# COLLIDING BEAMS - LIMITATIONS/INSTABILITIES

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## I. Fundamental Limitations on Luminosity

In principle, for moderate intensities, all coherent instabilities can be cured by modifying the impedance of the beam environment either passively or actively (feed-back). Self fields and beam-image forces, although extremely nonlinear, are not very rich in harmonics. They produce incoherent tune spreads that, if too large, will cause beam loss through resonances. The beam-beam forces, which are the most limiting, are however both nonlinear and rich in harmonics and cause the beam to blow up through a diffusion-like process. This stochastic process of beam growth can be counteracted only by some kind of cooling mechanism. For a crude measure of the magnitude of the beam-beam forces, one generally uses the linear tune shift. The achievable luminosity is then determined by the maximum beam-beam tune shift allowed.

For head-on collision of bunched beams, the luminosity is given by

$$L = \frac{n_p n_{\bar{p}}}{2\pi \left(\sigma_p^2+\sigma_{\bar{p}}^2\right)_H^{1/2} \left(\sigma_p^2+\sigma_{\bar{p}}^2\right)_V^{1/2}} fN,$$

where $n_p$ and $n_{\bar{p}}$ are the numbers of particles (p or $\bar{p}$) per bunch, $\sigma$ is the rms beam half-width (subscripts p for proton, $\bar{p}$ for antiproton, H for horizontal, V for vertical), f is the revolution frequency, and N is the number of bunches in each beam (same number in both beams). If parameters are identical in the horizontal and the vertical planes, we can write

$$L = \frac{f n_p n_{\bar{p}}}{2\pi \left(\sigma_p^2+\sigma_{\bar{p}}^2\right)} N,$$

or, in terms of the emittance $\epsilon \equiv 6\pi\sigma^2/\beta$, (assuming zero dispersion)

$$L = 3f \frac{n_p n_{\bar{p}}}{\epsilon_p+\epsilon_{\bar{p}}} \frac{N}{\beta^*}.$$

The beam-beam tune shifts per bunch are given by

$$\Delta\nu_p = \frac{3}{2}\frac{r_o}{\gamma}\frac{n_{\bar{p}}}{\epsilon_{\bar{p}}}, \quad \Delta\nu_{\bar{p}} = \frac{3}{2}\frac{r_o}{\gamma}\frac{n_p}{\epsilon_p},$$

where $r_o = 1.535 \times 10^{-18}$ m is the classical proton radius. Maximum luminosity is obtained when both beams are the same and limited by the same allowable tune shift. We can then drop the subscripts p and $\bar{p}$ and express both L and n in terms of $\Delta\nu$. This gives

$$\begin{cases} L = \dfrac{2f}{3}\dfrac{(\Delta\nu)^2}{r_o^2}\gamma\epsilon_o\dfrac{N}{\beta^*} \\ n = \dfrac{2\Delta\nu}{3r_o}\epsilon_o \end{cases}$$

($\epsilon_o \equiv \beta\gamma\epsilon$ = normalized emittance; $\beta \simeq 1$)

If no cooling is applied during collision, a traditionally acknowledged safe upper limit for $\Delta\nu$ is 0.005, for a beam lifetime of few tens of hours. If cooling is applied during collision through heat exchanging with a cold electron beam, as suggested during the Workshop by C. Rubbia,[1] the upper limit of the tolerable $\Delta\nu$ can be raised before encountering a strong resonance effect or the stochastic limit (overlapping of resonances in one dimension). Here we will continue to take $\Delta\nu$ = 0.005. The following table gives the parameters for the Fermilab Tevatron and the CERN SPS.

| | Tevatron | SPS |
|---|---|---|
| f | 48 kHz | 43 kHz |
| $\gamma$ | 1067 (1000 GeV) | 288 (270 GeV) |
| $\beta^*$ | 2.5 m | $\sqrt{1\text{ m} \times 5\text{ m}}$ |
| $\epsilon_o$ | $15\pi \times 10^{-6}$ m-rad | $14\pi \times 10^{-6}$ m-rad |
| $\Delta\nu$ | 0.005 | 0.005 |
| n | $1.0 \times 10^{11}$ | $1.0 \times 10^{11}$ |
| L | $(0.68 \times 10^{30})$ N cm$^{-2}$sec$^{-1}$ | $(0.17 \times 10^{30})$ N cm$^{-2}$sec$^{-1}$ |

To get L = $10^{31}$cm$^{-2}$sec$^{-1}$ we need approximately

| | Tevatron | SPS |
|---|---|---|
| N | 15 | 60 |
| nN | $1.5 \times 10^{12}$ | $6 \times 10^{12}$ |

We condude that a pp luminosity of $10^{31}$cm$^{-2}$sec$^{-1}$ is obtainable with moderate effort in the Tevatron, but is more difficult to attain in the SPS because of the lower beam energy. We next investigate the incoherent tune spread and the coherent instabilities that may be encountered by the somewhat intense proton beam bunches in the Fermilab scheme.

## II. Transverse Incoherent Tune Spread

This is given by the Laslett formula (round beam)

$$\delta\nu = \frac{r_o\lambda}{\gamma}\left(\frac{\pi R}{\gamma\epsilon_o B} + \frac{G}{\nu}\frac{R^2}{g^2}\right),$$

where the first term in the parentheses is due to the self field, and the second term is due to the image field. For the Tevatron, the largest $\delta\nu$ occurs at 100 GeV. With

- $\gamma$ = 107.6
- R = ring radius = 1000 m
- $\nu$ = betatron tune = 19.4
- g = aperture half-gap = 1 in. = 0.0254 m
- B = bunching factor, ratio of the bunch length to the bunch separation $\simeq$ 0.3
- G = geometrical factor = $\pi^2/12$ (rectangular beam pipe)

$\epsilon_0$ = normalized emittance = $15\pi \times 10^{-6}$m
$\lambda$ = average particle linear density
= $(1.0 \times 10^{11})/5.6\text{m} = 0.18 \times 10^{11}\text{m}^{-1}$
(rf wave length = 5.6 m),

we get

$$\delta\nu = 0.017,$$

which is a tolerable spread.

## III. Longitudinal Individual - Bunch (Microwave) Instability

A beam bunch can experience second-order self-bunching at harmonics of the fundamental synchrotron frequency. The instability is described by a longitudinal impedance $|Z_n/n|$ in analogy to the coasting-beam case, where now the mode number n does not really have a precise meaning. If we give $|Z_n/n|$ the meaning of an equivalent impedance after summation over the beam spectrum, the following criterion for stability is commonly used.

$$|Z_n/n| \leq \frac{E/e}{eI_p} \Lambda \left(\frac{\Delta p}{p}\right)^2, \qquad (1)$$

where $\Lambda = \gamma_t^{-2} - \gamma^{-2} = 0.00276$, $I_p$ is the peak current in the bunch and $\Delta p/p$ the full width at half maximum of the momentum distribution.

Equation (1) is a consequence of the Landau damping coming from a spread in the synchrotron frequency that is proportional to the square of the beam height.

By conservative extrapolation from beam observations in the Main Ring, we take

$$|Z_n/n| \sim 50 \text{ ohm}$$

For a Gaussian distribution with rms energy spread $\delta$ and rms length $\sigma$,

$$\frac{\Delta p}{p} = 2.355 \frac{\delta}{E}$$

and

$$I_p = \frac{R\sqrt{2\pi}}{\sigma} I_b,$$

with $I_b$ the average current per bunch. Combining these, we have the result that Eq. (1) transforms to

$$\sigma\delta^2 \gtrsim 6 \times 10^{15} \text{ eV}^2\text{ m}$$

which we have calculated for 1000 GeV and for the parameters of section I, since this is the worst case for a bunched beam.

For instance, if $\delta/E = 10^{-4}$, then

$$\delta \gtrsim 60 \text{ cm}$$

or, in terms of bunch area S (= $6\sigma\delta/c$),

$$S \geq 1.2 \text{ eV-sec.}$$

This condition should be easily satisfied.

Another critical situation is at 100 GeV just after a single proton beam pulse is debunched, when $\Delta p/p$ is smallest. At this time, one should use the coasting beam criterion, which is still of the form (1) if we replace the peak current $I_p$ with the total beam average current $I_o$.

We have $I_o = 0.17$ A, and by taking $|Z_n/n| = 50$ ohm again, the stability condition gives

$$\frac{\Delta p}{p} \gtrsim 1.7 \times 10^{-4},$$

which is about three times larger than one would expect from a longitudinal emittance of 0.1 eV·s. The required final momentum is obtained by letting the bunches blow up in a controlled fashion (with a bunch spreader) up to approximately 0.35 eV·s as is presently done in the Main Ring.

## IV. Longitudinal Bunch-To-Bunch Instability

Each beam bunch can oscillate in various modes, m, of the longitudinal oscillation. The wake field of the oscillation of one bunch can affect all following bunches and cause a bunch-to-bunch instability at some mode number $\mu$. This instability is stabilized by Landau damping from a spread $\delta\Omega$ in phase oscillation frequency within a bunch.

Sacherer[2] has calculated the complex shift $\Delta\omega_m$ of the angular synchrotron frequency. It depends on the longitudinal coupling impedance and on the spectrum of the beam. By the approximation of an impedance that increases linearly with frequency, namely when $Z_n/n$, as defined in the previous section, is constant,[3] one has simply

$$\left|\frac{\Delta\omega_m}{\Omega}\right| < \sqrt{m}\, \frac{I_o\, |Z_n/n|\, N^2}{2\pi h B^3 V \cos\phi_s}, \qquad (2)$$

where $\Omega$ is the angular phase oscillation frequency, $I_o$ the average current in N bunches, h the harmonic number, V the peak rf voltage and $\phi_s$ the stable synchronous phase. Equation (2) applies in the limit in which spurious sharp resonances have been eliminated or shifted; otherwise the shift is given by [4]

$$\frac{\Delta\omega_m}{\Omega} = \frac{Z_s\, I_o\, N}{2\pi h B V \cos\phi_s} F_m, \qquad (3)$$

where $Z_s$ is the resonance shunt impedance, and $F_m$ a form factor that measures the excitation of the beam. At worst, $F_m$=1. We take also $\phi_s = 0$.

The stability condition [4] is

$$\frac{\sqrt{m}}{4}\delta\Omega \geq |\Delta\omega_m| \qquad \text{for (2)}$$

and

$$\frac{\delta\Omega}{4} \geq |\Delta\omega_m| \qquad \text{for (3)}$$

The spread in $\delta\Omega$ arises from the nonlinearities of the particle motion within a rf bucket. If the bucket is not full, one has

$$\frac{\delta\Omega}{\Omega} \simeq \frac{\phi_o^2}{16}$$

for a bunch with half-length $\phi_o$ expressed in rf radians.

At the same time,

$$B = \frac{2\phi_o N}{2\pi h}$$

Taking $|Z_n/n| = 50\Omega$, V= 1 MV and 0.75 mA per bunch, we derive from (2) the following condition on the bunch length.

$$\phi_o \gtrsim 1.7 \text{ rad}$$

which, because of the way we have derived it, does not depend on the number of bunches.

From our experience in the Main Ring, spurious modes occur mostly in the rf cavities and can be easily damped down to a few tens of k$\Omega$. If we take $Z_8$ = 30 k$\Omega$, we derive from (3) the following condition on the bunch length.

$$\phi_o \gtrsim 0.1\, N^{1/3} \text{ rad},$$

which is less stringent than the previous one.

If shorter bunch length is desired, one can either stablize the shorter bunch by using a Landau cavity or at least eliminating the $N^{1/3}$ multiplier by spacing the N bunches asymetrically around the ring, so that their wake fields do not add constructively.

## V. Transverse Individual-Bunch Instability (Head-tail Instability)

This instability can generally be controlled by properly adjusting the chromaticity $\xi = \Delta\nu/(\Delta p/p)$ Above transition, setting $\xi < 0$ will make most of the m> 0 modes stable (depending on the impedance structure) leaving only the monopole mode m = 0 unstable. The m= 0 mode is that in which the beam bunch oscillates transversely as a rigid body and can easily be damped with a feedback circuit as is done in the Main Ring. In the absence of Landau damping, the growth time is proportional to $n/\gamma$. Although the number of particles per bunch n is increased by a factor 5 compared with the present operation of the Main Ring, this factor is compensated by the increase in $\gamma$. Therefore we do not expect that the feedback damper will have to be very different from the one now used in the Main Ring.

## VI. Transverse Bunch-to-Bunch Instability

The dipole case (m = 1) was studied long ago by Courant and Sessler[5] and the throbbing modes (any m) by Lee, Mills, and Morton.[6] More recently, Sacherer[7] has unified the theories of the transverse instabilities including also the head-tail effect by combining the effects of short and long-range wake fields and taking into account the non rigidity of the bunches. The result of this general theory is that the instability causes a complex shift of the betatron angular frequency which is given by

$$\Delta\omega_m = \frac{1}{1+m}\,\frac{i}{2\nu\omega_o}\,\frac{eI_b}{\gamma m_o L}\,\frac{\sum_p Z_\perp(\omega)\, h_m(\omega-\omega_\xi)}{\sum_p h_m(\omega-\omega_\xi)}, \tag{4}$$

where m is the internal bunch mode, $i = \sqrt{-1}$, $\omega_o$ the angular revolution frequency, $\nu$ the betatron tune, e the particle charge, $I_b$ the average current per bunch $\gamma m_o$ the relativistic mass, L the full bunch length, $Z_\perp$ the "transverse impedance" of the surroundings which has to be calculated at the angular frequencies

$$\omega = (p+\nu)\,\omega_o$$

where $-\infty < p$, integer $< +\infty$ for a single bunch or several bunches oscillating independently, and p $= \mu + kN$, $-\infty < k$, integer $< +\infty$ for coupled motion of N bunches, $\mu$ being then the bunch to bunch mode number. In addition,

$$\omega_\xi = \nu\omega_o \frac{(\Delta\nu/\nu)/(\Delta p/p)}{|\gamma_t^{-2} - \gamma^{-2}|}$$

and $h_m(\omega)$ are Sacherer's functions[7] which give weights for the contribution of the beam sprectrum.

The transverse impedance $Z_\perp$ can be approximated for circular geometry in terms of the longitudinal impedance $Z_n/n$ used in section III above[7] as

$$Z_\perp = \frac{2c}{b^2}\,\frac{Z_n/n}{\omega/n}, \tag{5}$$

where b is the vacuum-chamber radius.

The beam is made stable by providing a spread in time $\delta\nu$ (Landau damping) such that

$$\omega_o\, \delta\nu \gtrsim \Delta\omega_m\ . \tag{6}$$

In the approximation that $Z_n/n$ is constant, and anomalous, parasitic modes have been reasonably damped, $Z_\perp$, (5), is a constant and can be taken out of the summation at the r.h.s. of (4). This gives[8], by taking the worst case, m=0 and by making use of (2),

$$\Delta\nu \gtrsim \frac{eI_b}{\nu\omega_o^3}\,\frac{c^3|Z_n/n|}{E\,L\,b^2}\,. \tag{7}$$

If this condition is satisfied, the beam is certainly stable, provided spurious impedance resonances are properly damped, but the opposite is not necessarily true. With this very conservative procedure, the stability condition (7) does not depend on the two instability mode numbers m and $\mu$. Inserting the same numbers as before, we have

$$\Delta\nu \gtrsim \frac{0.03}{L\text{ (in m)}} \qquad \text{(at 100 GeV)} \tag{8}$$

If the bunch length L > 2m as required in section IV, the corresponding tune spread should be attainable. Eventually a slow damper similar to the one presently used in the Main Ring can be used to damp dipole bunch-to-bunch modes. Higher modes m require less spread than (8).

## VII. Conclusion

The overall conclusions are, for the high-intensity proton bunches in the Fermilab scheme:

1. If the longitudinal emittance is larger than 0.35 eV-sec per bunch, there will be no trouble in adiabatic debunching in the Main Ring at 100 GeV.

2. The head-tail instability can be controlled in the usual manner by adjusting the chromaticity and using the feedback damper.

3. Harmful spurious resonances in the rf cavities must be shorted to impedances below, say, 30 kΩ. Then, as long as the beam bunches are longer than approximately 2 m, there will be no trouble with either longitudinal or transverse bunch-to-bunch instabilities. Eventually a Landau cavity can be used to shorten the bunches.

4. A bunch spreader is required for the Energy Doubler to adjust the final bunch area to the threshold of instability.

## References

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