Design of High Energy Proton Storage Rings

S. Ohnuma

Fermi National Accelerator Laboratory
Batavia, Illinois, U.S.A.

I. Introduction

In 1969, Matthew Sands opened a series of lectures on the physics of electron storage rings with a statement "Electron storage rings have now come of age."[1] If this sentiment were not shared by other physicists - we must remember that neither DORIS at Hamburg nor SPEAR at SLAC was completed at that time - storage rings of not only electrons but also of protons are here to stay as one of the most important tools of high energy physics research. In fact, achievements in particle physics with DORIS, SPEAR and CERN ISR (for pp colliding) have been so dazzling that most if not all major projects under way or in the proposal stage are for colliding beam facilities. Energy Doubler at Fermilab, which will double the proton energy from the presently available value of 500 GeV to 1 TeV using superconducting magnets, seems to maintain its glamor by incorporating colliding beam capabilities.

It is not difficult to understand why colliding beam facilities are now so popular compared with conventional fixed-target accelerators. Physicists engaged in the research of "elementary" particles are fascinated by the inner structure of these particles and the fundamental interactions that can explain their behavior. For the study of these, they always demand very high energies available to produce massive new particles and to reveal new types of interactions. The available energy (the center-of-mass energy) of a beam is usually called $\sqrt{s}$, a notation which is strange like so many others in high energy physics. With fixed-target accelerators, targets are at rest in the laboratory and the bulk of the beam energy is simply used for the forward motion of the entire system. The value of $\sqrt{s}$ increases only as the square root of the beam energy. When "target" particles are also moving in the laboratory toward the incoming beam, we convert a larger fraction of the total energy into $\sqrt{s}$ until the momentum of the beam is the same as the target momentum. The available energy $\sqrt{s}$ is then just the total energy. Some kinematic relations are given in Appendix A.

Since accelerators are merely tools for research, accelerator builders are obligated to provide whatever are most useful for experiments. At the same time, it is natural for them to question the wisdom of this rush to colliding beam facilities. Are conventional accelerators already obsolete in high energy physics? Do we get a complete picture of elementary particles and fundamental laws from reactions involving only electrons, positrons and protons (possibly antiprotons also in future)? I am not qualified to make any comments on these questions but a talk given by Leon Lederman sometime ago[2] is particularly interesting as one coming from a man who has been very active both at Fermilab (fixed-target accelerator) and at CERN (colliding beam).

The talk I have prepared here is definitely not for specialists. There are numerous reports and proposals on colliding beam facilities. It is not possible for me to even give a decent list of references - accelerator physicists are notorious for keeping many of their valuable works unpublished. Those of you who want more details on storage rings should probably consult proposals for PEP,[3] ISABELLE[4] and POPAE[5] and find references for individual topics. My aim is simply to tell you a story on the design of proton storage rings based on personal experiences. As such, it is unavoidable to make prejudiced remarks here and there and to say very little on some topics which are important for the design but are not well digested by me. A design principle adopted at Fermilab may not be considered "right" at CERN or Brookhaven. Each laboratory has by now established a certain distinctive style and often reaches the same goal in an entirely different way. In building accelerators, end almost always justifies means as long as one stays within a reasonable cost.

I have decided not to talk on electron storage rings for various reasons. For one thing, I have never participated in any electron storage ring projects. The beam dynamics of electrons in a circular machine is dominated by synchrotron radiation and its quantum fluctuations. As a consequence, it is not easy for any design to substantially deviate from a number of well established procedures. One can obtain a fairly detailed picture from a small number of parameters and these can be chosen by extrapolation from many existing machines.[6] The lecture note by Matthew Sands[1] is the best material for beginners and I have copied in Appendix B a few formulae from the note. Together with Keil's article,[6] these formulae should be sufficient for specifying major parameters of electron storage rings.

II. Energy

Energy is the most important parameter of any storage ring. For users, the beam energy should be as high as possible but there are of course a number of factors that can influence the design value.

a) cost As long as we build a strong focusing synchrotron as the storage ring, with or without acceleration, the cost of the ring must go up with the energy. Even for an affluent community like Europe, this seems to be the overriding factor in deciding the energy. It is generally believed in U.S. now that the total cost of any proton colliding beam facility should not exceed ~\$300 million.

b) site It is natural to build the facility at an existing high energy laboratory. Since higher energy requires a larger machine, the site is an important factor.

c) bend field A study at CERN to build 400 GeV proton storage rings with conventional iron magnets has concluded that the circumference is uncomfortably large and the power consumption (~ 120 MW) is prohibitively high.[7] The highest field value presently available from superconducting dipoles for a reliable accelerator operation is believed to be 45 kG. For a storage ring without acceleration, 60 kG proposed for POPAE (1 TeV × 1 TeV) may become reasonable within a few years. These values apply also to the highest field at conductors of quadrupoles.

d) injector We have abundant information on electron storage rings which has been accumulated in the past ten years or so at various places. Compared with this, there is only one proton storage ring, the CERN ISR. Although ISR magnets are mostly of conventional type, proton storage ring designers must depend to a large extent on the ISR experience. Since there is no acceleration in the ISR, it is by no means clear whether an intense, bunched proton beam can be reliably accelerated in superconducting magnets. We must remember that a new type of accelerator usually produces a new type of beam instability. Superconducting magnets for accelerating storage rings are also more difficult to build than dc magnets. If the energy of the available injector is much lower than the top energy of the storage ring, the transition energy may be dangerously close to the injection energy and this becomes a severe limiting factor in the overall performance. With superconducting magnets, the change of field is so slow that the passage of transition region is something no accelerator builder is willing to face. The case of ISABELLE clearly demonstrates this difficulty.[8] The injector (AGS) energy is 29.4 GeV. For the top energy of 200 GeV at 43 kG, the transition energy is 20.3 GeV, very close to the injection energy but still lower than that. If the top energy is raised to 350 GeV, still at 43 kG, the transition energy goes beyond the injection energy to 32 GeV. Even with the field increased to 60 kG, the injection energy will be lower than the transition energy when the top energy of the storage ring goes beyond ~350 GeV. One solution is to have a "booster" which uses conventional magnets.[8] This can be used to increase the injection energy of the main storage ring to a comfortable value, and to do beam stacking as well. It is certainly an added trouble to the already complex system.

III. Luminosity

For a storage ring, the luminosity L is probably as important as the beam energy. If the luminoisty is too low, the energy usable for experiments may not be as high as $\sqrt{s}$, the theoretically available value.[2] This is especially true for rare production processes with very small cross sections. The definition of L is such that the number of events N per second is ($L\sigma$) when the cross section is σ. The conventional unit of σ is cm^2 and the unit of L is usually $cm^{-2}sec^{-1}$. For example the luminosity of the ISR can go as high as 10^{31} in this unit. There seems to be a universal value of luminosity for pp storage rings, $(1 - 3) \times 10^{33}$, which accelerator designers are aiming at. The corresponding number for ep and e^+e^- is 10^{32}. One limitation of the luminosity is purely a practical one, how to handle the enormous amount of the stored energy of the beam. For POPAE with 1 TeV and 5 amp, the stored energy in each ring is more than 90 MJ. The energy in each ISABELLE ring with 200 GeV and 10 amp is 20 MJ. These should be compared with a few MJ in the Fermilab main ring or in the ISR. The stored energy of the beam must be dumped in emergency cases and the design of the beam dump is non-trivial. Even during a stable operation, the natural beam loss deposits a certain amount of energy on superconducting magnets and this may become a significant heat load.

Detailed derivation of the luminosity is beyond the scope of this talk. There are several articles on the subject[9] and a few useful expressions are given in Appendix C. Here we will simply see the general dependence of luminosity on various parameters when two unbunched beams are identical. The luminosity per unit volume in the collision between two beams is

$$dL/dV = n^2 \; |\vec{v}_1 - \vec{v}_2|$$

where n is the common volume density of two beams travelling with velocities $\vec{v}_1$ and $\vec{v}_2$. Since the crossing angle α is usually very small compared to unity and $|\vec{v}| \simeq c$, $|\vec{v}_1 - \vec{v}_2| \simeq 2c$. Let us assume that the crossing of two beams is on the horizontal plane and each beam has a rectangular cross section of w (horizontal) and h(vertical) within which the particle density is uniform. The volume density is, in terms of the beam current I, $n = 1/(ecwh)$. The length of the interaction region (luminous region) is $2w/\alpha$ and the volume is w^2h/α so that

$$L = 2c(I/ec)^2(1/\alpha h)$$
$$= 2c(I/ec)^2(1/wh)(w/\alpha).$$

When α is zero (head-on collision), this relation gives an infinite luminosity. In reality, two beams must be separated at some places and the length of the luminous region is always finite. Even for an infinitely long length, the luminosity is finite when we take into account the increase of beam dimension. Relations derived here show that the (integrated) luminosity is independent of the horizontal beam size w. On the other hand, the length of the luminous region, $2w/\alpha$, increases directly as w. The luminosity per unit length, dL/dz (the coordinate z bisects the angle α), takes the maximum value

$$(dL/dz)_{max} = 2c(I/ec)^2(1/wh)$$

at the center and decreases linearly along z-axis. For most experiments, the length of the luminous region should be reasonably short. Design of high luminosity insertion tries to reduce w, h and α keeping w/α not too large. As will be explained later, there are conflicting requirements like beam-beam interactions and the maximum beam size at the quadrupoles near the interaction region and, as for many other machine parameters, the final design values of these parameters have to be a compromise.

Storage rings dhould of course be useful at energies below the top design value. Study of the energy dependence of various reactions is often very important. A question arises as to how the luminosity scales with the beam energy. If the betatron parameters β_H^* and β_V^* at the interaction point are specified by the design of experimental insertion independent of energy and if the effect of the momentum dispertion on the beam size is negligible there, beam sizes w and h are determined from the horizontal and vertical emittances, respectively, of the beam. The emittance of proton beams varies with energy E as 1/E (in contrast with the radial electron beam emittance which varies as E^2 due to quantum fluctuation). We have

$$w = 2\sqrt{\beta_H^* \varepsilon_H} \propto 1/\sqrt{E}, \qquad h = 2\sqrt{\beta_V^* \varepsilon_V} \propto 1/\sqrt{E}$$

where $\pi\varepsilon_H$ and $\pi\varepsilon_V$ are horizontal and vertical emittances, respectively. The length of luminous region varies as $1/\sqrt{E}$. If the same current can be maintained,

$$L \propto 1/h \propto \sqrt{E}.$$

In order to maintain the same length of luminous region, one may change the crossing angle α as $1/\sqrt{E}$. Then $L \propto E$. At some sufficiently low energy, the current may have to be reduced. The radial size of the beam increases at a point in the ring where the momentum dispersion is non-zero since the relative momentum spread of the beam varies as 1/E. If the current is varied as E, $L \propto E^{5/2}$. If, in addition, the length of the luminous region is to remain unchanged, $\alpha \propto 1/\sqrt{E}$ and $L \propto E^3$.

IV. Lattice and Insertions

IV. A. Superperiod

It may be an overstatement to say that resonances are a nightmare for many accelerator builders. Nevertheless, they instinctively shrink away from designing a machine with a low periodicity. For example, there are six superperiods in the Fermilab main ring, 24 in the booster (why so many I have never understood) and 12 in AGS. Single period machine like one at Cornell (12 GeV electron synchrotron) is indeed a rare specimen. In a ring containing N superperiods, there is in principle no driving force of the resonance

$$n\,\nu_H + m\,\nu_V = k; \quad n, m, k = 0 \text{ or integers}$$

unless k is a multiple of N. If one takes a real machine with imperfection in construction, resonance with any integer k can be excited since the ring periodicity is then 1. However, these so-called imperfection resonances are much weaker than structure (intrinsic) resonances associated with the symmetry. For a single period machine, all resonances are intrinsic and are therefore potentially dangerous. Of many resonances, by far the most serious are half-integer resonances, $|n| + |m| = 2$, arising from chromatic aberration in quadrupoles. In an ordinary synchrotron, the relative momentum spread of the beam is so small that it is not difficult to avoid half-integer stopbands. The beam in a storage ring must be stacked in momentum space and this results in a large momentum spread. Moreover, demand for higher luminosities at beam crossing points makes the betatron oscillation function β abnormally large at strong quadrupoles nearest crossing points. Even when the beam with a large momentum spread is contained between two adjacent half-integer stopbands, the nearby existence of stopbands increases the momentum dependence of β.[10)]

Storage rings constructed to be useful for a large variety of experiments with several different types of insertions will necessarily be operated in a single period mode. This is true even for a ring with an apparent periodicity in lattice structure since quadrupoles and dipoles may have to be excited differently in each insertion. Accelerator builders must sooner or later face the problem of single period operation. There are two different attitudes - there may be others which I am not aware of - among U.S. accelerator designers in trying to cope with this problem.

ISABELLE designers try to maintain the basic symmetry of eight superperiods as long as possible.[4)] The symmetry will be preserved not only during stacking and acceleration but in the initial phase of experiments also. All crossing points would have the same beam and consequently the same luminosity. The maximum luminosity is then 4×10^{32} cm^{-2}sec^{-1}. After that, they will gradually break the symmetry, feeling each step cautiously. In order to get the advertized maximum luminosity of 1×10^{33} cm^{-2}sec^{-1}, they will install large aperture bending magnets which will reduce the crossing angle from the standard 11.4 mrad to 4.8 mrad. They may in future even change some magnet locations to allow the introduction of apparatus such as large spectrometer into the free space around the collision point. This certainly would make a single period machine.

POPAE designers decided to face the problem squarely from the beginning. In Phase I design, partly because of site consideration, they chose a ring of race-track shape without any symmetry. In the subsequent revision, the shape was changed to six-sided geometry but there was no change in philosophy; the ring must work in a single period mode. They have made an extensive study on how to rectify the trouble associated with such a structure.[11)] Various arrangements of correction sextupoles and the introduction of special phase-adjusting sections are two main results of the study. It is undeniable that they felt confident in their ability to run a complex system of correction elements. After all, the Fermilab main ring is the most "corrected" accelerator in the world. In the initial phase of operation, they will probably have to start with a rather gentle beam size variation in experimental insertions and gradually squeeze the beam.

It is quite possible that they may end up doing the same thing in the real operation of the machine. Perhaps the difference in attitude at present may simply be a reflection of the difference in the beam characteristics. During stacking, ISABELLE needs a momentum aperture of almost 2% compared to POPAE's 0.57%. The maximum momentum spread of the ISABELLE beam during the acceleration is 1.6% due to the necessary bunching. The final momentum spread of the stacked beam is 0.7% for ISABELLE and 0.05% for POPAE.

It should be mentioned that the recent successful operation at the ISR with a single low-β insertion[12)] is a welcome event for all accelerator builders.

IV. B. Normal Cells

It is by now a matter of common practice to use a simple separated FODO cell with 90° betatron phase advance as the normal cell in curved sections. The choice of 90° is not really the optimum one if the smallest beam size is wanted (for that one takes ~75°) but the difference is usually insignificant compared to the convenience of 90° phase advance for general beam manipulations. This choice is also convenient in arranging correction sextupoles; by taking four cells as a group, one can make combinations which act selectively on the chromaticities, off-momentum gradient stopbands, and third-integer resonances, respectively. Various parameters of the cell can be easily calculated as a function of the cell length and the total bend angle in the cell using the so-called thin-lens approximation. The approximation is accurate enough for all practical purposes in the initial stage of design. Tom Collins provided convenient relations during the POPAE design work which are reproduced almost verbatim in Appendix D.

There are a few factors important in deciding details of the normal cell configuration. One naturally wants to have as large a packing factor (bend filed length/total length) as possible for the sake of economy. Remember that, regardless of its shape, the ring must have 360° (or more) bend in total; a smaller packing factor means a longer, that is, a more expensive curved section. A longer dipole is better for this purpose but there is a limit, especially with superconducting magnets. The choice of 6 m for POPAE dipoles is probably too long to be comfortable. The sagitta, which is proportional to the length squared, is another problem of a long dipole. (Can we make a curved superconducting dipole without sacrificing field qualities?) In Phase I of POPAE design, a 3 m long space is left after every quadrupole and the importance of this is strongly emphasized. When the consideration of cost became overwhelming, the space was reduced to 0.8 m and all correction elements were incorporated in dipoles and quadrupoles. I am sure many POPAE designers still share the sentiment expressed in Phase I Summary Report:

> "A versatile and easily manipulated set of correction magnets is an essential system in the storage rings that we outline here. Further study will aid in defining the scope of this system. However, we doubt that the correction requirements can be fully analyzed without operation experience, and we feel that an early reduction in the space allocated for this purpose would prove to be a very poor economy indeed."

Maybe it is up to magnet designers to decide whether one can realistically include a complex but reliable correction system in dipoles and quadrupoles without unduly affecting other equally important considerations.

IV. C. Dispersion Elliminating Cells

There used to be a lot of arguments on what type of beam geometry is most favorable from experimenter's and builder's point of view: horizontal beam crossing <u>vs</u> vertical crossing, one ring on top of the other or two rings side by side. In early days, people somehow liked to have one ring on top of the other. Presumably this is to reduce the tunnel size. Crossing can still be either horizontal or vertical but a horizontal crossing of two vertically separated beams is very awkward as one can see from the initial design of ISABELLE.[13)] With the vertical crossing, the necessity of eliminating both horizontal and vertical dispersions at the crossing point often results in a rather contorted design of experimental insertion.[14)] Beam lines going up and down are certainly not comfortable to builders. It is safe to say that, unless there are other compelling reasons to do otherwise, the prefered style is now to have a horizontal crossing with two rings placed side by side. Whether two rings should be close together with accessible spaces on both sides or should be separated to have an access in between is a matter of taste. ISABELLE design chose the former and POPAE design the latter. In any case, there is no vertical bend in the ring and no vertical dispersions to worry about.

For experiments with very small cross sections, the luminosity and the length of the source (luminous region) are of primary importance. In order to reduce the beam size as much as possible, dispersion and its derivatives must be eliminated entirely at the end of curved sections. Experiments requiring high angular resolution are usually accompanied with large cross sections and the luminosity is not a problem. One would rather make the beam size large (hence the name high-beta insertion) and reduce the angular spread of the beam. Dispersion (of displacement) does not have to vanish but its derivative which contributes to the angular spread of the beam should be zero. In practice both dispersion and its derivative are eliminated in all straight sections. There are two ways of doing this, each having advantages and disadvantages.

One can take out some dipoles and rearrange others in one or more cells near the end of each curved section. The difference in total bend angle of the inner arc and that of the outer arc then becomes the crossing angle. One can eliminate dispersions without changing the quadrupole strengths from the standard value in the normal cell. Betatron oscillation functions $\beta_{H,V}$ are essentially unchanged since the focusing action of dipoles is much weaker compared to quadrupoles. This scheme has the disadvantage of creating a relatively large number of missing dipole gaps, thereby reducing the overall packing factor. For example, ISABELLE design calls for 56 missing dipoles out of 320 lattice positions in each ring. The other method of eliminating dispersion is to take out only one dipole each near both ends of the inner arc. With N crossings, the total number of missing dipoles is just N, a small number. The beam crossing angle is equal to the bend angle of one dipole. In this scheme, the strengths of all quadrupoles in end cells of each curved section, both inner and outer arcs, have to be modified from the standard value. The resulting changes in $\beta_{H,V}$ make it necessary to design two different matching systems between a curved section and its adjacent straight section, one for inner arcs and another for outer arcs. One should probably add in defence of the first scheme that many missing gaps will be valuable for various machine functions such as injection, ejection, rf systems and beam diagonostics. Thin-lens approximation is convenient in quickly finding proper arrangements of dipoles or necessary changes in quadrupole strengths. One traces the off-momentum beam trajectory x_p and x_p'; the action of a quadrupole with the focal power f is to change x_p' by $-f\ x_p$ (f positive for focusing quadrupoles) and the action of a dipole with the bend angle θ is to change x_p' by θ at the bend center. The change in x_p is only in free spaces, x_p changed by $x_p'\ell$ where ℓ is the free space length. For a detailed calculation, computer programs like TRANSPORT[15] or MAGIC[16] are easily available at most places.

IV. D. Phase Adjusting Insertions

POPAE designers felt strongly from the beginning that there should be special insertions to adjust the betatron phase advance over a range of ~100°. There were three such insertions in Phase I design, each 90 m long, with the provision that one or more may eventually be eliminated when enough experience is gained in the operation of the machine. The number was subsequently reduced to one in the final design in order to satisfy the greed of experimental physicists for many experimental insertions. Although limited in performance (luminosity a few times $10^{31}\text{cm}^{-2}\text{sec}^{-1}$) this insertion could be used for less-demanding experiments.

There are a number of reasons for including the phase adjusting section in storage rings. First, operating tunes can be adjusted to the optimum point without breaking the 90° phase advance in normal cells. Until a real machine is built, one never knows the optimum tune precisely.

This has been amply demonstrated in the Fermilab main ring and in the KEK main ring. Second, it allows us to change freely the tuning of any experimental insertion without worrying about the resulting change in tunes. This is important since experimental insertions are meant to be versatile and be usable for many different types of experiments. Third, one may reduce the effects of beam-beam interactions by changing the phase advance between two crossing points. After all, the advantage of high periodicity is the cancellation of driving forces for some resonances. One can achieve the same thing by adjusting the phase advance between two sources of resonance driving force.

The importance of phase adjusting insertions does not seem to be taken seriously by other accelerator designers. They must feel they can build a machine so well that such a precaution is unnecessary. It may be just a difference in style; there is no way to decide one is "right" and the other "wrong" before we have a real machine.

IV. E. Experimental Insertions

Design of experimental insertions should always be a joint venture of accelerator designers and high energy experimentalists. Important parameters are: luminosity, source length, length of free space available to experimenters around the crossing point, tuning range of the beam size and crossing angel. For small angle pp experiments requiring a good angular definition, there should be a space for detectors at 90° phase-advance locations to insure a parallel-to-point geometry. Some experimentalists demand the capability of operating the two rings at unequal energies. They say one can vary the center-of-mass angle of the emitted particle without moving the detector by simply changing the ratio of two energies. Quadrupoles shared by two beams are then out of question. We may have to install a pair or more of large aperture (18 cm is mentioned in the POPAE design) superconducting dipoles to reduce the standard crossing angle, even down to head-on collision. The symmetry of these dipoles should be such that there will be no dispersion at the crossing point. It has been pointed out by Montague and Zotter[17)] that, for unequal energy beams, the collision line which bisects the angle between beams is tilted from that for equal energy, the angle of inclination becoming larger for larger energy ratio. As some quadrupoles in experimental insertions tend to be strong, results obtained by thin-lens approximation is not always reliable. TRANSPORT and MAGIC are most widely-used tools for designing experimental insertions. Even with these powerful computer programs, a good design is a painful, time-consuming task. The work is not suited to feeble-minded machine designers or whimsical experimentalists.

Since there are so many possibilities in the design of insertions, designers generally make their work easier by imposing symmetry conditions. The operation of the rings may become easier in symmetric optics than otherwise. Two alternative symmetry arrangements are possible in the focusing sequence of each beam: symmetric and antisymmetric. In a symmetric arrangement, focusing actions of quadrupoles have reflection symmetry about the midpoint. Since the space there is free of quadrupoles, the bema size is at its minimum at the crossing point for both horizontal and vertical directions ($\alpha_{H,V} = 0$ in beam optics language). If the insertion begins with a horizontally focusing quadrupole of a normal cell, it must end with another horizontally focusing quadrupole. In general, there is no simple relations between horizontal and vertical beam optics; two phase advances of betatron oscillations could be quite dissimilar. In an antisymmetric arrangement, quadrupole actions have reflection symmetry about the midpoint with change of sign. The beam optics in the horizontal and vertical directions are midpoint-reflection of each other. This insures that two phase advances are identical. On the other hand, the beam size does not necessarily take its minimum value at the crossing point. The insertion started with one quadrupole of the normal cell ends up with another of opposite focusing action.

Novices often forget that magnets must have a certain minimum size. First two quadrupoles (or dipoles not shared by two beams) on the same side of the crossing point are often very close to each other. The distance between two beams is 23 cm in the ISABELLE design and 30 cm (the closest case) in the POPAE design. These values probably represent the minimum one can realistically think about. Quadrupoles may have to be of special construction, for example sharing a common dewar. One can of course stagger these quadrupoles if one is willing to perturb the symmetry. When there are both horizontal and vertical bends, one must take into account geometries and optics in both planes, a nerve-racking task indeed. When dipole bend angles are not too large, one can get geometrical positions of the beam by turning off all quadrupoles in the calculation. Resulting dispersions from dipoles alone give the geometry of the beam.

V. Injection, Stacking, Ejection

V. A. Injection

Design of injection lines is a relatively easy part of the colliding beam facility. Total bending angles of two beam lines should be as small as possible to save the cost. Magnets can be of conventional type since the transport lines are used only during the beam stacking. Last elements of the lines are a series of septum magnets and the kicker. If the space between magnets in the ring is at liquid He temperature (the so-called cold-bore system like POPAE and Energy Doubler at Fermilab), there may be non-trivial technical problems which are not completely solved by designers of these projects. Before getting extracted from the injector, the beam shape in the longitudinal (energy-phase) space is adiabatically changed by reducing the injector rf voltage such that the resulting beam shape is matched to the waiting rf bucket of the beam stacking system. ISABELLE designers considered all kinds of rf gymnastics in the initial stage of design but settled on the "standard" procedure. The injected beam must be matched in transverse directions to the storage ring lattice and the beam dispersion should be zero at the septa. The kicker is placed at 90° or 270° phase advance from the end of septum magnets. The dispersion at the kicker position must be large enough to have an adequate distance between the injected beam closed orbit and the already-stacked beam orbit. Betatron oscillation function should not be too small at the kicker and at the septum positions; otherwise the amount of necessary kick may become excessive. A kicker with a mechanically moving shutter to shield the previously stacked beam from the kicker field has been in use for many years in the ISR. Both ISABELLE and POPAE design employ the ISR scheme. The distance between the full stacked beam and the injected beam (edge to edge) is 14 mm in ISABELLE and 15 mm in POPAE.

It is possible to use a full-aperture kicker without kicker-shield for the injection but the kicker timing precision becomes critical.[14),18)] The scheme can best be understood from a specific case, POPAE Phase I design. Before the arrival of a beam pulse from the injector, two pulsed kicker dipoles perturb the closed orbit in the storage ring such that the beam finds itself on the closed orbit appropriate to its momentum, in other words the closed orbit is on the "other" side of the septum.

Two kickers K_1 and K_2 are, respectively, at 270° upstream and downstream of the septum S. The dispersion is maximum at S and very small at K_1 and K_2. The duration of the beam pulse is 21 μsec and the revolution time is 28 μsec. Kickers must be turned off in 7 μsec before the next passage of the injected beam. After acceleration (or deceleration) of the injected beam to the stack position (the beam now has a different momentum from that of the injected beam), kickers are again turned on to move the closed orbit of the next pulse to the outside of the septum. Both the turn-on and the turn-off of two kickers must be done such that the downstream kicker K_2 is delayed by the beam transit time between two kickers, 1.2 μsec in this case. Error in the amount of necessary kicks or in the delay times will leave a residual betatron oscillation in the stacked beam, thereby effectively increasing its emittance. The distance between two closed orbits, one for the injected beam and the other for the stacked beam, is taken to be 13 mm corresponding to 0.3 % difference in two momenta with 4.5 m dispersion function.

V.B. Stacking

Stacking of the beam is done in the momentum space. It is difficult to imagine a stacking in the betatron phase space. The beam with a large transverse emittance will never have a lifetime of many hours and the resulting luminosity will be unusably low. There is a technical jargon, "stacking on the top" or "stacking at the bottom". "Top" and "bottom" here refer, respectively, to the momentum edge of the stack farthest and nearest to the inejction momentum. Stacking on the "top" is a repetitive process. The energy difference between the injection and the stacking orbits is identical for each pulse and the amount of frequency modulation during the stacking remains the same. The injected beam is synchronously captured by the matched stationary bucket of the stacking rf system and accelerated (or decelerated) to the existing stack. Before approaching the stack, the rf voltage is reduced to shrink the beam buckets to a tight fit around the beam bunches. This minimizes the phase space dilution of the stacked beam when the buckets go through the stack. Stacking at the "bottom" is a non-repetitive process. Although one may be able to reduce the stacking time somewhat, demands on the programming of the frequency and voltage of the stacking rf are more difficult to be met. It is also theoretically a less stable mode of stacking than the other one. If the beam is to be accelerated to the final energy (ISABELLE, for instance), the stacked beam which is completely debunched must be rebunched by another rf system for the acceleration. With superconducting magnets, acceleration is a rather slow process. For ISABELLE, it takes three minutes to accelerate the stacked beam from 30 GeV to 200 GeV.

The longitudinal emittance ε_ℓ of each rf bunche is usually defined to be the area in $(\Delta E/\omega_{rf})$ - $(\Delta\psi)$ phase space where ω_{rf} is the rf angular frequency (= 2πx harmonic number x revolution frequency), ΔE and $\Delta\psi$ are respectively the energy and the phase spreads. For ISABELLE, ε_ℓ = 1 eVsec and for POPAE, ε_ℓ = 0.1 eVsec. The relative momentum spread, $\Delta p/p$ (= $\Delta E/E$) of the debunched beam is ($\Delta\psi = 2\pi$)

$$(\Delta p/p)_{bunch} = \varepsilon_\ell \times (\text{rf frequency/energy}).$$

If N pulses from the injector are needed to get the full design current in the storage ring, the final momentum spread of the stacked beam is

$$(\Delta p/p)_{stack} = N\ (\Delta p/p)_{bunch}\ .$$

However, there is always a certain amount of phase space dilution during the stacking and the real momentum spread is larger than this expression. The factor to be multiplied has been obtained from the ISR experience and it is written in the phenomenological form[19)]

$$1 + \frac{2\ \sin\psi_s}{3\ \alpha\sqrt{n}} \qquad (>1)$$

where ψ_s is the synchronous phase of acceleration (or deceleration) through the stack, n is the total number of pulses stacked, and $\alpha(\psi_s)$ is the ratio of the moving bucket area with ψ_s to the stationary bucket area for the same rf voltage.[20)] For POPAE, ψ_s = 50°, α = 0.121, and n = 66 pulses give 1.5 (or 66 % stacking efficiency). Since $\alpha(\psi_s)$ is a monotonically decreasing function of ψ_s ($\alpha(0°) = 1$, $\alpha(90°) = 0$), ψ_s should be small to get a good stacking efficiency. On the other hand, as the bucket must fit tightly around the beam when it goes through the stack, the rf voltage becomes lower for smaller ψ_s and one gets a longer acceleration (or deceleration) time. (Remember that the bucket area

$\propto\sqrt{\text{rf voltage}} \times \alpha$, $d\alpha/d\psi_s<0$, and the rate of energy change $\propto$(rf voltage)x sin ψ_s.) A compromise must be made between the requirements of shorter stacking time on the one hand and better stacking efficiency on the other. The selection of 50° for ψ_s seems a reasonable one.

For synchronous capture of every rf bunch, the stacking rf frequency must be locked to the beam bunch frequency. It may be best to use the injected beam itself to obtain the bunch phase information on each pulse. A phase detector is then placed in the injection line and the rf cavity is at a short distance upstream of the injection point. There will be a beam scraping system in the injection line to minimize the beam loss in the storage ring during injection and stacking. It is essential to avoid a large heatload on the superconducting magnet system.

V.C. Ejection

In principle, beam ejection is a simple matter. An extraction channel made up of pulsed septum magnets is fully excited. The beam is then kicked into the channel by a fast kicker. There are of course many technical problems associated with the ejection, some of which are not yet solved in a satisfactory manner. The design of the beam dump is going to be a nontrivial problem considering the magnitude of the beam energies one must handle. There should be scraper blocks downstream of the septum magnets to shield the superconducting magnets from the spray of radiation. The kicker must have a fast rise time, much faster than the revolution time of the beam, in order to minimize the inevitable beam loss on the septum. A rise time of $\sim$100 nsec may be possible if one is willing to pay for that. If the total deflection needed is 20 mm and the septum thickness 2 mm, the septum will in effect intercept the entire beam current for $\sim$10 nsec. For POPAE, the nergy deposited on the septum will then be 50 KJ.

The most serious situation arises when one must do a totally unplanned ejection due to a sudden malfuction of some ring compoennts. The detection and the kicker excitation can be fast enough but the rise time of the pulsed magnets are likely to be so long that the entire beam can be lost in the ring. One may be able to localize the damage (although attempts to localize the beam loss have not yet been successful in the ISR) by dumping the beam partially on internal absorber blocks and to eject the rest after the extraction channel is excited to the full field. It is not practical to keep the septa fully excited at all times unless they are also superconducting magnets. The stored energy of the beam may indeed become the ultimate limiting factor of future storage rings.

VI. Vacuum

My ignorance on the subejcts of vacuum and magnet is so glaring that I should probably refrain from talking on them. After giving some thoughts on this, I decided not to say anything on superconducting magnets; the subject must be left to specialists since half-baked discussions are likely to contribute more to a confusion than to an enlightenment. The vacuum, on the other hand, is so important in storage ring designs that I found it impossible not to include here albeit my poor qualification. The improvements of the vacuum in the ISR have always contributed sustantially to the increase in current. There are broadly speaking three problems in the storage ring vacuum. They are: 1) life-time and background for experiments, 2) trapping of electrons, and 3) abnormal pressure bump phenomenon.

Any storage ring must have a life-time of at least a few hours. If one calculates the life-time from some standard formulae on the multiple Coulomb scattering, one finds that the beam size growth is negligible at the average pressure of 10^{-10} - 10^{-11} Torr (mm of Hg). According to the ISR experience, experimentalists start screaming about the intolerable background long before any vacuum-related beam instability sets in. Requirements from the tolerable background level are apparently more difficult to satisfy than those from the life-time. Background events can come from the circulating beam interacting with the residual gas in the chamber and from beam particles striking the chamber wall. The usual design value of the pressure is based on the ISR experience; 3×10^{-11} Torr in the curved section and less than 1×10^{-11} in the interaction region. It is also important to scrape out the beam halo frequently and to have a proper disposal system. One needs a collimator placed at 90° phase advance away from the scraping block in order to catch the particles scattered back into the chamber.

Electrons (and negative ions) are created by the beam-gas collisions and are trapped in the potential well of the (positively charge) beam.

Two different phenomena harmful to the beam stability can occure. One is the neutralization of the beam space charge and the other is the transverse coherent motion of the protons coupled to the electron oscillations in the potential well of the beam (ep instability). Without neutralization, the change of betatron oscillation tunes arising from the beam self-field is negligible at high energies due to the cancellation of the electric ($\propto 1/\gamma$) and magnetic ($\propto -\beta^2/\gamma$) contributions. If there are N_e electrons and N_p protons, the electric field is modified by a factor $1 - (N_e/N_p)$ and the effect becomes proportional to

$$1/\gamma^3 - (N_e/N_p)(1/\gamma)$$

instead of $(1 - \beta^2)/\gamma = 1/\gamma^3$. If the production rate of electrons is R_p electrons per second for each proton and the electron clearing rate is R_c ($R_c\ N_e$ electrons cleared per second), the equlibrium value of the neutralization is $(N_e/N_p) = R_p/R_c$. The clearing of electrons is achieved by clearing electrodes placed between magnets and at some intervals in straight sections. They are ~10 cm long and operated at 5 - 10 kV. The degree of neutralization, N_e/N_p, is of the order of 10^{-4} in most designs. Electrons are driven toward the location of clearing fields by three mechanisms; a) beam heating - electrons undergo successive collisions with the circulating protons and gain logitudinal energy, b) logitudinal electric field of the beam due to the change in beam size, and c) in dipoles, there are crossed electric (transverse beam field) and magnetic fields which give rise to the longitudinal drift velocity. The phenomenon of coherent ep instability is not yet understood completely although there is an estimate on the threshold value of (N_e/N_p).[21] The calculated threshold for the ISR is $\sim 10^{-2}$ when the beam is 20 amp. With the clearing system, the neutralization is reduced to a level below 10^{-3} but the ep oscillations can still be observed occasionally. There are in addiiton mysterious connections between the ep oscillations and nonlinear resonance.[22] The calculation cited here assumes that the neutralization is fairly regularly distributed around the circumference rather than being concentrated in a few localized spots. Because of aperture variation, there may be pockets where neutralization is abnormally large.

The most serious limitation on the ISR current has been set by a dynamic pressure (pressure in the presence of beam) phenomenon which is now known as the "pressure bump".[23] The effect is caused by ionized gas molecules, which are driven into the chamber wall by the beam field, liberating (desorbing) the adsorbed molecules from the surface of the wall. At a certain value of current, I_{crit}, the resulting increase in the gas pressure progresses rapidly in a manner of chain reaction and causes the destruction of beam. The desorption coefficient η is defined as the net number of molecules created for each positive ion that impinges on the chamber wall. If one molecule is returned for each ion, there is no net change and $\eta = 0$. If all the ions are trapped by the wall and no molecules are released, $\eta = -1$ and the dynamic pressure will be lower than the static pressure (pressure in the absence of beam). One calculates the product ηI_{crit} and measures η (which depends critically on the surface preparation of the wall material, and the mass and energy of the bombarding ions) to find I_{crit}. The quantity (ηI_{crit}) is a function of pumping speed, distance between pumps, and radius of the chamber (which essentially determines the unit conductance).[24] The design parameters of ISABELLE are such that $(\eta I_{crit}) \lesssim 30$ amp; the corresponding number for the ISR started at ~25 amp and increased to ~100 amp. The desorption coefficient η can be reduced to 3 or less with a careful surface preparation such as chemical polish, bake-out in a vacuum and glow discharge in argon and oxygen.[25]

With the use of superconducting magnets, there are two possibilities, a cold bore at the liquid He temperature and a warm bore at room temperature. Arguments on pros and cons of both systems have been going on, esepcially in connection with the pressure bump henomenon, and they will undoubtedly continue for sometime to come. With a warm bore, one needs a superinsulation layer between superconducting coils and the warm bore tube, thereby reducing the aperture. Advantages are, on the other hand, numerous. Even with a cold bore, interaction regions, internal beam dumps, cavities and septa must be at room temperature and warm to cold transitions are not trivial. It is generally more difficult to install necessary elements like pick-up electrodes and clearing electrodes in a cold bore system. Expected complexities in the mechanical design are so much that one vacuum engineer is rumored to have said "Cold bore over my dead body'". The particular advantage of the warm bore system is the availability of a wealth of information gained at the

ISR on surface preparation. If the beam is to be accelerated in the storage ring, there will be a heat load from the intense bunched beam or from eddy currents and the warm bore system may very well be the only practical solution.

The tremendous appeal of the cold bore system for storage ring designers is not so much the elimination of the superinsulation (this advantage is minor) as the existence of a continuously-distributed cryo-pumping. The chamber itself now presents a pumpng action, molecular adsorption pumping. As a result, the pumping speed is large and independent of aperture. (In the warm bore system, the magnet aperture is essentially dictated by the vacuum requirement.) Distance between pumps can be shorter (better packing factor since dipoles can be longer) and the cost of the vacuum system will be lower than a conventional vacuum system for the same vacuum quality. One small caution here; what determines the gas-beam interaction rate is not the pressure but the gas density and 3×10^{-11} Torr at room temperature corresponds to 4×10^{-13} Torr at 4.5°K. The use of a vacuum chamber operating at liquid helium temperature had been considered originally for ISABELLE[26)] but abandoned later in view of uncertainties on η at that temperature and other practical engineering considerations. The quantity (ηI_{crit}) is of the order of (5,000 - 30,000) × (radius of the bore in cm).[27)] Detailed analyses of cold bore system by Calder[28)] and by Benvenuti[29)] have been encouraging to POPAE designers and they have decided to use the cold bore system. A remark prepared by D.A. Edwards on these analyses during the POPAE design work is reproduced in Appendix E in a condensed form. Any errors in it are of course my responsibility.

The quantity η is the most controversial in the whole discussion of the cold bore system. This is clearly a place where specialists on surface physics could give a very valuable help. Only available data on desorption from cold surfaces are those by Erents and McCracken[30)] who have studied the desorption of various gases (H_2, He, N_2 and Ar) condensed on a copper surface. They found that for hydrogen, which has the largest yield, $\eta \simeq 5 \times 10^4$ with a coverage of a few monolayers (one monolayer $\simeq 3 \times 10^{15}$ molecules/cm^2) while for coverage of one-tenth of a monolayer η = 400 ~ 2,000 depending on the experimental conditions. However, the energy of the bombarding protons was 5 and 20 keV and these numbers could be an overestimate for ISABELLE (~2 keV) or for POPAE

($\sim$0.5 keV).[31] More systematic studies and experiments on η are clearly needed before building a storage ring with a cold bore.

VII. Problems in Beam Dynamics

There are numeroud problems in beam dynamics that can impose limitations on the performance of storage rings. A general survey of these problems given by L.C. Teng[32] is a good summary with many relevant references. ISABELLE Summer Study[4] in 1975 includes many articles on specific topics. Although they tend to be more an application to ISABELLE than a general treatment, references given there are useful for designers of any machine. Other interesting and valuable articles can be found in IEEE Trans. Nucl. Sci., vols. NS-20 (1973) and NS-22 (1975), and in the Proceedings of the 1974 International Conference on High Energy Accelerators at SLAC. Here I will just comment on a few topics which, I believe, are more important than others in the design of pp storage rings.

VII. A. Beam-Beam Tune Shift

This is one of the problems which do not exist in conventional fixed-target accelerators. When two beams cross each other, electromagnetic fields generated by one beam act on particles of the other beam as a focusing or a defocusing lens in both transverse directions. The field is highly nonlinear and azimuthally localized at the crossing points so that there will be not only shifts in betatron tunes but driving forces of nonlinear resonances as well. The parameter which is now universally used as a measure of the effects is the called "linear tune shift".[33] The name is somewhat unfortunate in that it is not exactly the linear (that is, for an infinitesimally small oscillation) tune shift when the tune is near integre or half-integer values.[34]

Linear tune shifts are usually calculated with the Gaussian distribution of particles in the beam.[35),36] Let us assume that a particle in the beam 1 is going through the beam 2 crossing on the horizontal plane. It is easy to see that on the crossing plane the focusing action of the beam 2 changes its sign at the crossing point. As a consequence, the horizontal tune shift is much smaller than the vertical. One first calculates the electric field E_y of the beam 2 and evaluates its gradient $\partial E_y/\partial y$ at the trajectory of the particle in the crossing plane. The linear tune shift is

$$\Delta\nu_V = (1/4\pi) \int_{-\ell/2}^{\ell/2} \beta_{1V}(s)\, k(s)\, ds; \ \ell = \text{free space length},$$

where

$$k(s) = (2\, e/m_1 c^2 \gamma_1)\, (\partial E_y/\partial y)_{y=0}\ ,$$

$m_1 c^2\, \gamma_1$ = total energy of the particle in the beam 1, $\beta_{1V}(s)$ = vertical betatron oscillation parameters of the beam 1. The factor 2 in the numerator of k(s) comes from the addition (instead of cancellation for self-field) of electric and magnetic contributions when two beams are moving toward each other. When the beam 2 is not round, $\sigma_H \neq \sigma_V$, the field E_y cannot be written in a simple closed form.[36] Even for a round beam, the integral along s requires, in general, numerical calculations but the following expressions[35] are adequate for most cases:

$$\Delta\nu_V = (I_2/ec)(r_p \beta_1^* \ell/2\pi\ \gamma_1\ \sigma_2^{*2}) \text{ for a head-on collision,}$$

$$= \Delta\nu_0\ [1 + (\sigma_2^* \ell/\sqrt{2\pi}\ \beta_1^{*2}\ \alpha)]$$

where

$$\Delta\nu_0 = (I_2/ec)(\sqrt{2}\ r_p\ \beta_1^*/\sqrt{\pi}\ \gamma_1\ \alpha\sigma_2^*),$$

$\beta_1^* = \beta_{1V}(s=0)$, σ_2^* = rms radius of the beam 2 (round) at s=0, α = crossing angle, r_p = classical proton radius = 1.53×10^{-18} m.

Once the lienar tune shift is calculated for each insertion, one asks: "Is this value safe or too large for a stable operation of my storage ring?" The sad truth is that nobody can answer the question in a definite manner for pp and ep storage rings. We know by experience (but not by any theoretical argument) that the linear tune shifts should not exceed 0.05 $\sim$ 0.1 for electron storage rings. We also know from the ISR experience that 5×10^{-4} is a safe number for ptoon storage rings. An attempt to find the limit for proton storage rings by artificially creating large tune shifts with a strong nonlinear element gave inconclusive (to me at least) results at CERN.[37] There is a term "stochastic limit" which used to be very popular among accelerator theoretists several years ago but is now almost forgotten. The idea is to calculate the widths of all resonances corresponding to the beam size in a square region of the tune diagram. If the sum of all widths is larger than the area of the square, betatron oscillations will become essentially random in amplitude and in phase. However, there is no unique way of calculating the width where two or more resonances overlap[38] and the stochastic limit varies from one theorist to the next. Then there is something called "Arnol'd diffusion".[32), 39] For two-dimensional (and higher) motions, the familiar closed invariant curves in (x-x') or (y-y') phase space are replaced by invariant tori in many dimensional phase space. Mathematically there is a possibility that a particle may escape through an intricate system of channels, thereby causing the beam to diffuse slowly. A theoretical analysis[40] of this process predicted the limit to be $\Delta\nu \sim 0.005$. There were some numerical works which "confirmed" this value. Ever since, the value 0.005 has been faithfully observed by almost all designers of proton storage rings. Later numerical works[41] using a better representation of the beam action seem to indicate this limit to be as large as 0.04 or even 0.05. In all numerical works, there in only one interacting point in the ring so that any possibel change when there are two or more interacting points is totally unknown.

So, where are we now? Perhaps, to those who are willing to design a proton storage ring with the linear tune shift larger than 0.005, the following advice given in an entirely different discipline may not be so inappropriate:

> "He knows that he will be able to defend himself if he is condemned for it, but also that, until he has done so, he will be condemned."[42]

VII. B. Incoherent Tune Shift*)

In accelerator physics there are not many papers that are considered classic but the article by Laslett on the incoherent (and coherent) tune shifts[43] is certainly one of them. He pointed out that, while the tune

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* "Incoherent" tune shift is the change of tune of individual particle in the beam. "Coherent" tune shift expresses the change when the beam oscillates as a whole.

shift from the direct space charge goes down as $1/E^3$, contributions from the images due to the presence of vacuum chamber and magnet iron go down only as $1/E$. Neglecting the contribution from the direct effects, one can write the Laslett formula as

$$\Delta\nu = (Nr_p/\pi\gamma)\bar{\beta}(\varepsilon_1/h^2 + k\cdot\varepsilon_2/g^2)$$

where N = total number of particles, r_p = classical proton radius, $\bar{\beta}$ = average betatron oscillation function, h = half aperture of the vacuum chamber, g = half height of the magnet gap, k = fraction of circumference occupied by magnets. For the parallel plate geometry, $\varepsilon_1 = \pi^2/48$ and $\varepsilon_2 = \pi^2/24$. Since the beam size is ignored in this approximation, a circular geometry will make $\varepsilon_1 = \varepsilon_2 = 0$ from ths symmetry and from the fact that, outside the beam, the image coefficients obey Laplace's equation and hence are of equal magnitude but opposite sign in two orthogonal directions. Thus, with circular superconducting magnets, there will be practically no incoherent tune shifts if the beam is at the center.

The problem in storage rings associated with the incoherent tune shift is that the beam may not always be centerd, especially during stacking, and that the beam is wide radially at places where the dispersion is large. Tune shifts become non-uniform across the beam in the radial direction. The parabolic term in ν versus radius, combined with the resistive wall instability effects, gives rise to the so-called "brick wall effect" which hampered the ISR operation in early days.[44] The remedy for this, known as "prestressing", is to compensate for the effects dynamically with correction coils during the stacking.[45] In doing this, analytic expressions derived by Zotter[46] have been quite useful, predicting the necessary amount of prestressing accurately. Zotter's formula for circular vacuum tube is as follows. Since the magnetic boundary (iron) is generally farther away than the electric boundary (vacuum tube) in superconducting magnets, ε_1 is more important. Let R be the radius of the wall and 2a the radial beam width. The beam height is assumed to be small. The image coefficient for a particle at x in the beam which is centered at x_0 is

$$\varepsilon_{1,H} = -\varepsilon_{1,V} = (1/2\eta^2)[1 + 1/(u^2-v^2) + (1/v)\ell n \frac{u - v}{u + v}]$$

where $\eta = x/R$, $\eta_0 = x_0/R$, $u = 1 - \eta \cdot \eta_0$ and $v = (a/R)\eta$.

Although neither ISABELLE nor POPAE expects any serious trouble of this type, it is certainly one of the problems machine designers must be aware of. For some complicated geometries of the beam and magnets, one may even have to use a computer program.[47]

VII. C. Transverse Resistive Wall Instability

The effect of resistive walls on the transverse stability of coasting beams[48] has been a well-known probelm for many years now. Since there is always a small coherent oscillation of the beam in a real machine, there will be an oscillating field induced by the beam. Depending on the electromagnetic properties of the enclosure (vacuum wall) on which the induced field must satisfy certain boundary conditions, there will be in-phase component and out-of-phase component in the induced field. In the terminology of electrical engineering, one can say that there is a complex impedance Z . The induced field acts back on the beam (Lorentz force) and this may either enhance or damp the beam oscillation. The problem is to find the magnitude of this impedance below which the coherent transverse motion will become stable. The simplest coherent oscillation of the beam is of the dipole type which can be written as

$$A\, e^{\pm i\nu f t}\, e^{i\, n(\theta - f\, t)}, \quad n = \text{integer}$$

where A is the amplitude of oscillation, f is the beam revolution frequency, ν is the betatron tune, and θ is the azimuthal angle around the machine. At a cretain time (t = 0, for example), there are n waves in the ring and the wave number n is usually called the mode number. Note that, at a given location, the frequency of the oscillation is $(n \pm \nu)f$.

Theoretical analyses of this process predict that the motion will be suppressed, if there are enough spreads in the frequency, through the mechanism of Landau damping.[49] In the simplest terms, Landau damping is an exchange of energy between the beam oscillation wave and the induced electromagnetic wave. The stability condition takes the form[50] for a mode number n

$$|Z_\perp| < (mc^2\gamma)(\pi\nu/IR)\,\frac{1}{e}\,|(n-\nu)\eta + \xi| \cdot (\Delta p/p)$$

where $mc^2\gamma$ is the total energy of the beam particle, $2\pi R$ is the machine circumference, $(\Delta p/p)$ is the full momentum spread of "resonable" distributions, $\xi = d\nu/(dp/p)$ is the absolute chromaticity, I is the current, and $\eta = 1/\gamma_{tr}^2 - 1/\gamma^2$ [note that $\Delta f/f = \eta(\Delta p/p)$]. All mode numbers n less than ν are stable so that $(n - \nu) > 0$. The transverse impedance $Z_\perp$ can be calculated for a smooth resistive vacuum chamber of constant cross section (radius = b),

$$|Z_\perp| = (2/b^3)\sqrt{[R^3 Z_0/\ (n-\nu)]}$$

where $Z_0 \simeq 377$ ohms is the vacuum impedance and σ is the conductivity of the chamber material. For aluminum, $\sigma = 6.25 \times 10^7$ /(ohm-m) (but don't forget the temperature dependence for a cold-bore system). In real machines, there will be elements like bellows, clearing and pick-up electrodes, and detectors for experiments and Z will in general be larger than this.

For low mode numbers, $(n - \nu)\eta$ is small and we have to depend on ξ to satisfy the stability condition. This, however, may make the total tune spread of the beam dangerously large. It is better to use a feed-back damper system. For a given value of ξ allowed from other considerations, there is a minimum value of n, n_0, above which all modes are stable. The feed-back system then must have a bandwidth

$$(n_0 - \nu) \times (\text{revolution frequency}).$$

A fast feed-back system of 50 MHz bandwidth is now working at the Fermilab main ring and one with ∿100 MHz seems technically feasible. It is nevertheless a good policy to take ξ as large as other conditions permit. An instability of the bunched beam called head-tail effect[51)] is also suppressed, at least for low mode numbers, by taking a large, positive ξ. When the stability condition is not satisfied, the beam is unstable with the growth rate[48)]

$$1/\tau = (IRr_p/eb^3\nu\gamma)\sqrt{\frac{2R}{Z_0\sigma(n-\nu)}}\ .$$

In a real storage ring, there is a severe limitation on the longitudinal coupling impedance $Z_{/\!/}$ (see next section) and this may dictate $Z_{\perp}$ which actually exists. For a smooth vacuum chamber and for cavity-like objects, the relation is[50]

$$Z_{\perp} \simeq (2R/b^2) Z_{/\!/} / (n - \nu).$$

VII. D. Longitudinal Resistive Wall Instability

Negative mass instability[52] is a well-known example of longitudinal instabilities. A small nonuniformity in the longitudinal particle desnity of a coasting beam is amplified by the induced longitudinal electric field, thereby causing the beam to self-bunch. The effect is suppressed by Landau damping arising from a spread in the revolution frequencies of particles in the beam.

The longitudinal stability criterion for a coasting beam is usually written in the form[53]

$$|Z/n| < F\ (E/e)\ (\eta/I)\ (\Delta p/p)^2$$

where Z = longitudinal coupling impedance for the mode number n, E = total energy of the beam, I = beam current, $\eta = (1/\gamma_{tr})^2 - (1/\gamma)^2$, $(\Delta p/p)$ = full momentum spread at half maximum. The form factor F is a quantity of the order of unity but its exact value depends on the shape of the distribution. It is not quite fair to take $(\Delta p/p)$ as FWHM of the momentum distribution since the stability comes from the shoulders and not from the flat portion in the center of the distribution. People generally assume that when I is increased by stacking in a storage ring, $(\Delta p/p)$ will also increase in such a way that F remains constant. The stability condition is thus most severe for the first few pulses of the stack.

The problem of longitudinal instability has become one of the hottest topics in accelerator physics. It is now believed[54], [55] that the instability inside a bunch with GHz range frequency (very large mode number) is also governed by the above stability criterion which was originally derived for coasting beams. The momentum spread and the current in the criterion must now be their instantaneous values in the bunch. The justification for this is that the growth time of the instability with such a high mode number becomes comparable to or even shorter than the phase oscillation period and the distinction of bunched and unbunched beam loses its meaning. Applied in this manner, the criterion is indeed very severe. For ISABELLE, $|Z/n| \lesssim 5$ ohms and for POPAE, $|Z/n| \lesssim 2$ ohms. It is not clear at this stage if one could build a machine with such a small total impedance. The whole story seems to have started at CERN when they contemplated debunching the beam in the CPS prior to the injection into the SPS (rf frequencies different in two machines). They observed an enormous blow-up in the momentum spread during debunching of the beam in the CPS.[55)] Frequencies between 1 to 5 GHz were observed with the rise time of the order of msec. They eventually decided to debunch the beam in the SPS and recapture adiabatically. Blow-up of a factor 2 $\sim$ 3 during stacking and microwave structures on the bunch were also observed in the ISR.[45)] Sometimes resulting clusters are expelled from the bunch.

The major contribution to the coupling impedance comes from any discontinuity in the vacuum chamber cross section. Objects such as cavities, pick-up and clearing electrodes, bellows, and detectors for experiments are all potentially dangerous. Cavities in particular may have to be shorted when they are not in use. Blow-up of a few pulses at the beginning of the stacking may be unavoidable and they may have to be scraped later. One can even take an optimistic view that the currently fashionable interpretation is all wrong and there is no need to reduce the coupling impedance that much.

It is clear from the expression that there is essentially only one parameter which can be "controlled" by the design and that parameter is the transition energy γ_{tr}. For a ring like POPAE, $\gamma >> \gamma_{tr}$ so that $\eta = 1/\gamma_{tr}^2$. If γ_{tr} is reduced by a factor 2, the impedance can be four times larger. Unfortunately, γ_{tr} is approximately proportional to the tune (see Appendix D) and a smaller tune means a weaker focusing, that is, a larger beam size. The beam will become more susceptible to nonlinear resonances. For ISABELLE, the injection energy and the transition energy are close to each other and η becomes very small. (See II. d.)

VIII. Concluding Remarks

There are many more topics I should have talked about - correction of resonances, various tolerances, choice of the operating lines, ... - there seems to be no end to what one has to do in the design of storage rings. Let me just emphasize again that accelerators and storage rings are merely tools for physics experiments. They have no intrinsic values until physics is produced. At the same time, accelerator builders should have a pride in what they do. I believe Leon Lederman expressed our sentiment beautifully when he said[2)]

> "The teamwork of the users and the builders is part of our profession. We work hard in mutual stimulation. You make the machines, and we try to use them so well that the need for more machines becomes self-evident. Both of us, of course, working so that Gell-Mann and his friends become more and more famous. (That is not really fair. It is really an unholy trinity, engaged in a more or less honorable endeavor, the significance of which for history and for the future we do not have to elaborate here.)"

Appendix A. Kinematics of Colliding Beams

Reference: K.G. Dedrick, Rev. Mod. Phys. 34 (1962), 429.

Consider two particles ($i = 1, 2$) with the rest mass m_i, the total energy E_i and the momentum $\vec{P}_i$ in a laboratory frame of reference. In the center-of-mass frame, energy and momentum are E_i' and $\vec{P}_i'$, respectively. By the definition of the c.m. frame

$$\vec{P}_1' + \vec{P}_2' = 0.$$

The total energy of the two particles is $W = E_1 + E_2$ in the laboratory frame and $W' = E_1' + E_2'$ in the c.m. frame. From the Lorentz transformation, one can show that the c.m. frame is moving with the velocity $\vec{v}_0$ relative to the laboratory frame,

$$\beta_0 \equiv |\vec{v}_0|/c = c|(\vec{P}_1 + \vec{P}_2)|/W.$$

To total energy W' in the c.m. frame (which is called $\sqrt{s}$) is

$$W' = \sqrt{1 - \beta_0^2} \cdot W \equiv W/\gamma_0.$$

For high energy colliding beams, $E_i \simeq c|\vec{P}_i|$ and $E_i' \simeq c|\vec{P}_i'|$. When the crossing angle is small, we get

$$\beta_0 = (E_1 - E_2)/(E_1 + E_2) \text{ and } \gamma_0 = (E_1 + E_2)/(2\sqrt{E_1E_2})$$

and $$\sqrt{s} \equiv W' = 2\sqrt{E_1E_2} .$$

The labotarory angle θ of a particle (for example intermediate boson W) is related to the c.m. angle θ' by

$$\tan \theta = (1/\gamma_0)\sin\theta'/(\cos\theta' + g)$$

where $g = c\,\beta_0/$ (velocity of the particle in the c.m. frame).
If the second particle is at rest in the laboratory frame, we have

$$\beta_0 = \sqrt{E_1^2 - (m_1c^2)^2}/(E_1 + m_2c^2),$$

$$\gamma_0 = \sqrt{(E_1 + m_2c^2)} / \sqrt{(m_1c^2)^2 + (m_2c^2)^2 + 2m_2c^2E_1}$$

$$\simeq \sqrt{E_1/(2m_2c^2)},$$

$$\sqrt{s} = W/\gamma_0 \simeq \sqrt{2m_2c^2E_1} \ .$$

Appendix B. Useful Formulae for Electron Machines

Formulae are taken from the lecture note by Matthew Sands.[1)]

It is assumed throughout that the machine is "isomagnetic", that is, the radius of curvature of the design orbit has everywhere the same value ρ_0 except where there is no bending field. Original equation numbers are also given so that more detailed explanations can be easily found.

1. Luminoisty L (each beam contains N particles in B bunches).

$$L = \frac{N^2 f}{4\pi B \sigma_x \sigma_z} \qquad (6.1) - (6.4)$$

f = revolution frequency, σ_x and σ_z = rms beam sizes of the Gaussian distribution. This formula is for a head-on collision.

2. Linear tune shift (head-on collision)

$$\Delta\nu_{x,z} = \frac{r_e \ N\beta^*_{x,z}}{2\pi B \ \sigma_{x,z} \ (\sigma_x + \sigma_z)\gamma} \qquad (2.122,123)$$

$\beta^*_{x,z}$ = Courant-Snyder betatron oscillation functions at the interaction point, γ = total energy/rest energy, r_e = classical electron radius, 2.818×10^{-15} m. Linear tune shifts should not exceed $\sim$0.06 at each intersection.

3. Synchrotron radiation

a) instantaneous power loss by one electron

$$P_\gamma = \frac{cC_\gamma E^2}{2\pi\rho^2} \qquad (4.4)$$

$$C_\gamma = \frac{4\pi}{3} r_e/(mc^2)^3 = 8.85 \times 10^{-5} \text{ meter}/(\text{GeV})^3 \qquad (4.2)$$

ρ = local radius of curvature, E = total energy of the electron, c = speed of light.

b) energy radiated by one electron in one revolution on the orbit

$$U_0 = C_\gamma E^4/\rho_0 \qquad (4.8)$$

c) average power radiated

$$\langle P_\gamma \rangle = U_0 f \qquad (4.10)$$

d) overvoltage

$$q = e\hat{V}/U_0 \qquad (3.60)$$

$\hat{V}$ = peak rf voltage necessary to achieve a quantum life time τ_q.

e) damping time constant

$$\tau = 2\, E/J\langle P_\gamma \rangle \qquad (4.53)$$

damping partition number J is

radial $J_x = 1 - D$, vertical $J_z = 1$ and energy oscillation $J_\varepsilon = 2 + D$.

For a separated function ring of cirumference $2\pi R$,

$$D = \alpha R/\rho_0 \qquad (4.22)$$

α = momentum compaction factor (3.11)

In general D is a positive number much less than 1.

4. Quantum fluctuation

a) quantum life time

$$\tau_q = (1/2)\ \tau_\varepsilon \frac{e^\xi}{\xi} \qquad (5.137)$$

τ_ε = damping time constant for the energy oscillation

$$\xi = [(J_\varepsilon E)/(\alpha k E_1)]\ F(q) \qquad (5.141)$$

k = harmonic number of the rf system

$$E_1 = (55\sqrt{3}\ \hbar c/64 r_e) \simeq 1.08 \times 10^8 \text{ eV} \qquad (5.142)$$

$$F(q) = 2[\sqrt{q^2 - 1} - \cos^{-1}(1/q)] \qquad (3.60)$$

b) bunch size (rms value σ of Gaussian distribution)

b.1) energy oscillation

$$\sigma_\varepsilon^2 = (C_q \gamma^2 E^2 / J_\varepsilon \rho_0) \qquad (5.48)$$

$$C_q = (55\ \hbar/32\ \sqrt{3}\ mc) = 3.84 \times 10^{-13} \text{ m} \qquad (5.46)$$

b.2) bunch length

$$\sigma_\ell^2 = (R/k)^2 \frac{F(q)}{\xi\sqrt{q^2 - 1}} \qquad (6.31)$$

The bunch length in high intensity electron storage rings is usually larger than this. The bunch lengthening is not well understood.

b.3) radial size (at a place where betatron oscillation function is β_x^*)

$$\sigma_r^2 \simeq \sigma_x^2 \left(1 + \frac{J_x}{J_\varepsilon}\right) \qquad (5.96)$$

$$\sigma_x^2 \simeq (\beta_x^* \alpha C_q\ R\gamma^2 / J_x \rho_0 \nu_x) \qquad (5.90)$$

ν_x = radial betatron tune

b.4) Natural vertical beam size is very small and energy independent. In storage rings, this is enlarged by introducing a coupling with the horizontal betatron oscillation in order to increase the luminosity. Note that, for $\sigma_x >> \sigma_z$,

$$\Delta\nu_z = r_e N\beta_z^* / (2\pi B\gamma\sigma_x\sigma_z)$$

$$L = \frac{Nf\gamma\Delta\nu_z}{2\gamma_e\beta_z^*} = \frac{3\rho_0^2\Delta\nu_z P}{8\pi r_e^2\ mc^2\ \gamma^3 R\beta_z^*}$$

where $P \equiv N\ P_\gamma$ = total power loss by N electrons. Since $\Delta\nu_z$ cannot exceed a certain value ($\sim$0.06), one must increase σ_z in order to increase the luminosity.

Appendix C. Luminosity with Gaussian Distribution

We consider two beams crossing on the horizontal plane. The crossing angle α (<< 1) is bisected by the longitudinal z-axis. The crossing point is at $z = 0$ and two beams are separated at $z = \pm\ell/2$, that is, the length of the luminous region is ℓ. Both beams are unbunched (pp colliding) or one is bunched and the other unbunched ($e^\pm p$). Although particle distributions are not precisely Gaussian for protons, they are generally closer to Gaussian than to rectangular. Beam size is specified by rms values σ_H and σ_V. The definition of emittances $\pi\varepsilon_H$ and $\pi\varepsilon_V$ is not unique but we take the phase space area in each transverse direction containing 95 % of the beam,

$$\pi\varepsilon_{H,V} = 6\pi\ \sigma_{H,V}^2 / \beta_{H,V}$$

where $\beta_{H,V}$ are betatron oscillation functions. Emittances are independent of z but $\beta_{H,V}$ and $\sigma_{H,V}$ change along z-direction. We assume that $\beta_{H,V}$ take their minimum values $\beta_{H,V}^*$ at $z = 0$. At other points,

$$\beta_{H,V} = \beta_{H,V}^* + z^2/\beta_{H,V}^* .$$

If $\beta^*_{H,V} << \ell$, $\beta_{H,V}$ at $z = \pm\ell/2$ become very large and this accentuates the effects of quadrupole chromatic aberration. In order to have adequate space for experimental equipments, ℓ should not be too small. In most cases, $\beta^*_{H,V} \sim$ a few meters and ℓ = 40 m (ISABELLE) $\sim$ 90 m (POPAE) for proton storage rings. For the sake of simplicity, all dispersion effects are assumed to be negligible. This is usually the case in almost all pp and $e^{\pm}p$ designs.

If the average currents of beam 1 and beam 2 are, respectively, I_1 and I_2, the luminosity per unit length along z-axis is

$$dL/dz = \frac{c}{\pi} \frac{I_1}{ec} \frac{I_2}{ec} \frac{1}{\sqrt{(\sigma^2_{H,1} + \sigma^2_{H,2})(\sigma^2_{V,1} + \sigma^2_{V,2})}} \times$$

$$\times \exp \left[- \frac{\alpha^2 z^2}{2(\sigma^2_{H,1} + \sigma^2_{H,2})} \right]$$

where $\sigma^2_{H,1} = \beta_{H,1} \cdot \varepsilon_{H,1}/6$ etc. are all functions of z. To get the total luminosity L, one must integrate the above expression from $z = -\ell/2$ to $\ell/2$ taking into account the variation of σ^2's. Fortunately, in many cases, the luminosity is concentrated near the center and one can integrate from $-\infty$ to ∞, ignoring the variation of the beam size. When two beams are identical, we have

$$dL/dz = \frac{c}{\pi} \left(\frac{I}{ec}\right)^2 \frac{1}{2\sigma^*_H \sigma^*_V} \exp \left[- \left(\frac{\alpha}{2\sigma^*_H}\right)^2 z^2 \right]$$

where σ^*_H (σ^*_V) is the rms horizontal (vertical) beam size at z = 0. The integrated luminosity is

$$L = 2\ c\ (I/ec)^2/(2\sqrt{\pi}\sigma^*_V \alpha).$$

It is intersting to see that this result is identical to what we get with a rectangular particle distribution if the beam height h is interpreted as $2\sqrt{\pi}\sigma^*_V$.

Appendix D. Cookbook for FODO Cells (by Tom Collins)

$(B\rho)$ in kG-m = (Beam momentum in GeV/c)/0.029979.

Focal power of a quadrupole $f = B'\ell_0/(B\rho)$ where B' = field gradient and ℓ_0 = effective length of the quadrupole.

$2L$ = cell length.

2ψ = phase advance in one cell.

2θ = total bend angle in one cell.

β_{max} and β_{min} = max. and min. values of $\beta_{H,V}$.

$x_{p,max}$ and $x_{p,min}$ = max. and min. values of the dispersion function.

ν = betatron tunes.

γ_T = transition energy/rest energy.

$2\pi R_c = L/\theta$ = total length of curved sections (assuming no irregular cells with dipoles).

$2\pi R$ = circumference of the ring including insertions.

1. $f = 2\sin\psi/L = \sqrt{2}/L$ for $2\psi = 90°$.
2. $\beta_{max} = 2L(1 + \sin\psi)/\sin(2\psi) = (2 + \sqrt{2})L$ for $2\psi = 90°$.
3. $\beta_{min} = 2L(1 - \sin\psi)/\sin(2\psi) = (2 - \sqrt{2})L$ for $2\psi = 90°$.
4. smallest β_{max} ($= 3.33\ L$) is at $2\psi = 76°$.
5. $x_{p,max} = (1 + 0.5\sin\psi)L\theta/\sin^2\psi = 2.707\ L\theta$ for $2\psi = 90°$.
6. $x_{p,min} = (1 - 0.5\sin\psi)L\theta/\sin^2\psi = 1.293\ L\theta$ for $2\psi = 90°$.
7. $\nu = (\psi/\theta)(R/R_c)$ if the average focusing is the same.
8. $\gamma_T = (0.73/\theta)\sqrt{R/R_c} = 0.93\nu\sqrt{R_c/R}$. This is for $2\psi = 90°$.
9. The following corrections for thick quadrupoels are useful and quite accurate.

   Given B', calculate quadrupole length ℓ_0 from thin lens approximation. Then, for a real quadrupole with B' and length ℓ,

$$\ell = \ell_0\,[1 + \{(\ell_0/L) + (\ell_0/L)^2\}/3]$$

$$\left.\begin{aligned} \beta_{max} &= (2 + \sqrt{2})L[1 - (2\ell_0/17L)] \\ x_{p,max} &= (2 + 1/\sqrt{2})L\,[1 - (\ell_0/36L)] \end{aligned}\right\} \quad \text{for } 2\psi = 90°\ .$$

Appendix E. Remarks on Pressure Stability in Cold Bore Vacuum Systems

Remarks presented here were originally prepared by D.A. Edwards during the POPAE design work. The purpose of these remarks was to explain quantitative differences between results given in ref.28 and in ref.29. Any errors in this note are of course entirely my responsibility.

Assume that the conductance along the bore tube is negligible. Let N be the number of molecules per unit length in the pipe and $n(r,\theta)$ be the distribution of number density in the cross section of the circular pipe with radius R,

$$N = \int n(r,\theta)\ r\ dr\ d\theta.$$

For each positive ion created and driven into the wall by the beam field, the net number of neutrals appearing in the beam will be denoted by η. The "luminosity" per unit length for ion creation is $c\ n(r{=}0)\lambda$ where λ is the line density of particles in the proton beam, $\lambda = I/ec$. For an ionization cross-section σ, the rate of ion production per unit length is

$$dN_i/dt = \lambda c\ n(0)\sigma$$

and the contribution to the rate of neutral production from this source is $\lambda c\ n(0)\sigma\eta$. The rate of absorption at the wall will be

$$\int n(R,\theta)\bar{v}_\perp(R,\theta)\alpha\ R\ d\theta$$

where $\bar{v}_\perp$ is the average of the perpendicular component of the speed at the wall and α is a sticking probability. Let Q represent the production rate of neutrals per unit length from other processes. Then

$$dN/dt = \eta\lambda c\ n(0)\sigma - \int n(R,\theta)\bar{v}_\perp(R,\theta)\alpha\ R\ d\theta + Q.$$

To obtain the expressions used by Benvenuit and Calder, take $n(R,\theta) = n$, a constant independent of r and θ. Then, in equilibrium,

$$\eta\lambda c\ n\sigma - \alpha n\ \bar{v}_\perp\ (2\pi R) + Q = 0$$

or

$$n = \frac{Q}{\alpha \bar{v}_\perp (2\pi R) - \eta\lambda c\sigma} = \frac{Q\alpha \bar{v}_\perp (2\pi R)}{1 - \dfrac{\eta\lambda c\sigma}{\alpha \bar{v}_\perp (2\pi R)}}$$

If we next use the Boltzman distribution relation between $\bar{v}_\perp$ and the average speed $\bar{v}$, $\bar{v}_\perp = \bar{v}/4$. We arrive at the criterion for stability used by both authors:

$$2\eta\lambda\sigma c/(\alpha\bar{v}\pi R) < 1$$

or converting to current

$$(\eta I_{crit}) = (\alpha\bar{v}\pi\ R\ e/2\sigma).$$

If we insert the ionization cross section for N_2, $\sigma(N_2) = 1.2 \times 10^{-18}\ cm^2$,

$$(\eta I_{crit}) = \alpha\bar{v}R\ \frac{\pi}{2}\ \frac{1.6 \times 10^{-19}}{1.2 \times 10^{-18}}$$

$$= \pi\alpha\bar{v}\ R/15 \quad (\bar{v}\ \text{in cm/sec, R in cm, I in amperes}).$$

Now the authors diverge. Benvenuti takes σ for H_2 as 1/7 that of N_2,

$$(\eta I_{crit}) = 7\pi\alpha\bar{v}\ R/15$$

and $\eta = 5 \times 10^4\theta$ where θ is the fraction of a monolayer of H_2 present. Then

$$(\theta I_{crit}) = 7\pi\alpha\bar{v}\ R/(15 \times 5 \times 10^4).$$

He also sets $\alpha = 1$, and using

$$\bar{v} = 14551\sqrt{T/M}\ \text{cm/sec} = 2.19 \times 10^4\ \text{cm/sec for } H_2 \text{ at } 4.5°K,$$

$$(\theta I_{crit}) = 7\pi \times 2.19 \times 10^4\ R/(15 \times 5 \times 10^4)$$

$$= 0.64\ R\ (R\ \text{in cm}).$$

He further argues that θ will not be greater than 0.1. Then I_{crit} = 6.4 R (R in cm).

Next, consider Calder. He stays with nitrogen cross section but uses the hydrogen speed and η = 100 for 0.1 monolayer. For α = 1,

$$I_{crit} = \pi \times 2.19 \times 10^4 \, R/(15 \times 100) = 46 \, R.$$

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