_1(0)$ is the wave function which also contains two distinct terms:

$$|\psi_1(0)|^2 = \frac{k_1^3}{\pi} \{1 + 4.93 \frac{m^2}{k_1^6} \langle \frac{\pi\alpha_s}{18} G^2 \rangle\} . \tag{24}$$

Substituting here the b-quark mass, $m_b = 4.8$ GeV, one gets approximately 90 MeV. Unfortunately with such quark mass the expansion parameter is bad, of order unity, and there are no reasons to believe that higher-order non-perturbative terms do not modify the answer.

Recently Voloshin has noted [30] that the leading effect reduces simply to renormalization of $|\psi_1(0)|^2$, and other non-perturbative contributions can be bounded from above under reasonable assumptions ($\lesssim$ 5 MeV for $b\bar{b}$) [32]. On the other hand, $|\psi_1(0)|^2$ is known phenomenologically, from $\Gamma(\Upsilon \to \mu^+\mu^-)$. In this way we get

$$M_\Upsilon - M_{\eta_b} \simeq 8\Gamma_{ee}(\Upsilon)\,\frac{\alpha_s(m_b)}{\alpha^2}\left(1+6.1\,\frac{\alpha_s(m_b)}{\pi}\right) + \langle\frac{\pi\alpha_s}{18}\,G^2\rangle(4mk_1^2)^{-1}\,\frac{688}{153} \simeq 36\text{ MeV},$$

where I included the $O(\alpha_s)$ correction of ref. 31. Amusingly, this formula works for J/ψ (gives $\sim 60+30 \simeq 90$ MeV), although it is not legitimate, of course, to use it in charmonium.

Of course, with such a small mass difference experimental search for η_b becomes an extremely difficult but honorable task.

As for the fine splitting between 1^3P_J bottonium states, it is expected to be even smaller [33]: $\Delta M(^3P_1 - {}^3P_0) \sim 20$ MeV, $\Delta M(^3P_2 - {}^3P_1) \sim 10$ MeV. For an alternative point of view see, however, ref. 34.

Returning to the psi-family, I would like to notice that now we have a new chance to check our theoretical abilities. As was announced by Dr. Scharre an η_c' candidate is found at SLAC, and $M_{\psi'} - M_{\eta_c'} = 92 \pm 5$ MeV. This number seems reasonable theoretically, but the detailed analysis based on modern views may reveal interesting aspects.

D and B Mesons

Although this is not quite my topic I cannot resist a temptation to say a few words about the spectroscopy of open charm and beauty states. Two recent works [35,36] based on the QCD sum rules are devoted to this subject.

Unlike charmonium or upsilonium levels the masses of D and B mesons are extremely sensitive to another fundamental characteristic of the QCD vacuum,

$$\langle vac|\bar{\psi}\psi|vac\rangle \neq 0 \tag{25}$$

where $\psi = u,d$ or s. The existence of the quark condensate (25) is known for a long time [37], however, over many years it manifested itself only in pion physics.

The role of the quark condensate in $(Q\bar{q})$ systems is striking - it induces a very large splitting between opposite parity states. The results of refs. 35, 36 are summarized in Table 1 (the numbers are somewhat rounded).

Quantum numbers J^{PC}	Mass (GeV) Ref. 35	Ref. 36
0^{-+} , 1^{--}	5.3	5.2
0^{++} , 1^{++}	6.1	6.0

Table 1

Masses of mesons with quark content $b\bar{u}$ (or $b\bar{d}$). Theoretical uncertainty is about 100 **MeV.**

Technical details are different - nonrelativistic borelization in one case and moment technique in the other - but the predictions nicely agree within theoretical uncertainty of about 100 MeV. The mass difference between negative and positive parity states is 0.8 GeV. For comparison let us notice that the potential model gives 0.5 GeV for the open charm system [38].

It is important that the analysis needs no model assumptions - fundamental vacuum parameters are immediately translated in the language of observable quantities.

II. LEPTONIC AND PHOTONIC DECAYS

What else do we know?

The physics of heavy quarkonium is simpler than that of "old" particles, but still it is much richer than, say, the problem of hydrogen atom. In hydrogen, if one can calculate the energy levels, one can calculate everything else, and every new question would be a mere repetition of the previous one. This is not the case with $Q\bar{Q}$. Therefore, I would like to discuss other issues, namely, leptonic and hadronic decay modes. Such field of research as photo- and hadroproduction of J/ψ, Υ and so on also attracts much attention at present [39]. Beyond any doubt these investigations are informative and promising; however, discussion of the corresponding problems would require a special talk.

Leptonic Widths

The same sum rules which are so useful in spectroscopy give simultaneously leptonic widths of ground levels. All relativistic effects, renormalizations, etc. are accounted for automatically. For J/ψ the result is known for already 3 years [4,7], it is in excellent agreement with experiment, and, perhaps, there is no need in further comments. The situation with $\Gamma(\Upsilon \to \mu^+\mu^-)$ is intriguing. Due to large Coulomb factors appearing in the sum rules this decay rate is very sensitive to the value of α_s. The theory requires [5]:

$$\Gamma(\Upsilon \to \mu^+\mu^-) = 1.15 \pm 0.20 \text{ keV} \tag{26}$$

and

$$\alpha_s(1 \text{ GeV}) = 0.3 \pm 0.03 \,. \tag{27}$$

It is impossible to violate these limits, at least if our basic concepts are correct.

These numbers were obtained a couple of years ago when the experimental situation was not clear, at least, not as clear as now.

The world average for $\Gamma(\Upsilon \to \mu^+\mu^-)$ quoted by Dr. Schamberger today is

$$\Gamma_{exp} = 1.17 \pm 0.05 \text{ keV} .$$

As for the quark-gluon coupling constant, eq. (27) is also welcome by latest data. If before there existed a controversy on the magnitude of the QCD scale parameter,

$$\Lambda = 100\text{-}200 \text{ MeV} \quad \text{or} \quad \Lambda = 500\text{-}700 \text{ MeV} ,$$

the question is definitely settled now. The current CERN muon experiment favors [41] smaller Λ's, and the data [40] on $BR(\Upsilon \to \mu^+\mu^-)$ $(3.3 \pm 0.5\ \%)$ shows in the same direction (for a detailed discussion see ref. 29). I think that we should forget about old estimates of Λ and accept that $\alpha_s(1\text{ GeV}) \simeq 0.3$ or $\Lambda_{\overline{MS}} = 100$-150 MeV. Actually, the QCD sum rules never admitted a larger value of α_s [7].

A few words about leptonic decays of D and B mesons, say, $D \to \mu\nu$ and $B \to \tau\nu$. It is convenient to introduce coupling constants f_D, f_B determining the decay amplitudes by analogy with the familiar f_π,

$$\langle D(B)|\bar{Q}\gamma_\mu\gamma_5 q|0\rangle = -if_{D(B)}p_\mu .$$

Estimates of f_D attracted attention not only in connection with the $D \to \mu\nu$ decay, but, mostly, in connection with various models explaining D^+/D^0 anomaly.

Ref. 36 presents the analysis of the two-point function induced by the current $\bar{Q}\gamma_\mu\gamma_5 q$. In this work it is found that[1)]

$$f_D \simeq 230 \text{ MeV}, \qquad f_B \simeq 140 \text{ MeV} . \tag{28}$$

Similar sum rules were considered in ref. 35 which gives a larger value of f_B; however, one of the assumptions there seems suspicious. In any case these numbers are much smaller than has been conjectured to explain the $D^+ - D^0$ lifetime difference. It is instructive to compare eq. (28) with the results based on other principles. Say, the predictions [42] of the naive bag model are several times larger. Future experiment will choose, of course, the right scheme.

From QCD sum rules one can extract other leptonic couplings, for instance, the two-photon widths of P levels. Some estimates already exist [23,4]. Actually, they reduce to a refined QCD duality - up to now effects due to the gluon condensate are not calculated explicitly.

Not to make a false impression it is worth mentioning that not every quantity is immediately calculable in this way. Take, for instance, radially excited states - neither their masses nor coupling constants are determined by the sum rules (in their present form) with sufficient accuracy. Fortunately, these parameters are more or less stable in the potential models. The energy levels of first radial excitations seem to be insensitive to non-potential effects; these effects also basically cancel in the ratios

$$\Gamma(n^3S_1 \to e^+e^-)/\Gamma(1^3S_1 \to e^+e^-) \quad \text{(see Table 2).}$$

State	Buchmüller and Tye, ref. 2	Martin, ref. 43	Experiment, ref. 40
2S,Υ'	0.44	0.51	0.46 ± 0.02
3S,Υ"	0.32	0.35	0.34 ± 0.02
4S,Υ"'	0.26	0.27	0.23 ± 0.02

Table 2

The ratios $\Gamma(n^3S_1 \to e^+e^-)/\Gamma(1^3S_1 \to e^+e^-)$ in the potential models.

Other Methods, Other Trends

The potential model has been already mentioned in connection with various aspects of quarkonium physics. Also often used is the traditional local duality [45] which says that

$$\int_{s_0-\Delta s}^{s_0+\Delta s} \sigma_{\text{physical}}(s)ds = \int_{s_0-\Delta s}^{s_0+\Delta s} \sigma_{\text{bare quarks}}(s)ds \ . \tag{29}$$

One should not demand from these models, however, more than they can really give.

It is important to realize that the potential fitting charmonium and upsilonium spectra is nothing else than an effective potential. It has nothing to do with the genuine static energy of infinitely heavy quarks which will be inferred, one day, by investigators of the Wilson loop. The true static energy might reveal itself in highly excited levels (slightly below the flavor threshold). Here the level spacings are small and the quark frequencies are much less than those characterizing the gluon medium. In other words, in this case the potential language is fully justified from the theoretical point of view.

The potential model is indispensible for orientation and it gives a nice overall picture; however, it cannot (and should not) answer all subtle questions, such as fine splittings, precise determination of decay rates and so on.

The naive duality relations like (29) are usually exploited in order to extract the couplings of mesons to various currents. It is well-known that the amplitudes

$$\langle 0|\bar{c}\gamma_\mu c|J/\psi\rangle \ , \quad \langle 0|\bar{c}\gamma_\mu c|\psi'\rangle, \ldots$$

are well reproduced in this way.

The origin of duality is quite transparent in quasiclassical treatment of the Schrödinger equation [46]; it is, actually, of a more general nature and is explained by a local character of interactions. Consider a virtual photon of high energy E which converts to a $Q\bar{Q}$ pair. The conversion takes place at distances

$\sim 1/E$ and its probability is proportional to $\sigma(e^+e^- \to$ bare quarks). Only at much larger distances, $\sim E/\Lambda^2$, confinement effects switch on in full. They play a role of a large box which make the spectrum discrete. The sum over close discrete levels, evidently, reproduces $\sigma(e^+e^- \to$ bare quarks) up to terms of order $(1/E)$ to a positive power.

Thus, the local duality for highly excited states is a rather trivial fact. Its validity for the lowest state, J/ψ, seems to be more surprising. May be, this shows that J/ψ is in a sense also "highly excited" with respect to current quarks, $M_{J/\psi} - 2m_c(\text{current}) \simeq 0.6$ GeV.

With increasing the quark mass the accuracy of relation (29) becomes worse. Actually, in purely Coulombic situation the Coulomb poles should be added by hand to the right-hand side of eq. (29) [4] so that the procedure becomes almost senseless.

I have checked also (by considering a toy model) that the procedure is completely inapplicable to high waves: the matrix elements

$$\langle 0|\bar{Q}\gamma_{\mu_1} \overleftrightarrow{D}_{\mu_2} \ldots \overleftrightarrow{D}_{\mu_n} Q|\text{spin-n meson}\rangle$$

obtained from naive dual formulas are qualitatively different from their real values.

In many cases instead of naive formulas (29) QCD suggests a refined version

$$\int_{\text{threshold}}^{\infty} ds\ \sigma_{\text{physical}}(s) f(s) = \int_{4m^2}^{\infty} ds\ \sigma_{\text{quark}}(s) f(s)$$

with specific weight functions $f(s) \sim s^{-n}$. Equations of this type are based on asymptotic freedom and dispersion relations [23,4]. The resonance masses and the position of the continuum threshold are put in by hand, the resonance coupling constants are the desirable output. One of the first dispersion calculations refers to photonic widths of C-even charmonium levels [23]. Now we have other examples, see, e.g. [47]. This work is devoted to raditive transitions $\psi' \to \chi\gamma$ and $\chi \to J/\psi\gamma$. The author starts with three-point functions

$$\int dx dy\ e^{-i(kx+qy)} \langle 0|T\{j_1(0) j_\mu^{em}(x) j_2(y)\}|0\rangle \tag{30}$$

where j_1 and j_2 are external charmed quark currents with appropriate quantum numbers, say, $j_1 = \bar{c}c$ and $j_\mu^{em} = \bar{c}\gamma_\mu c$. In euclidean domain there exist two alternative expressions for the matrix element (30) (fig. 6) which results in an overdetermined set of equations. Approximate solutions of these equations are displayed in Table 3.

For comparison this table contains also the numbers obtained with two different potentials. The difference between the predictions is especially large in the transitions $\psi' \to \chi_0\gamma$ and $\chi_2 \to J/\psi\,\gamma$.

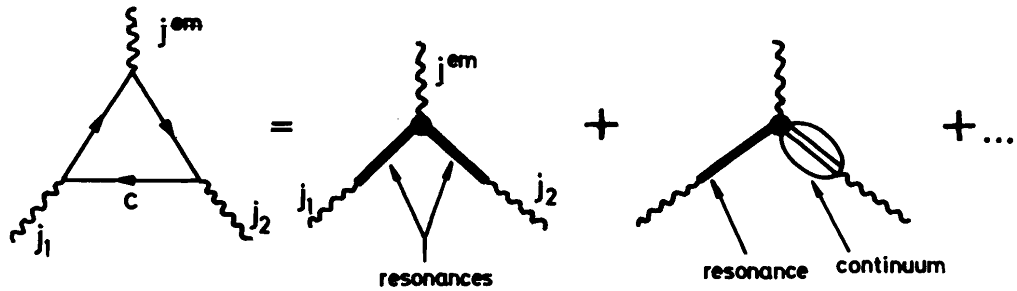

Fig. 6 Graphical representation of QCD duality relations

Decay mode	Γ, keV, [47]	Experim.[40]	Potential Models	
			Eichten et al.[2]	Buchmüller and Tye [2]
$\psi' \to \chi_0\gamma$	8	21 ± 1	50	58
$\psi' \to \chi_1\gamma$	31	18 ± 1	45	49
$\psi' \to \chi_2\gamma$	31	16 ± 1	29	38
$\chi_0 \to J/\psi\gamma$	108	100 ± 30	141	182
$\chi_1 \to J/\psi\gamma$	160	< 420	289	381
$\chi_2 \to J/\psi\gamma$	136	310 ± 110	398	496

Table 3 Radiative decay rates in charmonium

Photon Transitions

These play a distinguished role uncovering a rich world of C-even charmonium levels. As for electric dipole transitions nothing dramatic happened on this theoretical scene over last years. Results obtained in the potential models, dispersion approach and from non-relativistic sum rules [48] (Thomas-Reiche-Kuhn, etc) coexist peacefully awaiting for future development. Some of them are collected in Table 3, which contains also fresh experimental data from SLAC [40]. Theoretical predictions are off by a factor of 2.

The situation in the Υ family seems to be better. The photonic transitions here were observed in an indirect way [40]. The CUSB collaboration measured $BR(\Upsilon'' \to X \to 2 \text{ jets})$ with jets coming essentially from $^3P_0$ and $^3P_2$. They obtained the following numbers:

$$BR(\Upsilon'' \to \gamma\,^3P) \simeq 30 \pm 5\,\%, \quad BR(\Upsilon' \to \gamma\,^3P) = 12 \pm 3\,\% . \tag{31}$$

This is in agreement with the potential model calculations [2] which imply, for instance, 29 ± 6 % for the first branching ratio.

Unfortunately, this indirect measurement says nothing about the position of **the P** levels.

A few words about M1 transitions. Allowed decays like $J/\psi \to \eta_c\gamma$ or $\Upsilon \to \eta_b\gamma$ should be described to a very good approximation by the simplest formula

$$\Gamma(1^3S_1 \to 1^1S_0 + \gamma) = \frac{16}{3}\mu^2\omega^3 \tag{32}$$

where μ is the Dirac magnetic moment, μ = (quark charge)$\times\sqrt{\alpha}/2m$. Sometimes an ad-hoc assumption is made according to which c and b quarks may possess a large anomalous magnetic moment which, of course, invalidates eq. (32). This assumption is wrong. Not only eq. (32) can be derived in a controllable way, corrections to it are calculable and small. Actually, one can show that [49,50]

$$\Gamma(J/\psi \to \eta_c\gamma) = \frac{2}{9}\,\frac{\Gamma(\eta_c \to 2\gamma)}{\Gamma(J/\psi \to e^+e^-)}\,\alpha\,\frac{M_\psi^4}{M_{\eta_c}^3}\left(1 - \frac{M_{\eta_c}^2}{M_\psi^2}\right)^3(1 - 0.28\alpha_s) \tag{33}$$

where

$$\Gamma(\eta_c \to 2\gamma)/\Gamma(J/\psi \to e^+e^-) = 3(\text{quark charge})^2(1 + O(\alpha_s, \mu/m)),$$

$$\mu \equiv \mu_{\text{non-pert.}},$$

and similar relations hold for Υ. It is easy to understand why there are no large corrections to the magnetic moment in the transitions like $J/\psi \to \eta_c\gamma$. Consider the amplitude $\eta_c \to 2\gamma$ and represent it in the form of the dispersion integral in one of the photons. The dominant contribution to the dispersion integral comes from J/ψ. Other states are separated by a large gap, $\delta^2 \sim 2M\Delta M$ (where $\Delta M \sim M_{\psi'} - M_\psi$) and their contribution is of order $O(\alpha_s(\delta^2))$. In this way we arrive at eq. (33) with corrections being essentially determined by short distances [49,50].

Naively one might expect that $\Gamma(\eta_c \to 2\gamma)/\Gamma(J/\psi \to e^+e^-)$ is close to 4/3, and, then, $(J/\psi \to \eta_c\gamma) \simeq 2.5$ keV. Perturbative effects tend to increase the ratio of Γ's giving $\sim(4/3)1.13$ instead of 4/3; see footnote 2. On the other hand, according to the preliminary data of the Crystal Ball Collaboration,

$$\Gamma(J/\psi \to \eta_c\gamma)_{\text{exp}} = (0.7 \pm 0.2)\,\text{keV}.$$

Something is going wrong with this decay. In order to reproduce 0.7 keV theoretically it is necessary to assume that the ratio $\Gamma(\eta_c \to 2\gamma)/\Gamma(J/\psi \to e^+e^-)$ is suppressed by a factor of 3.6 (~ 0.37 instead of 1.33). It is impossible to ensure such a suppression without a deep revision of our basic notions. For theorists it would be much better if $\Gamma(J/\psi \to \eta_c\gamma)_{\text{exp}}$ would grow by 2-3 times, but I am afraid that the SLAC people do not share my hope. Notice, that with $\Gamma(\eta_c \to 2\gamma) \simeq \frac{1}{3}\Gamma(J/\psi \to e^+e^-) \simeq 1.6$ keV we get a lot of troubles in other places [4], in particular, enormous violations of the Appelguist-Politzer recipe, which can not be understood in any way.

In the $b\bar{b}$ family the small mass difference $M_\Upsilon - M_{\eta_b}$ hampers searches for the decay. The numbers look pessimistic, indeed

$$\Gamma(\Upsilon \to \eta_b \gamma) \sim 2\text{ eV} \quad \text{and} \quad BR(\Upsilon \to \eta_b \gamma) \sim 10^{-4} .$$

Unfortunately, it is not easier to reach η_b starting from Υ'.

Decays like $\Upsilon' \to \eta_b\gamma$ or $\psi' \to \eta_c\gamma$ are forbidden in the non-relativistic limit. The ψ' decay is seen experimentally [44] with the rate

$$\Gamma(\psi' \to \eta_c\gamma)_{exp} = 0.6 \pm 0.2 \text{ keV}. \tag{35}$$

Thus, deviations from the non-relativistic approximation should be essential. What does the theory say on that?

Recently it was argued [50] that the $\psi' \to \eta_c\gamma$ transition is basically due to the gluon admixture in the ψ' wave function. The argumentation is as follows. If the local duality is valid one can substitute the amplitude

$$A(\gamma \to \psi')A(\psi' \to \eta_c\gamma) ,$$

in the dual sense, by

$$A(\gamma \to \text{quarks, gluons})A(\text{quarks, gluons} \to \eta_c\gamma) . \tag{36}$$

The latter product is easily calculable, of course, which, in turn, fixes $A(\psi'\to\eta_c\gamma)$. The theoretical result is compatible with (35). What is more important, numerically dominant contribution to (36) comes from the intermediate state $c\bar{c}g$. It is natural to interpret this fact as the gluon admixture in ψ'. Simultaneously, $\Gamma(\psi'' \to \eta_c\gamma)$ is predicted to be large,

$$\Gamma(\psi'' \to \eta_c\gamma) \sim 1 \text{ keV} , \tag{37}$$

much larger than in standard potential models where $\psi'' \to \eta_c\gamma$ decay is strongly forbidden. Unfortunately, it is not easy to check the latter estimate experimentally because the corresponding branching ratio does not exceed $5\cdot10^{-5}$.

For the Υ' the decay rate is suppressed as compared to $\psi' \to \eta_c\gamma$ by at least the following factor:

$$\frac{1}{4}\left(\frac{M_\psi}{M_\Upsilon}\right)^2 \left(\frac{\alpha_s(\Upsilon)}{\alpha_s(\psi)}\right)^2 \cdot 0.8 \sim 1/100 . \tag{38}$$

Here the ratio of the coupling constants characterizes the gluon admixture and 0.8 reflects the phase space. Combining with eq. (35) we get

$$\Gamma(\Upsilon' \to \eta_b\gamma)_{theor} \lesssim 8 \text{ eV} . \tag{39}$$

The corresponding branching ratio is expected to be smaller than 5.10^{-4}.

III. HADRONIC DECAYS

Heavy Quarkonia and Old World

The issues discussed up to now refer mostly to heavy quarks and their relations with the surrounding vacuum medium. We proceed now to another fundamental aspect - coupling of heavy quarkonium to old hadrons. Theoretical and experimental investigations in this field give information on the structure of $Q\bar{Q}$ systems, glueballs and traditional old hadrons. In many cases the information is unique since it is impossible to get it in other ways.

Inclusive Hadronic Decays

The famous Appelquist-Politzer recipe [1] prescribes to calculate elementary lowest-order processes

$$Q\bar{Q} \to 2q,\ 3q \quad \text{or} \quad q\bar{q}g$$

instead of summing over a large number of exclusive channels. This brilliant invention is applicable, beyond any doubt, to asymptotically heavy $Q\bar{Q}$ states. We are interested, however, in charmonium and bottonium, and here various preasymptotic (non-perturbative) corrections may be important.

The Appelquist-Politzer recipe assumes an ideal gluon-hadron duality. For light quarks $9\ \mathrm{GeV}^2$ ($\simeq M^2_{\eta_c}$) is indeed an asymptotic domain where the hadron cross section coincides with the quark one. Is this valid for gluons as well?

The onset of asymptotic behaviour is determined by non-perturbative effects - one can hardly doubt this fact today. These are drastically different in quark and gluon channels. Gluonic currents are coupled to vacuum fields much stronger than quark ones (fig. 7, for details [51]), and, as a result, the asymptotic regime sets in for gluons at larger energies.

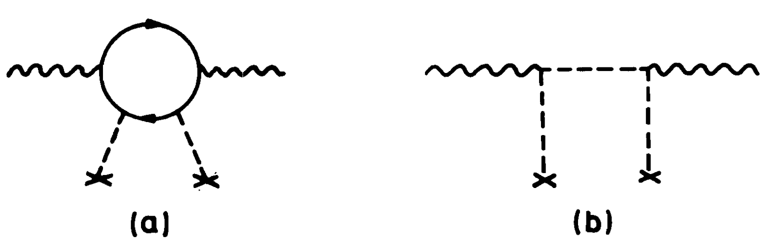

Fig. 7 $\langle G^2 \rangle$ effects in correlation functions of quark and gluon currents. In the gluon case (b) we deal with the Born graphs. In the quark case (a) diagrams necessarily contain loops. Each extra loop gives a suppression factor of order $1/16\pi^2$

It is not a simple task to find quantitatively what this energy really is. Still, some estimates exist in the literature. In [51] it was shown that

$$(s_0)_{\text{two gluons, } J^P = 0^-} \simeq 6\text{-}16 \text{ GeV}^2 \tag{40}$$

where s_0 denotes the boundary of asymptotic domain. If so, the charmonium family is in dangerous vicinity from the critical zone or, may be, even inside it. Therefore, I would not be surprised by some (moderate) deviations from perturbative formulas for $c\bar{c}$ annihilation. On the other hand, the $b\bar{b}$ annihilation should be described to a good accuracy by these formulas.

Of course, it is important to obtain reliable and accurate theoretical answers for $c\bar{c}$ as soon as possible. The best thing we can do at present is to check the Appelquist-Politzer prescription phenomenologically. Unfortunately, the phenomenological situation is rather controversial. On one hand, the recent measurement of $BR(\Upsilon \to \mu^+\mu^-)$ [40] nicely confirms, among other things, this recipe. Indeed, starting from $\Gamma(\Upsilon \to e^+e^-) = 1.7 \pm 0.05$ keV and $BR(\Upsilon \to e^+e^-) = 3.3 \pm 0.5$ % we get $\Gamma_{tot}(\Upsilon) = 35.5 \pm 7$ keV or $\Gamma_{\text{direct hadr}}(\Upsilon) = 27 \pm 6$ keV. Assuming that $\Gamma_{\text{direct hadr}} = \Gamma_{3g}$ we find

$$\frac{\Gamma_{3g}(\Upsilon)}{\Gamma_\mu(\Upsilon)} = \frac{10(\pi^2-9)\alpha_s^3(m_b)}{9\pi\alpha^2}\left[1 + (1.1 \pm 0.5)\frac{\alpha_s}{\pi}\right] = 23 \pm 5 \tag{41}$$

where the $O(\alpha_s)$ correction on the l.h.s. was found recently by Mackenzie and Lepage [52] (in $\overline{MS}$ scheme). It is easy to extract now the quark-gluon coupling constant, $\alpha_s(m_b) = 0.156 \pm 0.013$. Then the standard renormalization group formula yields $\alpha_s(m_c) = 0.210 \pm 0.028$ which implies, in turn,

$$\frac{\Gamma_{3g}(J/\psi)}{\Gamma_\mu(J/\psi)} = 15.5 \pm 6 \quad . \tag{42}$$

The agreement with the experimental number, 9.2 ± 2.4, is quite satisfactory.

On the other hand, due to efforts of the SLAC group (Crystall Ball) we know now the hadronic widths of η_c and χ_0:

$$\Gamma_{hadr}(\eta_c) = 12.4 \pm 4.1 \text{ MeV}, \quad \Gamma_{hadr}(\chi_0) = 16.3 \pm 3.6 \text{ MeV}. \tag{43}$$

Their photonic widths are more or less fixed theoretically [4] (4.5 - 6 and 4.5 - 5.5 keV, respectively), and we can compare the ratios

$$\Gamma_{hadr}(\eta_c)/\Gamma_{\eta_c\to 2\gamma} \quad \text{and} \quad \Gamma_{hadr}(\chi_0)/\Gamma_{\chi_0\to 2\gamma}$$
$$\text{with} \quad \Gamma(c\bar{c} \to 2g)/\Gamma(c\bar{c} \to 2\gamma) \quad . \tag{44}$$

A formidable work has been done in order to account for the first-order perturbative correction in the ratios like (44) [52,53]. In the η_c case, for instance, it was found [53] that

$$\frac{\Gamma(c\bar{c}\to 2g)}{\Gamma(c\bar{c}\to 2\gamma)} = \frac{9}{8}\left(\frac{\alpha_s(m)}{\alpha}\right)^2 \{1 + 9.2\,\frac{\alpha_s(m)}{\pi}\} \tag{45}$$

where m is the quark mass, and the coefficient of α_s actually depends on renormalization procedure. (Eq. (45) refers to the so-called $\overline{MS}$ scheme.) This coefficient can be minimized by a proper choice of normalization point [54].

With $\alpha_s(m_c) = 0.2$ the $O(\alpha_s)$ correction amounts to 50 % - essential, but not dangerous, in the sense that the perturbative series seems to be under control and does not blow up. Substituting the two-photon widths we get $\Gamma_{hadr}(\eta_c) \approx$ 6-8 MeV, and the analogous result for $\Gamma_{hadr}(\chi_0)$ is 6.2-7.6 MeV. The experimental data quoted in (43) seem to exceed systematically, by a factor of 2, these numbers. Notice that if $\Gamma(\eta_c\to 2\gamma) \approx 1.7$ keV as suggested by $\Gamma(J/\psi\to\eta_c\gamma)_{exp}$ (see above) then the discrepancy amounts to a factor of 4 in the η_c case - the possibility which is difficult to imagine. Notice that in the tensor channel (i.e. for χ_2) the Appelquist-Politzer recipe works perfectly and yields 1.7-2.3 MeV for $\Gamma_{hadr}(\chi_2)$ while the experimental width is 1.8 ± 0.6 MeV. There emerges an impression that the anomaly takes place only for spin zero. The possibility of such a situation was predicted theoretically [51].

Returning to technical aspects let me demonstrate a few useful expressions [52,53] whose derivation required a lot of computational work:

$$\frac{B(0^{-+})}{B(0^{++})} = \begin{cases} 1+0.9\,\frac{\alpha_s}{\pi}\ (c\bar{c}) \\ 1+2.1\,\frac{\alpha_s}{\pi}\ (b\bar{b}) \end{cases} ; \quad \frac{B(2^{++})}{B(0^{++})} = \begin{cases} 1+6.5\,\frac{\alpha_s}{\pi}\ (c\bar{c}) \\ 1+4.0\,\frac{\alpha_s}{\pi}\ (b\bar{b}) \end{cases} \tag{46}$$

where $B(J^{PC}) = \Gamma(J^{PC}\to 2\gamma)/\Gamma(J^{PC}\to \text{gluons})$. These ratios are convenient because they are independent of the renormalization scheme. Of much practical importance is the recent result of Mackenzie and Lepage [52]:

$$\Gamma(Q\bar{Q},\ 1^{-}\to 3g) = \Gamma_0 \begin{cases} 1-(3.8\pm 0.5)\,\frac{\alpha_s}{\pi}\ (c\bar{c}) \\ 1-(4.2\pm 0.5)\,\frac{\alpha_s}{\pi}\ (b\bar{b}) \end{cases} \tag{47}$$

Other details as well as numerical examples can be found in the paper by Barbieri et al. [31], which gives a nice review of the whole subject.

Exclusive Hadronic Decays

Here we are witnessing a considerable progress in theoretical understanding. First, it was demonstrated [55,56] that the well developed form-factor technique [57] is applicable to exclusive decays like $\chi_0\to\pi^+\pi^-$, $J/\psi\to\rho\pi$, ... in the limit of $m\to\infty$ (see fig. 8). This fact alone implies a lot of useful "selection rules": some of the decays turn out to be suppressed as compared to others [58] for instance

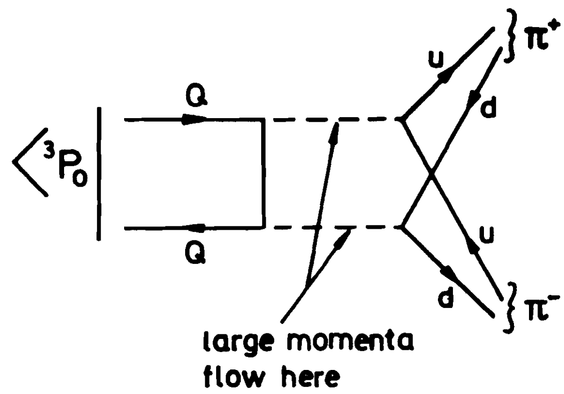

Fig. 8 Typical diagram describing exclusive annihilation of heavy quarkonium. Each gluon line carries momentum of order m. Initial and final quark legs are soft and should be substituted by corresponding wave functions.

$\bar{Q}Q$ level	Suppressed mode	Non-suppressed mode
$^1S_0$	$\rho\rho$	ρB
$^3S_1$	$\rho\pi$	πB
$^3P_0$	ρB	$\pi\pi$
$^3P_1$	$\rho\rho$	ρB
$^3P_2$	πA_2	$\pi\pi$

A typical prediction for a two-particle mode looks like [58]

$$BR(^3P_0 \to \pi^+\pi^-) = |\pi\tilde{\alpha}_s \frac{16\sqrt{2}}{27} \frac{f_\pi^2}{m^2} I_0|^2 \tag{48}$$

where $\tilde{\alpha}_s$ is an effective coupling constant and I_0 is an integral over the pion wave function $\varphi_\pi(\xi)$,

$$I_0 = \int_{-1}^{1} \frac{d\xi_1 \varphi_\pi(\xi_1)}{1-\xi_1^2} \int_{-1}^{1} \frac{d\xi_2 \varphi_\pi(\xi_2)}{1-\xi_2^2} \frac{1}{1-\xi_1\xi_2}\left[1 + \frac{1}{4}\frac{(\xi_1-\xi_2)}{1-\xi_1\xi_2}\right] .$$

Previous attempts to guess the wave function or substitute it by its asymptotic form invariably led to extremely small branching ratios - of order $10^{-2} - 10^{-3}$ % in charmonium - in sharp disagreement with experiment. Chernyak and Zhytnitsky extracted $\varphi_\pi(\xi)$ from QCD sum rules [59] similar to those I have discussed above. The following function saturates the sum rules;

$$\varphi_\pi(\xi) \simeq \frac{15}{4}\,\xi^2(1-\xi^2)\ . \tag{49}$$

(The result refers to a rather low normalization point figuring in charmonium decays.) The curve for $\varphi_\pi(\xi)$ is presented in fig. 9; in comparison with traditional models it has an unexpected shape. With this rather broad distribution they obtain, for instance, [58]

$$BR(\chi_0 \to \pi^+\pi^-) \simeq 1.1\ \%;\quad BR(\chi_2 \to \pi^+\pi^-) \simeq 0.24\ \%\ .$$

(The corresponding experimental numbers: 1 ± 0.3 % and 0.15 ± 0.07 %, respectively.) Let me also quote a typical prediction for $b\bar{b}$:

$$\Gamma({}^3P_0(b\bar{b}) \to \pi^+\pi^-)/\Gamma({}^3P_0 \to \text{hadr}) \simeq 1.8\cdot10^{-3}\ \%\ .$$

This result merely shows that it will be almost impossible to observe exclusive hadronic modes in the Υ family.

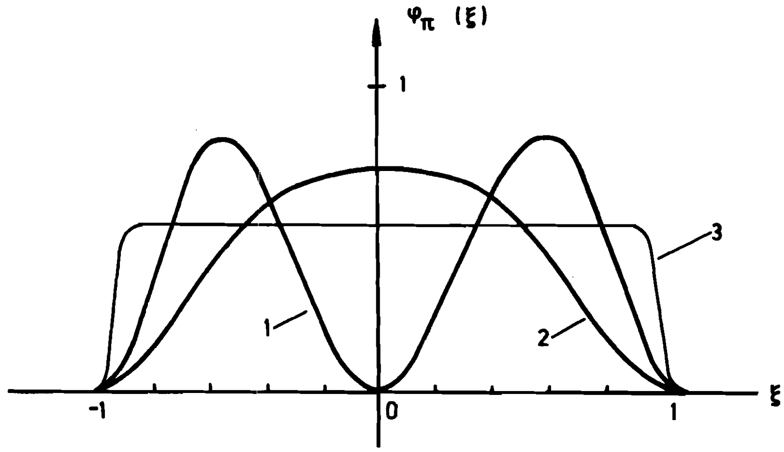

Fig. 9 Pion wave function (in the infinite momentum frame) versus ξ.
1 - CZ wave function extracted from QCD sum rules (normalization point $\mu \simeq 0.5$ GeV);
2 - asymptotic form of $\varphi_\pi(\xi)$ $(\mu \to \infty)$: $\frac{3}{4}(1-\xi^2)$;
3 - a popular model - flat distribution

Hadronic Transitions Between Quarkonium Levels

Decays like

$$\psi' \to J/\psi\pi\pi \quad \text{or} \quad \Upsilon'' \to \eta\Upsilon \tag{50}$$

and others of that type probe the gluonic content of ordinary hadrons, Gottfried was the first who emphasized [60] that transitions (50) can be viewed as a two-step process: first, emission of soft gluons by heavy quarks at relatively short distances, and then conversion of the gluons into light hadrons at relatively large distances. So long as quarkonium size is small as compared to that of "old" hadrons, one can consistently use the well-known multipole expansion to describe the gluon emission [60-63].

Factorization alone (plus symmetry properties of the transition amplitudes) yields a lot of predictions for relative rates, fcr instance [62]

$$\begin{aligned} d\Gamma(2^3S_1 \to 1^3S_1+2\pi) &= d\Gamma(2^1S_o \to 1^1S_o+2\pi), \\ d\Gamma(1^3D_3 \to 1^3S_1+2\pi) &= d\Gamma(1^1D_2 \to 1^1S_o+2\pi), \ldots \end{aligned} \tag{51}$$

More intriguing is a unique possibility of testing QCD low-energy theorems. Within the framework of the multipole expansion the following decomposition is valid [64]

$$\begin{aligned} A(n_i{}^3S_1 \to n_f{}^3S_1+\pi\pi) &= C_1\langle 0|\vec{E}^a\vec{E}^a|\pi\pi\rangle + \text{higher multipoles}, \\ A(n_i{}^3S_1 \to n_f{}^3S_1+\eta) &= C_2\langle 0|\vec{E}^a\times\vec{H}^a|\eta\rangle + \text{higher multipoles}, \end{aligned}$$

where $\vec{E}^a$ and $\vec{H}^a$ denote chromoelectric and chromomagnetic fields, and the coefficient functions C_1, C_2 code information on heavy quarkonium. These coefficients are proportional to each other and cancel in the ratio of the amplitudes; moreover, in particular quarkonium models they can be found explicitly [65].

At first sight it seems impossible to calculate such non-trivial matrix elements as

$$\langle 0|\vec{E}^a\vec{E}^a|\pi\pi\rangle \ , \quad \langle 0|\vec{E}^a\times\vec{H}^a|\eta\rangle \tag{52}$$

which reflect conversion of gluons to mesons at large distances. Surprisingly, we can do that, starting from first principles only. These matrix elements are related to the so-called triangle anomalies in the trace of the energy-momentum tensor and in the divergence of the axial-vector current. The answers are so attractive that I cannot help illustrating them by a few examples. Say, $\langle 0|\vec{E}^a\vec{E}^a|\pi\pi\rangle$ reduces to a combination of the following quantities [64,66]: $m^2_{\pi\pi}$, b, $\rho^G(\mu)$, $\alpha_s(\mu)$ (b is the first coefficient in the Gell-Mann-Low function, ρ^G is the gluon share of the pion momentum; analogous quantity for nucleon is measured in deep inelastic scattering; μ is a normalization point of order of the inverse quarkonium radius).

The ratio $\Gamma(\psi' \to J/\psi\pi\pi)/\Gamma(\psi' \to J/\psi\eta)$ was found in [64] in perfect agreement with experiment. For bottonium these authors predict

$$\frac{\Gamma(\Upsilon' \to \Upsilon\eta)}{\Gamma(\Upsilon' \to \Upsilon\pi\pi)} \simeq 4\cdot 10^{-3}\left(\frac{M_{\Upsilon'}-M_{\Upsilon}-M_{\eta}}{10\ \text{MeV}}\right)^{3/2}, \quad \frac{\Gamma(\Upsilon'' \to \Upsilon\eta)}{\Gamma(\Upsilon'' \to \Upsilon\pi\pi)} \simeq 2\cdot 10^{-2} \ . \tag{53}$$

The shape of the pion spectrum is also well understood [66]. Namely,

$$\frac{d\Gamma}{dq^2} \sim \left\{\left[q^2 - \kappa(\Delta M)^2\left(1+\frac{2m_\pi^2}{q^2}\right)\right]^2 + \frac{\kappa^2}{5}[(\Delta M)^2-q^2]^2\times\left(1-\frac{4m_\pi^2}{q^2}\right)^2\right\} \tag{54}$$

where

$$q^2 \equiv m_{\pi\pi}^2 \ , \quad \Delta M = M(Q\bar{Q}_i) - M(Q\bar{Q}_f)$$

and

$$\kappa = \frac{b}{6\pi}\alpha_s(\mu)\rho^G(\mu) \simeq 0.2 \text{ for charmonium.}$$

The second term $\sim\kappa^2/5$ in eq. (54) is due to the D-wave contribution, and thus, suppression of the D wave ($\kappa^2/5 \sim 1/125$) is explained theoretically. Notice that the value of κ is non-universal: in $\Upsilon' \to \Upsilon\pi\pi$ it is expected to be smaller by a factor of 2.

For very heavy quarks forming Coulombic levels the quarkonium coefficients C_1, C_2 are calculable, and, hence, the absolute rates are fixed unambiguously. With real c and b quarks, however, which are not heavy enough, one has to invoke models. One of them is developed in ref. 65. Perhaps, the most interesting finding here is an unexpected suppression of the $\Upsilon'' \to \Upsilon\pi\pi$ transition due to unfortunate cancellations in C_1. The authors argue that $\Gamma(\Upsilon'' \to \Upsilon\pi\pi)$ should be smaller than $\Gamma(\Upsilon' \to \Upsilon\pi\pi)$, while naive estimates [29] imply that $\Gamma(\Upsilon'' \to \Upsilon\pi\pi)/\Gamma(\Upsilon' \to \Upsilon\pi\pi) = 5 - 20$.

Meanwhile the $\Upsilon'' \to \Upsilon\pi\pi$ decay has been observed [44] with the branching ratio $\simeq$ several percent ($\Gamma(\Upsilon'' \to \Upsilon\pi\pi)$ of order of 1 keV; compare with [40,67] $\Gamma(\Upsilon' \to \Upsilon\pi\pi)_{exp} = 7.8 \pm 1.5$ keV). A strong suppression is quite evident, and this shows that the model of [65] is not bad, at least at the qualitative level.

The most important lesson supported by data is the applicability of the multipole expansion in bottonium and even charmonium families. If the audience is not entirely convinced by the facts already presented let me add a few words about $\Upsilon' \to \Upsilon\pi\pi$. The multipole expansion implies that

$$C_1 \sim \langle n_f\,{}^3S_1|(t_1^a - t_2^a)r_i G_{(8)}(\varepsilon_n) r_i (t_1^a - t_2^a)|n_i\,{}^3S_1\rangle$$

where $G_{(8)}$ is the non-relativistic Green function in color-octet state. In other words, one may expect that

$$\frac{\Gamma(\Upsilon' \to \Upsilon\pi\pi)}{\Gamma(\psi' \to J/\psi\pi\pi)} \simeq \left\{\frac{\langle r^2\rangle_{\Upsilon'}}{\langle r^2\rangle_{\psi'}}\right\}^2 \simeq 1/16 \tag{55}$$

or $\Gamma(\Upsilon' \to \Upsilon\pi\pi) \simeq 6$ keV. This expectation agrees very well with recent data [40] (see also [68]).

Hadronic transitions are promising in one more respect. The cascade

$$\Upsilon'' \to 1\,{}^1P_1 + \pi\pi, \qquad 1\,{}^1P_1 \to 1\,{}^1S_o + \gamma$$

seems to be the best way to discover at once two elusive $b\bar{b}$ levels: ${}^1P_1$ and ${}^1S_o$. The "bottle-neck" of this chain is the first decay whose branching ratio is not large. According to [65] it is about 1 %, but further efforts are needed in order to eliminate theoretical uncertainties inherent to this work. Once ${}^1P_1$ is produced the problem of η_b is solved: almost every second decay of ${}^1P_1$ is

$${}^1P_1 \to {}^1S_o + \gamma \,.$$

One last remark concerns isotopic symmetry violating decays $\psi' \to J/\psi\pi^o$ or $\Upsilon' \to \Upsilon\pi^o$. They measure directly the current quark masses. To be more exact, the following theorem holds [69]

$$\frac{\Gamma(\psi' \to J/\psi\pi^o)}{\Gamma(\psi' \to J/\psi\eta)} = \left(\frac{3\sqrt{3}}{4}\,\frac{m_d - m_u}{m_s}\right)^2 \left(\frac{\vec{p}_\pi}{\vec{p}_\eta}\right)^3 [1 + O(m_q/\mu)] \tag{56}$$

where m_q stands for the u,d,s-quark mass, μ is a characteristic scale of strong interactions (few hundred MeV), and the analogous relation is valid, of course, for Υ. The existing data [70] are compatible with the standard values of m_q: $m_d \simeq 7.5$, $m_u \simeq 4$ and $m_s \simeq 120$ MeV.

$J/\psi \to \gamma$ + Light Hadrons and $\Upsilon \to \gamma$ + Light Hadrons

In gluon physics these processes are analogs of the famous e^+e^- annihilation. In the quark-gluon language we say that

$$Q\bar{Q} \to gg\gamma, \quad gg \to \text{light hadrons} \tag{57}$$

(fig. 10). Regulating the photon energy we simultaneously regulate the invariant mass of the hadronic system,

$$m^2_{\text{light hadrons}} = M^2(1-x)\,, \quad x = 2E_\gamma/M\,,$$

so that, in principle, all the interval from zero up to M^2 is within reach. Practically, experimental difficulties block investigations in the domain $x \lesssim 0.5$.

The lowest-order QCD predicts [71] the inclusive probability

$$\frac{\Gamma_{\gamma\,+\,\text{light hadr.}}}{\Gamma_{3g}} = \frac{36}{5}\,(\text{quark charge})^2\,\frac{\alpha}{\alpha_s(m)} \tag{58}$$

Eq. (58) actually assumes a perfect gluon-hadron duality, which is, evidently, violated at large x (small $m^2_{\text{light hadr.}}$).

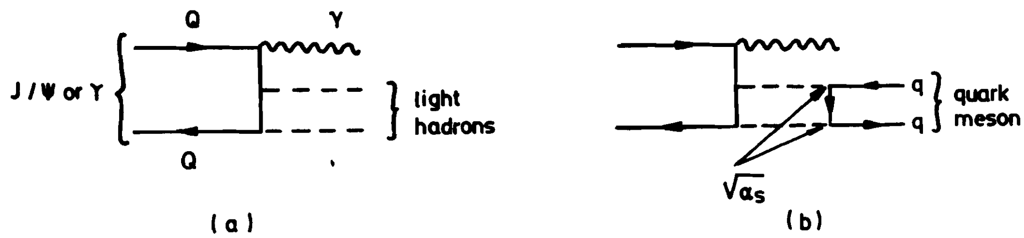

Fig. 10 Radiative annihilation of $J/\psi(T)$, (a) inclusive; (b) exclusive. In the latter case the extra factors $\sqrt{\alpha_s}$ is compensated by a large logarithm ($\sim \ln M^2 R^{-2}_{\text{conf}}$) emerging from the loop integration.

From SLAC data referring to J/ψ [72] we know that the total rate is more or less compatible with eq. (58), although the shape of the photon spectrum at $x > 0.5$ has nothing to do with the perturbative result. The questions number one are: "which states do actually saturate the total probability?" and "at which $m^2_{\text{light hadr}}$ does the parton-like regime set in?" The latter parameter, the boundary of asymptotic domain, is an important dynamical characteristic. For light quarks, as we know from e^+e^- annihilation, it is about $1.5\ \text{GeV}^2$, but there are good reasons to believe that this scale is non-universal. It was argued that the boundary shifts to higher values in the gluonic sector, $s_o \sim 6\ \text{GeV}^2$ (eq. (40) and ref. 51). If so, the genuine glueball continuum can hardly be investigated in the J/ψ radiative decays, and this will become the prerogative of the T physics ($x < 0.94$).

On the other hand, the resonance production is much easier to study starting with J/ψ. It is generally believed that the gluon pair in (57) materializes in the form of a glueball, basically 2^+ glueball. The conclusion seems to be based on the perturbative analysis of ref. 73. I would like to emphasize that in exclusive decays of the type

$$J/\psi \to \gamma + \text{a meson} \tag{59}$$

situation is far from being so simple. Indeed, in the $0^\pm$ channels direct non-perturbative fluctuations effectively mix the quark and gluon degrees of freedom, so that quark-meson production is not suppressed at all (ref. 51).

For 2^+ the non-perturbative mixing is small; however, there is another effect which is often forgotten about. The gluons are emitted in the annihilation process at distances $\sim 1/m$. In other words, the gluon source reduces to $\Theta^G_{\mu\nu}(m)$, where m denotes the normalization point and $\Theta^G_{\mu\nu}$ is the gluon piece of the energy-momentum tensor. On the other hand, the characteristic off-shellness in the meson wave function is of order R^{-1}_{conf} (a few hundred MeV); one should account for evolution from m down to R^{-1}_{conf} . As a result there emerges the standard ogarithmic mixing

$$\Theta^{G}_{\mu\nu}(m) \Rightarrow \Theta^{G}_{\mu\nu}(R^{-1}_{conf}) + \varepsilon\, \Theta^{quark}_{\mu\nu}(R^{-1}_{conf}) \tag{60}$$

the mixing parameter ε being of order unity (fig. 10b). This explains, in particular, why the classical quark meson f is produced in reaction (59) without noticeable suppression.

Still, beyond any doubt, the final hadronic system should be enriched by various unusual states. According to phenomenological analysis of Ishikawa [74] the observed γ spectrum assumes the existence of a broad tensor glueball with mass around 2 GeV (fig. 11). Tensor gluonium with such mass is welcome by the QCD sum rules [51].

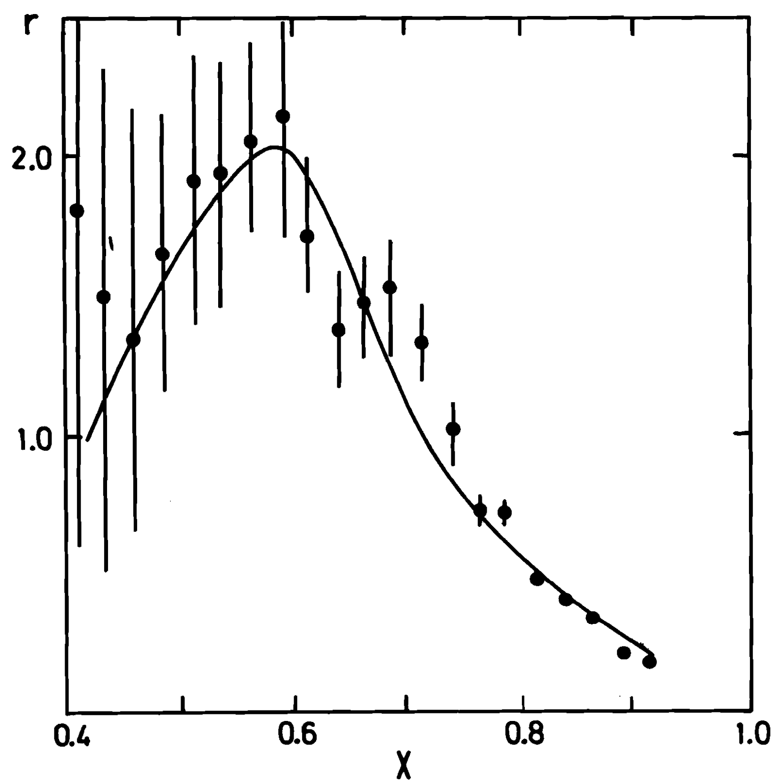

Fig. 11 Normalized photon spectrum in J/ψ radiative annihilation versus x . $r = (d\Gamma/dx)_{exp}/(d\Gamma/dx)_{lowest\text{-}order\ QCD}$. The Breit-Wigner curve represents Ishikawa's fit with $M = 2$ GeV and $\Gamma_{tot} = 0.6$ GeV. Data points are from ref. 72.

The resonance peak in the $K\bar{K}\pi$ mode at ~1.44 GeV [75] also attracted much attention during the last year. The parents of this resonance at SLAC named it ι (iota).

Some facts are already established for certain, for instance, [76]

$$M_\iota = 1440 \pm 10 \text{ MeV} \quad \text{and} \quad \Gamma_\iota = 60 \pm 20 \text{ MeV}, \quad J^P = 0^- .$$

Moreover, iota does not coincide with the so-called E meson, known from strong interactions. However, the main question of its gluonic nature is still open. There are arguments pro and con gluonium interpretation [77.78]. Personally I do not think this is a glueball because I expect gluonium states to be heavier. QCD-based analysis [51] indicates that the pseudoscalar gluonium mass is higher than 2 - 2.5 GeV. If so, the 1.4 peak might be an exotic state, say $q\bar{q}g$ or $q\bar{q}q\bar{q}$. Further analysis is needed in order to prove or reject this conjecture. Rich potential possibilities of the J/ψ radiative annihilation became obvious with the discovery of the next new meson [76], $\Theta(1640)$, which was announced by Dr. Scharre. Presumably, its quantum numbers are 2^+, and this may well be the first glueball we have ever seen. It is obvious that it is not the end of the story, rather, it's the beginning.

Among other decay modes of J/ψ or Υ I would like to mention $\gamma\eta$ and $\gamma\eta'$. The ratio of the corresponding widths reduces to [79]

$$\frac{\Gamma(J/\psi \to \eta'\gamma)}{\Gamma(J/\psi \to \eta\gamma)} = \left|\frac{\langle 0|\alpha_s G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}|\eta'\rangle}{\langle 0|\alpha_s G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}|\eta\rangle}\right|^2 \times (\text{phase-space factors}). \tag{61}$$

While the denominator here is fixed by symmetry properties alone, the numerator codes highly non-trivial information on gluon coupling to η'. Several models suggest their own answers for $\langle 0|\alpha_s G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}|\eta'\rangle$ [80 - 83,54]. The experimental result of Partridge et al. [84]

$$\Gamma_{\eta'\gamma}/\Gamma_{\eta\gamma} = 5.9 \pm 1.5$$

implies that

$$\langle 0|\frac{3\alpha_s}{4\pi} G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}|\eta'\rangle = M^2_{\eta'} \, (120\text{-}160 \text{ MeV}) \tag{62}$$

in agreement with QCD-based estimates [85].

Super-Heavy Quarks

Superheavy quarkonium may represent a fascinating world where the roles of weak, electromagnetic and strong interactions are reversed as compared to those we get used to. Weak-interaction effects negligible in charmonium and bottonium may turn out to be essential or even dominant in $t\bar{t}$. Everything is transparent here from the theoretical point of view - the Coulomb description is valid within limits determined by the theory - but the decay properties are very peculiar, indeed [86].

For moderate masses $m_t \gtrsim 20$ GeV weak decays of quarks forming quarkonium levels show up while the role of three-gluon annihilation goes down. At $m_t = 30$ GeV the corresponding branching ratio does not exceed 30 %. With increasing the quark mass the interplay of various forces becomes even more spectacular. For instance, in the vicinity of Z the leptonic width of a higher excited state which is closer to the Z pole should be larger than that of a lower state [87]. The list of funny things can be easily continued further.

Concluding remarks

After the discovery of J/ψ and the pioneering paper of Appelquist and Politzer heavy quarkonium theory has gone far away. It is a popular business now with its specific problems and methods. In my talk I tried to discuss only a few topics leaving untouched such issues as

(i) exotic states $Q\bar{Q}$ + a gluon excitation;

(ii) broad levels above flavor thresholds;

(iii) static energy from fundamental QCD;

and some others. I am less familiar with them and havn't heard of any decisive progress in these directions.

The progress in this field of high energy physics was surprisingly rapid. The list of major theoretical and experimental findings of the last year includes no less than 10-15 items, and, what is more important, the resources are far from being exhausted. I am sure that all new efforts invested in the field will be repayed.

This talk would probably be impossible without permanent contacts with my colleagues from the theoretical department of ITEP. My point of view on many questions considered here resulted from discussions with them and common work.

References

[1] The pioneering paper on heavy quarkonium is: T. Appelquist and H.D. Politzer, Phys. Rev. Lett. 34, 43 (1975). First detailed calculations in potentials: T. Appelquist et al., Phys. Rev. Lett. 34, 365 (1975); E. Eichten et al., Phys. Rev. Lett. 34, 369 (1975).

[2] K. Gottfried, in Proc. of the Int. Symp. on Lepton and Photon Interactions at High Energies, Hamburg 1977; J.D. Jackson, in Proc. of the 1977 European Conf. on Particle Physics, Budapest, 1977, V. 1, p. 603; T. Appelquist, R.M. Barnett and K.D. Lane, Ann. Rev. Nucl. Part. Sci. 28, 387 (1978); J.D. Jackson, C. Quigg and J.L. Rosner, in Proc. of the 19th International Conf. on High Energy Physics, Tokyo, 1978, p. 824; M. Krammer and H. Krasemann, Acta Physica Austr. Suppl. XXI, p. 259 (1979); C. Quigg and J.L. Rosner, Phys. Rep. 56, 169 (1979); E. Eichten et al., Phys. Rev. D21, 203 (1980); K. Gottfried, Comm. on Part. and Nuclear Sci. (1981); W. Buchmüller and S. Tye, Phys. Rev. D24, 132 (1981); H. Krasemann, preprint CERN TH 3036 (1981).

[3] A. Martin, Talk at the 1981 European Conf. on Particle Physics, Lisbon, July 1981

[4] V.A. Novikov et al., Phys. Rep. 41, 1 (1978).

[5] M.B. Voloshin, preprint ITEP-21 (1980).

[6] M. Shifman, A. Vainshtein, M. Voloshin and V. Zakharov, Phys. Lett. 77B, 80 (1978);

[7] M. Shifman, A. Vainshtein and V. Zakharov, Nucl. Phys. B147, 385, 448 (1979).

[8] L.J. Reinders, H.R. Rubinstein and S. Yazaki, Nucl. Phys. B186, 109 (1981).

[9] J. Bürger, talk at this Conference.

[10] C. Callan, R. Dashen and D. Gross, Phys. Rev. D19, 1826 (1979); E. Shuryak, Phys. Lett. 79B, 135 (1978).

[11] See e.g. D. Beavis et al., Phys. Rev. D20, 743 (1979); H. Krasemann, Phys. Lett. 101B, 259 (1981); J. Arafune and M. Fukugita, Phys. Lett. 102B, 437 (1981); R. McClary and N. Byers, Relativistic effects in quarkonium decays, submitted to this Conference.

[12] R. Crewther, Phys. Rev. Lett. 28, 1421 (1972); M. Chanowitz and J. Ellis, Phys. Lett. 40B, 397 (1972); J. Collins, A. Duncan and S. Joglekar, Phys. Rev. D16, 438 (1977).

[13] A. Vainshtein et al., JETP Lett. 27, 55 (1978)

[14] S. Eidelman et al., Phys. Lett. 82B, 278 (1979).

[15] R.A. Bertlmann, preprint UWThPh-81-16 (1981).

[16] M. Voloshin, preprint ITEP-54 (1979).

[17] H. Leutwyler, Phys. Lett. 98B, 447 (1981).

[18] M. Voloshin, preprint ITEP-30 (1981).

[19] C. Quigg and J.L. Rosner, Phys. Lett. 72B, 462 (1978).

[20] See e.g. S. Pakvasa et al., Phys. Rev. D20, 2862 (1979);
J. Ellis, report CERN 79-01 (1979);
G. Goggi and G. Penso, Nucl. Phys. B165, 429 (1980);
W. Buchmüller and S. Tye, ref. 2
[21] T. Appelquist and H. Georgi, Phys. Rev. D8, 4000 (1973);
A. Zee, Phys. Rev. D8, 4038 (1973).
[22] E. Poggio, H. Quinn and S. Weinberg, Phys. Rev. D13, 1958 (1976).
[23] V. Novikov et al., Phys. Rev. Lett. 38, 626 (1977); Phys. Lett. 67B, 409 (1977).
[24] D.J. Gross and F. Wilczek, Phys. Rev. Lett. 30, 1343 (1973);
H.D. Politzer, Phys. Rev. Lett. 30, 1346 (1973).
[25] M. Voloshin, Yad. Fiz. 29, 1368 (1979).
[26] L. B. Okun and M.B. Voloshin, preprint ITEP-95, Moscow 1976;
A. DeRujula, H. Georgi and S. Glashow, Phys. Rev. D12, 147 (1976);
for a recent discussion see E. Eichten and F. Feinberg, Phys. Rev. D23, 2724 (1981).
[27] M. Voloshin, Nucl. Phys. B154, 365 (1979).
[28] W. Buchmüller, Y.J. Ng, S. Tye, preprint FERMILAB-PUB-81/49-THY (1981)
[29] M. Voloshin and V. Zahkarov, DESY internal report F-15-80/03 (1980).
[30] M. Voloshin, to be published.
[31] One-loop correction to the first term in eq. (23) is calculated by W. Buchmüller et al., ref. 28, and R. Barbieri, R. Gatto and E. Remiddi, to be published.
[32] Analogous assertions are scattered in the literature. See, e.g. C. Callan et al., Phys. Rev. D18, 4684 (1978). In this paper instanton-induced forces are considered.
[33] D. Beavis et al., Phys. Rev. D20, 2345 (1979);
D. Pignon and C.A. Pickety, Phys. Lett. 74B, 108 (1978);
M. Krammer and H. Krasemann, Phys. Lett. 73B, 58 (1978);
A. Billoire and A. Morel, Nucl. Phys. B135, 131 (1978).
[34] A. Khare, preprint M/C TH 81/17, Manchester 1981.
[35] L.J. Reinders, H.R. Rubinstein and S. Yazaki, preprint RL-81-049 (1981).
[36] E. Shuryak, Novosibirsk preprint INP-81-67 (1981).
[37] See M. Gell-Mann, R. Oakes and B. Renner, Phys. Rev. 175, 2195 (1968) and references therein.
[38] E. Eichten et al., ref. 2.
[39] See, e.g. papers No. 33, 58, 197 submitted to this Conference:
(M. Binkley et al., K. Hidaka, E.L. Berger and C. Sorensen).
[40] D. Schamberger, talk at this Conference
[41] D. Bollini et al., preprint CERN-EP/81-58 (1981).
[42] R.L. Jaffe and J. Kiskis, Phys. Rev. D13, 1355 (1976);
J.F. Donoghue and E. Golowich, Phys. Rev. D14, 1386 (1976).
[43] A. Martin, Phys. Lett. 100B, 511 (1981).
[44] Crystal Ball Collab., preliminary data.

[45] A. Bramon, E. Etim and M. Greco, Phys. Lett. 41B, 609 (1972);
J.J. Sakurai, Phys. Lett. 46B, 207 (1973);
For a recent review see R.A. Bertlmann, preprint UW ThPh-38 (1980).
[46] M. Krammer and P. Leal-Perreira, Rev. Bras. Fis. 6, 7 (1976);
C. Quigg and J. Rosner, Phys. Rev. D17, 2364 (1978);
J.S. Bell and Pasupathy, Phys. Lett. 83B, 389 (1979).
[47] A. Khodjamirian, Erevan preprint EFI-427(34)-80 (1980).
[48] J.D. Jackson, Phys. Rev. Lett. 37, 1107 (1976).
[49] M. Shifman, Z. Phys. 4, 345 (1980);
[50] M. Shifman and M. Vysotsky, Z. Phys. C10, 131 (1981)
[51] V. Novikov et al., Nucl. Phys. B191, 301 (1981)
[52] R. Barbieri et al., Nucl. Phys. B154, 535 (1979);
K. Hagiwara et al., Nucl. Phys. B177, 461 (1981);
P.B. Mackenzie and G.P. Lepage, Cornell preprint CLNS/81-498 (1981).
[53] R. Barbieri, M. Caffo, R. Gatto and E. Remiddi, Phys. Lett. 95B, 93 (1980);
preprint CERN TH-3071 (1981);
[54] G. Parisi and R. Petronzio, Phys. Lett. 94B, 51 (1980).
[55] V.L. Chernyak, in Proc. of the XV School of Physics LINP, V. 1, p. 65, Leningrad 1980.
[56] A. Duncan and A.H. Mueller, Phys. Lett. 90B, 159 (1980).
[57] D.R. Jackson, Ph.D.Thesis, Caltech, Pasadena 1977;
V.L. Chernyak et al., ZhETF Pisma, 26, 760 (1977).
A. Efremov and A. Radyushkin, Dubna preprint E2-11535 (1978);
S.J. Brodsky and G.P. Lepage, Phys. Lett. 87B, 359 (1979);
G.R. Farrar and D.R. Jackson, Phys. Rev. Lett. 43, 246 (1979);
A. Efremov and A. Radyushkin, Teor. i Mat. Fiz. 42, 147 (1980);
I. Aznauryan et al., Phys. Lett. 90B, 151 (1980);
V.L. Chernyak and A.R. Zhitnitsky, Yad. Fiz. 31, 1053 (1980).
A. Duncan and A.H. Mueller, Phys. Rev. D21 , 1636 (1980).
S.J. Brodsky et al., Phys. Lett. 91B, 239 (1980);
for reviews see: V. Chernyak, ref. 55; G.P. Lepage and S.J. Brodsky, preprint SLAC-PUB-2478 (1980); M. Shifman and M. Vysotsky, Nucl. Phys. B186, 475 (1981)
[58] V.L. Chernyak and A.R. Zhitnitsky, Novosibirsk preprint INP 81-75 (1981).
[59] V.L. Chernyak and A.R. Zhitnitsky, Novosibirsk preprint INP 81-74 (1981).
[60] K. Gottfried, Phys. Rev. Lett. 40, 538 (1978).
[61] M. Voloshin, Nucl. Phys. B154, 365 (1979);
M. Peskin, Nucl. Phys. B156, 365 (1979);
G. Bhanot and M. Peskin, Nucl. Phys. B156, 391 (1979);
[62] T.M. Yan, Phys. Rev. D22, 1652 (1980).
[63] K. Shizuya, Phys. Rev. D23, 1180 (1981).
[64] M. Voloshin and V.I. Zakharov, Phys. Rev. Lett. 45, 688 (1980).
[65] Y.-P. Kuang and T.-M. Yan, preprint CLNS 81/495 (1981).
[66] V. Novikov and M. Shifman, Z. Phys. C8, 43 (1981).
[67] T. Böhringer et al., paper No. 47 submitted to this conference.
[68] G. Mageras et al., Phys. Rev. Lett. 46, 1115 (1981);

J.J. Mueller et al., Phys. Rev. Lett. 46, 1181 (1981);
B. Niczyporuk et al., Phys. Lett. 100B, 95 (1981).
[69] B.L. Ioffe and M.A. Shifman, Phys. Lett. 95B, 99 (1980)
[70] T.M. Himmel et al., Phys. Rev. Lett. 44, 921 (1980).
[71] M. Chanowitz, Phys. Rev. D12, 918 (1975).
L.B. Okun and M.B. Voloshin, preprint ITEP-95, Moscow 1976.
[72] G.S. Abrams et al., Phys. Rev. Lett. 44, 114 (1980).
M.T. Ronan et al., Phys. Rev. Lett. 44, 367 (1980).
D.L. Scharre et al., preprint SLAC-PUB-2513 (1980).
[73] A. Billoire et al., Phys. Lett. 80B, 381 (1979).
[74] K. Ishikawa, paper No. 79 submitted to this Conference.
[75] D.L. Scharre et al., preprints SLAC-PUB-2514 (1980), SLAC-PUB-2538 (1980).
[76] D.L. Scharre, Talk at this Conference.
[77] K. Ishikawa, paper No. 80 submitted to this Conference;
M. Chanowitz, Phys. Rev. Lett. 46, 981 (1981).
[78] Y.M. Cho, J.L. Cortes and X.Y. Pham, paper No. 131 submitted to this Conf.
[79] V. Novikov et al., Nucl. Phys. B165, 55 (1980).
[80] E. Witten, Nucl. Phys. B156, 269 (1979);
G. Veneziano, Nucl. Phys. B159, 213 (1979);
D. Dyakonov and M. Eides, Leningrad preprint LINP-639 (1981).
[81] V. Novikov et al., Phys. Lett. 86B, 347 (1979).
[82] H. Goldberg, Phys. Rev. Lett. 44, 363 (1980).
[83] K.A. Milton et al., Phys. Rev. D22, 1647 (1980).
[84] R. Partridge et al., Phys. Rev. Lett. 44, 712 (1980).
[85] For a brief review see ref. 54.
[86] Theoretical estimates are scattered in numerous publications.
There exists a recent review: J.P. Leveille, University of Michigan preprint UM HE 81-11 (1981).
[87] W. Buchmüller and S. Tye, ref. 2.

Footnote:

1) A close number for f_D was also obtained by V.S. Mathur and M.T. Yamawaki (to be published).

2) However, non-perturbative effects work in the opposite direction. The minimal value which seems to be admitted by the modern theory is $\Gamma(\eta_c \to 2\gamma)/\Gamma(J/\psi \to e^+e^-) = 0.9$. Then $\Gamma(\eta_c \to 2\gamma) = 4$ keV and $\Gamma(J/\psi \to \eta_c\gamma) \simeq 1.7$ keV.

Discussion

A. Ali, DESY: I have just a remark to make about the value of Λ from jet distributions in e^+e^-. We have recently checked the calculation of Ellis, Ross and Terrano of the $O(\alpha_s^2)$ jet distributions and compared it with the high energy jet data. We get a value

$$\alpha_s = 0.125 \pm 0.01$$

for $\sqrt{s} = 35$ GeV, corresponding to a value $\Lambda_{\overline{MS}} = 110^{+70}_{-50}$ MeV.

P. Hasenfratz, Budapest: I have two short questions. The first is: What is the definition of the GG-expectation value in the light of a divergent perturbation series (this series is at best asymptotic)? The second question concerns the Λ parameter. You said that it was predicted by the ITEP group three years ago that it is 100 MeV and it compares favourably to the $\Lambda_{\overline{MS}}$. The question is: Did you use the same $\overline{MS}$-scheme so that this comparison makes sense?

M.A. Shifman: As to your first question: It is a rather difficult one and I am afraid that I will have no time to answer it in full. There is a problem of subtraction of perturbative series. This quantity, the vacuum expectation value of G^2 normally diverges in perturbation theory, so one should subtract this divergent contribution in order to keep only the nonperturbative terms, which are convergent. In principle it is possible to invent a well-defined procedure involving a few Pauli-Villars regulator fields, which makes the subtraction automatically. But, of course, one would like to have a more physical procedure. However, when you consider the sum rules, there are no questions because the whole computation is very definite and in each step you understand quite clearly which perturbative contribution is subtracted and which is included. Practically everything is determined quite certainly, and there is no place for ambiguities. As to your second question, about the value of Λ: In our analysis, which was performed three years ago, we actually used another definition of Λ, namely the so-called $\Lambda_{e^+e^-}$, which is defined in such a way as to make the $O(\alpha_s^2)$ term in R vanishing,

$$R = 3(\text{quark charge})^2(1 + \frac{\alpha_s}{\pi} + 0).$$

This definition of Λ somewhat differs from $\Lambda_{\overline{MS}}$, but the difference is not very important numerically, it is $\Lambda_{e^+e^-} = 1.2\ \Lambda_{\overline{MS}}$. So our prediction refers to this Λ, but it almost coincides with the more conventional $\Lambda_{\overline{MS}}$.

H. Terazawa, INS, Tokyo: Is our approach effective for the di-quark condensation in baryon physics?

M.A. Shifman: It is impossible to incorporate such a notion as the "di-quark" condensation in the fundamental chromodynamics. If it actually occurs within baryons it should appear as a result of dynamical calculation. I am aware of no such calculation. However, the QCD sum rules were successfully used in baryon spectroscopy. At least three works (C. Chung et al., B.L. Ioffe, E. Shuryak) give masses of N, Δ and so on in reasonable agreement with experiment. They use only fundamental parameters, such as $\langle\bar{\Psi}\Psi\rangle$.