Theoretical Developments II

H. Abarbanel, Fermilab
Session Organizer

REVIEW OF RECENT RESULTS ON QCD AND CONFINEMENT

S. Mandelstam

Department of Physics
University of California
Berkeley, California

General Features of the Confinement Problem

I should like to begin my talk by discussing some general features of the confinement problem, following which I shall outline a selection of contributions made during the past year.

Nature of the Vacuum in the Confined Phase

Most if not all workers on the subject are now in agreement as to the general properties of the vacuum in a confined system. At the present time, various mechanisms have been suggested whereby the vacuum actually acquires such properties but I believe it is fair to say that none of these mechanisms has received general recognition as being the correct one.

The confinement vacuum is characterized by the inability of color electric flux to lose energy by spreading out, as was originally suggested by 't Hooft[1] and Kogut and Susskind.[2] If two quarks are separated

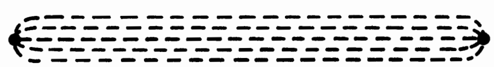

FIG. 1. Tube of flux between two quarks.

by a large distance, they will thus be joined by a tube of flux. Since the flux carries energy, the system will possess an energy proportional to the distance between the quarks, and we have confinement. The tube of flux represents the "string" of the dual model--a model found to be successful qualitatively but not quantitatively, in explaining many features of hadron physics.

The behavior just discussed is precisely the analogue, with electric and magnetic quantities interchanged, of the behavior in an ordinary superconductor or, what is the same thing, in a system with complete Higgs symmetry breaking. (By Higgs breaking we do not necessarily imply that actual Higgs particles are present; we refer to the type of symmetry breaking exemplified thereby.) An ordinary superconductor repels magnetic flux. It is possible to force magnetic flux into a superconductor and, if one does so, it is squeezed into quantized vortices containing $\frac{2\pi n}{g}$ units, n being an integer. (For a non-Abelian theory, n is only defined modulo N.) In fact, magnetic vortices in the ordinary superconducting system were studied first, by Nielsen and Olesen[3] and Nambu.[4] The extension to electric vortices was made later.

Within the magnetic vortex, the vacuum is in a normal, not a superconducting phase. Similarly, the vacuum within an electric vortex is in the normal, not the confined phase. The situation represented by Fig. 1 is for well-separated quarks; it describes resonances of high angular-momentum, where the centrifugal force keeps the quarks apart. For light hadrons the length of the vortex may be comparable to its width. In any case, we have a region of normal phase containing the quarks and the electric flux associated with them, surrounded by a superconducting region into which the flux cannot penetrate. This is the structure described phenomenologically by the M.I.T. or S.L.A.C. bag, and in the related, somewhat less phenomenological model, of Lee and Friedberg.

An ordinary superconductor may be regarded as a coherent superposition of charged objects (Cooper pairs or Higgs particles). Mandelstam[5] and 't Hooft[6] proposed that the confined phase might be realized as a coherent superposition of magnetic monopoles. (A non-Abelian theory differs from an Abelian theory in that monopole-like states may be constructed from the fields themselves; they do not have to be introduced explicitly.) I should now like to refer to some more general work by 't Hooft,[7,8] which explores the superconductor-confinement analogy.

A number of years ago, Wilson[9] proposed characterizing the confinement vacuum by the operator

$$W = \frac{1}{N}\,\mathrm{Tr}\!:\ \mathrm{Exp}\ \{ \oint dx^{\mu}\ \tau^{\alpha}\ A^{\alpha}_{\mu}(x) \}: \tag{1}$$

the integration to be taken round a large closed curve. The τ^{α}'s are the SU(N) generalizations of the Pauli matrices. The symbol : : indicates that the exponential is to be expanded and the τ's ordered along the path. The operator (1) simply creates an electric flux tube, of strength equal to the color charge on one quark, along the path of integration. We have noted that such tubes exist <u>as physical objects</u> in a confined vacuum. Hence, if the tube were infinitely long in a given direction, one could characterize the confinement vacuum as being an eigenstate of the number of such tubes. (The number is, in fact, only defined modulo N.) The matrix element $\langle 0|W|0\rangle$ would therefore vanish in a confinement vacuum. If the tube, instead of being infinite, were along a large loop, we would expect the matrix element to be small. Closer examination shows that "small" means Exp. {-const. A}, "not small" means Exp. {-const. P}, where A and P refer to the area and perimeter of the loop. Thus

$$\langle 0|W|0\rangle = \mathrm{Exp}\ \{-\mathrm{const.}\ A\}, \qquad \text{Confinement}, \tag{2a}$$

$$\langle 0|W|0\rangle = \mathrm{Exp}\ \{-\mathrm{const.}\ P\}, \qquad \text{No Confinement}. \tag{2b}$$

't Hooft proposed constructing a similar operator (M) for creating a tube of magnetic flux. The analogue of (2) would be

$$\langle 0|M|0\rangle = \mathrm{Exp}\ \{-\mathrm{const.}\ A\} \qquad \text{Complete Higgs breaking} \tag{3a}$$

$$\langle 0|M|0\rangle = \mathrm{Exp}\ \{-\mathrm{const.}\ P\} \qquad \text{No complete Higgs breaking} \tag{3b}$$

As far as we are aware, Eqs. (3) provide the <u>only</u> gauge-invariant definition of complete Higgs symmetry breaking. Physically, a system with complete Higgs breaking is one which can support vortices of magnetic flux.

For an SU(2) gauge theory <u>in a phase without massless particles</u>, 't Hooft showed that the combinations (2a), (3b) or (2b), (3a) were the only ones possible. In other words, we have complete Higgs symmetry breaking or confinement.

If we allow massless particles, two more phases are possible (as far as we know). The four possible phases are thus

i) The "perturbation theory" phase with physical massless gluons.
ii) The "Georgi-Glashow" phase, with partial Higgs symmetry breaking and a remaining U(1) invariant group. The phase contains charges and 't Hooft monopoles as well as photons.
iii) The phase with complete Higgs symmetry breaking which can support vortices of magnetic flux. Such vortices would appear as particles or resonances.
iv) The phase with confinement, which can support vortices of electric flux.

In all this work, we notice a striking duality between electric and magnetic quantities. Phases i) and ii) are symmetrical under electric-magnetic interchange, phases iii) and iv) transform into one another under such interchange. I have studied the question of electric-magnetic duality and have come to the conclusion[10] that there is complete duality as long as one asks kinematic questions, i.e., as long as one does not ask for the precise form of the Hamiltonian. As far as we know, there is no dynamic duality. The elementary particles are electrically, not magnetically changed.

't Hooft also studied the case of (2+1) dimensions in detail. The system is now simpler, since the magnetic vortices are replaced by particles, and the characterization (3) is replaced by the presence or absence of a global conservation law (modulo N) for such particles. Transformation between the Higgs and combined phase corresponds to the particles becoming tachyonic. 't Hooft constructed a simple (2+1)-dimensional model in which these features were realized.

Let us finally emphasize that the above remarks, and much of the work to be described later, refer to a system without actual quarks. In most treatments of confinement, quarks are introduced later, on the assumption that quark couplings are weak. If quark couplings had been strong, the quark model would probably not have worked; a baryon, for example, would have consisted of a large number M of quarks, and M-3 antiquarks. The question of why quark couplings may be regarded as weak brings up to our next general topic.

The 1/N Expansion

't Hooft[11] has pointed out that non-Abelian SU(N) theories simplify if we consider the limit

$$N \to \infty, \; g \to 0, \; g^2 N \text{ fixed.} \qquad (4)$$

In that limit planar diagrams, no matter how complicated, are all equally important, but non-planar diagrams are down by factors of N^{-1}. Furthermore, diagrams containing quark loops are also down.

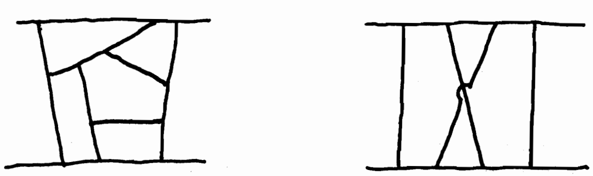

FIG. 2. Planar and non-planar diagrams.

In lowest order, a "string" consisting of a flux line stretched between a quark and an anti-quark would not separate into two strings. Hence if we could understand confinement in the 1/N approximation, we should also understand several predictions of the string model, such as

i) The narrowness of meson resonances,
ii) The existence of exchange-degenerate meson trajectories,
iii) The Iizuka-Okubo-Zweig rule.

In general, we should understand why quark couplings are weak. Lipkin had, in fact, suggested the 1/N approximation as an explanation for weak quark couplings in 1968.

Since nature appears to possess the features mentioned above, we may suppose that the 1/N expansion is reasonably accurate for N = 3.

In two dimensions (1 space + 1 time), one can solve Q.C.D. without making any approximations besides the 1/N approximation.[12] The solution exhibits many of the features found in the real world, and provides a model useful for many purposes. In two dimensions confinement is automatic; the attractive Coulomb potential between a quark and an anti-quark increases proportionally to the distance. The two-dimensional model can therefore not help us understand confinement in four dimensions.

At present it does not appear possible to solve Q.C.D in four dimensions without making approximations in addition to the 1/N approximation. Nevertheless, the 1/N limit will probably still provide a simplification, and this limit certainly helps us to understand many features qualitatively.

Following these general remarks, I should like to summarize some contributions made during the past year. My selection criteria can at best be arbitrary, as I am forced to omit several interesting contributions. I shall limit my selection to papers concerning Q.C.D as such. Two-dimensional or lattice models will be excluded except where their results are immediately relevant to Q.C.D.

The Instanton Vacuum

Polyakov had originally hoped to understand confinement by considering the vacuum as a Euclidean four-dimensional plasma of instantons. Unfortunately the short-range nature of instantons in four dimensions appears to rule out such a possibility. As far as I am aware, there is general agreement on this point. In the above statements, it has been assumed that the infra-red divergence has been cut off by limiting the instanton size. One cannot say that infinitely large instantons could not give confinement but, at present, it appears that one cannot handle such instantons until the confinement problem is solved.

In 1977, Callan, Dashen and Gross[13] suggested that it might be possible to understand confinement by supposing the vacuum to be a four-dimensional plasma of merons. These objects are obtained by smoothing out the singular solutions of De Alfaro, Fubini and Furlan[14] and, unlike instantons, they do not satisfy the field equations everywhere. Two merons are topologically equivalent to one instanton, hence their name. Merons are long-range objects. The fields associated with them fall off like r^{-2} at large r. A meron plasma may therefore confine; in fact, a fixed-time cross-section of a meron plasma is essentially the monopole plasma mentioned earlier.

At the present time, quantitative calculations involving merons have not been performed. More recently, C.D.G.[15] suggested that there might be an intermediate scale of distances where semi-weak-coupling calculations (perturbation theory + instantons) were adequate, and which might include the radii of low-lying hadrons. At larger distances, where confinement forces came into play, perturbation theory would be completely inadequate.

C.D.G. based their treatment on the instanton plasma in an external electric field. As instantons are the four-dimensional analogue of magnetic dipoles, the plasma would be paramagnetic-μ and ε^{-1}, which are equal by Lorentz invariance, would be greater than one. The external field therefore reduces the instanton density. The reduction is most important for large instantons, and a sufficiently strong field cuts off the well-known infra-red divergence.

The curve of D against E is shown in Fig. 3.

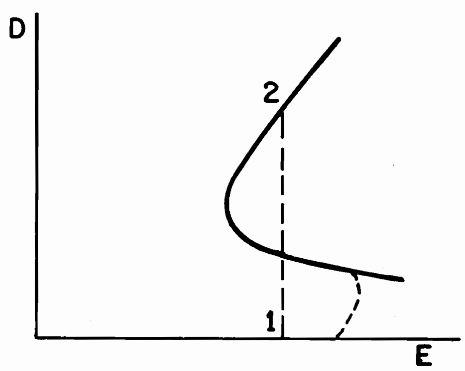

FIG. 3. Plot of D against E in the C.D.G. calculation.

When E is large the instanton density is small, and $D \approx E$. As D and E are reduced, E becomes larger than D; ultimately the infra-red divergence takes over and E approaches infinity. With an infra-red cutoff, the curve would turn back and pass through the origin.

For a reasonable cutoff, C.D.G. find that D/E, or ε, approaches 1/10 as D and E approach zero. Recalling that confinement is the electric-magnetic dual of superconductivity, which is sometimes defined as perfect diamagnetism, we can interpret a zero value of ε as confinement. C.D.G. conjecture that a plasma in which an instanton was allowed to "split" into two merons would give a value ∞ instead of 1/10 for ε. The curve of D against E would then be represented by the dotted line for small E.

The curve in Fig. 3 now recalls the PV curve for a fluid and, in fact, it implies the existence of a first-order phase transition. If the vertical dashed line in Fig. 3 is drawn so that the two areas between it and the DE curve are equal, the system could exist in two phases represented by the points 1 and 2. Phase 2 is the normal phase ($\varepsilon \approx 1$), with non-zero electric flux density, which exists within the hadron; phase 1 is the confining phase ($\varepsilon = 0$) with zero flux which surrounds the hadron.

In order to calculate the energy density of the confining phase (the "bag constant"), it would be necessary to handle high-density plasmas of instantons and merons. C.D.G. do not attempt such a calculation. For the rest, they regard their work as a justification of the M.I.T. bag calculations, but do not feel that they have yet taken it to a point where they can refine the calculation. They make an estimate of the thickness of the layer which separates the two phases at the boundary of a hadron, and conclude that it is small compared with the hadronic radius. In a more recent paper,[16] C.D.G. calculate the running coupling constant, as a function of distance, given by the instanton plasma. They find that their calculation provides a reasonably good interpolation between the short-distance renormalization-group behavior and the large-distance behavior given by the strong-coupling lattice-gauge theory. They thus conclude that instanton effects are sufficient to take us into the strong-coupling regime.

Baryons in the 1/N Approximation

The fact that a baryon consists of N quarks makes the 1/N approximation less straightforward for baryons than for mesons. Each vertex gives rise to a factor $N^{-\frac{1}{2}}$, but there are N such vertices for each interaction involving one baryon.

Witten[17] has shown that it is nevertheless possible to treat baryons in the 1/N approximation; I may remark in passing that his paper provides a very clear outline of the 1/N approximation itself. The wavefunction of a baryon is symmetric in all variables except color. One may therefore neglect color and, instead, pretend that the quarks were bosons. As one can put all the particles in the same state, the density increases with N. The interaction of a given quark with any other quark is small, but the interaction with all quarks together is not. Under these circumstances, quantum fluctuations average out, and one may replace the (N - 1) quarks interacting with the given quark by a (color) charge cloud. Witten shows that such a Hartree-like approximation becomes exact in the limit $N \to \infty$.

As with mesons, one can perform calculations without further approximation in two-dimensional Q.C.D., and one can obtain qualitative information about four-dimensional Q.C.D. The average kinetic energy and the average potential energy of each quark approaches a constant limit as N approaches infinity, so that the baryon mass becomes proportional to N.

Witten also considers baryon interactions. The baryon-baryon and the baryon-meson S-matrices remain finite as N approaches infinity, unlike the meson-meson S-matrix which approaches zero. (The question of baryon widths is still unsettled.) According to the string picture, the interactions involving each string are

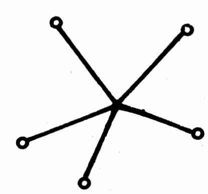

FIG. 4. String model of a baryon.

weak, but there are a large number of strings. As the form of the baryon itself depends on N, it is difficult to see how we could obtain a string picture as a 1/N limit. This may perhaps explain why the original dual model was so much more successful in dealing with mesons than with baryons.

Witten shows that the cross-section for the reaction meson + meson $\leftrightarrow$ baryon + anti-baryon, at fixed baryon velocity, behaves like e^{-N} for large N. This is because N $Q\bar{Q}$ pairs have to be created simultaneously. On the other hand, the cross-section for the reaction at fixed baryon momentum approaches a constant; annihilation at rest is not forbidden. The behavior of the scattering and annihilation reactions do not contradict crossing symmetry.

Phases of Gauge Theories with Fields in the Fundamental Representation

The next contribution concerns calculations by Fradkin and Shenker[18] and by Banks and Rabinovici[19] in a lattice-gauge theory with fields which transform like the fundamental representation of the gauge group, i.e., quark-like fields. Recall that the conventional treatment of confinement assumes that there are no particles in the fundamental representation, i.e., no quark-like particles. Actual quarks are later introduced perturbatively.

If quark-like particles are present, a vortex at electric flux can break up by the creation of $Q\overline{Q}$ pairs (Fig. 5). The vacuum can no longer support

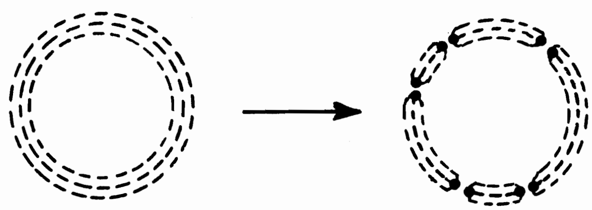

FIG. 5. Break-up of a vortex of electric flux by the creation of $Q\overline{Q}$ pairs.

such vortices, and the Wilson integral behaves like e^{-P}—a feature that has been stressed by Susskind. In fact our ability to recognize confinement experimentally depends either on the existence of conserved quantum numbers external to Q.C.D. (baryon number and flavor) or on the weakness of quark couplings. The latter feature gives rise to near-linear Regge trajectories associated with strings of flux.

In the Weinberg-Salam model, the quark-like fields acquire non-zero vacuum-expectation values. If the coupling is weak we understand the meaning of this statement and can deduce quantitative results from it. But, in general, no one has found a gauge-invariant, and therefore physically meaningful, definition of the Weinberg-Salam vacuum. The criterion used for complete Higgs symmetry breaking by adjoint-representation (non-quark) fields, namely the ability to support vortices of magnetic flux (Nielsen-Olesen vortices) does not apply here. <u>In the presence of actual fundamental-representation (quark-like) fields without "external" conserved quantum numbers, there appears to be no fundamental distinction between the confinement and Weinberg-Salam phases.</u>

Fradkin and Shenker and Banks and Rabinovici examined, <u>inter alia</u>, an Abelian lattice gauge model with the action

$$S = S_{\text{Wilson}} + \frac{\beta}{2} \sum_{(\underset{\sim}{r},\mu)} [\phi(\underset{\sim}{r}) D\{\underset{\sim}{r},\ \underset{\sim}{r}+\underset{\sim}{e}_\mu\} \phi^+(\underset{\sim}{r}+\underset{\sim}{e}_\mu)]\,, \quad |\phi| = 1$$

where D is the Wilson link operator. If β is large, ϕ has a preferred phase and we have a Higgs vacuum; if β is small we do not. This model had previously been studied by Einhorn and Savit,[20] Israel and Nappi,[21] and Sinclair,[22] who showed that the Wilson criterion no longer applied.

In Fig. 6, the weak-coupling Higgs region is

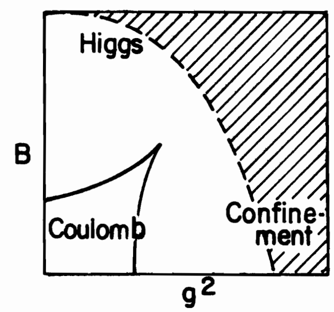

FIG. 6. Phase diagram for the Abelian lattice Higgs model.

the large β, small g region; the strong-coupling confinement region is the small β, large g region. Fradkin and Shenker showed that, within the shaded region, which joins the Higgs and confinement regions, all Green's functions are analytic. The Weinberg-Salam and confinement phases are thus not differentiated. There is a second phase, the ordinary perturbation theory phase with real photons.

In the case where the matter field is multiply charged, which corresponds to a non-Abelian theory with non-quark-like Higgs particles, Fradkin and Shenker found that the Weinberg-Salam and confinement phases were distinct.

Thus, in an SU(2) theory with quark-like field and no extraneous conserved quantities, the phase classification will be

i) the perturbation theory phase,

ii) the Georgi-Glashow phase,

iii) the phase with complete Higgs symmetry breaking by non-quark-like fields. This phase supports Nielsen-Olesen vortices.

iv) the Weinberg-Salam-confinement phase.

Phase iv) has no long-range order. This has led Banks and Rabinovici[19] to re-examine the possible "deconfinement" which is expected to occur at high temperatures. In a lattice model with quark-like fields, they conclude that such deconfinement does not occur. It is an open question whether the existence of external conserved quantum numbers changes this last conclusion.

Non-Abelian Gauge Theories and the Dual String

During the past year, there has been considerable activity in the direction of formulating gauge-theory dynamics in terms of Wilson loop operators. Progress has been made by Gervais, Jaeckel and Neveu,[23,24,25] Polyakov,[26,27] Makeenko and Migdal[28,29] and Eguchi.[30] The obvious question is whether these operators satisfy equations similar, or possibly identical, to those of the string creation operators of the dual model.

The string operators are functionals of the path $P \equiv x^\mu(s)$, whose s is an arbitrary parameter which varies along the string. They satisfy the equations (see Fig. 7)

$$\frac{\delta^2}{\{\delta x_\mu(s)\}^2}\,\psi(P) + \frac{1}{4\pi\alpha^2}\left\{\frac{\partial x_\mu(s)}{\partial s}\right\}^2 \psi\{P\} = \text{Inter-action terms,} \qquad (5a)$$

$$\frac{\partial x_\mu(s)}{\partial s}\,\frac{\delta}{\delta x_\mu(s)}\,\psi\{P\} = 0 \qquad (5b)$$

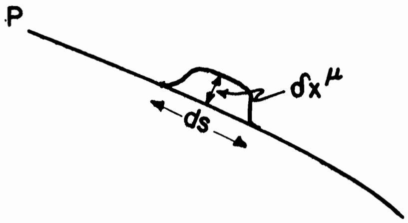

FIG. 7. Definition of quantities in Eqs. (5).

The constant α is the slope of the Regge trajectories. The interaction terms, which have not been written down, represent the breaking, joining and recombination of strings.

We first treat the non-Abelian field without quarks. For the classical non-Abelian field, Gervais and Neveu show that Eqs. (5), without the second term on the left and the interaction term, are satisfied. In the quantized theory, Migdal and Polyakov obtain, formally, an extra term,

$$g^2 N \int dt \quad \delta\{x_\mu(s) - x_\mu(t)\} \frac{\partial x_\mu(s)}{\partial s} \frac{\partial x_\mu(t)}{\partial t} \,\mathrm{Tr}\{ \frac{1}{N}\, \psi(P_1) \times \tau^\alpha \frac{1}{N}\, \psi(P_2)\, \tau^\alpha \}$$

where $\psi(P_1)$ and $\psi(P_2)$ are the untraced Wilson-loop operators for the paths between s and t and between t and s respectively. There may be additional δ-function terms due to short-distance singularities of operator products. The delta functions could give a contribution when s and t are adjacent points and when a string crosses itself. With suitable interpretation, these two contributions give the missing terms in Eq. (5) for closed strings. However, the manipulation of singular quantities is not straightforward and, at this time, no group claims to have a definitive result.

If the closed-string model is derived from the pure gauge theory, with couplings of order N^{-1}, it will certainly give us much more insight into the $N = \infty$ limit, but it will not represent a solution of the theory in this limit, at any rate without further work. The simple solution of the string equation has a tachyon and violates the Froissart bound, so it must correspond to the wrong vacuum. The solution with the correct vacuum has not yet been achieved. The difficulties which occur when $D \neq 26$ are obviously also very pertinent.

As we have already mentioned, any approximate solution of the theory without quarks can hopefully be used as a starting-point for actual Q.C.D., the quarks being treated semi-perturbatively. To the extent that the theory without quarks corresponds to the closed-string model, it might be expected that the full theory corresponds to the open-string model, the string being a flux tube between a $Q\bar{Q}$ pair. Gervais and Neveu show that this is partly but not completely correct, the difference being that the quarks carry a finite amount of momentum at the ends of the string.

Disappearance of the Instanton Gas

I should now like to mention another model calculation, this time on a two-dimensional continuum model known as the CP^{N-1} model. It was performed by Witten[31] with the aim of studying the importance of instantons in a confined theory.

The instanton number is a topological invariant defined by the integral

$$\frac{g^2}{8\pi^2}\,\mathrm{Tr}\int d^4x,\ F^{\mu\nu}\tilde{F}_{\mu\nu}. \tag{6}$$

However, the expression (6) is a topological invariant only if we impose the boundary condition $Fr^2 \to 0$, $r \to \infty$. In a confining theory, the vacuum fluctuations of the fields cannot satisfy this requirement, otherwise the Wilson criterion would not be fulfilled. One may therefore seriously question the importance of instantons in a confined theory.

Witten proposed studying the question by examining the dependence on the coupling constant of effects usually associated with instantons. The only variable parameter is the N of SU(N); if $N \to \infty$, $g^2N \to$ const., g^2 would be proportional to N^{-1}. A real or effective term $\theta F^{\mu\nu}F_{\mu\nu}$ could be present in the Lagrangian due to instantons, in which case its effects would be proportional to $\exp(-g^2)$ or $\exp(-N)$. It could also occur due to the failure of the boundary condition $Fr^2 \to 0$, when its effects would depend on a power of N. Examples of such effects are CP violation and θ-dependence of the vacuum (in a system without massless quarks), or the η mass (in a system with massless quarks).

The CP^{N-1} model possesses features analogous to all those just mentioned for Q.C.D. without quarks and in addition, it can be solved in the 1/N approximation. The Lagrangian is

$$\mathcal{L} = \partial_\mu n_i^* \partial^\mu n_i + (n_i^* \partial_\mu n^i)(n_j^* \partial^\mu n^j) \tag{7}$$

with $n_i^* n_i = 1$. The integral

$$\frac{1}{2\pi i}\int d^2x\,\partial_\mu (n_i^* \varepsilon^{\mu\nu} \partial_\nu n^i) \tag{8}$$

is a topological invariant provided $n_i \to$ const. at large x. There are instanton solutions with non-zero topological charge.

D'Adda, DiVecchia and Lüscher[32,33] have solved the CP^{N-1} model in the 1/N approximation. We shall not discuss their results, except to mention that the boundary condition $\phi \to$ const., $r \to \infty$, turns out not to be fulfilled. The solution is thus analogous to the confined phase in Q.C.D.

Witten has shown that an effective Lagrangian, which may be used to solve the theory in the 1/N approximation, has no instantons. He also showed that the θ-dependence of the masses involved a factor $N^{-2/3}$ and not e^{-N}. The CP^{N-1} model thus appears to indicate that instantons may not be relevant in the confined phase.

Berg and Lüscher[34] have succeeded in solving the problem of the exact (non-dilute) instanton gas for the CP^{N-1} model, and in eliminating the infra-red divergence. Their results are the same as those of the instanton-free calculations just mentioned. The instanton gas is infinitely dense, and the large instantons have evidently destroyed the topological distinction between states of different instanton number. Such an approach appears to be impossible in Q.C.D., where exact classical solutions exist for only a very limited class of multi-instanton solutions. In any case, explicit introduction of instantons is unnecessary in the CP^{N-1} model and, if one does attempt to parametrize the important configurations by means of instantons, one cannot use the dilute-gas approximation and one must be able to handle the infra-red divergence.

Use of the CP^{N-1} model as a "guinea-pig" for Q.C.D. has been criticized and, at the moment, there is no general agreement on this question.

Approximation Scheme for Confinement

I should like to conclude by outlining an approximation scheme for hadronic structure and confinement which I have proposed.[35] The fact that light hadrons are not far away from the linear Regge trajectories due to the confining forces suggests to us that the problems of hadron structure and confinement should be treated together. As our scheme makes use of the Schwinger-Dyson equation, I shall refer to a study of these equations by Anishetty, Baker, Ball, Kim and Zachariasen.[36]

As usual we begin with Q.C.D. without quarks. Our approach is motivated by the vacuum instability, i.e., the fact that the vacuum energy is lowered by giving the color magnetic field a non-zero vacuum-expectation value. This result was first noticed, as far as I am aware, by Saviddy[37] and Wilczek[38] and was studied in more detail by Nielsen and Olesen.[39]

A non-zero value of $\langle \mathcal{H} \rangle$ violates Lorentz invariance. We should like to suggest that the vacuum instability implies, not that the zero-frequency component is non-zero, but that the amplitudes of the low-frequency components are enhanced. Such a feature is closely linked to confinement.

We wish to keep as close to the Nielsen-Olesen calculation as possible in order to obtain a manageable approximation. They consider diagrams such as Fig. 8(a). We replace the static field by a virtual field and, if we neglect non-planar diagrams, we obtain the diagrams of Fig. 8(b). In other words, we have to solve the non-linear integral equation of Fig. 8(c). Faddeev-Popov ghosts may be included in the obvious way.

We are thus motivated towards a particular truncated form of the Schwinger-Dyson equations for the gluon propagator. We have solved the equation to about 5% accuracy using a desk computer; we found that a solution exists and that it behaves like $(p^2)^{-2}$ at low momentum. This is the behavior which naive power counting associates with confinement. The 5% error in our calculations could possibly have misled us, but we believe that to be unlikely. It appears to us that the Baker-Ball-Zachariasen equations do not have a solution with a pure $(p^2)^{-2}$ behavior, but we should not like to assert this fact too strongly.

A $(p^2)^{-2}$ singularity in momentum space corresponds to a $\ln x$ behavior at large distances in coordinate space. With such a behavior, the first term in the Wilson integral

$$\frac{1}{4} \mathrm{Tr}\ \tau^\alpha \tau^\beta \int dx^\mu dy^\nu \langle T\{A^\alpha_\mu(x) A^\beta_\nu(y)\}\rangle \qquad (9)$$

is proportional to the area and independent of the shape, in agreement with the conventional wisdom about confinement.

In any approximation scheme in any field theory, one has to make an assumption about the n-point Green's functions. The usual assumption that they are given by the sum of disconnected diagrams is not applicable in a confined theory where clustering does not hold. Instead we assume that the shape-independence of the area-dependent term in (9), which resulted from our calculations, was not a coincidence and that it is true for the Wilson loop as a whole. Such an assumption, together with the use of disconnected diagrams for the non-leading terms at large distance, enables us to calculate the Wilson-loop integral from the gluon propagator; the Wilson-loop integral is all that is required for color-singlet quark calculations. The assumption of shape-independence implies a form exp {-const. A} for large Wilson loops, as may be seen by considering a loop consisting of two other well-separated loops.

The Wilson-loop formula exp {-const. A} tells us that, for large separations, we may replace our non-Abelian gluons by Abelian gluons whose propagator behaves like $\ln|x|$ at infinity. Crossed and uncrossed diagrams must be included; the crossed diagrams replace diagrams with interacting non-Abelian gluons. Neglect of crossed diagrams in the confinement region is completely inadequate; such diagrams dominate the higher terms in the expansion of the Wilson integral for large loops. Summation of diagrams with crossed and uncrossed gluon lines appears to be a formidable problem. However, it is possible to approximate the area of the loop by a kind of "non-covariant area," and thereby to replace the co-variant gluon propagator by a static potential proportional to the distance. Corrections to this approximation probably involve the higher modes of the dual string, the static force just mentioned corresponding to the zero mode.

It is now a straightforward matter to treat the equations for the quark propagator and the $Q\bar{Q}$ bound state. Since, at large distances, we have a static linear potential, it is not surprising that we obtain trajectories which rise linearly at large ℓ and $E(\ell \propto E^2)$. We take such trajectories to imply confinement.

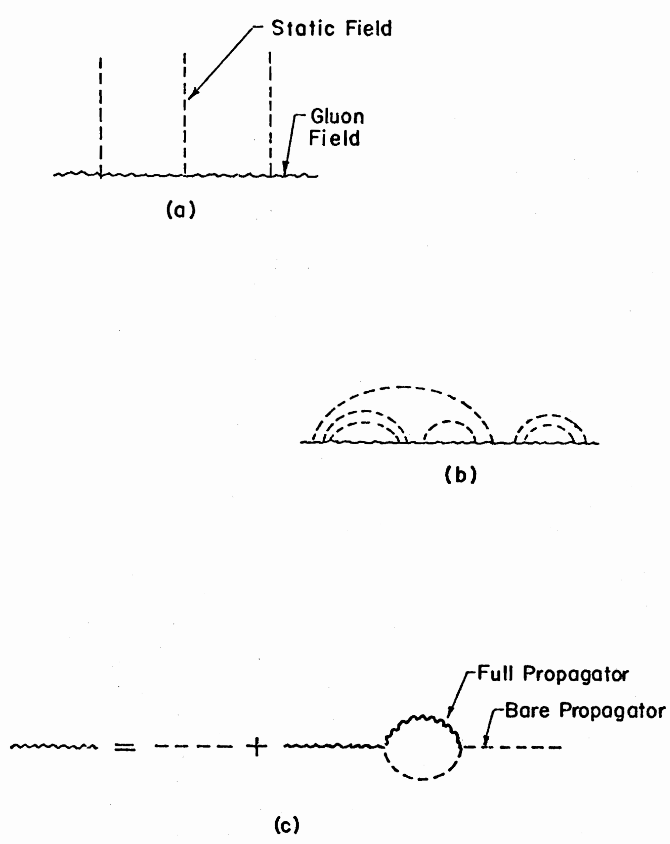

FIG. 8. Diagrams for the gluon propagation.

Our equations contain the possibility of chiral symmetry breaking, along the lines indicated by previous workers.[40,41,42] We then obtain a zero-mass pseudo-scalar $Q\bar{Q}$ bound state. If closed quark loops are included the η acquires a mass. In agreement with Witten's work, such a mass appears without explicit introduction of instantons, and it is proportional to N^{-1}.

Research supported by the National Science Foundation under grant PHY77-23512.

References

1. G. 't Hooft, CERN preprint.
2. J. B. Kogut and L. Susskind, Phys. Rev. D11, 395 (1975).
3. H. B. Nielsen and P. Olesen, Nucl. Phys. B61, 45 (1973).
4. Y. Nambu, Phys. Rev. D10, 4262 (1974).
5. S. Mandelstam, Phys. Rep. 23C, 245 (1976).
6. G. 't Hooft in High Energy Physics, Proceedings of the European Physical Society International Conference, Palermo, 1975, edited by A. Zichichi (Editrice Compositori, Bologna, 1976), p. 1225.
7. G. 't Hooft, Nucl. Phys. B138, 1 (1978).
8. G. 't Hooft, Nucl. Phys. B153, 141 (1979).
9. K. Wilson, Phys. Rev. D10, 2445 (1974).
10. S. Mandelstam, Phys. Rev. D19, 2391 (1979).
11. G. 't Hooft, Nucl. Phys. B72, 461 (1974).
12. G. 't Hooft, Nucl. Phys. B75, 461 (1974).
13. C. G. Callan, R. F. Dashen and D. J. Gross, Phys. Rev. D7, 2717 (1978).
14. V. deAlfaro, S. Fubini and G. Furlan, Phys. Lett. 65B, 1631 (1977).
15. C. G. Callan, R. F. Dashen and D. J. Gross, Phys. Rev. 19, 1826 (1979).
16. C. G. Callan, R. F. Dashen and D. J. Gross, Instantons as a Bridge between Weak and Strong Coupling, Princeton preprint.
17. E. Witten, Baryons in the 1/N Expansion, Harvard preprint.
18. E. Fradkin and S. Shenker, Phase Diagrams of Lattice Gauge Theories with Higgs Fields, SLAC preprint.
19. T. Banks and E. Rabinovici, Finite Temperature Behavior of the Lattice Abelian Higgs Model, Berkeley preprint.
20. M. Einhorn and R. Savit, Phys. Rev. D10 2583, (1978).
21. R. B. Israel and C. R. Nappi, Comm. Math. Phys. 64, 177 (1979).
22. D. R. T. Jones, J. Kogut and D. K. Sinclair, Phys. Rev. D19, 1882 (1979).
23. J-L Gervais and A.Neveu, Phys. Lett. 80B, 255 (1979).
24. J-L Gervais and A. Neveu, Nucl. Phys. B153, 445 (1979).
25. J-L Gervais, M. T. Jaekel, and A. Neveu, Nucl. Phys. B155, 75 (1979).
26. A. M. Polyakov, Phys. Lett. 82B, 247 (1979).
27. A. M. Polyakov, Lecture given at Aspen Summer Institute.
28. Y. M. Makeenko and A. A. Migdal, Exact Equation for the Loop Average in Multicolor Q.C.D., ITEP preprint.
29. A. A. Migdal, Properties of the Loop Average in Q.C.D., Landau Institute preprint.
30. T. Eguchi, Strings in Lattice Gauge Theories, Chicago preprint.
31. E. Witten, Instantons, the Quark Model, and the 1/N Expansion, Harvard preprint.
32. M. Lüscher, Phys. Lett. 78B, 465 (1978).
33. A. D'Adda, P. DiVecchia and M. Lüscher, Nucl. Phys. B146, 63 (1978).
34. B. Berg and M. Lüscher, Computation of Quantum Fluctuations around Multi-Instanton Fields from Exact Green's Functions: the CP^{N-1} Case, Hamburg preprint.
35. S. Mandelstam, Approximation Scheme for Q.C.D., Berkeley preprint.
36. R. Anishetty, M. Baker, J. S. Ball, S. K. Kim and F. Zachariasen, Infrared Properties of the Coupling Constant in Non-Abelian Gauge Theories, Seattle preprint.
37. G. K. Saviddy, Phys. Lett. 71B, 133 (1977).
38. F. Wilczek, private communication from D. Gross.
39. N. K. Nielsen and P. Olesen, Nucl. Phys. B144, 376 (1978).
40. A. D. Maris, V. E. Herscovitz and G. Jacob, Phys. Rev. Lett. 12, 313 (1964).
41. H. Pagels and P. Langacker, Phys. Rev. D9, 3413 (1974).
42. K. Lane, Phys. Rev. D10, 2605 (1974).