FLAVOR DYNAMICS

Makoto KOBAYASHI
National Laboratory for High Energy Physics
Oho-machi, Tsukuba-gun, Ibaraki-ken, 305, Japan

§1 Introduction

The Weinberg-Salam theory of unified weak and electromagnetic interactions[1)] has achieved a remarkable success in the neutral current phenomena, and no one can deny its excellent features as a fundamental theory. In contrast to the simplicity of gauge coupling, however, this theory contains too many parameters in the interactions with the Higgs boson, which generate the masses of quarks and leptons, the Cabibbo mixing angle etc, through the spontaneous breaking of the gauge symmetry. Furthermore we do not know how many quark or lepton flavors exist. Thus, it can be said that our understanding of the nature still lacks some fundamental physical principle which regulates this complication. Recent discovery of the Υ particle, which indicates the existence of the b-quark, has raised the importance of this problem.

The purpose of this lecture is to present a introduction to overcome the above problem. We concentrate ourselves to quark masses and mixing angles. How to treat them in a certain extended version of the Weinberg-Salam (W-S) model will be explained in detail. A basic knowledge about the W-S theory is assumed, but not necessarily required. All discussions will be made in the tree approximation, and no renormalization problem will be considered since it does not change the essential features of the following discussion except finite corrections.

In §2, the construction of quark mass terms is discussed and the number of mixing parameters in the sequential scheme is given. §3 is devoted to some phenomenological analyses in the six-quark case. In §4, an attempt to obtain relations between masses and mixing angles is explained.

§2 Quark masses and the generalized Cabibbo mixing

2.1 Gauge fields and Higgs fields

First, we recall the basic quantities of the W-S model. The model is the SU(2) × U(1) gauge theory, which requires four gauge fields;

$$\begin{aligned} &A_\mu^a \ \cdots\cdots \ SU(2), \qquad Y = 0, \quad (a = 1,2,3) \\ &B_\mu \ \cdots\cdots \ U(1), \qquad Y = 0, \end{aligned} \tag{2.1}$$

where Y is the weak hypercharge which describes the transformation property under U(1). Spontaneous symmetry breaking is realized by introducing a Higgs doublet;

$$\phi = \begin{bmatrix} \phi^+ \\ \phi^o \end{bmatrix}, \qquad Y = 1. \tag{2.2}$$

It is useful to notice that

$$\tilde{\phi} = \begin{bmatrix} \phi^{o*} \\ -\phi^- \end{bmatrix}, \qquad Y = -1, \tag{2.3}$$

is also transformed as a doublet.

The Lagrangian of this partial system is given by

$$L = -\frac{1}{4}(F_{\mu\nu}^a)^2 - \frac{1}{4}F_{\mu\nu}^2 + |(\partial_\mu - ig\frac{\tau^a}{2}A_\mu^a - ig'\frac{1}{2}B_\mu)\phi|^2 - V(\phi) , \tag{2.4}$$

where

$$\begin{aligned} F_{\mu\nu}^a &= \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g\epsilon^{abc}A_\mu^b A_\nu^c , \\ F_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu, \end{aligned} \tag{2.5}$$

and

$$V(\Phi) = \mu^2\Phi^+\Phi - \lambda(\Phi^+\Phi)^2. \tag{2.6}$$

In the tree approximation, the vacuum expectation value of Φ is given by

$$\langle\Phi\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \tag{2.7}$$

where

$$v = \mu^2/\lambda = (\sqrt{2}G_F)^{-1/2} \tag{2.8}$$

The last equality follows from the equivalence with the Fermi's theory at the low energy. Then, the shifted fields are expressed as follows;

$$\Phi = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} + \frac{1}{\sqrt{2}} \begin{pmatrix} i\chi^1 + \chi^2 \\ \phi - i\chi^3 \end{pmatrix}, \tag{2.9}$$

where χ^i are the Goldstone modes to be absorbed into the weak bosons and ϕ describes the physical Higgs boson. The physical gauge fields are given by

$$W_\mu^\pm = \frac{1}{\sqrt{2}} (A_\mu^1 \mp iA_\mu^2),$$

$$Z_\mu = \frac{1}{\sqrt{g^2 + g'^2}} (gA_\mu^3 - g'B_\mu),$$

$$A_\mu = \frac{1}{\sqrt{g^2 + g'^2}} (g'A_\mu^3 + gB_\mu), \tag{2.10}$$

where $W_\mu^\pm$, Z_μ and A_μ are the charged, neutral weak bosons and the photon, respectively.

2.2 Quarks --- four-quark case.

In this subsection and the next one, we describe the way of constructing the quark parts of the Lagrangian, especially their mass terms. We first illustrate it in a simple four-quark case, so called G.I.M. model[2], so that notational complexity of the general case which is discussed in the next subsection may not prevent understanding of physical implications.

Interactions between the gauge bosons and the quarks are determined by the transformation property of the quarks under SU(2) × U(1). Since we are considering parity violating interactions, there is no need for the left and right components of the quark fields to have the same transformation properties. The left and right components are defined by

$$\psi_{\substack{L\\R}} = \frac{1}{2}(1 \mp \gamma_5)\psi, \quad \bar{\psi}_{\substack{L\\R}} = \bar{\psi}\,\frac{1}{2}(1 \pm \gamma_5). \tag{2.11}$$

Note that we have simple but useful relations;

$$\bar{\psi}_L\psi_L = \bar{\psi}_R\psi_R = \bar{\psi}_L\gamma_\mu\psi_R = \bar{\psi}_R\gamma_\mu\psi_L = 0 \tag{2.12}$$

In the G.I.M. model, the assignments of transformation properties are as follows

$\begin{pmatrix} u \\ d' \end{pmatrix}_L$, $Y = 1/3$, $d' = \cos\theta_c d + \sin\theta_c s$,

$\begin{pmatrix} c \\ s' \end{pmatrix}_L$, $Y = 1/3$. $s' = -\sin\theta_c d + \cos\theta_c s$,

u_R, c_R, $Y = 4/3$,

$$d_R,\ s_R, \qquad Y = -2/3. \tag{2.13}$$

where u, d, s, c are the usual quark fields which diagnalize the quark mass terms. For later convenience we note that we can represent two left-handed doblets equivalently as

$$\begin{pmatrix} u' \\ d \end{pmatrix}_L , \qquad Y = 1/3, \qquad u' = \cos\theta_c u - \sin\theta_c c ,$$
$$\begin{pmatrix} c' \\ s \end{pmatrix}_L , \qquad Y = 1/3, \qquad c' = \sin\theta_c u + \cos\theta_c c , \tag{2.14}$$

since any linear conbination of doublets is also transformed as a doublet.

Writing down the Kinetic parts of the Lagrangian, which include the couplings to the gauge fields, is straight forward from the above assignment;

$$\begin{aligned} L = & \ i[\bar{u}, \bar{d}']_L \gamma_\mu (\partial_\mu - ig \frac{\tau^a}{2} A^a_\mu - ig' \frac{1}{6} B_\mu) \begin{pmatrix} u \\ d' \end{pmatrix}_L + \begin{Bmatrix} u \to c \\ d' \to s' \end{Bmatrix} \\ & + i\bar{u}_R \gamma_\mu (\partial_\mu - ig' \frac{2}{3} B_\mu) u_R \qquad + \{u \to c\} \\ & + i\bar{d}_R \gamma_\mu (\partial_\mu + ig' \frac{1}{3} B_\mu) d_R \qquad + \{d \to s\}. \end{aligned} \tag{2.15}$$

Since, as is seen in (2.13), the left and the right components have different transformation properties, any scalar density of quark bilinear form can not be gauge invarient. Therefore, quark masses arise only from the Yukawa coupling to the Higgs field, through the spontaneous breaking of the symmetry. Then, we find that the proper form of the Lagrangian is

$$\begin{aligned} L = & - g_u \{ \bar{u}_R \tilde{\phi}^+ \begin{pmatrix} u \\ d' \end{pmatrix}_L + [\bar{u}\ \bar{d}']_L \phi u_R \} - (u \to c) \\ & - g_d \{ \bar{d}_R \phi^+ \begin{pmatrix} u' \\ d \end{pmatrix}_L + [\bar{u}'\ \bar{d}]_L \phi d_R \} - (d \to s), \end{aligned} \tag{2.16}$$

where

$$g_u = \sqrt{2}\, m_u/v, \quad g_d = \sqrt{2}\, m_d/v, \quad \text{etc.} \tag{2.17}$$

Replacing the Higgs field by its vacuum expectation value, we can easily check the adequacy of (2.16). Note that other gauge invariant forms, such as

$$\bar{u}_R \tilde{\phi}^+ \begin{pmatrix} c \\ s' \end{pmatrix}_L , \quad \text{etc.} \tag{2.18}$$

do not appear, since they violate the asumption that u,d,s,c diaganalize the mass terms.

An important fact is that, in this shceme, the requirement of gauge invariance does not impose any constraints on the values of masses and mixing angles.

2.3 Quarks --- generalized sequential scheme.

We have already had experimental indications of the existence of more than four quarks. The Υ and Υ' are interpreted as $\bar{q}q$ states of a heavy quark, b. A simple and attractive assignment of b-quark is such that b forms another left-handed doublet with t-quark which is not yet discovered. In view of this proliferation of the quark flavour, we give here a general frame work for the sequential left-handed doublet scheme.

Let us assume that we have N sequences of doublet and therefore 2N quark flavours. Then, they are denoted as follows;

$$L^i = \begin{bmatrix} \psi_i \\ \sum_{j=\{d,s,b..\}} U_{ij} \psi_j \end{bmatrix} , \qquad Y = 1/3 ,$$

$$R^i = \psi_{iR}, \qquad Y = 4/3,$$

$$R_j = \psi_{jR}, \qquad Y = -2/3, \tag{2.19}$$

where ψ_i and ψ_j stand for the physical quark fields and i runs over u,c,t... and j runs over d,s,b... Note that the position of suffices of L and R indicates which set of the flavour indices they should take. The generalized Cabibbo-like mixing is expressed by an N × N unitary matrix U, whose ij-component is denoted as U_{ij}. The mixing matrix U must be unitary in order that the kinetic parts of the quark fields have a properly normalized diagonal form (See below). Corresponding to (2.14), we introduce the following:

$$L_j = \begin{pmatrix} \sum_{i=\{u,c,t...\}} U^{-1}_{ji}\psi_i \\ \psi_j \end{pmatrix}_L = \sum_{i=\{u,c,t...\}} U^{-1}_{ji} L^i$$

$$j = d,\ s,\ b,\ \ldots \tag{2.20}$$

In this notation, the Lagrangian has a simple form:

$$L = i\sum_i \bar{L}^i\gamma_\mu(\partial_\mu - ig\frac{\tau^a}{2}A^a_\mu - ig'\frac{1}{6}B_\mu)L^i + i\sum_i \bar{R}^i\gamma_\mu(\partial_\mu - ig'\frac{2}{3}B_\mu)R^i$$

$$+ i\sum_j \bar{R}_j\gamma_\mu(\partial_\mu + ig'\frac{1}{3}B_\mu)R_j$$

$$- \sum_i g_i\{\bar{R}^i\tilde{\phi}^+L^i + \bar{L}^i\tilde{\phi}R^i\}$$

$$- \sum_j g_j\{\bar{R}_j\phi^+L_j + \bar{L}_j\phi R_j\}, \tag{2.21}$$

where

$$g_k = \sqrt{2}\, m_k/v. \tag{2.22}$$

The range of summation will be selfevident from the above notational convention.

To make a further investigation, we rewrite the Lagrangiam in terms of the physical fields:

$$L = \sum_{k=\{u,d,s,c...\}} \{i\bar{\psi}_k\gamma_\mu\partial_\mu\psi_k - m_k\bar{\psi}_k\psi_k(1 + \frac{\phi}{v})$$

$$+ eQ_k\bar{\psi}_k\gamma_\mu\psi_k A_\mu + \frac{1}{2}\sqrt{g^2 + g'^2}\,\bar{\psi}_k\gamma_\mu\tilde{Q}_k\psi_k Z_\mu\}$$

$$+ \frac{g}{2\sqrt{2}} \sum_{\substack{i=\{u,c...\}\\ j=\{d,s...\}}} \{U_{ij}\bar{\psi}_i\gamma_\mu(1-\gamma_5)\psi_j W^+ + U^*_{ij}\bar{\psi}_j\gamma_\mu(1-\gamma_5)\psi_i W^-_\mu\},$$

$$\tag{2.23}$$

where Q_k is the electric charge of ψ_k and

$$\tilde{Q}_k = \begin{cases} \frac{1}{2} - 2Q_k\frac{g'^2}{g^2+g'^2} - \frac{1}{2}\gamma_5 & k = u,c... \\ \frac{1}{2} - 2Q_k\frac{g'^2}{g^2+g'^2} + \frac{1}{2}\gamma_5\,. & k = d,s... \end{cases} \tag{2.24}$$

We have taken the unitary gauge, so that no Goldstone modes appear in (2.23).

An important fact in this scheme is that flavour changing interactions arise only from the last line in (2.23). In such a case, some of the phases of U_{ij} can be absorbed into the phase convention of the quark fields and do not have physical significance. As an example, let us consider the case of N = 2. The most general form of 2 × 2 unitary matrix is given by

$$U = \begin{array}{c} \\ u \\ c \end{array}\begin{pmatrix} d & s \\ \cos\theta\, e^{i(\alpha+\beta)} & \sin\theta\, e^{i(\alpha+\gamma)} \\ -\sin\theta\, e^{i(\alpha-\gamma)} & \cos\theta\, e^{i(\alpha-\beta)} \end{pmatrix} \tag{2.25}$$

Then, we change the phase convention of quark fields as

$$\psi_k \to e^{i\phi_k}\psi_k \tag{2.26}$$

with

$$\phi_u - \phi_d = \alpha + \beta, \qquad \phi_u - \phi_s = \alpha + \gamma, \qquad \phi_u - \phi_c = -\beta - \gamma\,. \tag{2.27}$$

In the new convention, U looks like

$$U = \begin{array}{c} \\ u \\ c \end{array}\begin{pmatrix} d & s \\ \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{2.28}$$

This is nothing but the G.I.M. model. So, we find that the G.I.M. model does not lack any generality in the four-quark (two-doublet) case.

Now, let us turn to an arbitrary N. An N × N unitary matrix has N^2 parameters. Among 2N phase parameters of 2N quark fields, 2N-1 are responsible for the reduction of number of parameters of U. Note that the overall phase of quark fields does not affect to U. As a result, we find that the number of parameters of the Cabibbo-like mixing is given by

$$N^2 - (2N - 1) = (N - 1)^2 \tag{2.29}$$

2.4 CP violation.

Here, we investigate the CP properties of the Lagrangian (2.23). Let us consider the following CP-transformation;

$$\begin{aligned} \psi_{kL(R)} &\to \gamma_o C\, \bar{\psi}^T_{kL(R)}, \\ \bar{\psi}_{kL(R)} &\to -\psi^T_{kL(R)} C^{-1}\gamma_o, \\ W^{\pm}_{\mu} &\to \eta_\mu W^{\mp}_{\mu}, \\ Z_\mu &\to \eta_\mu Z_\mu, \\ \phi &\to \phi, \end{aligned} \tag{2.30}$$

where C is the usual charge conjugation matrix with

$$\begin{aligned} C^T &= -C\,, \\ C\gamma_\mu C^{-1} &= -\gamma^T_\mu\,, \end{aligned} \tag{2.31}$$

and

$$\eta_\mu = \begin{cases} -1 & \mu = 0 \\ 1 & \mu = 1,\,2,\,3. \end{cases}$$

It is easy to check that the first bracket in (2.23) is invariant under the transformation (2.30). Our main concern is the charged current interaction part, which is

$$\frac{g}{2\sqrt{2}} \sum_{\substack{i=\{u,c\ldots\} \\ j=\{d,s\ldots\}}} \{U_{ij}\bar{\psi}_i\gamma_\mu(1-\gamma_5)\psi_j W_\mu^+ + U_{ij}^*\bar{\psi}_j\gamma_\mu(1-\gamma_5)\psi_i W_\mu^-\} \qquad (2.32)$$

Under (2.30), this is transformed into

$$\frac{g}{2\sqrt{2}} \sum_{\substack{i=\{u,c\ldots\} \\ j=\{d,s\ldots\}}} \{U_{ij}\bar{\psi}_j\gamma_\mu(1-\gamma_5)\psi_i W_\mu^- + U_{ij}^*\bar{\psi}_i\gamma_\mu(1-\gamma_5)\psi_j W_\mu^+\} \qquad (2.33)$$

Therefore, if all U_{ij} are real, (2.32) is CP-invariant. Here, however, we should remind the problem of the phase convention. In general, we can multiply an arbitrary phase factor to the right hand side of the transformation of each quark field in (2.30). But this phase factor can be removed by changing the phase convention of quark fields. This implies conversely that we can assume CP-transformation of the form (2.30) for the field with phase convention whatever we like. Therefore, if we can make all U_{ij} to be real by suitably adjusting the phases of quark fields, then, defining the CP-transformation as (2.30) for the new fields which make U_{ij} real, we can see that the system is CP-invariant.

As we have seen, in the case of N = 2, U_{ij} can be made real. Therefore we have no CP-violation in this case. For a general N, a unitary matrix whose elements are real is nothing but an orthogonal matrix. The most general N × N orthogonal matrix has N(N - 1)/2 parameters. This number should be compared with $(N - 1)^2$ in (2.29). Thus the number of the parameter responsible for CP-violation is given by

$$(N-1)^2 - N(N-1)/2 = (N-1)(N-2)/2 \qquad (2.34)$$

From this we can conclude that for $N \geq 3$ we have a possibilities of CP-violation through the Cabibbo-like mixing.[3)]

In the above arguments, our assumption that the Higgs field is only one doublet has played an essential role. If we have many Higgs fields, we have possibilites of CP-violation through the Higgs boson exchange, even for N = 2. (For example, see ref. 4) To determine what mechanism is true, detailed phenomenological analyses are necessary.

§3 Phenomenology in the six-quark scheme

3.1 Parametrization

As seen from (2.29), we have four parameters to describe the Cabibbo-like mixing for N = 3. Here we adopt the following explicit parametrization;[3)]

$$U = \begin{matrix} & d & s & b \\ u & c_1 & -s_1c_3 & -s_1s_3 \\ c & s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_kc_2c_3 + s_2c_3e^{i\delta} \\ t & s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta} & c_1s_2c_3 - c_2c_3e^{i\delta} \end{matrix}, \qquad (3.1)$$

where

$$c_i = \cos\theta_i\,, \quad s_i = \sin\theta_i\,, \qquad i = 1, 2, 3.$$

The phase convention is such that the first low and the first column of U are made real. If δ vanishes, then U is an orthogonal matrix and we have no CP-violation.

In the following we investigate to what extent presently available experimental informations can determine the parameters. Systematical analysis has been done first in ref. 5).

3.2 Nuclear β-decay and $|\Delta s| = 1$ semileptonic decays.

Recently, Shrock and Wang[6)] made reanalysis of the above processes and obtained

$$|U_{ud}| = |\cos\theta_1| = 0.9737 \pm 0.0025$$
$$|U_{us}| = |\sin\theta_1 \cos\theta_3| = 0.219 \pm 0.011 \tag{3.2}$$

from which they concluded

$$|\sin\theta_3| = 0.28 {}^{+0.21}_{-0.28}. \tag{3.3}$$

We note that θ_1 is essentially the Cabibbo angle, and

$$\theta_1 \approx 13.2° \tag{3.4}$$

The magnitude of u-b transition is determined by

$$|U_{ub}| = \sqrt{1 - |U_{ud}|^2 - |U_{us}|^2}$$
$$\approx 0.06 \pm 0.06 \tag{3.5}$$

3.3 K_L-K_S mass difference

Direct measurement of the other components of U_{ij} useful to determine the rest of parameters are not yet available. Therefore, we must resort to some model dependent arguments. The neutral K-meson system has been well investigated for this purpose.

Let us consider the mass matrix M of K^o-$\bar{K}^o$ system. It can be decomposed into the dispersive and absorptive parts;

$$M = m + \frac{i}{2}\Gamma, \tag{3.6}$$

where m and Γ are 2 × 2 Hermitian matrices. Since we are cosidering a

local field theory, CPT invariance should hold, and therefore we have

$$\langle K^o|M|K^o\rangle = \langle \bar{K}^o|M|\bar{K}^o\rangle \tag{3.7}$$

Then, in our phase convention, K_L-K_S mass difference will be approximately given by

$$\Delta m = m(K_S) - m(K_L) \approx 2\mathrm{Re}\langle K^o|m|\bar{K}^o\rangle. \tag{3.8}$$

An estimation of $\langle K^o|m|\bar{K}^o\rangle$ has been made by considering a simple diagram of Fig. 1. The result is given by [7)8)]

$$\langle K^o|m|\bar{K}^o\rangle = -\frac{Bf_k^2 m_k G_F \alpha}{12\sqrt{2}\sin^2\theta} \sum_{i,j=u,c,t} U_{is}U^*_{id}U_{js}U^*_{jd}A_{ij} \tag{3.9}$$

where

$$A_{ij} = \frac{1}{(1-x_i)(1-x_j)} + \frac{1}{(x_i - x_j)}\left[\frac{x_i^2 \ell n x_i}{(1-x_i)^2} - \frac{x_j^2 \ell n x_j}{(1-x_j)^2}\right] \tag{3.10}$$

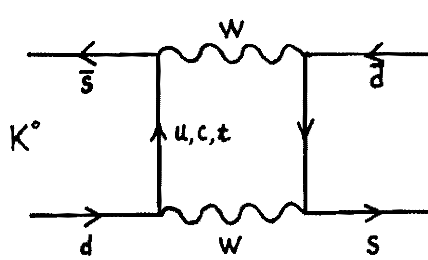

Fig. 1

with

$$x_i = m_i^2/m_W^2 , \tag{3.11}$$

and B is a model dependent factor. The bag model calculation gives $B \approx 0.4$.

In order to see how this formula works, we consider the limited case of $\sin\theta_3 = 0$. Assuming $x_j \ll 1$, we have

$$\Delta m_k \approx -2Bf_k^2 m_k \frac{G_F}{\sqrt{2}} \frac{\alpha}{4\pi} \frac{1}{3\sin^2\theta_W} \frac{m_c^2}{M_W^2} c_1^2 s_1^2 \times \left\{ c_2^4 + s_2^4 \frac{m_t^2}{m_c^2} + c_2^2 s_2^2 \frac{1}{1 - m_c^2/m_t^2} \ln \frac{m_t^2}{m_c^2} \right\} \tag{3.12}$$

which yields

$$c_2^4 + s_2^4 \frac{m_t^2}{m_c^2} + c_2^2 s_2^2 \frac{1}{1 - m_c^2/m_t^2} \ln \frac{m_t^2}{m_c^2} \approx 2.5 \tag{3.13}$$

where we have used $B \approx 0.4$, $f_k \approx 170$ MeV, and $m_c \approx 1.5$ GeV. For example, setting $m_t = 15$ GeV, we have

$$|s_2| \approx 0.33 \tag{3.14}$$

The value of $|s_2|$ decreases with increasing m_t.

As seen from (3.3), $\sin\theta_3$ in not necessarily small. Therefore the required numerical analysis becomes complicated. This, combined with the constraint arising from the CP violation effects, has been done recently in refs. 9) and 10) (See below).

3.4 CP violation

As we have seen in §2-3, in the six-quark scheme we have a possibility of CP violation through the Cabibbo-like mixing. In the following we investigate how the observed CP violating phenomena are described in this scheme. Allowed regions of the parameters are also discussed.

It is worth while reminding the follwoing point. Since the effect of CP violation is very small, the observed CP violaton may arise from other additional interaction which is very weak and practically does not change the ordinary weak interaction phenomenology. In such a case, the following arguments should be changed substantially.

Now, we consider again the K^0-$\bar{K}^0$ system. The mass matrix M contains informations on the CP violation. However, we can not say anything about CP violation from the dispersive part alone. The relevant quantity is the phase difference between $\langle K^0|m|\bar{K}^0\rangle$ and $\langle K^0|\Gamma|\bar{K}^0\rangle$. The latter is given by

$$\langle K^0|\Gamma|\bar{K}^0\rangle = 2\pi\sum_F \rho_F \langle K^0|T|F\rangle\langle F|T|\bar{K}^0\rangle \tag{3.15}$$

where ρ_F is the phase volume factor. Therefore we must investigate first the nonleptonic decay amplitudes.

The $K^0 \to \pi\pi$ amplitudes can be written as

$$\langle I|T|K^0\rangle = e^{i\delta_I} A_I, \quad \langle I|T|\bar{K}^0\rangle = e^{i\delta_I} A_I^*, \tag{3.16}$$

where $\langle I|$ denotes the final $\pi\pi$ state with isospin I, and δ_I is the strong interaction phase shift. Since we have already fixed the phase convention, A_0 is no longer a real quantity in general. It is necessary not to confuse with the usual Wu Yang convention in which A_o is real.

Let us consider the quark diagrams which contribute to the $K^o \to \pi\pi$ decay. The contribution of Fig. 2(a) to A_I is real, because no complex mixing parameters appear in this diagram. Diagrams (b) and (c) contribute only to the $\Delta I = 1/2$ part, and give rise to the imaginary part of A_o. Therefore, we can consider that in our phase convention only A_o is complex and A_2 is real. It has been argued that the contribution of (b) may be small because of the OZI rule. Then the diagram (c), so called penguin diagram, is important for the phase of A_o, as well as for the $\Delta I = 1/2$ rule in nonleptonic decays. Theoretical estimate, however, seems to have some uncertainty. Here, we consider an experimental upper bound of the phase of A_o.

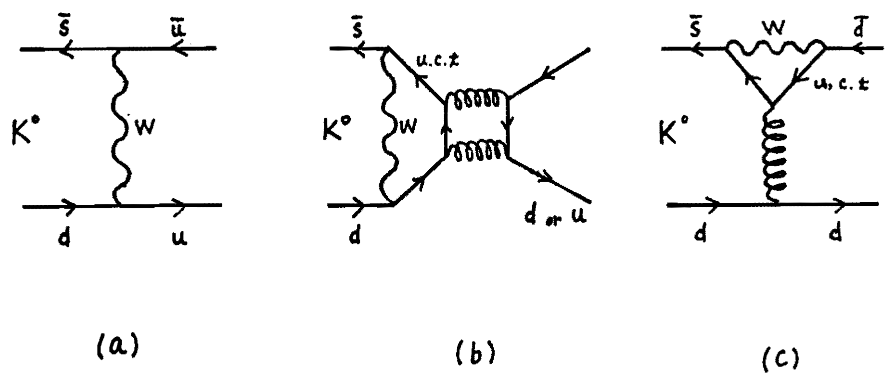

Fig. 2

Let us consider the ratio

$$\left|\frac{\eta_{oo}}{\eta_{+-}}\right| = \left|\frac{\varepsilon - 2\varepsilon'}{1 - 2\omega} \Big/ \frac{\varepsilon + \varepsilon'}{1 + \omega}\right|, \tag{3.17}$$

where

$$\varepsilon = \frac{<0|T|K_L>}{<0|T|K_S>}, \qquad \varepsilon' = \frac{<2|T|K_L>}{\sqrt{2}<0|T|K_S>}, \qquad \omega = \frac{<2|T|K_S>}{\sqrt{2}<0|T|K_S>}, \tag{3.18}$$

Since $\varepsilon' << \varepsilon$, we have

$$\left|\frac{\eta_{oo}}{\eta_{+-}}\right| \approx \left|1 - 3\frac{\varepsilon'}{\varepsilon} + 3\omega\right|$$

$$= \left|1 + i\frac{3}{\sqrt{2}}\left|\frac{A_2}{A_o}\right| e^{i(\delta_2 - \delta_o)} \frac{\alpha}{\varepsilon}\right| \tag{3.19}$$

where α is defined by

$$A_o = |A_o|\, e^{i\alpha}. \tag{3.20}$$

It is known that the contribution of the penguin diagram gives $\alpha < 0$,[11) which implies $|\eta_{oo}/\eta_{+-}| < 1$.

From the present experimental value of

$$\left|\frac{\eta_{oo}}{\eta_{+-}}\right| = 1.02 \pm 0.04, \tag{3.21}$$

we have

$$\left|\frac{\alpha}{\varepsilon}\right| < 0.2 \tag{3.22}$$

It is critical whether this bound is compatible with the contribution of the penguin diagram. Improvement of data on $|\eta_{00}/\eta_{+-}|$ will provide more stringent test of this scheme.

Now we turn to the estimation of the mixing parameters. In terms of the mass matrix elements. ε is given by

$$\varepsilon = \frac{\mathrm{Im}\langle K^0|m|\bar{K}^0\rangle - \frac{i}{2}\mathrm{Im}\langle K^0|\Gamma|\bar{K}^0\rangle}{i\Delta m + \frac{1}{2}\Delta\Gamma} + i\alpha$$

$$\simeq \frac{\mathrm{Im}\langle K^0|m|\bar{K}^0\rangle - \Delta m\alpha}{i\Delta m + \frac{1}{2}\Delta\Gamma} \qquad (3.23)$$

where we have used the approximate relation

$$\mathrm{Im}\langle K^0|\Gamma|\bar{K}^0\rangle \simeq \alpha\Delta\Gamma\ , \qquad (3.24)$$

which holds because the main contribution to Γ comes from the I = 0 $\pi\pi$ state. From (3.22), α in (3.23) may be negligible. In this case, ε is given by in terms of the mixing parameters by using (3.9), and experimental constraint on them can be obtained. References 9) and 10) made detailed numerical analyses. Results are seen in Fig. 3, which show regions allowed by Δm and ε. According to the sign of $\cos\delta$, $\xi = \cos\delta/|\cos\delta|$, we have two solutions for each m_t and s_3.

Although the constraints obtained above are rather loose, we can extract interesting consequences on the heavy quark decays. For example, we can say, that the branching ratio of b → c decay will be larger than b → u. Further discussion is seen in ref. 9) and 10).

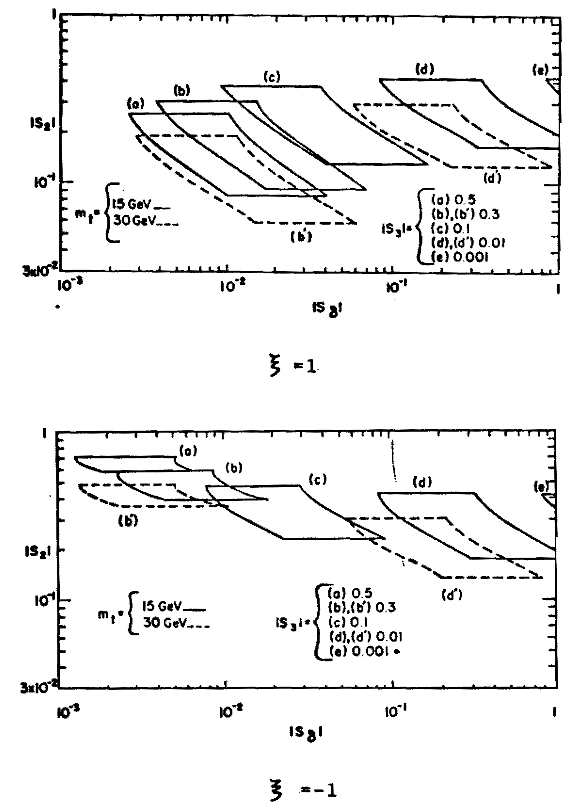

Fig. 3

Allowed regions of $|\sin\theta_2|$ and $|\sin\delta|$. (From ref. 10))

§4 Relations between masses and mixing angles.

4.1 Diagonalization of the mass matrix

In order to understand the close relation between quark masses and mixing angles, we investigate again the origin of quark mass terms. In §2 we started with the gauge multiplets which are written in terms of the physical quark fields. Here we go back to a more fundamental step.

Let us consider N left-handed doublets and 2N right-handed singlets:

$$\begin{pmatrix} p_i \\ n_i \end{pmatrix}_L, \qquad p_{iR}, \qquad n_{iR}. \qquad i = 1,\dots, N$$
$$Y = 1/3 \qquad Y = 4/3 \qquad Y = -2/3 \tag{4.1}$$

where p_i and n_i are not yet related to the physical quark fields. Relations should be determined by diagonalizing the mass terms. The most general Yukawa interaction with the Higgs field invariant under the $SU(2) \times U(1)$ transformation is given by

$$L = -\sum_{i,j=1}^{N} \{g_{ij}\bar{p}_{iR}\tilde{\Phi}^{+}\begin{pmatrix} p_j \\ n_j \end{pmatrix}_L + g^{*}_{ij}[\bar{p}_j\bar{n}_j]_L\tilde{\Phi}p_{iR}\}$$
$$-\sum_{i,j=1}^{N} \{h_{ij}\bar{n}_{iR}\Phi^{+}\begin{pmatrix} p_j \\ n_j \end{pmatrix}_L + h^{*}_{ij}[\bar{p}_j\bar{n}_j]_L\Phi n_{iR}\}. \tag{4.2}$$

Replacing Φ with its vacuum expectation value, we have

$$L = -\frac{v}{\sqrt{2}}\sum_{i,j=1}^{N}\{g_{ij}\bar{p}_{iR}p_{iL} + g^{*}_{ij}\bar{p}_{jL}p_{iR}\}$$
$$-\frac{v}{\sqrt{2}}\sum_{i,j=1}^{N}\{h_{ij}\bar{n}_{iR}n_{jL} + h^{*}_{ij}\bar{n}_{jL}n_{iR}\}$$
$$= -\{\bar{p}_R M_p p_L + \bar{p}_L M_p^{+} p_R + \bar{n}_R M_n n_L + \bar{n}_L M_n^{+} n_R\} \tag{4.3}$$

where we have used a matrix notation with $N \times N$ matrices M_p and M_n defined by

$$(M_p)_{ij} = \frac{v}{\sqrt{2}}g_{ij}, \qquad (M_n)_{ij} = \frac{v}{\sqrt{2}}h_{ij}. \tag{4.4}$$

Note that p_R and p_L, as well as n_R and n_L, are independent quantities and therefore M_p and M_n are not necessarily Hermitian. Diagonalization is performed by the unitary transformations

$$p'_L = U_p p_L, \qquad p'_R = V_p p_R,$$
$$n'_L = U_n n_L, \qquad n'_R = V_n n_R. \tag{4.5}$$

The unitary matrices U_p, V_p, U_n and V_n are determined so that M_p and M_n become diagonal;

$$V_p M_p U_p^{+} = \begin{pmatrix} m_u & & 0 \\ & m_c & \\ 0 & & \ddots \end{pmatrix}$$
$$V_n M_n U_n^{+} = \begin{pmatrix} m_d & & 0 \\ & m_s & \\ 0 & & \ddots \end{pmatrix} \tag{4.6}$$

This is possible, since any complex matrices can be diagonalized by two unitary matirces as shown below.

Let us consider an arbitrary matrix M. Since $M^{+}M$ is Hermitian, usual diagonalization can be performed by a unitary matrix U as

$$UM^{+}MU^{+} = \begin{pmatrix} \mu_1 & & 0 \\ & \ddots & \\ 0 & & \mu_N \end{pmatrix} \equiv \mu \tag{4.7}$$

It is easy to see that $\mu_i \geq 0$. For the purpose of simplicity, let us assume here that no zero eigenvalues exist. Then, we can define

$$V \equiv \mu^{1/2} U M^{-1} \tag{4.8}$$

Since we have

$$\begin{aligned} VV^{+} &= \mu^{1/2} U M^{-1} (M^{-1})^{+} U^{+} \mu^{1/2} \\ &= \mu^{1/2} (U M^{+} M U^{+})^{-1} \mu^{1/2} = 1 \,, \end{aligned} \tag{4.9}$$

V is unitary. Equation (4.8) can be rewritten as

$$V M U^{+} = \mu^{1/2}, \tag{4.10}$$

which completes the proof.

Now the original left-handed doublets are expressed in terms of the physical quark fields as

$$\begin{bmatrix} p_\ell \\ n_\ell \end{bmatrix}_L = \begin{bmatrix} \sum_{i=\{u,c\ldots\}} (U_p^{+})_{\ell i} \psi_{iL} \\ \sum_{j=\{d,s\ldots\}} (U_n^{+})_{\ell j} \psi_{jL} \end{bmatrix}, \qquad \ell = 1, \ldots\ldots N \,. \tag{4.11}$$

By taking linear combinations of them, we can rewrite them in the following way;

$$\begin{bmatrix} \psi_i \\ \sum_{j=\{d,s,\ldots\}} (U_p U_n^{+})_{ij} \psi_j \end{bmatrix}_L \qquad i = u,\ c, \ldots \tag{4.12}$$

This is nothing but (2.19) with

$$U = U_p U_n^{+} \,. \tag{4.13}$$

From the above arguments, we can see that the original Higgs coupling (4.2) completely determines both the quark masses and mixing angles. As seen in (4.13), the Cabibbo-like mixing arises from the difference between U_p and U_n.

In the present scheme, however, no constraints are imposed on the mass matrices M_p and M_n by the SU(2) × U(1) gauge invariance alone. Accordingly it has no predictive powers on the quark masses and the mixing angles. It is hard to think that a fundamental theory has so many free parameters. We believe that there exists a missing principle which governs Higgs interactions.

An attempt, which may provide a hint to this problem, is to derive some relations between masses and mixing angles by imposing higher symmetries. This will be discussed in the next subsection.

4.2 Discrete symmetry and Cabibbo angle

Gauge symmetry larger than SU(2) × U(1) have been considered from many motivations. Another possibility of a larger symmetry is a dicrete symmetry, which is fully utilized in the following discussions. The basic idea is to obtain constraints on the mass matrices as a result of certain discrete symmetries. Below we illustrate it in a simple model.[12)]

Let us consider the $SU(2)_L \times SU(2)_R \times U(1)$ gauge theory instead of SU(2) × U(1).*) To make a discussion simple, we consider the four-quark case. Then, we have

---

*) We have interesting models within the SU(2) × U(1) gauge theory.[13)] They, however, require rather involved calculations.

$$L_i = \begin{pmatrix} p_i \\ n_i \end{pmatrix}_L , \qquad R_i = \begin{pmatrix} p_i \\ n_i \end{pmatrix}_R , \qquad i = 1, 2$$

$$(\underline{2}, \underline{1}) \qquad\qquad (\underline{1}, \underline{2}) \tag{4.15}$$

where $(\underline{2}, \underline{1})$, for example, implies $SU(2)_L$ doublet and $SU(2)_R$ singlet. Higgs fields are two complex 2×2 matrices;

$$\phi \quad , \quad \chi$$
$$(\underline{2}, \underline{2}) \quad (\underline{2}, \underline{2}) \tag{4.16}$$

Note that $\tilde{\phi} \equiv \tau_2 \phi^* \tau_2$ and $\tilde{\chi} \equiv \tau_2 \chi^* \tau_2$ are also transformed as $(\underline{2}, \underline{2})$ representation. In this system, the most general Higgs-quark coupling invariant under $SU(2)_L \times SU(2)_R \times U(1)$ is

$$L = - \sum_{i,j=1}^{2} \{ g_{ij} \bar{R}_i \phi L_j + g'_{ij} \bar{R}_i \tilde{\phi} L_j + h_{ij} \bar{R}_i \chi L_j + h'_{ij} \bar{R}_i \tilde{\chi} L_j \} + \text{h.c.} \tag{4.17}$$

At this stage we have no constraints on the mass matrices for general vacuum expectation values of ϕ and χ. First we assume a discrete symmetry under the following transformation;

$$L_j \to iL_j , \qquad R_j \to R_j , \qquad j = 1, 2$$
$$\phi \to -i\phi , \qquad \chi \to -i\chi . \tag{4.18}$$

From this invariance we have

$$g'_{ij} = h'_{ij} = 0. \tag{4.19}$$

Next, we require a further discrete symmetry under the transformation,

$$L_1 \to iL_1 , \qquad R_1 \to -iR_1 ,$$
$$L_2 \to L_2 , \qquad R_2 \to R_2 ,$$
$$\phi \to \phi , \qquad \chi \to -i\chi , \tag{4.20}$$

from which we obtain

$$g_{11} = g_{12} = g_{21} = h_{11} = h_{22} = 0. \tag{4.21}$$

Finally we assume the invariance under parity transformation:

$$L_i \leftrightarrow R_i , \qquad \phi \to \phi^+, \qquad \chi \to \chi^+. \tag{4.22}$$

Then (4.17) becomes

$$L = -g\bar{R}_2 \phi L_2 - h\bar{R}_1 \chi L_2 - h^* \bar{R}_2 \chi L_1 + \text{h.c.} \tag{4.23}$$

where g is a real constant.

General vacuum expectation values of ϕ and χ are given by

$$\langle\phi\rangle = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 \end{pmatrix}, \qquad \langle\chi\rangle = \begin{pmatrix} x_1 & 0 \\ 0 & x_2 \end{pmatrix} \tag{4.24}$$

where v_i and x_i are complex numbers. Replacing ϕ and χ in (4.23) by (4.24), we have quark mass terms of the following form:

$$L = -(\bar{p}_1\ \bar{p}_2)_R \begin{pmatrix} 0 & a' \\ a & b \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}_L - (\bar{n}_1\ \bar{n}_2)_R \begin{pmatrix} 0 & c' \\ c & d \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}_L + \text{h.c.}$$

$$= -\bar{p}_R M_p p_L - \bar{n}_R M_n n_L + \text{h.c.} \qquad (4.25)$$

where a, a', b, c, c' and d are complex numbers with

$$|a| = |a'|, \qquad |c| = |c'| . \qquad (4.26)$$

Special forms of mass matrices should be noted.

Now we diagonalize these mass matirces in the following way:

$$V_p M_p U_p^+ = \begin{pmatrix} m_u & 0 \\ 0 & m_c \end{pmatrix} \qquad V_n M_n U_n^+ = \begin{pmatrix} m_d & 0 \\ 0 & m_s \end{pmatrix} \qquad (4.27)$$

It is convenient to devide the transformation into two steps:

$$U_p = U'_p \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi_p} \end{pmatrix}, \qquad V_p = V'_p \begin{pmatrix} e^{i\xi_p} & 0 \\ 0 & e^{i\zeta_p} \end{pmatrix} \qquad (4.28)$$

with

$$\psi_p = \arg.(b) - \arg.(a), \qquad \zeta_p = -\arg.(a)$$
$$\xi_p = \arg.(b) - \arg.(a) - \arg.(a'). \qquad (4.29)$$

Then we have

$$V_p M_p U_p^+ = V_p'^+ \begin{pmatrix} 0 & |a| \\ |a| & |b| \end{pmatrix} U_p'^+. \qquad (4.30)$$

Now it is easy to see that

$$V'_p = U'_p = \begin{pmatrix} \cos\theta_p & -\sin\theta_p \\ \sin\theta_p & \cos\theta_p \end{pmatrix} \qquad (4.31)$$

with

$$\tan\theta_p = \sqrt{m_u/m_c} . \qquad (4.32)$$

Similarly we have

$$U_n = \begin{pmatrix} \cos\theta_n & -\sin\theta_n \\ \sin\theta_n & \cos\theta_n \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi_n} \end{pmatrix}, \qquad (4.33)$$

with

$$\tan\theta_n = \sqrt{m_d/m_s}$$

$$\phi_n = \arg.(d) - \arg.(c). \qquad (4.34)$$

Then, the Cabibbo mixing matrix is given by

$$U = U_p U_n^+$$
$$= \begin{pmatrix} c_p c_n + e^{i\Delta} s_p s_n & c_p s_n - e^{i\Delta} s_p c_n \\ c_n s_p - e^{i\Delta} c_p s_n & s_p s_n + e^{i\Delta} c_p c_n \end{pmatrix} \qquad (4.35)$$

where

$$\Delta = \phi_p - \phi_n , \qquad c_p = \cos\theta_p , \qquad s_p = \sin\theta_p , \quad \text{etc. .} \qquad (4.36)$$

Reminding the arguments in §2-3, we have

$$|\tan\theta_c| = \left|\frac{\tan\theta_n - e^{i\Delta}\tan\theta_p}{1 + e^{i\Delta}\tan\theta_p\tan\theta_n}\right| . \qquad (4.37)$$

Since $\sqrt{m_u/m_c} << 1$, we can write this as

$$|\tan\theta_c| \gtrsim \sqrt{m_d/m_s} .$$

It is well known that this relation is phenomenologically well-satisfied. From the arguments on the chiral symmetry breaking we have an estimation

$$m_d/m_s \gtrsim 1/20 , \qquad (4.38)$$

which should be compared with

$$|\tan\theta_c|^2 \gtrsim 0.055. \qquad (4.39)$$

In this way, we have obtained a remarkable relation by requiring discrete symmetries. Similar discussions on the six-quark case are seen in ref. 14). Although these arguments are very suggestive, the discrete symmetries which we have considered are rather artificial and their physical meanings are not clear. Further studies are required to reveal a new physical principle.

References

1) S. Weinberg; Phys. Rev. Letters 19 (1967) 1264;
A. Salam; in "Elementary Particle Theory" edited by N. Svartholm (Almquist and Wiksell, Stockholm, 1968) P.367.

2) S. L. Glashow, J. Iliopoulos and L. Maiani; Phys. Rev. D2 (1970) 1285.

3) M. Kobayashi and T. Maskawa; Prog. Theor. Phys. 49 (1973) 652.

4) S. Weinberg; Phys. Rev. Letters. 37 (1976) 657.

5) J. Ellis, M. K. Gaillard and D. V. Nanopoulos; Nucl. Phys. B109 (1976) 213.

6) R. E. Shrock and L. L. Wang; Phys. Rev. Letters 41 (1978) 1692.

7) M. K. Gaillard and B. W. Lee; Phys. Rev. D10 (1974) 897.

8) Poggio and H. Schnitzer; Phys. Rev. D15 (1977) 1973;
T. P. Cheng and L. F. Li; Phys. Rev. D16 (1977) 1425.

9) V. Barger, W. F. Long and S. Pakvasa; Phys. Rev. Letters 42 (1979) 1585

10) R. E. Schrock, S. B. Treiman and L. L. Wang; Phys. Rev. Letters 42 (1979) 1589.

11) F. J. Gilman and M. B. Wise; Phys. Letters 83B (1979) 83.

12) S. Weinberg; The problem of mass, Harvard preprint (1979);
H. Fritzsch; Phys. Letters 70B (1977) 436.

13) S. Pakvasa and H. Sugawara; Phys. Letters 82B (1979) 105.

14) H. Fritzsch; Phys. Letters 73B (1978) 317;
T. Hagiwara, T. Kitazoe, G. B. Mainland and K. Tanaka; Phys. Letters 76B (1978) 602;
A. Ebrahim; Phys. Letters 76B (1978) 605.