BEAM STABILITY CONSIDERATIONS DURING ELECTRON COOLING

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I. Introduction

The method of cooling protons (and antiprotons) with an electron beam with the same velocity substitutes the damping effect of the synchrotron radiation for electron beams. At first sight one might think that proton beams can be cooled down to practically vanishing size. The only limit that was taken in the past (and, I would say, rather roughly) is the temperature of the electron beam.

Actually operation of electron cooling at low momentum is expected to be very sensitive to the final beam density, because space-charge and beam-stability limits are rather rapidly reached at low energies. Therefore, in this paper we calculate the beam-stability and space-charge limits for the electron-cooling experiment that is planned for Fermilab at 200 MeV kinetic energy. We find that the beam can be cooled down only to densities (longitudinal as well transverse) comparable to those presently obtainable with a proton beam circulating in the Booster at 200 MeV. Thus for very small intensities, one can also expect very small emittances, but if the beam current is raised the final equilibrium emittance must also increase. The equilibrium is given by the balancing two effects: the cooling damping and the space charge and collective phenomena that would make the beam blow up again. These effects do not interfere with the collection of antiprotons, because electron cooling makes space for new beam pulses. But once intensities similar to those we operate the proton beam are reached one should also expect the same emittance value for the antiproton beam.

In the following we estimate the beam stability against well-known theories.

II. Self-Bunching of the Coasting Beam

The scheme for $\bar{p}$ collection at Fermilab considers a combination of electron cooling and rf stacking. In this section we deal with stability against self-bunching of the stack. A crucial parameter for this sort of instability is the longitudinal impedance. The contribution from the self-field is

$$\frac{Z}{n} = i\,\frac{Z_0}{2\beta\gamma^2}\left(1 + 2\,\lg\frac{b}{a}\right) \sim 1 \text{ k ohm}, \qquad (1)$$

where Z is the equivalent impedance at the harmonic number n, Z_0 = 377 ohm, and a and b are respectively the beam and pipe radii. Observe that this impedance is large and therefore likely predominant compared with any other wall contribution. It is a pure positive reactance (anti-inductance) and does not cause any instability below transition energy, provided that it is the only existing one.

The stability diagram[1] is shown in Fig. 1, where,

$$U' - iV' = -i\,\frac{2eI\beta^2 Z/n}{\pi\,|\eta|\,E\left(\frac{\Delta E}{E}\right)^2}$$

and

I = beam current
$\beta = v/c$
$\eta = 1/\gamma_T^2 - 1/\gamma^2$
E = total particle energy
$\Delta E/E$ = full energy spread at half maximum of the energy distribution

The various curves correspond to different distributions:

1. Lorentzian
2. Gaussian
3. 5th-order parabola
4. 4th-order parabola
5. 3rd-order parabola
6. Squared cosine
7. 2nd-order parabola
8. Truncated cosine
9. 1st-order parabola

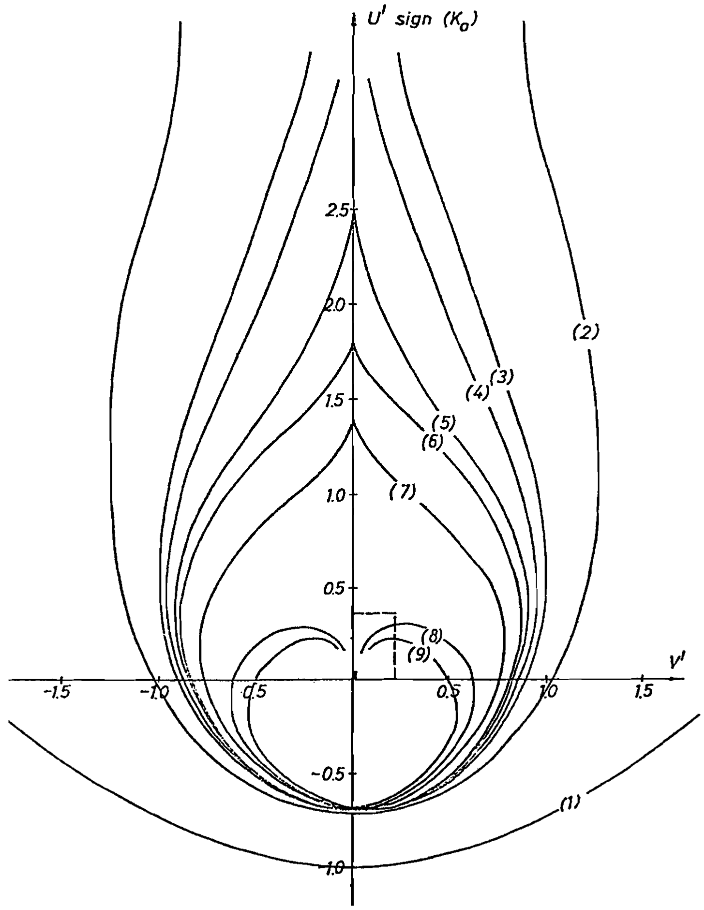

Fig. 1. Longitudinal stability diagram.

In our case $\gamma < \gamma_T$; thus one takes $k_o > 0$.

It is hard to make a judgement on beam stability since this depends very much on the energy distribution. The self-impedance (1) is very large and can hardly be reduced by an inductance. The only possible effect which can cause an instability is a resistance which moves the impedance along the V'-axis. In fact one has instability for a given distribution when the impedance point lies outside the boundary curve for that distribution shown in Fig. 1. Since U' is rather large, a small amount of resistance can move the point outside the stability diagram for practically all the distribution functions except the Lorentzian one.

In any case, the stability criterion is [2]

$$\left| \frac{Z}{n} \right| < \frac{E|\eta|}{eI\beta^2} \left(\frac{\Delta E}{E}\right)^2 .$$

If we take I= 2mA, equivalent to $N = 10^{10}$ particles $\Delta E/E = 10^{-5}$, corresponding to a longitudinal electron-beam temperature

$$T_{\|} = 0.4 \text{ eV}$$

we obtain

$$|Z/n| \sim 120 \text{ ohm}.$$

Thus the beam could be very unstable.

For 1 KΩ impedance, the threshold is

$$(\Delta E/E)_{th} = 3 \times 10^{-5}. \tag{2}$$

Since the cooling is a relatively slow process and one begins with a spread much larger than (2), it would also represent the final beam spread. If the operation is less adiabatic, it is proper to make use of the overshoot formula

$$\left(\frac{\Delta E}{E}\right)_{final} \left(\frac{\Delta E}{E}\right)_{initial} = \left(\frac{\Delta E}{E}\right)^2_{th}$$

to calculate the final spread $(\Delta E/E)_{final}$. If one takes $(\Delta E/E)_{initial} = 10^{-5}$, then

$$(\Delta E/E)_{final} = 10^{-4},$$

quite a reasonable number.

III. Transverse Stability of the Stacked Beam

Because of the very small momentum of the beam, in this case the self-field is also predominant.

Define the transverse impedance

$$Z_{\perp} = \frac{iRZo}{\beta^2\gamma^2a^2} + \frac{2ZR}{b^2(n-\nu)\beta}$$
$$= \text{self-field} + \text{wall impedance}$$

Uncooled Beam

Let us consider first the case in which the beam is not cooled transversely, but it is eventually in momentum. Then we take

R = machine radius ~ 22 m
a = 1 cm
b = 3 cm
ν = betatron tune ~ 4

and we have

$$Z_{\perp} = \left(i\, 1.7 \times 10^8 + 8.5 \times 10^4 \frac{Z}{n-\nu}\right) \Omega/\text{m}$$

where Z is the wall impedance at $(n-\nu)$ times the revolution frequency. The wall effect is negligible provided that

$$\frac{Z}{n-\nu} << 2K\Omega,$$

which we expect to be the case.

The tranverse impedance $Z_{\perp}$ has the effect of causing a shift of the betatron oscillation angular frequency [3]

$$\Delta\omega = i \frac{ecZ_{\perp}I}{\nu\gamma 4\pi E_o}$$

E_o = 938 MeV, rest energy. With only the self-field contribution, the shift, though large, is nevertheless real and the beam is stable against collective instabilities. Yet a very small resistive-wall impedance mades the shift complex and, eventually, the beam unstable. One observes here an analogy of behavior with the longitudinal case discussed in the previous section.

If we take I = 2mA and $|Z_{\perp}| = 170$ MΩ/m then

$$|\Delta\omega| = 2 \times 10^3 \text{ s}^{-1}.$$

The beam is made stable by providing enough spread at the offending frequency $(n-\nu)\,\omega_o$, ω_o being the angular revolution frequency, so that [3]

$$\Delta|(n-\nu)\omega_o| > |\Delta\omega|.$$

(i) Stabilization from revolution frequency spread. This requires

$$(n-\nu)\frac{\omega}{\beta^2}|\eta|\frac{\Delta E}{E} > 2 \times 10^3 \text{ s}^{-1}.$$

The smallest number we can conceive for $\Delta E/E$ is 10^{-5}, which equals the longitudinal temperature of the electron beam. In practice, $\Delta E/E$ will be larger either because of intra-beam scattering or because the beam is longitudinally unstable. In this last case, we have an upper limit of 1×10^{-4} from the overshoot criterion. All the modes

$n > \nu + 12$ for $\Delta E/E = 10^{-5}$
$n > \nu + 1$ for $\Delta E/E = 10^{-4}$

are stable. The lower modes could be damped with electronic feedback. The bandwidth required is at most 15 MHz. With a slightly larger $\Delta E/E$, the entire mode spectrum can be made stable and there will not be any need of a damper system that can ultimately interfere with the stacking operation.

(ii) Stabilization from tune spread. This requires

$$\Delta\nu > \frac{|\Delta\omega|}{\omega_o} = \frac{2\times 10^3}{\omega_o} = 2.5 \times 10^{-4}.$$

This is a rather small spread. The tune shift due to the crossing with the electron beam is as large as 0.06 and a fraction of this is presumably a spread across the beam. There are also contributions from the beam emittance ϵ and the non-linearities of the guide field around the ring.

(iii) Stabilization with chromaticity. The amount of chromaticity

$$\xi = (\Delta\nu/\nu)/(\Delta p/p)$$

required depends on the energy spread as is shown in the following table.

Beam	$\Delta E/E$	$\xi \to \Delta\nu = 2.5\times 10^{-4}$
Long. stable and cooled	10^{-5}	-2.2 (too large)
At the threshold of long. stabil.	3×10^{-5}	-0.73
Overshoot	1×10^{-4}	-0.22

It is required to blow up the beam somewhat longitudinally to get moderate chromaticity.

In general, the stability condition is

$$\Delta\left|(n-\nu)\frac{\omega_o}{\beta^2}\,\eta\,\frac{\Delta E}{E} + \nu\,\frac{\omega_o}{\beta^2}\,\xi\,\frac{\Delta E}{E} + \omega_o K\epsilon\right| > |\Delta\omega|.$$

To avoid cancellation, all the three terms must have the same sign. Since only the terms $n > \nu$ are unstable and $\eta < 0$ below the transition energy, the chromaticity ξ and the octupole strength K ought to be negative. This of course is relevant only when the three contributions to the spread are of the same order.

Transversally Cooled Beam

The radius of such a beam could be much smaller than what we have considered above. A final size a = 1 mm corresponds to an electron-beam temperature of 0.4 eV. In this case, $Z_\perp$ and $\Delta\omega$ are one-hundred times larger and pose serious concerns about the stability of the cooled beam. The amount of time spread which is now required is about 0.025, too large to be attained with reasonable chromaticity or octupole. An electronic feedback damper should have a band width larger than 60 MHz, rather hard to make.

Things can improve considerably if one lets the beam blow up further longitudinally, say up to

$$\frac{\Delta E}{E} = 1 \times 10^{-3},$$

which is the maximum the Booster rf system can accept. With this spread, all the unstable modes are given by

$$n - \nu \lesssim 12$$

which could be stabilized with a 15 MHz damper.

Of course, the other alternative is to give up transverse cooling completely and rely only on longitudinal cooling.

IV. Growth Rates

Most of the concern of course goes to the case in which the instabilities grow so fast that one can disregard the electron cooling itself. Eventually electron cooling might have the nice feature that it damps those instabilities which grow slowly compared to the cooling time.

The instability growth rate, in the absence of Landau damping is essentially given by the resistive part R_n of the impedance. In the limit when R_n is small compared with Z_n, one has

$$\frac{1}{\tau_n} \sim \frac{R_n}{|Z_n|}\,|\Delta\omega|,$$

where $|\Delta\omega|$ is the frequency shift. This would suggest that as long as $\tau_n >> \tau$, the cooling time, the beam should be stable.

There are some uncertainties about τ; thus we take

$$\tau_n \gtrsim 1 \text{ second.}$$

This condition, applied to the transverse case, is actually independent of the beam size. For the transverse case

$$|\Delta\omega| = 2\times 10^3\ s^{-1}$$

$$\frac{Z_n}{n} = \frac{b^2\beta Z_\perp}{2R} \sim 2\ K\Omega$$

and one derives

$$\frac{R_n}{n} < 1\Omega.$$

If this condition is satisfied, the transverse cooling process is presumably fast enough to damp any coherant oscillation.

V. Bunched Beam

There are only two situtation where the beam is bunched:

(i) During stacking - Each pulse has a much lower intensity, approximately $2\mu A$, which corresponds to 10^7 ppp. The spreads of each of these pulses that are not yet cooled, are also considerably larger. Thus we do not expect any transverse or longitudinal instability in this situation. Bunch-to-bunch instabilities should also not play a major role.

(ii) rf capture of the beam after stacking. If the coasting beam criteria are met when the average current is replaced by the peak current, the individual bunch modes as well as the bunch-to-bunch modes are stable. It is thus important to bunch the beam at a resonably low bunching factor, possibly 2 or 3, and extract the beam as rapidly as possible. Unfortunately, injection of the beam in the Booster will not soften the situation. The beam spreads are bound to increase to overcome the instabilities.

A final momentum spread up to 10^{-4}-10^{-3} can be expected. The transverse emittance will also grow but will be likely to be within the Booster acceptance.

VI. Incoherent Space Charge Limit

The incoherent space charge induces a betatron tune shift that is given by[4]

$$\Delta\nu = -\frac{rFN}{(\pi\epsilon_V\beta\gamma)\left(1+\sqrt{\frac{\epsilon_H}{\epsilon_V}}\right)\beta\gamma^2 B}$$

$$F = 1 + \frac{b(a+b)}{h^2}\left\{\epsilon_1\left[1+B(\gamma^2-1)\right] + \epsilon_2 B(\gamma^2-1)\frac{h^2}{v^2}\right\}$$

N = total number of particles in the ring
$r = 1.5347\times10^{-18}$ m
B = bunching factor (< 1)
b = mean semi-minor beam axis (vertical)
a = mean semi-major beam axis (horizontal)
2h = vertical vacuum-chamber aperture
2w = horizontal vacuum-chamber aperture
2v = height of magnet gap.

ϵ_1 (~ 0.2) and ϵ_2 (~ 0.4) are the Laslett image coefficients. ϵ_V and ϵ_H respectively the vertical and horizontal emittance.

For our case it is a good approximation to take

$$F = 1$$

$$\epsilon_H \sim \epsilon_V = \epsilon$$

$$\epsilon = a^2/\bar{\beta} \text{ with } \bar{\beta} = R/\nu \sim 5 \text{ m.}$$

For a transversally-cooled beam a ~ 1 mm (equivalent to the electron beam temperature $T_{\|}$ = 0.4 eV) and

$$\epsilon = 0.2\times10^{-6} \text{ m,}$$

which gives

$$\Delta\nu = 2.1\times10^{-12}\ N/B.$$

For a 10 mA beam

$$\Delta\nu = 0.02/B.$$

If during the final rf capture of the stack B ~ 1/5, then $\Delta\nu$ = 0.1 which may be reasonable.

VII. Conclusion

It seems that the cooled beam is too unstable to reach the spreads which are in equilibrium with the electron beam. Because of longitudinal and transverse instabilities the final spreads (momentum spread and transverse emittance) will be somewhat larger. The final values will probably equal the threshold values of the various instabilities, provided the cooling process is adiabatic enough. Otherwise "overshoot" will occur. Nevertheless, even in this case the final growth should be small enough to lead to emittances and spreads easily accepted by the Booster.

References

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