NONLINEAR ELECTRON-PROTON INTERACTIONS DURING ELECTRON COOLING

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I. Introduction

Electron cooling is the method by which antiprotons will be stored and stacked in the Fermilab $p\bar{p}$ scheme. In the storage ring, either protons or antiprotons will be under the influence of an intense electron beam which occupies a small fraction of the ring circumference. Thus each particle will receive a kick turn after turn which can in good approximation be taken as lumped. The cooling is due to the microscopic structure of the electron beam, that is, to scattering by a large but finite number of electrons. The electron beam can also be regarded as a solid, continuous charge distribution with which one associates a rigid, continuous field. We are interested in the effect of this field on the stability of the motion of the p or $\bar{p}$ particles. This is also called the beam-beam effect which is measured by the beam-beam tune shift. In first approximation, the electron beam can be regarded as a quadrupole with the same focusing action in both planes (focusing for protons, defocusing for antiprotons). Such a quadrupole, when regarded as a perturbation to the lattice of the storage ring, causes primarily a shift of the betatron-oscillation frequencies.

If the electron beam is assumed to have a uniform transverse charge and current distribution, its only effect is betatron tune shift, which is common to all the particles with oscillation amplitude smaller than the electron beam radius. For a non-uniform distribution, a non-linear effect is expected which might cause, in absence of cooling, stochastic behavior of the particle motion which can then be expressed as a diffusion process for the entire beam of hadrons. Such an effect might eventually limit the capability of the electron cooling itself.

According to the design specification, the cathode of the electron gun which generates the electron beam is carefully built to give the most uniform distribution possible. In this case, the linear tune shift can be easily compensated by retuning the quadrupoles of the storage ring. Here we are interested in the case when the electron beam does not have a perfect uniform distribution. We have intentionally exaggerated the beam shape to a Gaussian distribution with a standard-deviation size of 2 cm in both planes in order to test the sensitivity of the primary beam to nonlinearities.

Though the two beam travel in the same direction and the electric field and magnetic field counteract each other, nevertheless, the cancellation is minimal because of the low proton-beam kinetic energy (200 MeV). For an electron current of 25 A and an interaction length of 5 m, the tune shifts are

$$\Delta\nu_x = 0.065 \qquad (\beta_x^* = 20.4\ \mathrm{m})$$

$$\Delta\nu_y = 0.090 \qquad (\beta_y^* = 35.7\ \mathrm{m})$$

II. Method of Calculation

The beam-beam effect is difficult to calculate analytically, so we planned for a numerical simulation of the particle motion under the influence, turn after turn, of a nonlinear lens (the electron beam). The program was developed at Fermilab[1] to investigate beam-beam effects in other colliding-beam situations. After having checked that we do not lose much information from statistical fluctuation between 1000 and 100 particles, we have taken 100 particles for our computation. To each particle we associate four initial conditions: x, x', y, and y'. These are taken randomly with a distribution which describes the proton beam at the crossing location. Our simulation consists in applying simultaneously to all the particles a series of a large number of cycles. Each cycle simulates one revolution and is made of two steps. In the first step, we apply to the particle coordinates a linear transformation with a 4×4 matrix which describes the linear lattice of the storage ring. For its determination we supply β_x, β_y, α_x, and α_y at the crossing point and the two phase advances per turn, the fractional part of the two betatron tunes ν_x and ν_y. The second step simulates the nonlinear kick when crossing the electron beam. For each particle we change z' by

$$\Delta z' = -\frac{4\pi}{\beta_z^*}\Delta\nu_z \frac{1-e^{-u^2}}{u^2} z, \tag{1}$$

where z can be either x or y and

$$u^2 = \frac{x^2+y^2}{2\sigma^2}.$$

At the same time x and y are unchanged.

Equation (1) is derived in the approximations that β^* does not change across the electron beam length ℓ, and that $\beta^* >> \ell$.

Every 1,000 turns, four histograms of 20 channels corresponding to the four coordinates are prepared and displayed. Then averages, standard deviations, minimum and maxima are calculated and printed out. We always found that the histograms approximately reproduce a Gaussian distribution. Thus we take the standard deviation as a measure of the beam size. The tracking always takes 50,000 turns which correspond to 40 msec of actual time. At the end, the final beam size is taken by averaging over the last 5,000 turns. The damping due to the electron cooling is not applied during the simulation.

III. The Results

These are shown in Table I through Table VI (see following pages).

TABLE I: $\nu_x = 0.57$, $\nu_y = 0.52$. The final beam size σ and angle ψ are shown versus the initial emittance ϵ, which is defined for 95% of the beam. One observes a shrinking of the beam size at the cost of increasing the angles by a factor two. This is merely due to the betatron mismatch. The increase in angle should eventually be taken into account for setting the initial conditions of the electron cooling.

TABLE I.

ϵ_x ($\pi 10^{-6}$ m)	ϵ_y ($\pi 10^{-6}$ m)		σ_x mm	ψ_x mrad	σ_y mm	ψ_x mrad
40	20	(a)	11.0	0.56	10.7	0.30
		(b)	9.7	0.72	8.1	0.73
		(c)	9.4	0.71	8.2	0.72
20	20		6.8	0.51	8.4	0.71
10	10		4.6	0.37	5.8	0.54
5	5		3.2	0.27	4.0	0.40
1	1		1.5	0.12	1.8	0.18

[a] No beam-beam tune shift applied. One thousand particles taken.
[b] Beam-beam tune shift applied. One thousand particles taken.
[c] One hundred particles taken.

TABLE II: $\epsilon = 10\pi\ 10^{-6}$ m. The electron beam is not centered by x_B to the center of oscillation of the protons. No effect of the beam separation has been found.

TABLE II.

x_B mm	σ_x mm	ψ_x mrad	σ_y mm	ψ_y mrad
-	4.6	0.37	5.8	0.54
2.5	4.7	0.37	5.8	0.54
5	4.9	0.36	5.8	0.53
7.5	4.9	0.38	6.0	0.51
10.	5.0	0.38	6.0	0.51

TABLE III: The electron beam is displaced from +2 cm to -2 cm in N turns. There is no variation in the beam size and angle. The small differences between the numbers shown in this table and in Table II are due to a minor change of β_x^* and β_y^* in our simulation.

TABLE III.

N	σ_x mm	ψ_x mrad	σ_y mm	ψ_y mrad
1000	5.9	0.40	5.9	0.49
4000	6.7	0.43	5.9	0.49
10000	6.3	0.42	5.9	0.49
20000	6.7	0.48	5.9	0.49
50000	6.7	0.47	6.1	0.48
∞	5.9	0.40	5.6	0.50

TABLE IV: Same simulations of Table III but now the tunes have been changed to

$$\nu_x = 0.70 \qquad \text{and} \qquad \nu_y = 0.456.$$

TABLE IV.

N	σ_x mm	ψ_x mrad	σ_y mm	ψ_y mrad
1000	30.4	1.43	79	1.7
4000	22.5	1.22	74	1.6
10000	28	1.45	71	1.6
20000	22.5	1.17	72	1.4
50000	19.2	1.05	78	1.2
∞	31.3	1.62	80	1.8

The beam size and angle increase are large, as is also shown in Fig. 1. One notices here a linear increase of σ with time at a rate of 5.3×10^{-4} mm/turn. Thus there is definitely a strong tune dependence. This may be caused by the nonlinear mismatch, nonlinear coupling and periodic crossing of resonances induced by the electron beam.

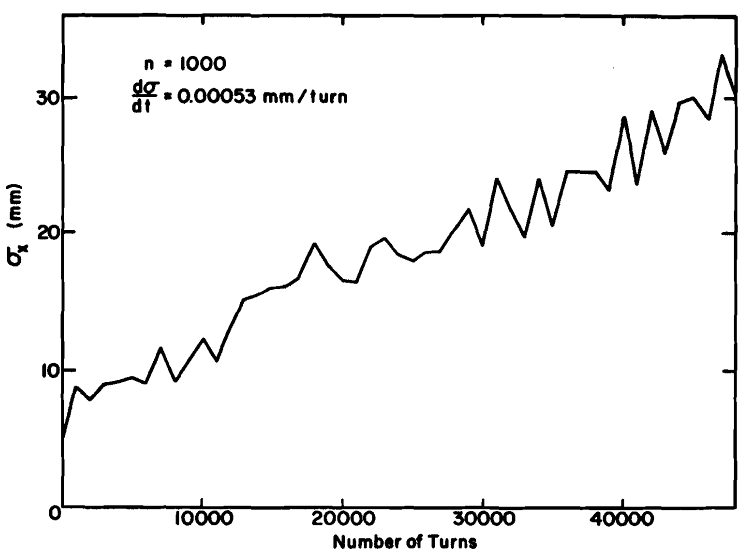

Fig. 1

Care should be taken to tune the storage ring properly to avoid these effects.

Another case is shown in Fig. 2. The electron beam is now moved more slowly (N = 50,000). The beam size at the end is down to the initial value, but one should note the increase in between.

TABLE V: The tunes are set to the original values. The electron beam center is made to oscillate horizontally according to the equation.

$$x_B = a \sin\left(2\pi \frac{n}{N}\right) + x_0.$$

No effect of the periodic movement of the electron beam, either centered or displaced, has been found.

TABLE V.

a mm	N	mm	σ_x mm	ψ_x mrad	σ_y mm	ψ_y mrad
-	-	-	4.5	0.38	5.9	0.55
1	10	-	4.4	0.40	5.9	0.55
1	100	-	4.6	0.37	5.9	0.56
1	1000	-	4.3	0.40	5.9	0.55
5	10	-	4.3	0.40	5.8	0.55
5	100	-	4.6	0.37	5.8	0.57
5	1000	-	4.5	0.38	5.9	0.56
1	10	1	4.5	0.38	5.8	0.56
1	100	1	4.6	0.38	6.0	0.54
1	1000	1	4.4	0.39	6.0	0.55

TABLE VI: The two beams are centered but the two betatron tunes change periodically according to the equation

$$\nu_x = 0.57 + \Delta\nu \cdot \sin(2\pi n/N)$$

$$\nu_y = 0.52 - \Delta\nu \cdot \sin(2\pi n/N).$$

This can either simulate phase oscillations and the machine chromaticity or power-supply ripple. Very large beam size (and angle) increases are now

TABLE VI.

$\Delta\nu$	N	σ_x mm	ψ_x mrad	σ_y mm	ψ_y mrad
-	-	4.5	0.38	5.9	0.55
0.010	10	6.6	0.48	10.0	0.86
	100	4.5	0.39	6.0	0.54
	1000	4.6	0.38	5.9	0.56
0.015	10	50	2.8	84	2.4
	100	4.6	0.38	6.0	0.57
	1000	5.0	0.47	5.2	0.52
0.020	10	76	3.9	108	3.0
	100	118	6.0	368	7.6
	1000	141	7.5	925	19.0
0.030	10	126	7.2	176	5.2
	100	207	10.9	208	10.6
	1000	187	10.0	664	20.5
0.100	10	116	6.7	150	3.9
	100	179	11.2	360	10.2
	1000	195	11.2	507	13.5

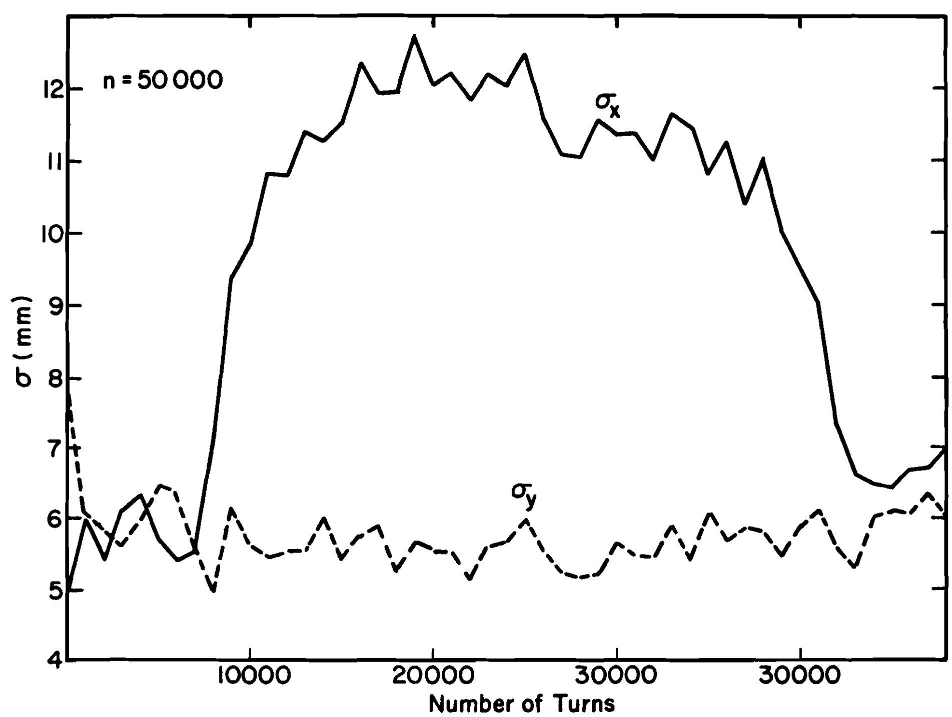

Fig. 2

noticed. Some cases are shown in Figs. 3 and 4. The beam size increases with $\sqrt{t}$ as is seen for the bottom curve of Fig. 4, where the dashed line is indeed a $\sqrt{t}$-curve. The diffusion coefficients $d\sigma^2/dt$ are shown in Table VII in units of m^2/sec. There is an increase of the diffusion with $\Delta\nu$ and N. For $\Delta\nu \lesssim 0.01$, no diffusion was observed, at least for the time explored during the computation.

TABLE VII.

$\Delta\nu$/N	10		100		1,000	
	x	y	x	y	x	y
0.010	-	-	-	-	-	-
0.015	65.1	186	-	-	-	-
0.020	149	305	366	3,562	523	22,515
0.030	417	814	1,127	1,137	920	11,601
0.100	353	587	843	3,409	1,000	6,776

References

[1] A. G. Ruggiero, Fermi National Accelerator Laboratory FN-293 and FN-293A, May and June 1976. Also, IEEE Trans. Nucl. Sci. NS-24, 1893 (1977).

IV. Conclusions

If proper care is taken, beam-beam effects can be greatly reduced. Of course our simulation was applied only for a very short period of time (40 msec) and we did not prove the stability of the beam over long times (several hours) at all. On the other hand, we used a model for the electron beam where non-linearities have been intentionally pronounced. Eventually one would expect a distribution close to flat. The other effect which still requires investigation is the limitation of the electron cooling process by nonlinear beam-beam interaction. This effect also should be easily simulated with our computer code and we plan to do so in the near future.

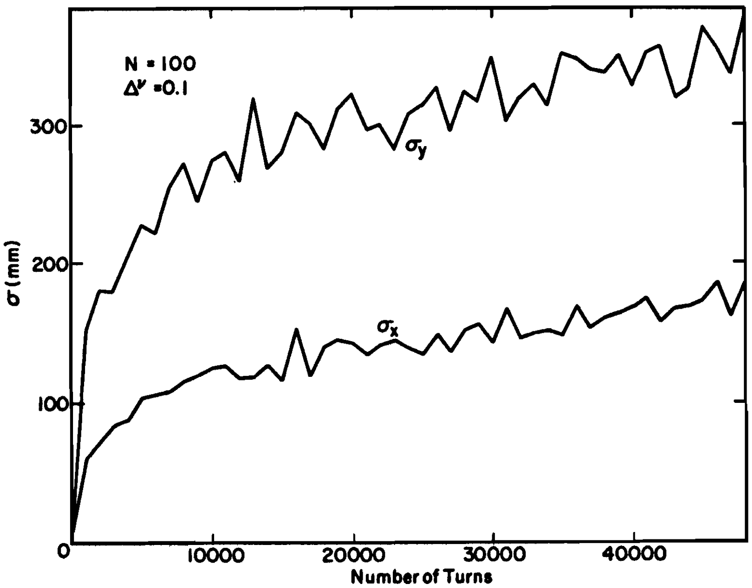

Fig. 3

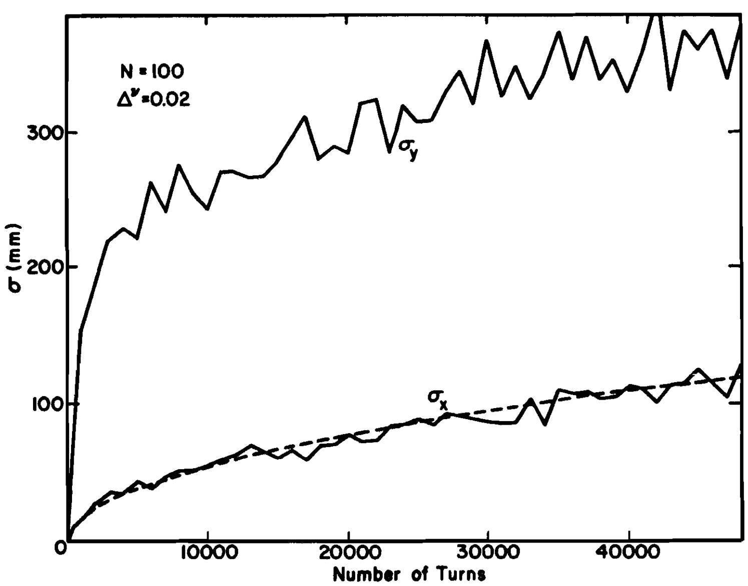

Fig. 4