LUMINOSITIES OF PROTON-ANTIPROTON COLLIDING BEAMS

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The purpose of this paper is to compare the luminosities achievable with the proton-antiproton collision schemes proposed by CERN and by Fermilab. Estimates have been made by both CERN[1] and Fermilab[2,3] groups but these estimates have not been made with a consistent set of assumptions. A comparison of the potential performance of the two schemes at present is therefore not possible. We are motivated not only by the need for a realistic assessment of the many details entering but also by a hope that a deep understanding of the factors contributing to the luminosity may lead to improvements which in turn could result in increased luminosity. Using the antiproton schemes proposed,[1,2] we find that the luminosity at 1000 GeV/c of the Fermilab Doubler as a $\bar{p}$-p collider is 3.4×10^{29} and the luminosity of the SPS at 270 GeV/c as a $\bar{p}$-p collider is 1.0×10^{30}.

I. Accumulation of Antiprotons

The number of antiprotons produced in a stationary target and accepted in a collection channel is

$$N_{\bar{p}} = \frac{d^2N}{dP\,d\Omega}\,\Delta p\,\Delta\Omega\,\eta_{tgt}\,N_p,$$

$\frac{dN}{dP\,d\Omega}$	number of antiprotons produced per unit solid angle per unit momentum per interacting proton
Δp	momentum acceptance of the collector
$\Delta\Omega$	solid angle acceptance of the collector
η_{tgt}	target efficiency
$N_{\bar{p}}$	number of antiprotons produced
N_p	number of protons incident on target.

To estimate the antiproton production cross section we use the review of Cronin[4] which discusses antiproton production through $pp\rightarrow\bar{p}x$. No measurements exist in the kinematic region to be used and the work of Cronin contains the best available information and extrapolations. The antiproton production rate is related to the invariant cross section

$$\frac{d^2\sigma}{dp\,d\Omega} = \frac{p^2}{E}\left(E\frac{d^3\sigma}{dp^3}\right)$$

p and E are the antiproton momentum and energy. The antiproton cross sections are believed to be largest when the antiproton is at rest in the barycentric system of the incident nucleons.

The solid angle which can be accepted by the channel depends upon the target dimensions and the desired beam emittance. The beam emittance (phase space area) is

$$\epsilon_V = \pi\theta_V h$$
$$\epsilon_H = \pi\theta_H w$$

h, w	are the target half height and half width
θ_V, θ_H	are the vertical and horizontal half angles of emission

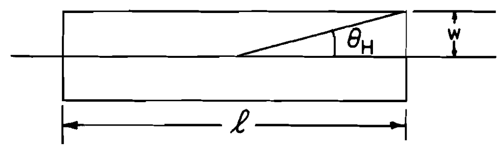

Fig. 1

The target width should be no larger than the antiproton beam size at the end of the target as shown in Fig. 1:

$$w \simeq \tfrac{1}{2}\ell\theta_H.$$

Similarly,

$$h \simeq \tfrac{1}{2}\ell\theta_V.$$

Under these simple assumptions

$$\theta_H^2 = \frac{2}{\ell}\frac{\epsilon_H}{\pi}$$
$$\theta_V^2 = \frac{2}{\ell}\frac{\epsilon_V}{\pi}.$$

The solid angle is

$$\Delta\Omega = \pi\theta_H\theta_V,$$

or

$$\Delta\Omega = \frac{2}{\ell}\sqrt{\epsilon_H\epsilon_V}$$

We use the emittances of the SPS design report[1] and the existing Fermilab Booster.[5]

The momentum acceptance of the channel depends upon the properties of the accumulator. In the CERN case, the momentum acceptance of the transfer lines and machine are designed to be maximum and there are simply no other restrictive constraints. The Fermilab case is quite different. The primary constraint here is the Booster rf acceptance, its operating cycle and the rf manipulation needed to achieve the design goal of $\Delta p/p = \pm 0.15\%$ at 6.1 GeV/c. However, given that the Booster can accept $\Delta p/p = \pm 0.15\%$ at 6.1 GeV/c with a bunching of a factor of 10 "on the fly" and "without significant change in the booster operating modes for p acceleration and $\bar{p}$ deceleration," then it appears possible to accomplish this and still end up with a "coasting beam momentum spread" of $\Delta p/p = \pm 0.15\%$ at the low energy end, this being the Booster acceptance at 200 MeV. The difficulties though should not be underestimated. Some change in the rf cycle could be required and recall that this must be done at short intervals, there being about 1 sec between the p acceleration phase and the $\bar{p}$ deceleration phase. Also handling, observing and controlling high intensity p's and low intensity $\bar{p}$'s alternately could pose problems.

The target efficiency, η_{tgt}, depends upon the target length. When the target length is one interaction mean free path and when reabsorption of the antiprotons in the target is taken into account, the maximum target efficiency is ~ 30%. The acceptance of the channel will not be uniform over the length of the target and over the transverse dimensions of the target. The additional losses may lower the efficiency by a factor 2. We therefore choose

$$\eta_{tgt} = 0.15.$$

The parameters of the CERN and Fermilab designs are shown in Table I.

From these parameters the number of antiprotons is calculated. The ratio $N_{\bar{p}}/N_p$ is shown in Table II (a). Assuming that the Fermilab accelerated proton intensity will double by the time the $\bar{p}p$ collider is built and assuming that the PS intensity also increases from its present value we have calculated the number of antiprotons which would be produced per day. These are shown in Table II (b).

The ratios of the factors used in the antiproton production rate calculation are also shown in Table II. Fermilab has considerable advantage in using high energy protons to produce the antiprotons but the Booster, in which the antiprotons are collected, has a small aperture and works to the disadvantage of Fermilab. For completeness, we show in Table III the parameters involved in $\bar{p}$ accumulation that we consider reasonable compared with those that have been assumed by Fermilab.[3]

II. Luminosity and Tune Shift

The luminosity of two bunched beams colliding head on is given

$$L = \frac{2 N_p N_{\bar{p}} f}{M \left[\beta_H \beta_V \left(\epsilon_{Hp} + \epsilon_{H\bar{p}}\right)\left(\epsilon_{Vp} + \epsilon_{V\bar{p}}\right)\right]^{\frac{1}{2}}}$$

N_p	number of protons stored
$N_{\bar{p}}$	number of antiprotons stored
M	number of bunches
f	frequency of revolution
β_H, β_V	horizontal and vertical β functions at the interaction region
ϵ_{Hp}, ϵ_{Vp}	horizontal and vertical proton beam emittances
$\epsilon_{H\bar{p}}$, $\epsilon_{V\bar{p}}$	horizontal and vertical antiproton beam emittances

The luminosity is computed with the parameters listed in Table IV. We find a luminosity of 3.4×10^{29} cm^{-2} sec^{-1} for the Fermilab scheme at 1000 GeV/c and 1.0×10^{30} cm^{-2} sec^{-1} for the CERN scheme at 270 GeV/c. Our calculation of the luminosity for the CERN ring is in agreement with their result given in their design report.[1]

To increase the luminosity, the number of particles in the beams can be increased, the number of bunches decreased, the β values decreased or the beam emittances decreased. This can be done until the beam-beam tune shift for the collisions exceeds the value where stable beam storage can be sustained and instabilities result in a rapid loss of luminosity. The tune shift is given by

$$\Delta\nu_{V,H} = \frac{2 r_p N}{\gamma} \sqrt{\frac{\beta_{V,H}}{\epsilon_{V,H}}} \frac{1}{\left(\sqrt{\epsilon_V \beta_V} + \sqrt{\epsilon_H \beta_H}\right)}$$

$\Delta\nu$	linear beam-beam tune shift at a given collision point
r_p	classical proton radius
N	number of particles in each bunch of the "other" beam
γ	beam energy in units of the proton rest mass

We note the horizontal and vertical beam $\frac{1}{2}$ - sizes are

$$a = \sqrt{\beta_H \epsilon_H / \pi}\ ,$$
$$b = \sqrt{\beta_V \epsilon_V / \pi}$$

We note that the beam-beam limit can be taken roughly to be given by

$$(\Delta\nu)_{limit} < 0.01.$$

The tune shifts are computed with the parameters of Table V. The tune shifts per collision are about a factor of 2 below the maximum allowed. Without the antiproton emittances, the beam-beam tune shifts for protons cannot be computed. For equal numbers of p's and $\bar{p}$'s the proton tune shift for Fermilab would of course be much higher than for antiprotons since the $\bar{p}$ emittances are so much smaller. However, in this paper we have found the $\bar{p}$ collection rate to be sufficiently low at Fermilab that this is probably not an issue. Since the number of p's exceeds the number of $\bar{p}$'s by a factor of 18, the emittances of the $\bar{p}$ beam would have to be less than 18 times smaller for the proton beam tune shift to become significant compared to the antiproton tune shift.

III. Conclusions

The luminosity which would be produced by the CERN $\bar{p}$-p colliding beam is about 3 times that proposed by Fermilab. The major advantage of the CERN proposal is that the acceptance of the cooling ring is 60 times that of the Fermilab proposal. The damping time for stochastic cooling, used at CERN is independent of oscillation amplitude. The momentum and solid angle acceptance can be made as large as the practical limit determined by the magnetic storage ring. At Fermilab the aperture of the Booster is roughly matched to the volume in momentum space that can be cooled by the electron beam during one acceleration cycle.

The limitation on acceptance at Fermilab is therefore imposed nearly equally by the Booster acceptance and the electron cooler.

The antiproton production cross section is larger when the antiprotons are produced with high-energy incident protons. The Fermilab proposal has the advantage of higher incident proton energy.

REFERENCES

[1] Design Study of a Proton-Antiproton Colliding Beam Facility, CERN report CERN/PS/AA 78-3, 1978.

[2] Design Report - Fermilab Cooling Experiment, Fermilab, November 1977.

[3] L. Teng, Introductory talk at this Workshop.

[4] J. W. Cronin - Antiproton Production at Rest in the Center of Mass, 1977 Summer Study on Colliding Beam Physics at Fermilab, Volume I, p. 269.

[5] D. Cline, Introductory talk at this Workshop.

Table I. Antiproton Production and Acceptance Parameters.

	E_p GeV/c^2	$p_{\bar{p}}$ GeV/c	$E\frac{d^3\sigma}{dp_{\bar{p}}^3}$ mb/GeV^2	$\Delta p/p$	ϵ_H mm-mr	ϵ_V mm-mr	ℓ_{tgt} cm	$\Delta\Omega$ μsr
Fermilab	80	6.1	0.7	3×10^{-3}	2.6π	1.3π	5	73.5π
CERN	26	3.5	0.2	1.5×10^{-2}	100π	100π	4.5	4444π

Table II.(a) Antiproton Collection Rates - Collection Factors

	$\frac{d^2N}{dp d\Omega}$ sr^{-1}GeV^{-1}	Δp GeV/c	$\Delta\Omega$ sr	η_{tgt}	$\frac{N_{\bar{p}}}{N_p}$
Fermilab	0.111	0.0186	$73.5\pi\times10^{-6}$	0.15	7.15×10^{-8}
CERN	0.0175	0.0525	$4444\pi\times10^{-6}$	0.15	192.4×10^{-8}
Fermilab/CERN	6.34	1/2.82	1/60.46	1	1/26.9

Table II. (b) Antiproton Collection Rates - Rates

	N_p/pulse	$N_{\bar{p}}$/pulse	sec/pulse	$\bar{p}$/hr	$\bar{p}$/day
Fermilab	3.3×10^{13}	2.36×10^{6}	6	1.42×10^{9}	3.40×10^{10}
CERN	1.0×10^{13}	1.92×10^{7}	2.6	2.66×10^{10}	6.38×10^{11}
Fermilab/CERN	3.3	1/8.1	2.3	1/18.8	1/18.8

Table III. Comparison of Parameters Used in $\bar{p}$ Accumulation.

Parameter	Fermilab (optimistic)	This Paper
p (GeV/c)	6.1	6.1
$\Delta p/p$	3×10^{-3}	3×10^{-3}
Δp (GeV/c)	0.018	0.018
ϵ_H (mm-mrad)	4π	2.6π
ϵ_V (mm-mrad)	2π	1.3π
$\Delta\Omega$ (μsr)	111.4π	73.5π
$d^2\sigma/dpd\Omega$ (mb/sr/GeV/c)	8.07	4.33
η_{tgt}	0.33	0.15
σ_{tot} (mb)	40	39

For $N_{\bar{p}}/N_p$ Computation

	$\frac{d^2N}{dpd\Omega}$ ($\bar{p}$'s/sr/GeV/c/int·p)	Δp GeV/c	$\Delta\Omega$ sr	η_{tgt}	$N_{\bar{p}}/N_p$
This Paper	0.11	0.18	73.5π	0.15	7.1×10^{-8}
Fermilab	0.20	0.18	111.4π	0.33	4.3×10^{-7}
Fermilab/This Paper	1.82	1.0	1.52	2.20	6.1

Table IV. Luminosity

Parameter	Fermilab	CERN SPS
E (GeV)	1000	270
N_p	6×10^{11}	6×10^{11}
$N_{\bar{p}}$ (1 day accumulation)	3.4×10^{10}	6×10^{11}
f_{rev} (kHz)	47.8	43.4
M (no. of bunches)	6	6
β_H (m)	2.5	4.7
β_V (m)	2.5	1.0
ϵ_{Hp} (mm-mrad)	$1.42\pi \times 10^{-8}$	$6.9\pi \times 10^{-8}$
ϵ_{Vp} (mm-mrad)	$1.03\pi \times 10^{-8}$	$3.5\pi \times 10^{-8}$
$\epsilon_{H\bar{p}}$ (mm-mrad)	$\ll \epsilon_{Hp}$	$3.8\pi \times 10^{-8}$
$\epsilon_{V\bar{p}}$ (mm-mrad)	$\ll \epsilon_{Vp}$	$1.9\pi \times 10^{-8}$
L ($cm^{-2}sec^{-1}$)	3.4×10^{29}	1.0×10^{30}

Table V. Beam Beam Tune Shift in a Single Collision

Parameter	Fermilab	CERN SPS	
	$\Delta\nu_H = \Delta\nu_V$ (antiproton)	$\Delta\nu_V$ (proton)	$\Delta\nu_H$ (proton)
N (particles/bunch)	1×10^{11} (proton bunch)	1×10^{11}	1×10^{11} (antiproton bunch)
β_V (m)	2.5	1.0	1.0
β_H (m)	2.5	4.7	4.7
b (mm)	0.16	0.138	0.138
a (mm)	0.19	0.423	0.423
γ	1066	288	288
$\Delta\nu$	4.1×10^{-3}	4.4×10^{-3}	6.8×10^{-3}