# PROTON COOLING BY RADIATION

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The emission of electromagnetic radiation by electrons moving in magnetic fields (synchrotron radiation) is well understood and has many beneficent effects, such as in the damping of synchrotron and betatron oscillations in circular accelerators or in storage rings. Because the radiation depends inversely upon the fourth power of the mass of the radiating particle (protons radiate less than electrons by a factor of about $10^{13}$) synchrotron radiation has as yet been negligible in proton accelerators. However, as proton energies grow ever higher, we eventually will reach a level at which the radiation will become large enough to be effective in damping orbit oscillations.

We can directly take over the theory of synchrotron radiation by electrons as given by Schwinger.[1] The rate of emission is given by

$$E_r = (2\omega e^2/3R)\,|\beta^3/(1-\beta^2)^2|\,, \tag{1}$$

which for protons becomes

$$E_r = 7.8 \cdot 10^{-12}\, E^4/R, \tag{2}$$

where the energy emitted per turn $E_r$ and the proton energy E are measured in TeV and the radius of curvature R due to the magnetic field is in kilometers. For the Tevatron, where the radius of curvature is about 0.8 km, the energy radiated per turn is only about 10 eV per turn. At 10 TeV, assuming twice the magnetic field of the Tevatron, $E_r$ would be 200 keV per turn, which is beginning to be significant.

Can such radiation provide cooling of betatron oscillations? The damping coefficient of synchrotron oscillations is given approximately by $2E_r/E$, and of the vertical oscillations by $E_r/2E$. The radial oscillations grow exponentially by the same factor, but can be coupled to synchrotron oscillations to provide overall damping. Thus for the 1 TeV case, if only this damping obtained, then the vertical size of the beam would be reduced by half in about 50 days. For the 10 TeV example, the halving time would be about 5 hrs, which is more significant.

We must also not forget the possibility that coherent effects will enhance the radiation.[2,3] Coherence will occur when the packets of protons are comparable in size to the wavelength of the radiation. The characteristic wavelength, $\lambda_c$, of the rather broad distribution of frequencies of the radiation by protons is given in centimeters by

$$\lambda_c = 3.5 \cdot 10^{-4}\, R/E^3. \tag{3}$$

For the Tevatron with a proton energy at 1 TeV, $\lambda_c$ is about $3\mu$.

L. I. Schiff calculated[4] the additional loss per particle per revolution due to coherent radiation in the absence of metallic shielding and found that it is given by

$$\Delta E \approx \frac{e^2}{R}\; N\,\phi\, 4/3, \tag{4}$$

for N protons in a gaussian distribution having an angular width of $\phi$ between the 1/e points. Much of the enhanced radiation is in the microwave region and will be reduced by surrounding metal shielding. Schiff also made a calculation that would apply when the orbit of the particle is midway between two parallel metallic plates. He found for this case that the above result (4) should be reduced by a factor of $5(a/R)^2$ where a is the distance from orbit to plate.

In the Doubler each "bucket" of protons is a narrow pencil, typically about 30 cm long and a few mm in diameter. Hence we might expect some coherent enhancement of the radiation. There are about one thousand buckets, each of which will contain about $10^{10}$ protons. Hence the extra coherent radiation by each proton will be about 100 eV/turn. Although this may be significant in producing a quite measurable signal, it is not at all clear that it will contribute very much to the damping of random betatron oscillations. Of course, if the center of mass of the coherent bunch is executing a coherent betatron oscillation then that will be dampened. Because any coherent bunch will have a randomly distributed component of protons (proportional to $\sqrt{n}$) executing coherent betatron oscillations, the coherent radiation of the fluctuation from the average will produce a kind of "stochastic" cooling, although of negligible magnitude.

Let us consider briefly the limitation by synchrotron radiation of the growth of a proton beam caused by multiple coulomb scattering of the proton by residual gas. The mean squared height of the beam, $\overline{y}^2$, grows by scattering at a constant rate, i. e., $d\overline{y^2}/dt = a$. That growth is damped by radiation as $dy/dt = -by$, which can be rewritten $dy^2/dt = -2by^2$. Without making a distinction between $y^2$ and $\overline{y}^2$, the sum of the two terms can be set to zero as a rough condition for the asymptotic size, $y_o$, which is then given by $y_o^2 = a/2b$. Evaluating the constants a and b by the usual approximate relationships gives

$$y_o^2 \approx 10^9\, \frac{R^4_{km}}{Qp\; E^5_{TeV}}$$

where Q is the number of betatron oscillations per turn and the pressure p is in Torr. For a room temperature equivalent pressure of about $10^{-8}$ Torr of $N_2$, the beam in the Tevatron would damp down to a height of about 1 mm if it had an infinite lifetime. For larger machines, say E > 5 TeV, or for a better vacuum, the radiation damping would become really effective in reducing the asymptotic size of the beam - a more exact calculation is indicated.

## REFERENCES


[1] J. Schwinger, Phys. Rev. 75, 1912 (1949).

[2] A. G. Ruggiero, Fermi National Accelerator Laboratory Internal Report TM-730, April, 1977. The author has considered the Main Ring and the Energy Doubler as a source of synchrotron radiation with special application to the measurement of beam size.

[3] R. Coissen, Opt. Commun. 22, 135 (1977) and Nucl. Instrum. Methods 143, 241 (1977). The author has studied modifications of the spectrum due to non-uniform fields.

[4] L. I. Schiff, Rev. Sci. Instrum. 17, 6 (1946).