RELATIVISTIC ELECTRON COOLING FOR HIGH LUMINOSITY PROTON-ANTIPROTON COLLIDING BEAMS AT VERY HIGH ENERGIES

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Electron cooling has been introduced by Budker[1] in order to extend to heavy particle beams most of the benefits of damping by synchrotron radiation, which is a very powerful tool in the process of accumulation and collision in e^+e^- storage rings. Assume an intense electron beam in contact with heavy particles (e.g. protons or antiprotons) stored in a ring. Provided the average electron and proton velocities are adjusted to be closely equal and the electron temperature (e.g. the residual kinetic energy of electrons in the frame moving with the common average velocity) is sufficiently small, in favorable conditions the proton temperature will decrease up to about twice the electron temperature. This means that the angular divergence of the stored proton beam θ_p will decrease eventually up to a value

$$\theta_p = \sqrt{\frac{2m_e}{M}}\ \theta_e ,$$

where θ_e is the angular divergence of the electrons, and m_e and M are respectively the electron and proton masses. A similar damping is expected to occur in the longitudinal motion.

As is well known, extensive experiments[2] carried out at Novosibirsk with the storage ring NAP-M have demonstrated cooling of 65 - MeV protons by electrons trapped in a solenoidal magnetic field. Their results are in satisfactory agreement with theory, once the specific properties of the proton and of the electron motions have been taken into account.

It is generally believed that because of the extremely fast energy dependence of the formula giving the cooling time, which for constant θ_p goes approximately as $\gamma^5\beta^4$, cooling with electrons is only possible at very low energies. In the present note, we discuss a number of practical arrangements in which we succeed in overcoming the large power-law effects by the reduced beam sizes and most important, the incredibly large current densities of electrons which can be obtained by synchrotron damping of the electron beam. We propose feasible schemes in which a relatively modest device can continuously cool protons and antiproton beams even at ultra-high energies like those in the CERN-SPS (γ=300) and in the Fermilab-ED (γ=1000) with remarkable improvements of beam stability and luminosity.

In order to understand the implications of electron cooling for $\bar{p}$-p colliders at high energies, we start with the formula giving the luminosity L for head-on collisions

$$L = \frac{f n^2 b}{4\pi\, \sigma_x \sigma_y} \simeq \frac{f n^2 b}{4\pi \sigma^2} , \qquad (1)$$

where n is the number of particles in each of the b bunches, $\sigma_x = \sigma_y = \sigma$ is the beam rms cross section at the crossing point and f is the revolution frequency. On the other hand, the tune shift ΔQ coming from beam-beam interactions

$$\Delta Q = \frac{r_p\, n\, \beta^*}{4\pi\, \gamma\, \sigma^2} , \qquad (2)$$

where $\beta^* = \beta_x = \beta_y$ is the value of the beta function at the crossing point, $r_p = 1.53 \times 10^{-16}$ cm is the classical proton radius and γ the usual relativistic factor. Combining the two formulae we get

$$L = \frac{f\, \Delta Q\, \gamma\, nb}{r_p \beta^*}, \qquad (3)$$

where one can see that for a given ΔQ the luminosity is independent of the number of bunches and linearly proportional to the total number of particles N = bn. Setting $L = 10^{30}$ cm^{-2} sec^{-1}, $\Delta Q = 10^{-2}$, $\gamma = 280$, $\beta^* = 1$ m and f = 43.4 kc/s we get $N = bn = 1.25 \times 10^{11}$. The corresponding invariant emittance E_o of the two beams, defined as the two s.d. point of the Gaussian approximation is easily evaluated

$$E_o = \frac{r_p\, n\, \beta}{\Delta Q} . \qquad (4)$$

In practice, cooling can be achieved with an electron beam stored in a small ring of elongated race-track shape running tangent to the protons and antiprotons along a straight section. In the simplified case of an electron velocity distribution uniform in three dimensions in the moving-particles frame, the damping time in the laboratory frame is given by the following general formulae

$$\tau_e \simeq \frac{0.5}{r_p r_e c L} \; \frac{\gamma^2}{n_e\, \eta} \begin{cases} \theta_p^3\, \beta^3 \gamma^3 & \text{for } \theta_p > \theta_e \\ (T_e/mc^2)^{3/2} & \text{for } \theta_p < \theta_e \end{cases} \qquad (5)$$

where r_p and r_e are the classic radii for the two particles, c is the speed of light, L≃20 is the Coulomb logarithm,[1] n_e is the density of electrons in the laboratory system and η is the fraction of the storage ring circumference with electron cooling. T_e is the electron temperature in the moving frame expressed in units of kinetic energy.

It is immediately apparent from this formula that if the beam is initially very cold, it is easy to keep it cooled. For a proton emittance as low as the one required by Eq. (4), it is expected that the proton velocity in the particle frame will become lower than the rms velocity of the electrons. This, in turn is equivalent in the laboratory system to the condition $\theta_e \geq \theta_p$. The frictional force of the electrons on the protons is then proportional to the residual proton velocity in the particle frame and the damping constant τ_e due to the electron friction force is independent of the amplitude of the proton oscillations. For a velocity distribution of the electron beam spatially symmetric and Gaussian and a constant density over the cross section of the beam, the damping constant is given more precisely by the formula[2]

$$\tau_e = \frac{3}{2\sqrt{2\pi}} \; \frac{\beta \gamma^2 e}{r_p r_e \eta j L} \left(\frac{T_e}{mc^2}\right)^{3/2} , \qquad (6)$$

where $T_e = \beta^2\gamma^2 mc^2 \theta_e^2$ is the electron temperature and j is the current density.

Of course the central question is what electron current density and temperature we can achieve in practice. The proton beam has a very small (circular) cross section $\sigma_p = \beta_o (E_o/\gamma\beta)$ where β_o is the (average) value of the beta function at the cooling straight section. Inserting $\beta_o = 60$ m, $\gamma\beta = 280$ and for the value of the emittance given in Eq. (4) for $n = 2 \times 10^{10}$ we find $\sigma_p = 0.66$ mm^2. A practical electron beam will cover perhaps twice this area in order to insure convenient matching. For a reasonable number of bunches, the electron beam cross section can be only of order of a fraction of mm^2. Even relatively modest electron currents can be used to achieve substantial current densities.

A crucial feature of the cooling with relativistic electrons is the fact that the longitudinal temperature $T_{e\parallel}$ is insensitive to the laboratory energy spread. The Lorentz transformation from the laboratory to the moving particle frame is surprisingly favorable;

$$T_{e\parallel} = \frac{1}{2} m_e c^2 \left(\frac{\Delta p_e}{p_e}\right)^2_{rms} \qquad (7)$$

For a relatively large momentum spread $(\Delta p/p)_{rms} = 10^{-3}$ in the laboratory frame, we find $T_e = 0.25$ eV! Likewise, the condition that the proton velocity in the moving frame should be no greater than the rms electron velocity becomes the laboratory condition $|\Delta p/p| \lesssim [(\Delta p_e)/p_e]_{rms}$, which is easily satisfied by the proton beam. On the other hand, substantial momentum spreads are needed to insure stability of both proton and electron bunches. For a parabolic distributions in line density and a peak current I_o, bunches are stable[3] provided

$$\left|\frac{Z}{n}\right| \leq 1.92\, \eta \frac{p}{I_o} \left(\frac{\Delta p}{p}\right)_{rms} , \qquad (8)$$

where Z is the longitudinal impedance for the n mode and η is the usual factor relating the change in period to the change of momentum. For $|Z/n| \simeq 20$ ohm (which can probably be achieved with a small weak focusing electron ring), $\eta = 0.7$ and $p_e = 143$ MeV/c, we get $I_e \leq 10$A. Applying the corresponding expression to the protons, with $|Z/n| = 25$ ohm, $\eta = 1.72 \times 10^{-3}$, $\Delta p/p \simeq 10^{-3}$ we find $I_p \leq 36$A, which is a very safe value.

The transverse temperature $T_{e\perp}$ is directly related to the rms angular divergence. The requirement $T_{e\parallel} = T_{e\perp}$ gives $\theta_e^{rms} = 1/\sqrt{2}\,\beta\gamma)\ /(\Delta p/p)$. This equation requires very small angular divergence for the electrons and it is likely that at high energies one has to accept $T_{e\perp} \geq T_{e\parallel}$.

Cooling is needed at high energies in order to compensate for beam growth due to beam-gas scattering, higher-order resonances, longitudinal and transverse instabilities, intra-beam scattering and so on. In order to have a first-order estimate of the cooling rate which is required, we shall estimate simply the effect of the multiple scatterings with the residual gas. The time constant for beam growth due to multiple scattering in absence of cooling is given by [4]

$$\frac{1}{\tau_{ms}} = \frac{1}{\theta_p} \frac{d\theta_p}{dt} = \frac{K \beta_o}{\gamma \beta E_o} n_{ms} , \qquad (9)$$

where $K = 4\pi^2[(m_e^2/c)\; r_e^2\, c\, G_{N2}\,(1/M)] = 1.08 \times 10^{-23}$ m^3 sec^{-1}, G_{N2} is the absolute gas factor for N_2 and n_{ms} the equivalent density of nitrogen atoms for multiple scattering. It is related to the equivalent multiple scattering pressure p_{ms} (Torr) by

$$n_{ms} = 1.93 \times 10^{25} [\mathrm{m}^{-3},\ \mathrm{K}^o,\ \mathrm{Torr}] \frac{p_{ms}}{T}, \qquad (10)$$

where T is the absolute temperature of the residual gas. For instance, setting $\beta_o \simeq 60$ m, $P_{ms} = 10^{-9}$Torr, T = 300 K^o, $\gamma\beta_o = 280$ and $E_o = 3.2 \times 10^{-2}$ rad m, we calculate $\tau_{ms} = 2.14 \times 10^4$ sec.

The balance equation between the damping and diffusion processes (Langevin equation) has the form

$$\frac{d\theta_p^2}{dt} = -\frac{2\,\theta_p^2}{\tau_e} + \sum_1^n \left(\frac{d\,\theta_p^2}{dt}\right)_i , \qquad (11)$$

where τ_e is the cooling time constant and $[(d\theta_p^2)/dt_i]$ is the diffusion rate for the corresponding process. The solution in case of cooling competing with just multiple scattering has been given in Ref. 2 . The equilibrium value of the square of the mean proton angle is

$$\tilde{\theta}_p^2 = 2\frac{m_e}{M}\,\theta_e^2 + \sqrt{18\pi}\,\frac{e\, c\, Z^2\, n_{ms}}{j\,\eta\,\beta^2}\,\frac{L_z}{L}\,\frac{m_e}{M}\left(\frac{T_e}{m_e c^2}\right)^{3/2} \qquad (12)$$

where $L_z = \ln(133\, Z^{-1/3})$ is the Coulomb logarithm for scattering on the nucleus of charge Z.

Electron cooling will counteract Coulomb scattering if $\tau_s \simeq \tau_e$. The corresponding electron current density is easily calculated combining Eq. (6) and Eq. (9)

$$j = 3.31 \times 10^{-20} \frac{\beta_o n_{ms} \gamma T_e^{3/2} (eV)}{E_o \eta} A/m^2 \qquad (13)$$

The minimum current density is inversely proportional to the equilibrium emittance of the proton beam E_o. Since the proton beam cross section and therefore the electron beam cross section are proportional to E_o, the total current is independent of the beam emittance. This is easily understood since a very small beam has a faster growth rate and therefore also needs more efficient cooling. As a numerical example, we can take the emittance from Eq. (4) for six bunches and insert the following numerical values in Eq. (13): $\beta_o = 60$ m, $n_{ms} = 4.65 \times 10^{13}$ ($P_o = 10^{-9}$ Torr), $\eta = 5 \times 10^{-3}$, $\gamma = 280$, $E_o = 3.2 \times 10^{-6}$ rad-m and $T_e = 0.5$ eV. We find $j = 0.57$ A/mm^2. The proton beam rms radius is $r_p = 1/2 [(E_o \beta_o)/\beta\gamma]^{\frac{1}{2}} \simeq 0.47$ mm. A reasonably well matched electron beam could have twice the rms radius of the protons, that is a cross-sectional area of about $\sigma_e = 2.8$ mm^2 or a total current $I_e = 1.56$A.

A transverse temperature of $T_{e\perp} = 0.5$ eV correspond to rms angular divergence $\theta_e = 1/\beta\gamma [T_e/(M_e c^2)]^{\frac{1}{2}}$ and for a rms radius of 0.66 mm, we find an invariant emittance (2 s.d.) $E_o = 4.1 \times 10^6$ rad-m, which is comparable to that of the protons. On the other hand, longitudinal temperature $T_{e\parallel} = 0.5$ eV corresponds to about $\Delta p/p_{rms} = 1.4 \times 10^{-3}$ which is substantially wide. An identical condition holds also for the proton beam.

We note that the previously indicated longitudinal impedance of $|Z/n| \simeq 20$ ohm for the electron ring, when combined with Eq. (8), gives us a maximum electron current of 20A, which is about fifteen times what is required to counterbalance the beam-gas collisions. Although other forms of instability of the electron beam still need to be investigated, it is likely that we shall end up with a lot of spare cooling capacity to counteract, if necessary, more virulent instabilities.

The tune shift ΔQ produced by the electron current on the proton beam limits the current density to the value

$$j \leq 2\, Q(\Delta Q) \frac{e\, c\beta^3}{\ell\, R_o} \frac{\gamma^3}{r_p},$$

where $\ell \simeq 20$ m is the length of the cooling region, R_o is the radius and Q is the tune of the SPS and other symbols have the same meaning as in the previous formulae. Setting $\Delta Q = 10^{-4}$ we get $j \leq 4.6\ 10^3$ A/mm^2 which is safely beyond any practical value.

Lifetime of the beam in absence of other effects will be determined by nuclear collisions and single large-angle scatterings. The beam-gas lifetime for nuclear collisions is given by [5]

$$\frac{1}{\tau_{ns}} = \frac{1}{I} \frac{\sigma I}{dt} = 7.32 \times 10^2\, P_{ns}\, (\mathrm{Torr}),$$

where P_{ns} is the residual N_2 equivalent pressure. For $P_{ns} \simeq 10^{-9}$ Torr we find $\tau_{ns} = 1.37 \times 10^6$ s or about 16 days. The single Coulomb-scattering lifetime depends on the limiting aperture b_o.

The elastic Coulomb cross section of nuclei of charge Z for very small q^2 can be approximated as

$$\frac{d\sigma}{dq^2} = 270\, Z^2\, (\mathrm{nb/GeV}^2) \times \frac{1}{q^4\,(\mathrm{GeV})},$$

where $q^2 \simeq \theta^2 p^2$ is the q^2 in the scattering. Integrating the cross sections for all scattering angles larger than θ_o gives $\sigma = (270 \times 10^{-33} Z^2)/(p^2 \theta_o^2)$. Replacing variables p and θ_o with more convenient quantities, we get

$$\frac{1}{\tau_{ss}} = 6.33 \times 10^3\ \mathrm{m}^4\, \mathrm{s}^{-1}\, \mathrm{Torr}^{-1} \frac{Z^2}{\gamma^2 \beta^2 \beta_o^2 b_o^2} P_{ss}.$$

Setting $b_o = 2$ mm for instance, we find $\tau = 4.88 \times 10^6$ sec or about 56 days.

The applicability of the scheme to the $\bar{p}$-p in the SPS is examined in more detail. The longer lifetime of beams suggests a longer collision time and therefore a longer accumulation time of $\bar{p}$'s. Assuming that 48 hrs is the largest time period over which accumulation can be practically envisaged, for the design performance of the source, [5] we et N = nb $\simeq 1.2 \times 10^{12}$ $\bar{p}$. Inserting this number in Eq. (3) and for standard values of β^* and $\Delta Q = 10^{-2}$ we get

$$L = 10^{31}\ \mathrm{cm}^2\ \mathrm{sec}^{-1}.$$

Longitudinal instabilities amongst other reasons suggest that individual bunches should not contain more than approximately 10^{11} particles. A preferable value could be 2×10^{10}, which has already been achieved, giving b = 60. Bunches are separated by about 115 m or 0.38 μs, which is acceptable for manipulation. The invariant emittance during collisions (at $\gamma\beta = 280$) is held constant with an appropriate balance between cooling and gas scattering to the value of Eq. (4), namely $E_o = 3.1 \times 10^{-6}$ rad-m.

The longitudinal area of bunches could be as large as 1.4 rad. For $V_o = 4.4$ MV and other standard rf parameters, we expect $\Delta p/p|_{full} = 1.8 \times 10^{-3}$ and a bunch length which is about 0.4 of the bunch separation. The rms betatron beam cross section in a middle of a straight section is $\sigma = 0.17$ mm^2. Note that the momentum spread gives an additional contribution to the width, which is $\Delta x = \Delta p/p \cdot \alpha_p \simeq 3.6$ mm for $\alpha_p = 2$ m. We can either locate the cooling section in a straight section with $\alpha_p \simeq 0$ or take advantage of the dispersion by matching it to an energy-modulated electron beam. The cooling time (assuming $\alpha_p \simeq 0$) is given by Eq. (6)

$$\tau_e = \frac{4.81 \times 10^4\, T_e^{3/2}\,(\mathrm{eV})}{J(\mathrm{A/mm}^2)}.$$

Assume an electron beam cross section which is four times the rms of the proton beam. Let us also take a maximum electron current of 4A at $\gamma\beta = 280$, corresponding to $p_e = 143$ MeV/c. Then $j = 4/(0.17\times4) = 5.9 A/mm^2$. For an electron ring which is approximately 50 m in circumference and electron and proton bunches of matched lengths, the average circulating electron current is then 50 mA, which is within the range of achieved performances. If the electron temperature is taken to be $T_e = 0.5$ eV, we find a cooling time $\tau_e = 2891$ sec, to be compared to the multiple-scattering lifetime of 21,400 sec. Clearly there is about a factor ten of safety.

In order to obtain a beam with the indicated transverse emittance, the design emittance of the accumulator ring has to be reduced a few times. This can be done by the cooling itself during the first few hours of collisions or by precooling at somewhat lower energy where the cooling time is greatly diminished.

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References

1. G. I. Budker, Atomnaya Energiya 22, 346 (1967).
2. G. I. Budker et al., BNL-TR-635 (Translation) and Particle Accelerators to be published.
3. E. Keil and W. Schnell, CERN-ISR-TH-RF/60-48 (1969).
4. G. Guignard, Yellow Report CERN-77-10.
5. Design Study of a proton-antiproton colliding beam facility, CERN-PS-AA 78-3 (1978).