UNIFICATION OF FUNDAMENTAL FORCES

Frank Wilczek
Princeton University
Princeton, N.J. 08544

Abstract

Superunified theories of particle interactions are reviewed and some new results are presented. The classic SU(5) and SO(10) theories, and the classical applications to the calculation of $\sin^2\theta_W$ and the estimation of the lifetime of the nucleon are briefly recalled. More conjectural applications to the explanation of the matter-antimatter asymmetry in the universe, the calculation of quark-lepton mass ratios, and the motivation of the Coleman-E.Weinberg calculation of the Higgs particle mass are reviewed. Extensions of the superunification idea, motivated by the desire to understand the problem of families, are presented. The advantages of orthogonal gauge groups and spinor fermion representations in this connection are pointed out. A new analysis of the effective Hamiltonian for nucleon decay is given, and the resulting intensity rules are presented.

A kinship hypothesis (KH) and an extended kinship hypothesis (EKH) important in analyzing nucleon decay and neutrino oscillations respectively are formulated. The possible relevance of recent work on the transition from weak to strong coupling in QCD to the problem of gauge hierarchies is conjectured. We close with a set of questions whose elucidation would mean great progress. The curious possibility that a low-energy effective theory may look unstable is expemplified in an appendix.

We now have gauge theories of the strong interactions and of the electroweak interactions, based on SU(3) and SU(2) x U(1) respectively, for which the evidence is becoming persuasive. It is very tempting to investigate whether these mathematically similar theories may be united into a single larger gauge theory.

This step may be compared to Gell-Mann's introduction of SU(3) symmetry generalizing the SU(2) x U(1) of isospin x strangeness.

Attempts of this kind were first made by Pati and Salam.[1] Shortly thereafter Georgi and Glashow[2] presented the SU(5) theory which fits more smoothly into our understanding of the low-energy SU(3) x SU(2) x U(1) theory and is the foundation of all the discussion which follows.

I. The Classic Superunified Theories - SU(5) and SO(10)

A. SU(5)

The groups of the strong, electromagnetic, and weak interactions - SU(3) x SU(2) x U(1) - have altogether four additive quantum numbers. So to encompass them we need at least a rank four Lie group, and the only simple group of rank four with an SU(3) x SU(2) x U(1) subgroup is SU(5).

The fermions in the first family may be grouped into a $\underline{5}$ (vector) and $\overline{\underline{10}}$ (antisymmetric tensor) representation of SU(5) as follows:

$$U\begin{pmatrix} ad_1 \\ ad_2 \\ ad_3 \\ \nu \\ e \end{pmatrix} \qquad U^*\begin{pmatrix} 0 & au_3 & -au_2 & u_1 & d_1 \\ -au_3 & 0 & au_1 & u_2 & d_2 \\ au_2 & -au & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & ae \\ -d_1 & -d_2 & -d_3 & -ae & 0 \end{pmatrix}U^+$$
$$v^\mu \qquad\qquad\qquad A_{\mu\nu} \tag{1.1}$$

Here we have adopted the convenient convention that $af = \mathcal{C}(f_R) = i\gamma_2\,(1+\gamma_5)/2\ f^*$ for a fermion field, and the understanding that all the fields we write are left-handed. (We represent right-handed fields explicitly as charge-conjugates of left-handed ones.) For any transformation $U\epsilon SU(5)$, the transformation of the multiplets is as displayed in Eq. (1.1). Here 1, 2, 3 are the color indices.

We can easily see that this assignment conforms to some basic ground rules: the total charge of the fields in each multiplet is zero, the color multiplets are complete. If we embed SU(3) x SU(2) x U(1) as follows:

$$\begin{array}{c|cc} & 3 & 2 \\ \hline 3 & SU(3) & 0 \\ 2 & 0 & SU(2) \end{array} \qquad U(1) = \begin{pmatrix} -2 & 0 & 0 & & \\ 0 & -2 & 0 & & 0 \\ 0 & 0 & -2 & & \\ & 0 & & 3 & 0 \\ & & & 0 & 3 \end{pmatrix} \tag{1.2}$$

Then it is not difficult to verify that all the assignments are correct in detail, that the low-energy SU(3) x SU(2) x U(1) theory of vector bosons and fermions does indeed fit quite snugly into SU(5).

This is very remarkable. Also remarkable is that the anomalies generated by the chiral multiplets $\underline{5}$ and $\overline{\underline{10}}$ cancel one another, so that the full theory is anomaly-free.

The breaking of SU(5) symmetry is accomplished in two steps. In the first step, suggested by Eq. (1.2), we have an adjoint Higgs field H with vacuum expectation value

$$\langle H\rangle = \begin{pmatrix} -2 & & & & \\ & -2 & & 0 & \\ & & -2 & & \\ & 0 & & 3 & \\ & & & & 3 \end{pmatrix} V \tag{1.3}$$

This breaks SU(5) $\rightarrow$ SU(3) x SU(2) x U(1), as we have seen. At this stage all the fermions remain massless. In the second step, an SU(5) - vector Higgs field ϕ acquires a vacuum expectation value

$$\langle\phi\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} v \tag{1.4}$$

which corresponds to the symmetry breaking in the ordinary Weinberg-Salam model.

The fermions acquire a mass due to couplings of the form

$$(\mathcal{C}A)_{\mu\lambda}\, v^{\mu}\, \phi^{\lambda} \;;\quad (\mathcal{C}A)_{\alpha\beta}\, A_{\gamma\delta}\, \phi^{*}_{\eta}\, \varepsilon^{\alpha\beta\gamma\delta\eta} \tag{1.5}$$

when ϕ acquires a vacuum expectation value.

Two aspects of the symmetry breaking require special comment:

i) As we shall see shortly, the scales associated with $SU(5) \to SU(3) \times SU(2) \times U(1)$ and with $SU(2) \times U(1) \to U(1)$ breaking are vastly different, $V \approx 10^{16}$ GeV as against $v \approx 300$ GeV. This is peculiar, since if we start with a Higgs potential containing terms like $\eta\phi^{+}\phi\, \mathrm{Tr}H^{2}$ then after the first stage of symmetry breaking we have $30\,\eta\, V^{2}\, \phi^{+}\phi$. Then the "natural" scale for $<\phi>$ is $\eta^{1/2}V$ so to arrange $<\phi> = v$ we require that η be very small ($\sim 10^{-27}$) or that tremendous cancellations occur. To make these arrangements is perfectly consistent,[3] but most people regard it as ugly and unsatisfactory. This is known as the problem of gauge hierarchies.

ii) If there is only one SU(5) vector Higgs field giving masses to the fermions then we get

$$m_e = m_d \;;\quad m_\mu = m_s \;;\quad m_\tau = m_b \tag{1.6}$$

at the tree graph level. These interesting relations are of course subject to renormalization, as we shall discuss below.

B. SO(10)

The appearance of two kinds of fermion representation, $\underline{5}$ and $\overline{\underline{10}}$, and the apparently miraculous nature of the anomaly cancellation in SU(5) are annoying features that can both be removed by going to the larger group SO(10).[4] SO(10) is rank 5, it has an extra additive quantum number not in SU(5) (or SU(3) x SU(2) x U(1)). The extra generator is coupled to B-L.

SU(5) is embedded in SO(10) as follows: Five component complex vectors can be written as 10 - component real vectors: $v_j = x_j + i\, y$; $w_j = s_j + i\, t_j$. SU(5) is the group of linear transformations leaving the inner product $(v,w) = (x_j\, s_j + y_j\, t_j) + i\,(x_j\, t_j - y_j\, s_j)$ invariant. SO(10) is the group which leaves just the real part invariant (and by the way Sp(10) is the group leaving just the imaginary part invariant). So SU(5) is contained in SO(10) as the subgroup of SO(10) leaving an additional antisymmetric form invariant, or more explicitly as the 10 x 10 orthogonal matrices R satisfying:

$$R^{T} J R = J$$

$$J = \begin{pmatrix} 0 & -1 & & & & & & & & \\ 1 & 0 & & & & & & & & \\ & & 0 & -1 & & & & & & \\ & & 1 & 0 & & & & & & \\ & & & & 0 & -1 & & & & \\ & & & & 1 & 0 & & & & \\ & & & & & & 0 & -1 & & \\ & & & & & & 1 & 0 & & \\ & & & & & & & & 0 & -1 \\ & & & & & & & & 1 & 0 \end{pmatrix} \tag{1.7}$$

Orthogonal groups have special representations, the spinor representations, in addition to the normal tensor representations. For SO(10) we have a spinor representation Δ_+ which branches into $\underline{5} + \overline{\underline{10}} + \underline{1}$ under the SU(5) subgroup. This representation can be used to accommodate all the fermions in the first family into one multiplet. This "explains" why the anomalies in SU(5) cancel, since it is a general theorem[5] that representation of SO(n), $n \geq 7$ are anomaly free. In addition there is a singlet field which has neither strong, electromagnetic, nor weak interactions, which we shall discuss in I.C.

It is straightforward to generalize the symmetry breaking scheme adopted for SU(5) to SO(10). In the first step we suppose there is an adjoint Higgs H acquiring a vacuum expectation value

$$<H> = \begin{pmatrix} 0 & 2 & & & & & & & & \\ -2 & 0 & & & & & & & & \\ & & 0 & 2 & & & & & & \\ & & -2 & 0 & & & & & & \\ & & & & 0 & 2 & & & & \\ & & & & -2 & 0 & & & & \\ & & & & & & 0 & -3 & & \\ & & & & & & 3 & 0 & & \\ & & & & & & & & 0 & -3 \\ & & & & & & & & 3 & 0 \end{pmatrix} \tag{1.8}$$

This breaks $SO(10) \to SU(3) \times SU(2)\, U(1) \times U(1)$, the extra U(1) being B-L. If we add a single SO(10) vector Higgs we get the very bad relations $m_{\nu_e} = m_u = m_d = m_e$, etc. Note that the SU(5) singlet field has become the right-handed neutrino. With two vectors we can do better, getting $m_{\nu_e} = m_u$, $m_d = m_e$, etc. The relations among the charged fermions are as in SU(5), Eq. (1.6). The relations for neutrino masses look silly but may actually (in a modified form) be very significant, as we shall now discuss.

C. Symmetry Breaking According to GRS; Neutrino Masses

So far we have a massless B-L generator (bad for neutrino experiments!) and a heavy neutrino. An elegant method fo tipping the scale in the other direction has been proposed by Gell-Mann, Ramond and Slansky.[6] They point out that a Higgs $\underline{126}$ (self-dual five-index antisymmetric tensor) may acquire a vacuum expectation value which simultaneously gives the B-L generator a mass and allows the right-handed neutrino to acquire a Majorana mass.

Let us elaborate on the latter phenomenon in the simpler context of SU(2) x U(1). The relevant multiplets are

$$\begin{pmatrix} \nu' \\ e \end{pmatrix}_L \quad N'_R \quad e_R \tag{1.9}$$

where with the primes we acknowledge that ν', N' may not be mass eigenstates. Allowing for a Majorana mass term

$$M\, \overline{\mathcal{C}N'}\, N' \tag{1.10}$$

The mass matrix becomes (using our old relation $m_\nu = m_u$)

$$\begin{array}{c c} & \begin{array}{cc} \nu' & N' \end{array} \\ \begin{array}{c} \nu' \\ N' \end{array} & \begin{pmatrix} 0 & m_u \\ m_u & M \end{pmatrix} \end{array} \tag{1.11}$$

Diagonalizing, we find that the physical neutrino has a mass $m_\nu = m_u^2/m_N$, and that the electron couples to the mixture $\nu + \sqrt{m_\nu/m_N}$ by the weak current.

Demanding that the neutrino masses be ≤ 10 eV on cosmological grounds[7] we find that the N particles must be quite heavy; $m_N \gtrsim 10^3, 10^8, 10^{11}$ GeV for the N's associated with u, c, t respectively.

Academic lower bounds on the physical neutrino mass in this theory are found if we demand the lifetime of the N's be ≤ 1 sec., again on cosmological grounds. The main decay mode of the N's is $N \rightarrow \nu Z$ induced by ν-N mixing, giving the N a lifetime $\tau \simeq \left(\frac{\alpha}{\pi} m_\nu\right)^{-1}$. We therefore find $M_\nu \geq 10^{-12}$ eV, with the suggestion that ν_μ and ν_τ are substantially heavier.

It may be inconsistent[8] to push M_{N_τ} much higher than 10^{11} GeV, or m_ν many orders of magnitude below 10 eV. There is therefore some suggestion that neutrino oscillations into ν_τ may not be far beyond experimental reach. This situation will become especially interesting if proton decay suggests that SO(10) breaks directly into SU(3) x SU(2) x U(1) x U(1), not through SU(5). (See IV.)

There are other possible choices for the symmetry-breaking pattern in SO(10) - for instance we might break directly SO(10) $\rightarrow$ SU(5) with an adjoint Higgs $\langle H\rangle \propto J$ or with a spinor Higgs. We have detailed the GRS scheme because it illustrates the interesting possiblity of a Majorana neutrino mass and because of its economy.

II. The Classic Applications

The decoupling theorem of Appelquist and Carrazone[9] states that in a renormalizable theory containing some very heavy particles (masses $\sim$M) experiments at momenta p << M involving light particles of masses m << M may be described by an effective renormalizable theory containing only light particles. The light particle theory will be accurate up to powers of p/M or m/M. This is the reason why direct applications of superunification are so few - the particles really characteristic of superunification, the vector bosons which are in SU(5) but not SU(3) x SU(2) x U(1), are predicted to be very heavy by the theory itself.

The applications fall into three classes:

i) Classification and symmetry - The representation content of the effective theory may be dictated by the superunifying theory. Couplings which *a priori* are unrelated in the effective theory may be related by the superunifying symmetry. (Part I and A, B, C below)

ii) Very rare processes - Certain processes which are forbidden by the effective low energy theory will go (at rates $\sim$p/M or $\sim$m/M) by hypoweak interactions. (D, E below)

iii) Processes in extreme conditions - If momenta ever climb to p $\sim$ M, the effective low energy theory becomes useless and the full superunified theory comes into play. (F below)

There is another curious way in which superheavy particles might manifest themselves, which is described in the Appendix.

I shall be very brief in reviewing the applications.

A. Calculation of $\sin^2\theta_W$

In a simple gauge theory such as SU(5) or SO(10) the couplings of all gauge bosons are equal (universality). This is manifestly not the case for the observed generators of SU(3) x SU(2) x U(1). The key to reconciling superunification with even the crudest observations lies in the renormalization group, first applied to this problem by Georgi, Quinn, and Weinberg.[10] The effective couplings for the gauge particles are momentum-dependent. The SU(3) or higher symmetry can become manifest only at very large momenta. For a meaningful comparison we need to see how the coupling constants evolve from their measured low-momentum values to their (equal) large-momentum values.

This calculation is illustrated in the familiar fig. 1. From three input parameters g_3, g_2, g_1 we obtain two outputs - the characteristic momentum at which the couplings become equal and the coupling at this momentum - and a consistency condition, which determines $\sin^2\theta_W$. Graphically, the consistency condition comes from the requirement that all three curves meet at one point.

The value of $\sin^2\theta_W$ computed in this way for SU(5) is .19 ± .02, not incompatible with existing data but possibly on the low side. The characteristic unification mass is $\sim 10^{15}$ GeV and the coupling at this mass is $g^2/4\pi \sim .02$.

This calculation is critically dependent on the assumption that nothing happens between $\sim 10^2$ and $\sim 10^{16}$ GeV. It could be substantially modified even if the basic superunification idea is correct - compare fig.2. Nevertheless it is the most important quantitative prediction of superunification to date and its apparent success is strong encouragement for the whole enterprise.

B. Quark-Lepton Mass Ratios

Renormalization effects also change the effective masses of particles measured at different momentum scales. The relations $m_e = m_d$, $m_\mu = m_s$, $m_\tau = m_b$ obtained from simple symmetry-breaking schemes hold at very large momenta. The quark masses increase by a factor $\sim$3 in passing to laboratory momenta.[11] The relation $m_b \approx 3\, m_\tau$ is quite good, but its luster is considerably dimmed by the bad relation $m_e/m_u = m_d/m_s$ (1/200 = 1/20) which accompanies it.

C. Motivation of Coleman-E. Weinberg Mechanism

It has been argued[12] that since the bare effective couplings of the SU(5) vector Higgs scalar ϕ (in particular, the bare mass) must be strangely small, we should really make it zero as conjectured on slightly different grounds by Coleman and E. Weinberg.[13] I think this argument is very much a matter of taste. In any case, it predicts the physical Higgs mass $\sim$10.4 GeV.

D. Nucleon Decay

It is extremely interesting that the calculated masses for the heavy SU(5) bosons lead to a prediction for the rate of nucleon decay (due to their exchange) which is not far below the experimental limit.[10,11] The detailed calculations necessary for this conclusion require a full-scale review in themselves, and P. Langacker has recently supplied one. I will present some new results on these decays, emphasizing symmetry principles, in Sect. IV.

E. Neutrino Masses

Many superunified theories predict non-zero neutrino masses, though usually the exact magnitude is not well determined (as we have seen above).

F. Matter-Antimatter Asymmetry

Yoshimura[14] pointed out that the violation of baryon number is superunified theories, together with CP and C violations, could lead to the asymmetry between matter and antimatter in the universe. His calculation was not quite correct, but in correcting it Toussaint, Treiman, Wilczek, and Zee[15] pointed out another important advantage of superunified theories. We showed that a theory involving only massless particles cannot develop a matter-antimatter asymmetry in the big bang, so that to get an appreciable asymmetry one must have particles whose mass is comparable to or greater than the temperature while significant baryon-number violating processes are occurring. The heavy particles which abound in superunified theories meet this criterion.

Based on these observations, many attempts[16] have been made to estimate the ratio n_B/n_γ which is measured to be $\sim 10^{-9}$. These attempts are subject to severe uncertainties (especially in the theory of CP violation at ultra-high energies), and published estimates range from 10^{-4} to 10^{-18} or smaller.

III. The Family Problem; Spinors and Families[17]

The SU(5) theory is a remarkable and audacious achievement but it may not go far enough. The proliferation of fermion representations - at least three $\underline{5}$'s and three $\overline{\underline{10}}$'s - is certainly unacceptable in a fundamental theory. SO(10) is more economical, requiring just three $\underline{16}$'s, but it too leaves the question: "Why does Nature repeat Herself?", unanswered.

It is reasonable to seek the solution of this problem in a further extension of the gauge group, such that after symmetry breaking starting from an irreducible representation of the large group we find repetitive family structures under the smaller effective SU(5) or SO(10) group.

Attempts of this kind using tensor representations generally run into trouble. For example, consider the following simple-minded attempt. Extend SU(5) to SU(5+n), and let the fermions transform according to the three-index tensor $T^\rho{}_{[\mu\nu]}$ (antisymmetric in the bottom two indices and traceless). After breaking down to SU(5), we find n $\overline{\underline{10}}$ representations and $n(n-1)/2$ $\underline{5}$ representations. If we demand $n = n(n-1)/2$ to get a reasonable tehory at the SU(5) level, we find n = 3 families (!). However, this is not a good theory, because there are many additional, unwanted representations - a $\underline{45}$, 3 $\underline{24}$'s, 9 $\overline{\underline{5}}$'s and 9 singlets. To get a reasonable low energy theory, we would have to explain how most of this motley crew acquires a large mass. In addition, the full theory has anomalies.

A little experimentation quickly convinces you that similar problems afflict any attempt to put the fermions in a single tensor representation.

It is remarkable and suggestive that the spinor representations of orthogonal groups are much more satisfactory in this regard - they are automatically free of anomalies and have a simple, symmetrical decomposition under orthogonal subgroups strongly suggestive of the repetitive family structure probably needed to describe nature.

This is not the place to expound all the intricacies of spinor technology. We will only state a few facts:

i) SO(2n) has two inequivalent spinor representatives Δ_+, Δ_- of dimension 2^{n-1}.

ii) Δ_+ and Δ_- are complex conjugates for n odd, but are both real for n even.

iii) Either spinor representation Δ_+ or Δ_- of SO(10+2m) branches into 2^{m-1} Δ_+ and 2^{m-1} Δ_- representations under SO(10).

Since the Δ_+ representation of SO(10) gives a satisfactory theory of intrafamily representations, point iii) is already very nearly a candidate solution to the faily problem. However, the Δ_- representations give "conjugate" families, with leptons coupling by right-handed weak currents, etc. What is required is a method of getting rid of the Δ_- representations, a way of giving these fermions a large mass.

A simple-minded way of doing this was proposed by A. Zee and me. We noticed that a certain simple symmetry breaking pattern could do what is desired - raise the Δ_- fermion masses, leaving the Δ_+ fermion masses untouched. This mechanism always leads to a number of families which is a power of two - based on what we know, it would have to be four.

A possibly more profound mechanism, which however rests on conjectural dynamics, has been suggested by Gell-Mann et. al.[6] We will exemplify this sort of idea without attempting realism.

Start with a spinor representation of SO(18), and suppose that SO(18) breaks to SO(10) x SU(4) with the SU(4) representing a super-strong interaction. This line of thought is inspired by Susskind's "technicolor".[18] The 256-dimensional spinor representation of SO(18) branches into $(\Delta_-, 4) + (\Delta_-, \overline{4}) + (\Delta_+, 6) + 2(\Delta_+, 1)$ under SO(10) x SU(4). If following the technicolor philosophy we suppose that the fermions which are not singlets under the SU(4) are confined in superheavy, unobserved "technihadrons", then the remaining singlets give us two SO(10) $\Delta+$ families. This idea can be extended to make three (or more) families.

The details of either class of proposals are certainly questionable. However, I think the real moral of this story is that the assignment of fermions to spinor representations of orthogonal groups leads to some attractive classification schemes, and permits for the first time a plausible approach to the problem of family structure. Let me summarize this section by briefly recalling the advantages of these representations:

i) They are anomaly-free.

ii) They branch in a simple way to SO(10), involving only $\Delta+$ and $\Delta-$ representations. The $\Delta+$ representations of SO(10) yield families of a kind we can use for phenomenology.

iii) They shine by comparison with the competition - recall our discussion of the tensor $T^\rho{}_{[\mu\nu]}$.

iv) If we regard it as desirable to group all fermions in a single irreducible representation of some gauge group, the large number of observed fermions we need to accommodate is a major consideration. The property of spinor representations, that their dimension increases exponentially with the rank of the gauge group, seems to be an important advantage. The alternatives - gauge groups of very large rank, representations with many indices, or reducible representations - don't seem very appealing.

IV. A Close-Up on Nucleon Decay[19]

The effects of heavy particles on processes at low energies can be described by an effective Hamiltonian involving only light particles, according to standard methods.[20] The most important operators in this Hamiltonian will be those of minimal dimension, with the appropriate quantum numbers, which respect the symmetry of the light particle theory with the light particles regarded as massless.

The application of these methods to nucleon decay is straightforward and gives significant results. Operators changing baryon number necessarily have dimension ≥ 6 (at least three quark fields, and hence four fermion fields to make a Lorentz scalar). Thus we are lead to the problem of classifying operators of dimension 6 which change baryon number and are singlets under SU(3) x SU(2) x U(1). The following is an exhaustive list (up to Hermitean conjugates):

Vector Type:

$$O_1 = \bar{e}\gamma_\mu\, au\bar{u}\gamma_\mu ad + \bar{\nu}\gamma_\mu au\bar{d}\gamma_\mu ad$$

$$O_2 = \bar{u}\gamma_\mu ae\bar{d}\gamma_\mu au$$

Scalar Type:

$$O_3 = \bar{e}\mathcal{C}u\bar{u}\mathcal{C}d + \bar{\nu}\mathcal{C}d\bar{d}\mathcal{C}u$$

$$O_4 = \overline{\mathcal{C}au}\, ae\, \bar{u}\mathcal{C}d$$

$$O_5 = \bar{e}\mathcal{C}u\, \overline{\mathcal{C}au}ad + \bar{\nu}\mathcal{C}d\, \overline{\mathcal{C}au}ad$$

$$O_6 = \overline{\mathcal{C}au}ae\, \overline{\mathcal{C}au}ad$$

Tensor Type: similar to scalar type

The color indices, which may be routed in only one way, have been suppressed. For notational convenience we write only the lightest fermion fields, and for the moment ignore other families. It is very important to note that by a combination of color, Fierz and Fermi antisymmetry O_1 is antisymmetric in the exchange $au \leftrightarrow ad$, O_2 is antisymmetric in the exchange $u \leftrightarrow d$; and by a combination of color, Dirac, and fermi antisymmetry O_3, O_4 are antisymmetric in $u \leftrightarrow d$, O_5, O_6 are antisymmetric in $au \leftrightarrow ad$. The corresponding tensor type operators turn out to be symmetric.

Some selection rules follow immediately from this kind of analysis: $\Delta(B-L) = 0$[21] and $\Delta S \geq 0$ in nucleon decay, to order (light mass/heavy mass)$^{-2}$

Remarkable also is the simplicity of the structure for vector type operators. This structure will necessarily obtain in any superunified theory where nucleon decay is mediated primarily by vector exchange, up to higher-order corrections involving Higgs exchange (which could bring in O_3 - O_6 and the tensors). The three strong-interaction operators which appear in the two parts of O_1 and in O_2 are closely related: the quark pieces of O_1 from a strong isodoublet and the quark piece of O_2 is just the parity transform of the appropriate piece of O_1. We can, therefore, derive intensity rules testing the underlying superunified theory, disentangled from uncertainties about hadronic matrix elements.

For example, we get the following relations for $\Delta S = 0$ nucleon decay:

$$\tfrac{1}{2}(1 + |r|^2)\,\Gamma(p \to \pi^+\bar{\nu}) = \Gamma(p \to \pi^o e^+) = (\tfrac{1}{2})\,\Gamma(n \to \pi^- e^+)$$

$$= (1 + |r|^2)\,\Gamma(n \to \pi^o\bar{\nu})$$

$$\Gamma(p \to e^+ + \text{any}) = (1 + |r|^2)\,\Gamma(n \to \bar{\nu} + \text{any})$$

$$(1 + |r|^2)\,\Gamma(p \to \bar{\nu} + \text{any}) = \Gamma(n \to e^+ + \text{any})$$

$$\Gamma(p \to e^+ + \text{any}) \geq \tfrac{1}{2}(1 + |r|^2)\,\Gamma(p \to \bar{\nu} + \text{any})$$

and so forth. Here r is the ratio of the coefficient O_2 to that of O_1 in the effective Hamiltonian, which depends on the particular superunifying theory. For example:

a) In SU(5) at the tree graph level r=2. However, substantial, but precisely calculable renormalization effects will change this number. This calculation is being done by A. Zee and me.

b) In SO(10), if the symmetry breaking by the adjoint representation is more severe than the B-L breaking, then nucleon decay is mediated mainly by some bosons which are in SO(10) but not in SU(5). These bosons are then five times lighter than the SU(5) bosons and thus dominant by a factor of $5^4 = 625$. For exchange of these bosons r = 0.

c) It has often been conjectured that $SU(2)_L \times SU(2)_R \times U(1)$ might be the effective electroweak theory at masses well below the superunification scale. In this case, we find r = 1 to a first approximation.

Also of great interest are nucleon decays involving μ^+ or with $\Delta S = +1$. Jarlskog[22] has observed that in principle new Cabibbo-like angles appear in superunified theories: e.g. a 5 of SU(5) might be

$$\begin{pmatrix} as_1 \\ as_2 \\ as_3 \\ e\cos\phi + \tau\sin\phi \\ \nu_e\cos\phi + \nu_\tau\sin\phi \end{pmatrix} \quad \text{instead of} \quad \begin{pmatrix} as_1 \\ as_2 \\ as_3 \\ \mu \\ \nu_\mu \end{pmatrix}$$

and we couldn't discover it outside nucleon decay. We would like to think that this is unlikely, even bizarre. In view of its importance, I would like to formulate precisely, name, and partly motivate a form of the "common-sense" view:

Kinship Hypothesis (KH): The charge -1/3 quarks and the charge -1 leptons are rotated by the same angles relative to the charge 2/3 quarks, measurable in ordinary weak processes (e.g., the Cabibbo angle). In the pairings of negative fermions, the lightest quark goes with the lightest lepton, etc. Thus according to KH, O_1 should properly be written as

$$O_1^{KH} = \bar{e}\,\gamma_\mu a(u\cos\theta_c - c\sin\theta_c)\,\overline{(u\cos\theta_c - c\sin\theta_c)}\gamma_\mu ad$$

$$+ \bar{\nu}\gamma_\mu a(u\cos\theta_c - c\sin\theta_c)\,\bar{d}\gamma_\mu ad$$

and so forth.

The kinship hypothesis is motivated by the partial success represented by the calculation of m_b/m_τ, which suggests that negatively charged fermions all acquire masses from a single source. It is very much open to doubt, however.

Fortunately, KH is also open to experimental test. Its predictions depend in detail on the nature of the superunifying theory, and we will not go into details here. In the SO(10) scenario b) mentioned just above, for example, we obtain from KH

$$\Gamma(n \to \mu^+\pi^-) = \tan^2 \theta_c \; \Gamma(n \to e^+\pi^-).$$

One result of potential importance is that KH allows one to predict the polarization of the μ^+ in nucleon-decay. In SU(5) at the tree-graph level the μ^+ is unpolarized, but this will be modified by the renormalization effects mentioned above. In SO(10), broken as in b), the μ^+ is essentially always right-handed (independent of KH).

V. Dynamical Symmetry Breaking and Hierarchies

Many of the questions we would like to answer cannot be usefully addressed at present. The couplings of gauge particles to each other and to fermions is completely specified by the gauge principle. However, all the effects associated with breakdown of the gauge symmetry - i.e., masses of particles and the hierarchy of interactions (strong, electromagnetic, weak, hypoweak), are presently lumped into the Higgs sector where we have no powerful principle limiting the couplings.

The situation grows more and more unsatisfactory as we grow increasingly ambitious. I have shown you the very disturbing adjustment of parameters that is necessary in the SU(5) theory, and things get no better in the orthogonal theories. And we have the sorry spectacle of the Grand Synthesis yielding predictions for only one or two parameters from among the dozens we need at the SU(3) x SU(2) x U(1) level accessible to experiment.

The mathematical similarity of the Higgs mechanism with fundamental scalars to the Landau-Ginzburg theory of superconductivity, which of course was later shown to result from a microscopic fermion-fermion interaction, encourages us to expect a similar development of non-abelian gauge theories with only gauge particles and fermions.

Recent work in the gauge theory of strong interactions, showing how an instability leading to chiral symmetry breaking can arise from the microscopic interactions[23] certainly encourages us in this expectation. In the QCD case, it is a global chiral symmetry that is broken. What is needed is an extension of this work to local symmetries; a search for instabilities in gauge non-singlet directions.

The hard work that must be done to realize the dream of deriving symmetry-breaking parameters from fundamental gauge theories has not been done. However, an appealing qualitative consequence of this kind of thinking can, I think, already be discerned - a suggestion of how the huge hierarchies of symmetry breaking we seem to need might be generated.

Investigations carried out in semi-classical approximations to QCD[24] and in lattice gauge theories[25] strongly suggest two important, highly non-trivial conclusions: that a transition from weak to strong coupling behavior occurs in a very narrow range of couplings, and that the coupling at which this transition occurs is small. "Small" in this case means that ordinary perturbative radiative corrections are small, e.g. two-loop diagrams may be neglected. The transition is induced by semi-classical, non-perturbative effects. These semi-classical effects are almost entirely negligible for couplings slightly smaller than the transition coupling, then come roaring in.

Now consider the behavior of a gauge theory based on some large group SO(N). The effective coupling in this theory is not really $\bar{g}$ (the value of the three-point function), but $\bar{g}^2N$. This is shown e.g. by the formula for the β-function in the renormalization group. Here $\bar{g}$ is the scale-dependent, running coupling constant.

Let us suppose that strong coupling behavior sets in at $\bar{g}^2N \geq K$. In the strong-coupling regime we can expect symmetry breaking; let us suppose that the SO(N) symmetry breaks to SO(N') at the scale where $\bar{g}^2N' < K$ is "small" as defined above, it will take a large scale change for the effective coupling of the SO(N') theory to renormalize to the critical K again, and for further symmetry-breaking to occur. This discussion is grossly oversimplified (in particular, the effects of fermions are crucial), but it does exemplify how hierarchies might be expected to arise dynamically.

VI. Summary - Questions

It is easiest to summarize a talk like this one, where many possibilities have been discussed, in the form of questions - trying to imagine how we could sort out the possibilities. This is a very conservative set of questions, in that I am assuming the basic framework holds up and I have tried to be as specific and realistic as possible.

A) Questions concerning nucleon decay

a) Tests of superunification

1) Does the nucleon have a lifetime $10^{31\pm2}$ years? If so, simple superunified theories score a triumph.

2) Does nucleon decay obey the selection rules $\Delta(B-L) = 0$, $\Delta S \geq 0$? If not, the decay can have at most an indirect connection with superunification - scotch one triumph.

3) Do the $\Delta S = 0$ nucleon decays satisfy the isospin constraints discussed in Sect. IV? If so, what is the value of r? The first question tests the whole idea of superunification, the second allows us to discriminate among different superunified theories.

b) Tests of KH

4) Does nucleon decay obey the selection rule $\Delta(S+L_\mu) = 0$ to high ($\lesssim 5\%$) accuracy? This is the most accessible direct test of KH.

5) Do nucleon decays involving the antimuon conserve parity?

As is discussed above, precious information about the underlying theory is tied up with the polarization of the μ^+.

B) Questions concerning Higgs particles

6) Is the mass of the Higgs particle $\sim$ 10.4 GeV? If so, score a triumph for S. Coleman, E. Weinberg, and their idea of scale invariance broken by radiative corrections. A Higgs particle of this mass should be readily visible in $t\bar{t}$ decay.[26] A light Higgs particle may pose a problem for dynamical

symmetry breaking schems, which tend to like a heavy ($\gtrsim$ 1 TeV), strongly interacting Higgs. A cosmological speculation is connected with $m_H \sim 7.5$ GeV.[27]

7) What is the branching ratio $\Gamma(H \to$ light hadrons$)/\Gamma(H \to$ any$)$? This quantity affords a direct measure of the number of heavy quarks.[26]

C) Test of EKH

8) Do the electron and muon neutrino oscillate into one another with amplitude $\sin 2\,\theta_c$? Superunified theories more ambitious than SU(5) tend to give finite neutrino masses, and SO(10) with the Majorana mass scheme suggests these masses (especially for the τ neutrino) may be rather large, maybe not much smaller than the cosmological limit of 10 eV. I think this consideration gives neutrino oscillation experiments a new urgency. An extension of KH to the pairing of neutrinos and charge 2/3 quarks is called the extended kinship hypothesis (EKH). The question about electron and muon neutrinos may, of course, be generalized to ν_τ, where EKH says neutrino oscillation amplitudes should follow the quark mixing angles.

D) Questions for Theorists

9) Can we squeeze more predictive power from the superunification idea? I think this question is closely tied up with the problem of sharpening the SO(10 + m) classification schemes into real theories, and especially with controlling the symmetry breaking.

10) How long can we leave gravity to the side? Superunification scales are within shouting range of the Planck mass, and quantum gravity is in sad shape. This cannot be a coincidence, and I suspect a synthesis of really dazzling beauty awaits discovery.

Appendix: No positivity

The Appelquist-Carrazone theorem says that the low-energy limit of a renormalizable theory is itself renormalizable. It is interesting to note, however, that the low energy theory need not satisfy normal positivity constraints.[28]

A very simple toy model suffices to demonstrate this. Consider a Lagrangian with two scalar fields ϕ_1, ϕ_2 symmetrically coupled:

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu\phi_1)^2 + \tfrac{1}{2}(\partial_\mu\phi_2)^2 + \tfrac{1}{2}\mu^2(\phi_1^2 + \phi_2^2) - \frac{\lambda}{4}(\phi_1^2 + \phi_2^2) - \frac{\eta}{2}\,\phi_1^2\,\phi_2^2 \qquad (A.1)$$

where $0 < \eta \ll \lambda \ll 1$, $0 < \mu^2$. ϕ_2 spontaneously develops a vacuum expectation value

$$\langle\phi\rangle = \mu^2/\lambda \qquad (A.2)$$

and the masses of the quanta of fields ϕ_1, ϕ_2 are

$$m_1^2 = \frac{\eta}{\lambda}\;\mu^2 \qquad (A.3)$$

$$m_2^2 = 2\mu^2 \qquad (A.4)$$

When we write an effective theory for the light particles associated with ϕ_1 we must remember to include the graphs shown in Fig. 3. Although these graphs contain a heavy propagator, the trilinear vertex is explicitly proportional to the heavy mass and there is no suppression. Putting it all together, the effective 4-point ϕ_1^4 coupling becomes

$$\mathcal{L}_{\text{eff. int.}} = + \frac{(2\eta + \eta^2/\lambda)}{4}\,\phi_1^4 \qquad (A.5)$$

The sign of this interaction makes the effective theory unstable - for large values of $\langle\phi_1\rangle$. However, of course, the effective theory does not apply for large values of $\langle\phi_1\rangle$ and the complete theory is stable. What is interesting is that the effective theory itself reveals its own incompleteness.

So this is one way that, in principle, the heavy particles despite their decoupling assert their existence at low energies.

Acknowledgements: The work reported in Sections III-IV was done in collaboration with A. Zee, and his influence pervades the rest too. I thank S. Treiman for many discussions, especially of the material in Section III. I thank C. Callan for a helpful discussion of the remarks in Section V and J. Collins for many helpful discussions of the appendix. Finally I am grateful to M. Gell-Mann, T. Goldman and R. Shrock for brief but informative discussions.

References:

1. J. Pati, A. Salam, Phys. Rev. D10, 275 (1974).
2. H. Georgi, S. Glashow, Phys. Rev. Lett. 32, 438 (1974)
3. A. Buras, J. Ellis, M. Gaillard, D. Nanopoulos, Nucl. Phys. B135, 66 (1978); S. Weinberg, Phys. Lett. 82B, 387 (1979).
4. H. Georgi, in Particles and Fields 1975 (AIP Press, N.Y.); H. Fritzsch, P. Minkowski, Ann. Phys. 93, 193 (1975).
5. H. Georgi, S. Glashow, Phys. Rev. D6, 429 (1972).
6. P. Ramond, talk at Sanibel Symposium (1979), to be published; M. Gell-Mann, P. Ramond, R. Slansky, private communications, to be published.
7. R. Cowsik, J. McClelland, Phys. Rev. Lett. 29, 669 (1972).
8. We have in mind here instabilities as discussed by P.Q. Hung, Phys. Rev. Lett. 42, 873 (1979); H.D. Politzer, S. Wolfram, Phys. Lett. 83B, 421 (1979).
9. T. Appelquist, J. Carrazone, Phys. Rev. D11, 2856 (1975).
10. H. Georgi, H. Quinn, S. Weinberg, Phys. Rev. Lett. 32, 438 (1974).
11. A. Buras et al., Ref. 3.
12. E. Goldener, S. Weinberg, Phys. Rev. D13, 3333 (1976).
13. S. Coleman, E. Weinberg, Phys. Rev. D7, 2887 (1973).
14. M. Yoshimura, Phys. Rev. Lett. 41, 281 (1978).
15. D. Toussaint, S. Treiman, F. Wilczek, A. Zee, Phys. Rev. D19, 1036 (1979).
16. S. Barr, G. Segre, H. Weldon, Penn preprint UPR-0127T (1979); D. Nanopoulos, S. Weinberg, Harvard preprint HUTP/79/A023 and references therein.
17. This section is mostly based on joint work done with A. Zee (to be published).
18. L. Susskind, SLAC-PUB 2142 (1978). See also S. Weinberg, Phys. Rev. D13, 974 (1976).
19. See note 17.

20. As codified by J. Collins, F. Wilczek, A. Zee, Phys. Rev. D18, 242 (1978).
21. This consequence of SU(3) x SU(2) x U(1) symmetry, in the less general framework of one-boson exchange, was pointed out by S. Weinberg, Phys. Rev. Lett. 42, 850 (1979).
22. C. Jarlskog, Phys. Lett. 82B, 401 (1979).
23. C. Callan, R. Dashen, D. Gross, Phys. Rev. D17, 2717 (1978); R. Carlitz, Phys. Rev. D17, 3225 (1978).
24. C. Callan, R. Dashen, D. Gross, Princeton preprint (1979).
25. J. Kogut, R. Pearson, J. Shigemetsu, Phys. Rev. Lett. 43, 484 (1979).
26. F. Wilczek, Phys. Rev. Lett. 39, 1304 (1977).
27. F. Wilczek, in B. Lee Memorial Conference (Harwood, London, 1978).
28. Related phenomena have been discussed independently by L. Chan, T. Hagiwara, B. Ovrut (Brandeis preprint, 1979). They interpret the non-positivity as a failure of the decoupling theorem. I think the decoupling theorem is perfectly all right within its proper domain.

Figure Captions:

Fig. 1: The effective coupling constants $g_3(\mu)$, $g_2(\mu)$ $g_1(\mu)$ for SU(3) x SU(2) x U(1) meet at a single point in the simple SU(5) theory.

Fig. 2: There may be other interactions with characteristic scales between 10^2 and 10^{16} GeV.

Fig. 3: Unusual contribution to the effective 4-pt. coupling. See text.

Discussion

Q. (Oakes, Northwestern) These schemes seem to only accommodate an even number of generations. Does that mean there's a t' and a b' and that there are ultimately four generations?

A. The mechanism that Tony Zee and I have only allows the number of generations to be 2^n, which given what we know implies there are four. The one suggested by Gellmann, Ramond and Slansky could accommodate other numbers of generations. The right one presumably is yet to be found. The one that Tony Zee and I found does suggest that the number of generations has to be four.

Q. (Harari, Weizmann Inst.) I would like to make a remark directed to experimentalists and especially those who are working at relatively low energies. If any of these models that put several generations into a very very large group much larger than SU(5). If any of these models is correct, then there have to be vector bosons which connect states in different generations. No one has the slightest idea what the masses of these objects are. But pushing the limits of experiments such as $\mu \to e$ conversions and heavy nuclei and K_{long} decaying into μe and $K \to \pi\mu e$ and several other exotic objects like this pushing any of these limits will improve our lower limit of the masses of such animals. For instance, the existing limit on $K_{long} \to \mu e$ already says that such animals have to be above 30 TeV in mass. If that can be improved, for instance, it can teach us a lot. So there are lots of low energy experiments which can teach us a great deal about the problem of generations.

A. In my list of questions I should've mentioned that I tried to make the questions as definite as I could even if difficult. Because I didn't want to say, well look for something rare or look for unexpected bosons or something.

Q. (Lorella Jones, Illinois) It seems to me that all of the models that you discussed are rather complicated and that the sort of model which Harari valued at 30¢ the other day might lead to a super unified theory that had a considerably simpler group structure and perhaps far fewer of these particles that are hard to find. Are people working on that sort of thing?

A. Well, Harari is. I don't think you should judge the complexity of a theory by the number of particles. Although these theories have a large number of particles, they have few coupling constants in principle and more or less known dynamics. That's the difference.

Q. (Frampton, Harvard/Ohio) You emphasized mainly the orthogonal groups with several families and you only gave one example of the unitary group. I think it was SU(8) which had a lot of diseases. But there are models, especially I think in SU(9) which have no anomalies and do have only the observed particles at light mass. I think 6 flavors is the preferred number in SU(9).

A. That's not a question. Do you have a question?

Q. I was just commenting that the distinction between the unitary groups and orthogonal groups is not so clear because the orthogonal groups are limited to a power of 2 as the number of families, unitary groups can have, for example, 3 families.

A. Well, I pointed out what I think are the advantages of orthogonal groups. I don't have the final theory, so I can't exclude other possibilities.

Q. (M.K.Gaillard, LAPP, Annecy) I just wanted to clarify this remark about the kinship theorem. The kinship theorem is exact in the minimal SU(5) model with only Higgs 5-plets.

A. Sure.

Q. Okay, because there is some confusion about that.

A. I said it was motivated by that, but since the mass relations you get from that are only partly successful, I think the kinship hypothesis is very much open to doubt.

Q. (Minh, Los Alamos) In the SU(5) you have a representation. You have an antidown or down, are they really pure or actually d cos θ_C + s sin θ_C.

A. They're impure. When I was writing these things down for notation, I pretended there was only one family.

Q. (L.Sulak, Michigan) Some people chuckled when you asked experimentalists to measure muon polarization. It's not perhaps a chuckling matter because I think all the proposals that have been made for looking at a sensitive level to proton decay in fact have the capability of measuring the muon polarization.

A. I can vouch for the fact that they laughed at QCD! They laughed at scaling violations!

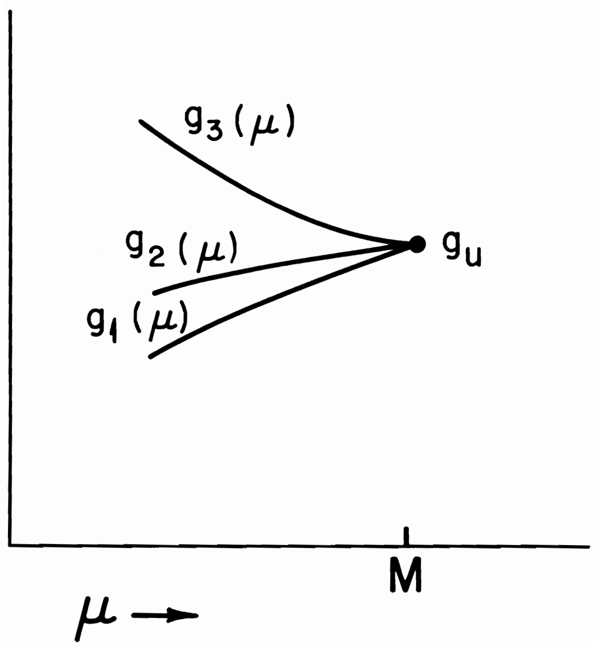

Fig. 1

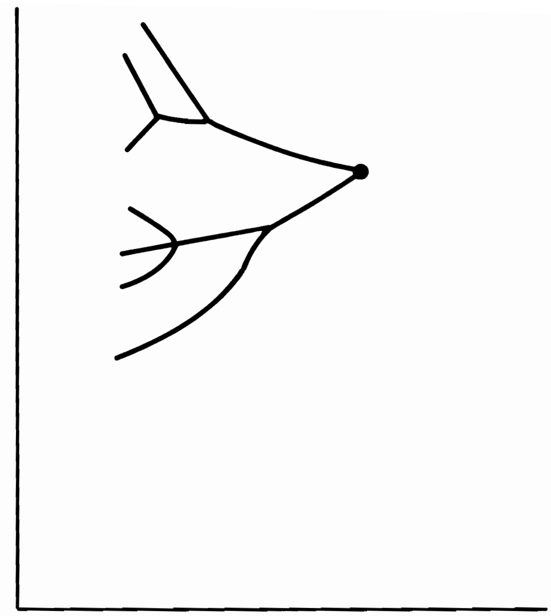

Fig. 2

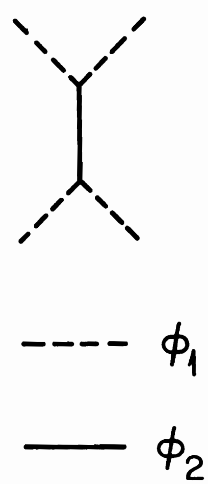

Fig. 3