NOTE ON BEAM-BEAM TUNE SHIFT IN SINGLE RING MULTI BUNCH MODE*

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If many identical counter-rotating bunches of protons and antiprotons are stored in a single ring, they will have identical orbits. Therefore, for n bunches, there will be 2 n collisions at 2n equally spaced locations around the ring. A particle of one beam will then be influenced by a periodic sequence of 2n strongly nonlinear forces caused by the n bunches in the other beam.

Let $\Delta\nu_I$ be the tune shift at some collision point I. Now the tune shift for the entire orbit is:

$$\Delta\nu\ (\text{Total}) = \sum_{I=1}^{2n} \Delta\nu_I .$$

The question is: Is this total tune shift relevant to the problem of beam stability? The answer is: not in general. What we are trying to describe is a nonlinear force and the nonlinear force is described by its "strength", $\Delta\nu_I$, for each bunch interaction individually. It is not at all clear that the sum of the individual $\Delta\nu_I$ is the significant quantity.

If, however, all the collisions are identical, meaning that the β-values are the same at all points where the bunches meet, then it might be argued that $\Delta\nu_O$, the beam-beam tune shift per collision is the true measure of the strength of the nonlinear force. The reasoning is that although there is more nonlinear force, this is cancelled by the effect of the symmetric distribution of the force along the orbit. Said another way, although the strength of resonances increases with more collisions, the density of resonances (in tune space) is decreased by the symmetry. This point is very difficult to verify in detail on theoretical grounds, but experiments at Adone have shown that there could be some validity to the claim. With 3 bunches, the beam-beam limit, the tune shift for rapid beam blow up, was not proportional to the total shift. It was also not proportional to the tune shift per bunch, but rather somewhere in between, suggesting a more complicated relationship between the beam-beam limit and the tune shift distribution around the orbit. This is true even in the case of a symmetric distribution.

On the other hand, the situation at both FNAL and the SPS is quite different. In these cases, the beam configuration at the collision points are not symmetric, but differ from one collision point to the other. The theoretical argument related to the decrease of the density of resonances (due to symmetry) does not strictly apply. We might expect, therefore, that the effect of the nonlinear forces would be better described by the total tune shift rather than the tune shift in any given bunch collision.

We might conclude that at FNAL or the SPS the addition of bunches can only provide increased performance at some collision point if the bunches are well separated at all other collision points!

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