RELATIVISTIC ELECTRON COOLING FOR HIGH LUMINOSITY PROTON-ANTIPROTON COLLIDING BEAMS AT VERY HIGH ENERGIES

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Electron cooling has been introduced by Budker\textsuperscript{4} in order to extend to heavy particle beams most of the benefits of damping by synchrotron radiation, which is a very powerful tool in the process of accumulation and collision in e\textsuperscript{+}e\textsuperscript{-} storage rings. Assume an intense electron beam in contact with heavy particles (e.g., protons or antiprotons) stored in a ring. Provided the average electron and proton velocities are adjusted to be closely equal and the electron temperature (e.g., the residual kinetic energy of electrons in the frame moving with the common average velocity) is sufficiently small, in favorable conditions the proton temperature will decrease up to about twice the electron temperature. This means that the angular divergence of the stored proton beam $\theta_p$ will decrease eventually up to a value

$$\theta_p = \sqrt{\frac{2m_e^3}{n_e M^2} \theta_e},$$

where $\theta_e$ is the angular divergence of the electrons, and $m$ and $M$ are respectively the electron and proton masses. A similar damping is expected to occur in the longitudinal motion.

As is well known, extensive experiments\textsuperscript{2} carried out at Novosibirsk with the storage ring NAP-M have demonstrated cooling of 65 MeV protons by electrons trapped in a solenoidal magnetic field. Their results are in satisfactory agreement with theory, once the specific properties of the proton and of the electron motions have been taken into account.

It is generally believed that because of the extremely fast energy dependence of the formula giving the cooling time, which for constant $\theta_p$ goes approximately as $\gamma^5\beta^4$, cooling with electrons is only possible at very low energies. In the present note, we discuss a number of practical arrangements in which we succeed in overcoming the large power-law effects by the reduced beam sizes and most important, the incredibly large current densities of electrons which can be obtained by synchrotron damping of the electron beam. We propose feasible schemes in which a relatively modest device can continuously cool protons and antiprotons even at ultra-high energies like those in the CERN-SPS ($\gamma=300$) and in the Fermilab-ED ($\gamma=1000$) with remarkable improvements of beam stability and luminosity.

In order to understand the implications of electron cooling for $\bar{p}-p$ colliders at high energies, we start with the formula giving the luminosity $L$ for head-on collisions

$$L = \frac{f n^2 b}{4\pi \sigma_x \sigma_y} = \frac{n^2 b}{4\pi \sigma^2},$$

where $n$ is the number of particles in each of the $b$ bunches, $\sigma_x = \sigma_y = \sigma$ is the beam rms cross section at the crossing point and $f$ is the revolution frequency. On the other hand, the tune shift $\Delta Q$ coming from beam-beam interactions

$$\Delta Q = \frac{\beta^*}{4\pi \gamma^2},$$

where $\beta^* = \beta_p = \beta_e$ is the value of the beta function at the crossing point, $r_p = 1.53 \times 10^{-16}$ cm is the classical proton radius and $\gamma$ the usual relativistic factor. Combining the two formulae we get

$$L = f \frac{\Delta Q \gamma \eta n \beta}{r_p \beta^*},$$

where one can see that for a given $\Delta Q$ the luminosity is independent of the number of bunches and linearly proportional to the total number of particles $N = bn$. Setting $L = 10^{30}$ cm$^{-2}$ sec$^{-1}$, $\Delta Q = 10^{-2}$, $\gamma = 280$, $\beta^* = 1$ m and $f = 43.4$ kc/s we get $N = 1.25 \times 10^{11}$. The corresponding invariant emittance $\varepsilon_0$ of the two beams, defined as the two s. d. point of the Gaussian approximation is easily evaluated

$$\varepsilon_0 = \frac{r_e \beta^*}{\Delta Q}.$$
velocity in the particle frame will become lower than the rms velocity of the electrons. This, in turn, is equivalent in the laboratory system to the condition \( \theta_e \geq \theta^* \). The frictional force of the electrons on the protons is then proportional to the residual proton velocity in the particle frame and the damping constant \( \tau_e \) due to the electron friction force is independent of the amplitude of the proton oscillations. For a velocity distribution of the electron beam spatially symmetric and Gaussian and a constant density over the cross section of the beam, the damping constant is given more precisely by the formula

\[
\tau_e = \frac{3}{2\sqrt{2\pi}} \frac{\beta \gamma^2 e}{r_p r_e \eta J L} \left( \frac{T_e}{mc^2} \right)^{3/2},
\]

where \( T_e = \beta^2 \gamma^2 \frac{mc^2}{2} \) \( \theta_e^2 \) is the electron temperature and \( j \) is the current density.

Of course the central question is what electron current density and temperature we can achieve in practice. The proton beam has a very small (circular) cross section \( \psi_0 = \psi_0 (E_0/\gamma) \) where \( \psi_0 \) is the (average) value of the beta function at the cooling straight section. Inserting \( \psi_0 = 60 \text{ m}, \gamma = 280 \) and for the value of the emittance given in Eq. (4) for \( n = 2 \times 10^{10} \) we find \( \psi_0 = 0.66 \text{ mm}^2 \). A practical electron beam will cover perhaps twice this area in order to insure convenient matching. For a reasonable number of bunches, the electron beam cross section can be only of order of a fraction of \( \text{mm}^2 \). Even relatively modest electron currents can be used to achieve substantial current densities.

A crucial feature of the cooling with relativistic electrons is the fact that the longitudinal temperature \( T_{eL} \) is insensitive to the laboratory energy spread. The Lorentz transformation from the laboratory to the moving particle frame is surprisingly favorable;

\[
T_{eL} = \frac{1}{2} m_e \epsilon^2 \frac{\delta p}{p_e} \frac{\delta p}{p_e} \frac{\delta p}{p_e},
\]

For a relatively large momentum spread \( \delta p/p_{rms} = 10^{-3} \) in the laboratory frame, we find \( T_e \approx 0.25 \text{ eV} \). Likewise, the condition that the proton velocity in the moving frame should be no greater than the proton velocity in the laboratory condition \( \frac{\delta p}{p} \ll \frac{\delta p}{p_{rms}} \) which is easily satisfied by the proton beam. On the other hand, substantial momentum spreads are needed to insure stability of both proton and electron bunches. For a parabolic distributions in line density and a peak current \( I_0 \), bunches are stable provided

\[
\left| \frac{Z}{n} \right| \leq 1.92 \eta \frac{P}{I_0} \left( \frac{\delta p}{p_{rms}} \right),
\]

where \( Z \) is the longitudinal impedance for the n mode and \( \eta \) is the usual factor relating the change in period to the change of momentum. For \( |Z/n| = 20 \text{ ohm} \) (which can probably be achieved with a small weak focusing electron ring), \( \eta = 0.7 \) and \( p_0 = 143 \text{ MeV/c} \), we get \( I_0 \leq 10^8 \text{ A} \). Applying the corresponding expression to the protons, with \( |Z/n| = 25 \text{ ohm} \), \( \eta = 1.72 \times 10^{-3} \), \( \delta p/p = 10^{-3} \) we find \( I_p \leq 36 \text{ A} \), which is a very safe value.

The transverse temperature \( T_{eL} \) is directly related to the rms angular divergence. The requirement \( T_{eL} = T_{eL}' \) gives \( \theta_e' \approx \theta_e \) \( T_{eL} \) is the electron temperature and it is likely that at high energies one has to accept \( T_{eL} \geq T_{eL}' \).

Cooling is needed at high energies in order to compensate for beam growth due to beam-gas scattering, higher-order resonances, longitudinal and transverse instabilities, intra-beam scattering and so on. In order to have a first-order estimate of the cooling rate which is required, we shall estimate simply the effect of the multiple scatterings with the residual gas. The time constant for beam growth due to multiple scattering in absence of cooling is given by

\[
\tau = \frac{1}{n_{rms}} \int \frac{d\phi}{\delta p} = \frac{1.08 \times 10^{-3} \text{ m}^2 \text{ sec}^{-1}}{\gamma \beta_o \text{ m}^2},
\]

where \( K = 4\pi^2 (m_e/c) \gamma^2 \text{ c G2} / \text{N2} \), \( \text{c G2} / \text{N2} \) is the absolute gas factor for \( \text{N2} \) and \( n_{rms} \) is the equivalent density of nitrogen atoms for multiple scattering. It is related to the equivalent multiple scattering pressure \( P_{rms} \) (Torr) by

\[
\frac{1}{n_{rms}} = 1.93 \times 10^{25} \left[ \text{m}^{-3} \right],
\]

where \( T \) is the absolute temperature of the residual gas. For instance, setting \( \beta_o = 60 \text{ m}, P_{rms} = 10^{-3} \text{ Torr}, T = 300 \text{ K}, \gamma \beta_o = 280 \) and \( P_0 = 3.2 \times 10^{-2} \text{ rad m sec}^{-1} \), we calculate \( n_{rms} = 2.14 \times 10^4 \text{ sec}^{-1} \).

The balance equation between the damping and diffusion processes (Langevin equation) has the form

\[
\frac{d\theta^2}{dt} + \tau_e \frac{d\theta^2}{dt} = \frac{2 \theta^2}{\tau_e} + \frac{n}{t} \left[ \frac{d\theta^2}{dt} \right]_1,
\]

where \( \tau_e \) is the cooling time constant and \( \left[ \frac{d\theta^2}{dt} \right]_1 \) is the diffusion rate for the corresponding process. The solution in case of cooling competing with just multiple scattering has been given in Ref. 2. The equilibrium value of the square of the mean proton angle is

\[
\frac{\theta^2}{\beta_p^2} = 2 \frac{m_e}{Me} \frac{\theta^2}{\beta_e^2} + \frac{18}{\pi} \frac{e}{\gamma \beta_e^2} \frac{Z}{n_{rms}} L_z \frac{m_e}{Me} \frac{T_e}{L \beta_e^3} \left( \frac{\theta^2}{\beta_p^2} \right)^{3/2},
\]

where \( L_z = \ln (133 Z^{-1/3}) \) is the Coulomb logarithm for scattering on the nucleus of charge \( Z \).

Electron cooling will counteract Coulomb scattering if \( \tau_e = \tau_e \). The corresponding electron current density is easily calculated combining Eq. (6) and Eq. (9).
The minimum current density is inversely proportional to the equilibrium emittance of the proton beam $E_0$. Since the proton beam cross section and therefore the electron beam cross section are proportional to $E_0$, the total current is independent of the beam emittance. This is easily understood since a very small beam has a faster growth rate and therefore also needs more efficient cooling. As a numerical example, we can take the emittance from Eq. (4) for six bunches and insert the following numerical values in Eq. (13): $\beta = 60 \text{ m, } \gamma = 4.65 \times 10^{-13}$ ($P_0 = 10^{-17}$ Torr), $n = 5 \times 10^{-3}$, $\gamma = 280$, $E_0 = 3.2 \times 10^{-16}$ rad-m and $T_e = 0.5$ eV. We find $j = 0.57 A/mm^2$. The proton beam rms radius is $r_p = \frac{1}{2}(P_0 \beta \gamma)^{1/2} = 0.47 \text{ mm}$. A reasonably well matched electron beam could have twice the rms radius of the protons, that is a cross-sectional area of about $a_e = 2.8 \text{ mm}^2$ or a total current $I_e = 1.5 \times 10^6 \text{ A}$. 

A transverse temperature of $T_{e,1} = 0.5$ eV correspond to rms angular divergence $\theta_e = 1/\beta \gamma [T_{e,0}/(M_0 e^2)]^{1/2}$ and for a rms radius of 0.66 mm, we find an invariant emittance (2 s.d.) $E_0 = 4.1 \times 10^{-6} \text{ rad-m}$, which is comparable to that of the protons. On the other hand, longitudinal temperature $T_{e,11} = 0.5$ eV corresponds to about $\Delta p/\gamma P_{\text{rms}} = 1.4 \times 10^{-3}$ which is substantially wide. An identical condition holds also for the proton beam.

We note that the previously indicated longitudinal impedance of $Z/n = 20 \text{ ohm}$ for the electron ring, when combined with Eq. (8), gives us a maximum electron current of 20A, which is about fifteen times what is required to counterbalance the beam-gas collisions. Although other forms of instabilities of the electron beam still need to be investigated, it is likely that we shall end up with a lot of spare cooling capacity to counteract, if necessary, more virulent instabilities.

The tune shift $\Delta Q$ produced by the electron current on the proton beam limits the current density to the value

$$j \leq 2 Q(\Delta Q) \frac{e c p^3 Y_3}{T R_0 \tau_p},$$

where $l = 20 \text{ m}$ is the length of the cooling region, $R_0$ is the radius and $Q$ is the tune of the SPS and other symbols have the same meaning as in the previous formulae. Setting $\Delta Q = 10^{-3}$ we get $j \leq 4.6 \times 10^3 A/mm^2$ which is safely beyond any practical value.

Lifetime of the beam in absence of other effects will be determined by nuclear collisions and single large-angle scatterings. The beam-gas lifetime for nuclear collisions is given by

$$\frac{1}{\tau_{ns}} = \frac{1}{I} \frac{\sigma I}{dt} = 7.32 \times 10^2 P_{ns} \text{ (Torr)},$$

where $P_{ns}$ is the residual $N_2$ equivalent pressure. For $P_{ns} = 10^4$ Torr we find $\tau_{ns} = 4.37 \times 10^6 \text{ s}$ or about 16 days. The single Coulomb scattering lifetime depends on the limiting aperture $b_0$.

The elastic Coulomb cross section of nuclei of charge $Z$ for very small $q^2$ can be approximated as

$$\frac{d \sigma}{dq^2} = 270 Z^2 (\text{nb/(GeV}^2)) \times \frac{1}{q^4 (\text{GeV})},$$

where $q^2 = \beta^2 \gamma^2$ is the $q^2$ in the scattering. Integrating the cross sections for all scattering angles larger than $\theta_\gamma$ gives $p = (270 \times 10^{-3} Z^2) / (\beta^2 \gamma^2)$. Replacing variables $p$ and $\theta_\gamma$ with more convenient quantities, we get

$$\frac{1}{\tau_{ss}} = 6.33 \times 10^3 m^4 s^{-1} \text{ Torr}^{-1} \frac{Z^2}{\gamma^2 P_{ns}} P_{ss}.$$ Setting $b_0 = 2 \text{ mm}$ for instance, we find $\tau = 4.98 \times 10^6 \text{ sec}$ or about 56 days.

The applicability of the scheme to the $\theta-P$ in the SPS is examined in more detail. The longer lifetime of beams suggests a longer collision time and therefore a longer accumulation time of $\theta-P$s. Assuming that 48 hrs is the largest time period over which accumulation can be practically envisaged, for the design performance of the source, we get $N = nb = 1.2 \times 10^{12}$ $\theta$. Inserting this number in Eq. (3) and for standard values of $\beta$ and $\Delta Q = 10^{-2}$ we get

$$L = 10^{31} \text{ cm}^2 \text{ sec}^{-1}.$$ Longitudinal instabilities amongst other reasons suggest that individual bunches should not contain more than approximately $10^{11}$ particles. A preferable value could be $2 \times 10^{10}$, which has already been achieved, giving $b = 60$. Bunches are separated by about 115 m or 0.38 $\mu$s, which is acceptable for manipulation. The invariant emittance during collisions (at $\gamma = 280$) is held constant with an appropriate balance between cooling and gas scattering to the value of Eq. (4), namely $E_0 = 3.1 \times 10^{-6} \text{ rad-m}$.

The longitudinal area of bunches could be as large as 1.4 m. For $V_o = 4.4 \text{ MV}$ and other standard rf parameters, we expect $\Delta p/P$ full = $1.8 \times 10^{-3}$ and a bunch length which is about 0.4 of the bunch separation. The rms betatron beam cross section in a middle of a straight section is $\sigma = 0.17 \text{ mm}^2$. Note that the momentum spread gives an additional contribution to the width, which is $\Delta x = \Delta p/P \cdot \sigma P = 3.6 \text{ mm}$ for $\sigma p = 2 \text{ m}$. We can either locate the cooling section in a straight section with $\sigma P = 0$ or take advantage of the dispersion by matching it to an energy-modulated electron beam. The cooling time (assuming $\sigma p = 0$) is given by Eq. (6)

$$\frac{3/2}{\tau_e} = \frac{4.81 \times 10^4 T_e}{J(a/mm^2)}$$

where $T_e$ is the electron beam temperature.
Assume an electron beam cross section which is four times the rms of the proton beam. Let us also take a maximum electron current of 4A at $\gamma_0 = 280$, corresponding to $p_e = 143$ MeV/c. Then $j = 4/(0.17 \times 4) = 5.9$ A/mm$^2$. For an electron ring which is approximately 50 m in circumference and electron and proton bunches of matched lengths, the average circulating electron current is then 50 mA, which is within the range of achieved performances. If the electron temperature is taken to be $T_e = 0.5$ eV, we find a cooling time $T_e = 2891$ sec, to be compared to the multiple-scattering lifetime of $21,400$ sec. Clearly there is about a factor ten of safety.

In order to obtain a beam with the indicated transverse emittance, the design emittance of the accumulator ring has to be reduced a few times. This can be done by the cooling itself during the first few hours of collisions or by precooling at somewhat lower energy where the cooling time is greatly diminished.

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References

2. G. I. Budker et al., BNL-TR-6335 (Translation) and Particle Accelerators to be published.