# ELECTRON COOLING OF HIGH ENERGY BEAMS*

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The purpose of this short note is to clarify the relationship between electron cooling time and beam energy[1] and also to see how the nature of the electron beam enters. In particular, we want to know whether it is electron total current or current density that is significant. There is no attempt here to include any measure of sophistication, such as the effect of a solenoid field, but simply to use Coulomb scattering and statistical equilibrium. For simplicity, we take the 3 phase spaces (horizontal, vertical and momentum) to have equal occupied areas and use the terminology for the transverse case.

Thus, we can use Skrinsky's expression for the e-folding cooling time,

$$\tau = \frac{\pi}{2}\left(\frac{m}{M}\right)\frac{e\,\beta\,\gamma^2}{L\,r_p^2\,\eta I a}\,E^3, \qquad (1)$$

where $\beta$, $\gamma$ are the usual relativistic parameters, the unitless velocity and energy,

- m, M are the electron and proton mass,
- e is the unit of electric charge ($1.6\times10^{-19}$C),
- $r_p$ is the classical proton radius ($1.54\times10^{-18}$m),
- L is the Coulomb logarithm depending on the minimum impact parameter; we take L = 20, and
- $\eta$ is the ratio of length of electron cooling region to the circumference of the proton (antiproton) ring.

We assume that the electron beam and proton beam are matched in space, this being the optimum configuration, where we expect the above expression for the cooling time to apply. Thus,

- a is the radius of the proton beam in the cooling region, and
- E is the invariant emittance of the proton beam.

E is defined without the notorious "$\pi$", i.e.,

E = Area/$\pi$ in phase space. In particular,

$$E = \beta\,\gamma\,\theta\,a,$$

with $\theta$ the 1/2-divergence of the proton beam, proportional to the square root of the beam temperature.

The variable I is the total electron current.

In the form (1), the basic nature of the electron cooling process is exposed:

1) Cooling depends most critically on the invariant emittance, $\tau \propto E^3$. This is a very important point, since E is generally a property of the source of particles and as the beam passes from machine to machine as it is accelerated to higher and higher energies, E is in principle invariant, but in practice the effective emittance will increase in response to the vagaries of the real world. Real beams tend to have $E > 20\times10^{-6}$ rad-m. In fact, one of the best high energy beams is to be found at the ISR, where with a great deal of vertical shaving, a value $E_V \lesssim 10\times10^{-6}$ rad-m is attained. This is all in accordance with Liouville's Theorem which tells us that the effective invariant emittance will not decrease in the real world-M- will not, that is, unless through some process which is unusual for particle beams, such as interaction with an electron beam having a thermal equilibrium level much lower than that of the proton beam. Thus, we have an interesting dilemma. If we take proton beams as nature provides, or if we accept antiproton beams at the production target of large solid angle, electron cooling will tend to be a very long process, since E will be a large number. It is the irony of electron cooling that it works best for beams that don't really need it!

2) The energy dependence is given by

$$\tau \propto \beta^{3/2}\,\gamma^{5/2}\left[\text{since } a \propto (\beta\gamma)^{-1/2}\right].$$

Thus, cooling times could conceivably be "reasonable" for very low energy proton beams.

3) The cooling time is increased as the ratio of cooling length to ring circumference is reduced. As the energy of the proton (antiproton) ring is increased, it takes a larger ring to store them. Thus, $\eta$ is smaller for higher energy, which further decreases the cooling rate.

4) The cooling rate is proportional to the total electron current and not the transverse current density. Once the total current is given, the transverse density needed can be deduced from the size of the proton beam which is to be cooled. Thus, in principle, very small high density electron beams available from electron storage rings do not appear to be useful in the context of the discussion given here.

To get a feeling for the order of magnitude of cooling times we might achieve, consider a 50 GeV beam with an invariant emittance equal to that of the best proton beam available; that is, $E = 20\times10^{-6}$ rad-m. Let us also take a cooling length of 20m in the Fermilab ring, giving $\eta = 20/(2\pi\times1000) = 3.2\times10^{-3}$. Assuming a β-function value, $\beta^* = 50\,\text{m}$ over the 20 m cooling length, we have:

$$a = \left(\frac{E\ \beta^*}{\beta\gamma}\right)^{1/2} = 4.3\ \text{mm},$$

with $\gamma = 53$. Thus, the cooling time is:

$$\tau = 4.6\times10^6\ \text{sec}/_{I\,(\text{amps})} = 53.4\ \text{days}/_{I\,(\text{amps})}.$$

Thus, for an electron beam of 50 A peak current matched to the proton beam, the e-folding cooling time is $\tau = 1.1$ days. This corresponds to a current densith in the electron beam, $n_e = I/\pi a^2 = 86\ \text{A/cm}^2$ and an average electron current of 5 A (if bunching is 10 to 1). This is a very high current indeed.

It thus appears to be impractical to consider using electron cooling at high energies for the purpose of cooling and accumulating antiprotons. However, if we already have "cold" proton and antiproton beams, it might be conceivable to use electron beams from storage rings for the purpose of sustaining constant luminosity and perhaps limiting beam loss and, therefore, background. Thus, if we managed to attain a cold beam of emittance 10 times less than we assumed - i.e., E = 2 μrad-m, then the cooling time for a 50 A electron current is reduced to 92 sec. Thus, in such a situation, blow-up processes occurring on a time scale of the order of 100 sec can, to some extent, be "damped". However, even such an application is not trivial and the conditions for stability of both electron and proton stored beams must be carefully studied. Furthermore, it should be remembered that these "high density" (low emittance) p and $\bar{p}$ beams must be formed at low energy where electron cooling is practical. Thus, space charge limitations at low energy become a factor.

We could imagine a system combining stochastic cooling at large amplitudes followed by electron cooling to maintain the small size reached. Such a system does not appear to be very promising; in any case, consideration of this subject is outside the intentions of this paper.

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1. Most of the results given here can be deduced from a 1971 BNL report, R. L. Gluckstern, Electron Cooling of Protons in ISABELLE, CRISP 71-24 (1971).