Some Aspects of Unified Theories of Weak and Electromagnetic Interactions*

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* Lectures given at the second KEK Summer School held at National Laboratory for High Energy Physics on July 17-21, 1973

§1. Introduction

It has been known for some time that in spite of an apparent disparity of the magnitude of the coupling constant, weak interaction, as formulated a là Cabibbo[1], shares a remarkable similarity with electromagnetism. Namely, currents which describe weak decay processes are almost conserved and their interactions with hadrons and leptons appear to be universal. As is well known, the fact that photon couples to the conserved current with universal couplings in one of the most remarkable properties of electromagnetic interactions and it is these features of weak currents that have led us to the notion of conserved vector current (CVC)[2] and partially conserved axial vector current (PCAC)[3].

Unlike electromagnetic interaction, however, it is not easy and straightforward to formulate the notion of universality of weak interactions in terms of local field theories. There are a number of reasons for this but the most serious one will be that weak currents, except for the strangeness conserving vector current, are conserved <u>only approximately</u>. This is also related to the fact that symmetries of strong interactions such as SU(2)⊗SU(2) or SU(3)⊗SU(3) are all approximate. To formulate such approximate strong interaction symmetries, Gell-Mann proposed in 1962 the algebra of currents[4], which is supposed to be exact in all orders of strong interactions, including the symmetry breaking interactions. Identifying then those currents which are associated with approximate SU(3)⊗SU(3) symmetry with components of hadronic weak currents, there was a hope that we can fix the normalization of coupling constants, since these current commutations are non-linear. This was achieved by the celebrated Adler-Weisberger relation which was derived in 1965[5] with an additional assumption of PCAC. It is to be noted here, though, that conceptually, the Adler-Weisberger relation is not the one which determines the renormalization of axial vector coupling constant ($-G_A/G_V$) but instead, it fixes the pion decay constant (f_π)[6]. The $-G_A/G_V$ ratio is then fixed by the Goldberger-Treiman relation[7].

It was soon realized that many successful results of current algebra can be best understood in terms of low energy theorems which are consequences of chiral SU(2)⊗SU(2) or SU(3)⊗SU(3) symmetry of strong interactions broken spontaneously[8]. If we assume the spontaneously broken chiral symmetry of strong interactions, the algebra of currents associated with the symmetry is a direct mathematical consequence of our starting assumption. <u>What is physical is the assumption that hadronic weak currents obey the same commutation relation rules</u>.

The notion of universality can also be formulated in terms of commutation relations; namely, both hadronic and leptonic weak charges generate the same algebra of $SU_L(2)$ or $O_L(3)$[9]. We could go one step further and derive this universality if we assume non-Abelian gauge invariance for weak interactions. However, since there is no massless gauge particles associated with these currents, it is apparent that we are not able to expect simple gauge invariance here. Also, one must remember that weak interactions violate parity and charge conjugation invariance maximally, while no such violations have been observed in electromagnetic interactions[10]. Nevertheless, perhaps, one of the most vital motivations which underlie the recent attmpts[11] to unifying weak and electromagnetic interactions will be that the current-current interactions, if taken very seriously within the framework of field theories, violate the unitarity at higher energies. In recent years, many physicists became more and more concerned about this fact and consider that they are at most phenomenological interactions and not basic ones. Just what is then the fundamental interactions was a very difficult question[12]. Because of bad asymptotic behaviour of massive vector meson propagators, it is clear that simple introduction of massive W bosons into the theory does not help the failure of the unitarity in the perturbation theory[13].

A major breakthrough to this problem came when Weinberg[14] and Salam[15] proposed a model of weak interactions based on spontaneously broken gauge invariance. In those theories, some of the gauge particles can be massive without destroying local gauge invariance and therefore, both interactions can now be treated on a unified basis. It is called the Higgs-Kibble mechanism[16] which gives rise to masses for gauge particles. This mechanism has an additional merit that we no longer have massless Nambu-Goldstone particles associated with the spontaneously broken symmetries.

It was already anticipated in 1967[14] that theories of this kind might be renormalizable even when the gauge symmetry is spontaneously broken but we had to wait until 1971 when 't Hooft[17] offered the explicit proof of renormalizability of spontaneously broken gauge theories based on the method of Fadeev and Popov[18]. The problem of renormalization was further elaborated by B.W. Lee[19] and Zinn-Justin[20] and 't Hooft and Veltman[21]. It will be worthwhile to note at this place that for a consistent theory of weak interactions, renormalizability might not be an essential requirement. As pointed out by Lee[19], what is really required for the theory of weak interactions is that higher order corrections are finite and small so that they are consistent with the well-established current-current interaction phenomenology at low energies. If the theory is not renormalizable, we must, of course, prepare a recipe to calculate higher order corrections, certainly not based on perturbation expansions. Within conventional local field theories, however, renormalizability will be the only way to achieve a consistent weak interaction theory. Also, a great success of renormalizable quantum electrodynamics is suggestive of describing weak interactions with similar approaches.

During the last two years, many models based on the spontaneously broken gauge symmetries have been proposed[22] and a general recipe for building such unified models of weak and electromagnetic interactions have now well understood[23]. However, it appears that we are still far from constructing models which describe the real world. As is well known, there are two outstanding features which characterize the models of this kind; namely, in general we need either heavy leptons and/or neutral currents. Whether or not these heavy leptons and/or neutral currents really exist in nature will have a direct bearing on forthcoming high energy neutrino experimenta at NAL[24] and at CERN and we are looking forward to hearing an exciting discovery on these facts in a few years ahead.

In this lecture, we pick up some features of the unified gauge models of weak and electromagnetic interactions and emphasize physical motivations behind them. A more systematic reviews on the subjects not covered in my lecture can be found in references (13) (69) and (19).

§2. Classical Theories

In this section, we will make a brief review on current-current interactions and point out some of the questions related to the unified theories of weak and electromagnetic interactions.

(2-1) The Cabibbo theory[1)]

The weak interactions which describe various decay phenomena can be summarized in terms of the effective Lagrangian of the form;

$$L = -\frac{G}{\sqrt{2}} J_\mu \cdot J_\mu^{+} \qquad (2\cdot 1)$$

where $$J_\mu = j_\mu^{h} + j_\mu^{\ell}, \qquad (2\cdot 2)$$

with $$j_\mu^{\ell} = i[\bar{\nu}_e \gamma_\mu (1+\gamma_5) e + \bar{\nu}_\mu \gamma_\mu (1+\gamma_5) \mu], \qquad (2\cdot 3)$$

and

$$j_\mu{}^h = \cos\theta\,[V_\mu(\Delta s=0,\ \Delta\underset{\sim}{I}=1) + A_\mu(\Delta s=0,\ \Delta\underset{\sim}{I}=1)]$$
$$+ \sin\theta\,[V_\mu(\Delta s=1,\ \Delta\underset{\sim}{I}=1/2) + A_\mu(\Delta s=1,\ \Delta\underset{\sim}{I}=1/2)] \qquad (2\cdot4)$$

Here, θ is the weak angle introduced by Gell-Mann and Lèvy[3] and Cabibbo[1] and distinguishes the strangeness changing weak decays from the strangeness conserving ones. Besides vector and axial vector characters, other important properties of these hadronic currents are that (a) they are charged currents [see Eq.(2·3)], (b) consist of pieces which transform as octet under SU(3), and (c) the weak charge associated with the currents generates the algebra of $SU_L(2)$;

$$[[Q^h,\ Q^h],\ Q^h[= 4Q^h \qquad (2\cdot5)$$

where $Q^h \equiv -i\int d^3x\ j_4{}^h(\underset{\sim}{x},\ t)$.

Unlike leptonic currents, the hadronic currents can not be expressed uniquely in terms of well-defined field operators unless models of strong interactions are specified. In the quark model, they are simply given by

$$\begin{aligned} V_\mu(\Delta s=0,\ \Delta\underset{\sim}{I}=1) &= F_\mu{}^{(1)} + iF_\mu{}^{(2)}, \\ A_\mu(\Delta s=0,\ \Delta\underset{\sim}{I}=1) &= F_{5\mu}{}^{(1)} + iF_{5\mu}{}^{(2)}, \\ V_\mu(\Delta s=1,\ \Delta\underset{\sim}{I}=1/2) &= F_\mu{}^{(4)} + iF_\mu{}^{(5)}, \\ A_\mu(\Delta s=1,\ \Delta\underset{\sim}{I}=1/2) &= F_{5\mu}{}^{(4)} + iF_{5\mu}{}^{(5)}, \end{aligned} \qquad (2\cdot6)$$

with $F_\mu^{(i)} = i\bar{q}\gamma_\mu(\frac{\lambda i}{2})q$ and $F_{5\mu}^{(i)} = i\bar{q}\gamma_\mu\gamma_5(\frac{\lambda i}{2})q$ $(i=1,2,3\cdots 8)$, satisfying all the properties mentioned above.

The fact that leptonic weak charge Q^ℓ defined by $Q^\ell = -i\int d^3x\, j_4{}^\ell(\underset{\sim}{x}, t)$ also generates the same commutation relation (2·5) can be regarded as expressing a kind of "universality" between leptons and hadrons[9], to which we will come back in the next section. Note also that $V_\mu(\Delta s=0, \Delta\underset{\sim}{I}=1)$ is the conserved current (CVC) and $A_\mu(\Delta s=0, \Delta\underset{\sim}{I}=1)$ is the partially conserved current (PCAC). Following Gell-Mann[4][9], we can then identify weak vector and axial vector charges with the generators of $SU(2)\otimes SU(2)$ or $SU(3)\otimes SU(3)$ of strong interactions. With the PCAC, many results of "current algebra" then follow and we refer to the literature[25] for more details.

(2-2) Experimental tests and open questions.

Intensive experimental investigations on various decay processes for testing the Cabibbo theory have been done over the last decade. And we are now confident that our overall understanding on the weak phenomena based on (2·1), if not completely correct, is in the right direction. Indeed, it has been shown[26] that selection rules which are direct consequences of (2·1), (2·2), (2·3) and (2·4), such as $\Delta\underset{\sim}{I} = 1/2$ and $\Delta Q = \Delta s$ rules for semileptonic decays, are in remarkably good agreement with experiments. Of course, this does not necessarily imply that there are no unsettled problems left in those decay processes. For leptonic processes, we still have no firm informations about the so called "diagonal processes"[12] such as ν-e elastic scattering.

Consider the following scattering process

$$\nu_e + e \rightarrow \nu_e + e \,. \tag{2·7}$$

According to (2·1), the effective interaction for the above process is

$$L_{eff} = -\frac{G}{\sqrt{2}}[\bar{\nu}_e\gamma_\mu(1+\gamma_5)e][\bar{e}\gamma_\mu(1+\gamma_5)\nu_e], \tag{2·8}$$

which is then Fierz transformed into

$$L_{eff} = -\frac{G}{\sqrt{2}}[\bar{\nu}_e\gamma_\mu(1+\gamma_5)\nu_e][\bar{e}\gamma_\mu(1+\gamma_5)e]. \tag{2·9}$$

An inspection of (2·9) shows that if there exists an additional neutral leptonic current, (2·9) is modified in general as

$$L'_{eff} = -\frac{G}{\sqrt{2}}[\bar{\nu}_e\gamma_\mu(1+\gamma_5)\nu_e][\bar{e}\gamma_\mu(C_V+C_A\gamma_5)e] \tag{2·10}$$

where C_V and C_A are parameters depending on the nature of the neutral current assumed. Thus, for instance, in the Weinberg-Salam theory[11)] (§4), they are given by

$$C_V = \frac{1}{2} + 2\sin^2\phi \,,$$

and (2·11)

$$C_A = \frac{1}{2} \,,$$

with ϕ being a mixing parameter introduced into the theory. It is therefore very important to determine those constants experimentally. Very preliminary analysis has been done by Chen and Lee[27)], using the data of reactor induced neutrino experiment and showed that the mixing parameter ϕ should satisfy

$$\sin^2\phi \leq 0.35$$

to be consistent with the present data. However, it will still be premature to draw any quantitative conclusions about the nature of neutral currents. Other diagonal process like $\nu_\mu + e \to \nu_\mu + e$ will also provide us important informations, since, according to (2·1), this is the forbidden process to the order of G and only proceeds with the neutral current, if it exists[28].

For semi-leptonic processes, possible existence of the neutral current for each $\Delta s = 0$ and $\Delta s = 1$ processes is also one of the current issues. According to the summary given by Lee[19] at the Batavia Conference (1972), experiments so far done show severe bounds for $\Delta s = 1$ neutral current [$K_L^0 \to \mu^+ + \mu^-$, $K^+ \to \pi^+ e^+ e^-$, etc.] and relatively moderate restrictions for $\Delta s = 0$ neutral current [$\nu + N \to \nu + N$, $\nu + N \to \nu + N^*$, etc.]. More definite informations will be provided by high energy neutrino experiments at NAL and at CERN[29].

Finally, we make a remark about non-leptonic processes. As is known, this is the most difficult ones to make quantitative predictions. Many questions are still unsettled[30] but the most basic one will be the origin of $\Delta \underset{\sim}{I} = 1/2$ rule. Experimentally, this rule is well established in both K and hyperon decays, although a small mixture of $\Delta \underset{\sim}{I} = 3/2$ interaction of the order of $5 \sim 6\%$ is necessary to account for the deviations from the $\Delta \underset{\sim}{I} = 1/2$ rule in $K \to 3\pi$ and 2π decays[31].

Now, to understand, in the simplest way, the $\Delta \underset{\sim}{I} = 1/2$ rule will be to assume quarks with Bose statistics and write the $j_\mu^{h} \cdot j_\mu^{h\dagger}$ in the form of local four Bose quark coupling[32]. It is then easy to show that the non-leptonic interaction transforms like an octet, from which the $\Delta I = 1/2$ rule immediately follows. A small $\Delta I = 3/2$ component will arise if heavy W bosons exist so that the current-current interactions are no more strictly local. Of course, we must face to serious difficulties once we admit such quarks; for example, charge conjugation of vector currents is opposite to the usual ones. But we must also realize that quarks are, after all, quite peculiar objects and no one knows about their reality yet[33].

As a more conservative approach, dynamical octet enhancement has been proposed by Dashen and Frautschi[34] and Gell-Mann et al.[35] and, from a somewhat different point of view, by Nishijima[36].

Their method is to assume small driving terms of $\underset{\sim}{8}$ and $\underset{\sim}{27}$ as inputs to their bootstrap equations and then look for feed-backed solutions with possible enhancements. In cases which they discussed, it was indeed shown that the octet component is enhanced compared to others. However, it appears that such enhancement will crucially depend upon the spin-parity and internal quantum numbers of initial and final particles involved. Thus, it is not clear whether or not similar mechanism works also for Ω^- decays, for instance. It is perhaps worthwhile to note that the phenomenon of octet enhancement is a common feature in the matrix element of current-current product, such as electromagnetic mass splittings among hadrons[33].

Another approach is to show the suppression of $\underset{\sim}{27}$ component by assuming strong interaction dynamics with duality and no exotic resonances[37). In this case, it is impotant to show that for octet component, the dual weak amplitudes can in fact be constructed such that they are in accord with the current algebra results[38).

§3. Synthesis of Weak and Electromagnetic Interactions

(3-1) Universality

As mentioned in §1, there is a remarkable similarity between weak and electromagnetic interactions. The electromagnetic current to which photon couples is strictly conserved and consists of hadrons and leptons in a universal manner, if they are charged particles. Likewise, weak current to which W boson, assuming they exist, will couple, is almost conserved and consists of hadronic and leptonic currents in much the same way as the electromagnetic current does. The weak coupling constant (g) can be as large as the electromagnetic coupling constant (e), if the mass of W bosons is sufficiently large ($G=g^2/M_W{}^2$).

Now, it is well known that for electromagnetic interactions, the above property of the current can be formulated in terms of (Abelian) gauge invariance of the first and the second kind. Here, existence of massless photons plays a crucial role for such formulation to work[39). When we try to understand weak interactions with a similar spirit, a great difficulty is that apparently there exist no such massless particles in nature associated with the possible gauge invariance. Also, another important difference between two interactions is that the weak current is the charged one, which couples to left-handed particles only. As a consequence, the weak charge $Q_W \equiv -i\int d^3\, J_4(\underset{\sim}{x}, t)$ is not hermitean and by commuting it with $Q_W^{\dagger}$, the algebra of $SU_L(2)$ (2·5) can be generated. It is based on this fact that Gell-Mann[9] suggested that universality may be expressed by requiring that both leptonic and hadronic weak charges satisfy the same algebraic relation. Such assumption can be valid even if there is no gauge principle behind it. However, we feel it more natural and satisfactory if it is possible to go one step further and derive the foregoing facts based on gauge principles. As we shall see later (§4), recent gauge theories provide a way to overcome some of the difficulties mentioned so far and thus we can formulate the weak interaction theory based on local gauge invariance, in which the symmetry is spontaneouly broken such that gauge particles become massive. This is, in my opinion, one of the most appealing aspects of unified gauge theories. From such point of view, we may understand better Gell-Mann's current algebra of $SU(2)\otimes SU(2)$ or $SU(3)\otimes SU(3)$, which is based on an ad hoc identification of weak and electromagnetic currents with conserved or nearly conserved currents of approximate symmetry of strong interaction[40].

(3-2) Parity

To combine weak and electromagnetic interactions, we must note that parity is conserved in electromagnetism, while it is maximally violated in weak interactions. As pointed out by Lipkin[41], in order to unify both interactions into larger invariance group, in which generators of the group are given by the charges of the currents involved, it is necessary to assume either neutral parity violating currents or currents which couple to right-handed particles. The latter possibility can be achieved if we introduce heavy leptons. Thus, it appears that presence of neutral parity violating currents and/or heavy leptons are quite general features in such unified gauge models.

(3-3) Higher order effects

One must distinguish two cases for higher order effects of weak interaction[12]: the one is related to quantum number changing effects, such as $\Delta s = 2$ and $\Delta s = -\Delta Q$ transition processes which are all forbidden to the order of G in (2·1). The other is related to unitarity bound for the weak scattering amplitude. Experimentally, the first effects are very small, as can be seen from the $K_S{}^0$-$K_L{}^0$ mass difference, for instance. Theoretically, therefore, it is necessary to keep these higher order effects to be weak. On the other hand, the latter effects are more closely related to the asymptotic behaviour of weak scattering amplitude.

Consider, for example, the following process[11]

$$e^- + e^+ \rightarrow W^- + W^+ \tag{3·1}$$

in the lowest order in G with the interaction

$$L_{int} = -g\bar{\nu}_e\gamma_\mu(1+\gamma_5)eW_\mu{}^\dagger, \tag{3·2}$$

where W_μ is W boson field with the mass M_W. It is then straightforward to show that for the helicity amplitude $f_{00;\frac{1}{2}-\frac{1}{2}}$, the high energy behaviour is[11)12)]

$$f_{00;\frac{1}{2}-\frac{1}{2}} \underset{E\to\infty}{\longrightarrow} \frac{G}{2\sqrt{2}\pi} E\sin\theta \qquad (3\cdot3)$$

where $s = 4E^2$ and $G/\sqrt{2} \equiv g^2/M_W{}^2$. Thus, at sufficiently higher energies, the amplitude must be modified to satisfy the unitarity bound.

Logically, there are many ways to cure the above diseases[12)13)]. One might say, for example, that the results based on perturbation theory is totally misleading and non-perturbative calculation is necessary to avoid the difficulties. Such possibility is not ruled out, of course, but it is not a convincing arguments, either. If we consider possible solutions within perturbation theories, we must perhaps introduce more diagrams such that when all diagrams are computed, bad asymptotic behaviours of each diagrams all cancel with each other. That this is indeed a possibility can be seen as follows: there are three classes of diagrams in which particles are exchanged in s, t, and u channels (Fig.1). For the process (3·1) we are discussing, neutral leptons like ν_e can be exchanged in t channel. Because of the definite sign for residues, it is not possible to cancel the bad asymptotic behaviours among t channel exchange diagrams themselves, even if we introduce other heavy leptons. It is possible, however, to cancel them with those of s and/or u channel exchange diagrams, since, for s channel exchange diagrams, signs of the coupling constants at each vertices can be appropriately chosen and for u channel exchange (doubly charged lepton exchange), the leading asymptotic behaviour has a sign opposite to those of t channel exchange diagrams. In the Weinberg-Salam model (§4), for example, photon and neutral Z boson are exchanged in s channel such that they cancel the leading term given in (3·3) to be consistent with the unitarity bound. The neutral current mediated by the Z boson plays a crucial role here. In Georgi-Glashow model[42], on the other hand, there is no neutral current nor doubly charged leptons. In this case, ν_e and a heavy lepton E^0 are exchanged in t channel, the leading terms of which then cancel with that from s channel photon exchange diagram. Such mechanism can work, since W boson couples to left-handed ν_e and right-handed E^0, resulting a parity conserving asymptotic amplitude, which cancels with the (parity conserved) photon exchange amplitude. As mentioned before, the charged current with right-handed particles plays a similar role as the neutral current.

Recently, it has been shown[43] that by requiring such cancellations systematically in all tree diagrams for the processes involving leptons and spin one and zero particles, one can uniquely derive constraints among coupling constants identical to those given by the unified gauge theories. Clearly, this shows that the requirement of renormalizability and unitarity bound in each tree diagrams is equivalent with each other for theories involving massive spin one particles. It will be very interesting to examine whether or not such equivalence can be extended to theories of spin two particles (gravitons).

Finally, we note that even if the theory is renormalizable or, equivalently, consistent with unitarity, the finite higher order effects may still produce troubles, such as possible parity violation effects of the order of α (and/or α^2) or induced $\Delta s = 1$ and $\Delta s = 2$ transitions of the order of G (and/or $G\alpha$)[44]. One must carefully examine such possibilities in each models of this kind. Also, to calculate various radiative corrections, one should keep in mind that it is essential to take into account both weak and electromagnetic corrections simultaneously[45].

§4. Unified Gauge Models for Leptons

There are a large class of unified gauge models for weak and electromagnetic interactions, which are all renormalizable and apart from possible neutral currents, give rise to the same current-current weak interactions (2·1)[46]. The standard recipe for such model building is also known[47]: an essential ingredient is the spontaneously broken non-Abelian gauge invariance——the Yang-Mills theory[48]—— with the so called Higgs-Kibble mechanism, for which we refer to the literatures for details[16)∿21)].

We shall now briefly discuss the Weinberg-Salam model for leptons as a prototype of such general theories. Conceptually, nothing very new is involved in other models so that once you become familiar with one poarticular model, the other models can be easily understood[49]. Also, our discussion on the W-S model will mostly be at the classical level and we omit various problems related to quantizations and renormalizations of gauge theories[50].

The gauge group of the W-S model is $SU_L(2) \otimes U_Y(1)$ and thus we have a triplet $\vec{A}_\mu$ and a singlet B_μ gauge fields, respectively, which physically correspond to $W^\pm$, Z^0 and γ. There are no new leptons assumed and for electron-type leptons, the $SU_L(2)$ gauge group acts on a left-handed doublet

$$L_e = \frac{1+\gamma_5}{2} \begin{pmatrix} \nu_e \\ e^- \end{pmatrix} , \qquad (4 \cdot 1)$$

and a right-handed singlet[51]

$$R_e = \frac{1-\gamma_5}{2} e^- \qquad (4 \cdot 2)$$

The electric charge operator is given by

$$Q = T_L^{(3)} + \frac{1}{2} Y \qquad (4 \cdot 3)$$

so that $Y = -1$ for L_e and $Y = -2$ for R_e. In addition, there is a complex scalar doublet

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \qquad (4 \cdot 4)$$

under $SU_L(2)$ with $Y = +1$.

This scalar doublet plays a role of spontaneously breaking the gauge symmetry down to U(1) of electric charge.

The total Lagrangian invariant under $SU_L(2) \otimes U_Y(1)$ involving the electron-type leptons is then of the form

$$
\begin{aligned}
L = &-\tfrac{1}{4}[\partial_\mu \vec{A}_\nu - \partial_\nu \vec{A}_\mu + g\vec{A}_\mu \times \vec{A}_\nu]^2 \\
&-\tfrac{1}{4}[\partial_\mu B_\nu - \partial_\nu B_\mu]^2 \\
&-\bar{L}_e \gamma_\mu (\partial_\mu - ig\vec{A}_\mu \tfrac{\vec{\tau}}{2} - i\tfrac{1}{2}g' B_\mu) L_e \\
&-\bar{R}_e \gamma_\mu (\partial_\mu - ig' B_\mu) R_e \qquad\qquad (4\cdot5) \\
&-\tfrac{1}{2}|\partial_\mu \phi - ig\vec{A}_\mu \tfrac{\vec{\tau}}{2}\phi + \tfrac{i}{2}g' B_\mu \phi|^2 \\
&+ F(\phi) \\
&+ G_e[\bar{L}\phi R + h.c.]
\end{aligned}
$$

where g and $g'/2$ are coupling constants associated with $SU_L(2)$ and $U_Y(1)$, respectively. $F(\phi)$ is an invariant polynomial of ϕ, which is responsible for the spontaneous breakdown of the gauge symmetry.

As a result, the following three become massive gauge fields,

$$
\begin{aligned}
W_\mu^\pm &= \frac{1}{\sqrt{2}}(A_\mu^{(1)} \pm iA_\mu^{(2)}) \\
Z_\mu^0 &= \frac{1}{\sqrt{g^2+g'^2}}(gA_\mu^{(3)} + g' B_\mu)
\end{aligned}
\qquad (4\cdot6)
$$

with $M_W{}^2 = \frac{1}{4}g^2 \langle\phi^0\rangle^2$ and $M_Z{}^2 = \frac{1}{4}(g^2+g'^2)\langle\phi^0\rangle^2$, respectively, while $A_\mu \equiv 1/\sqrt{g^2+g'^2}(gA_\mu^{(3)} - g' B_\mu)$ remains massless and can be identified with photon field. Here, $\langle\phi^0\rangle$ is a vacuum expectation value of the neutral component of scalar field and is of the order of

300 Gev, when determined from the relation $\langle\phi^0\rangle^2 \approx 1/\sqrt{2}G$.
The last term of (4·5) gives rise to the mass term for electrons with $m_e = G_e\langle\phi^0\rangle$. Since electromagnetic coupling constant should be identified with $e = -g\sin\phi$ with $\cos\phi \equiv g/\sqrt{g^2+g'^2}$, we have a famous prediction[52)]

$$M_W{}^2 \geq \frac{1}{4}\langle\phi^0\rangle^2 e^2 = \frac{\pi}{\sqrt{2}}\left(\frac{e^2}{4\pi}\right)\frac{1}{G} \qquad (4\cdot 7)$$
$$= (37.3\ \text{Gev})^2.$$

The interaction of these massive gauge fields with leptons can be read off from (4·5) and (4·6) and is

$$L^{(1)}_{\text{int}} = -\frac{g}{2\sqrt{2}}[j_\mu{}^{(+)}W_\mu + j_\mu{}^{(-)}W_\mu{}^\dagger]$$
$$-\frac{g}{\cos\phi}[j_\mu{}^{(0)} - \sin^2\phi j_\mu{}^{\text{e.m.}}]Z_\mu{}^{(0)} \qquad (4\cdot 8)$$
$$-ej_\mu{}^{\text{e.m.}}A_\mu$$

where[53)]

$$j_\mu{}^{(+)} = i\bar{\nu}_e\gamma_\mu(1+\gamma_5)e, \quad j_\mu{}^{(-)} = j_\mu{}^{(+)\dagger} \qquad (4\cdot 9)$$
$$j_\mu{}^{(0)} = \frac{1}{4}[\bar{\nu}\gamma_\mu(1+\gamma_5)\nu - \bar{e}\gamma_\mu(1+\gamma_5)e]$$

and

$$j_\mu{}^{\text{e.m.}} = i\bar{e}\gamma_\mu e.$$

Thus, $W^\pm$ couple to the left-handed charged current, while Z^0 couples to the neutral current. From (4·8) and (4·9), we obtain

$$\frac{G}{\sqrt{2}} \approx \left(\frac{g}{2\sqrt{2}}\right)^2 \frac{1}{M_W{}^2} = \frac{g^2}{8M_W{}^2} \,, \tag{4·10}$$

and

$$G_{neutral} \approx \left(\frac{g}{\cos\phi}\right)^2 \frac{1}{M_Z{}^2} = 8\frac{G}{\sqrt{2}} \,, \tag{4·11}$$

where $G_{neutral}$ is the effective coupling strength for the neutral current-current interaction. When we calculate $\nu_e + e \rightarrow \nu_e + e$ scattering (2·7)using (4·11), one indeed finds the effective interacgion given in (2·11). This is due to a huge effect of the neutral current, which you will see by comparing (4·10) and (4·11). Such neutral current, however, plays an essential role for cancelling the unitarity violating amplitudes in each orders of the perturbation expansion. As frequently emphasized, whether such neutral current really exists or not is a key point for the theories of this kind.

On the other hand, the first term of (4·5) gives rise to the coupling of gauge particles among themselves of the form.

$$\begin{aligned} L^{(2)}_{int.} = {} & i\cos\phi[gZ^0_\nu - g'A_\nu][W_\mu(\partial_\mu W_\nu{}^\dagger - \partial_\nu W_\mu{}^\dagger) \\ & -W_\mu{}^\dagger(\partial_\mu W_\nu - \partial_\nu W_\mu) + \partial_\mu(W_\mu W_\nu{}^\dagger - W_\nu W_\mu{}^\dagger)] \\ & -\cos^2\phi W_\mu W_\nu{}^\dagger[gZ^0_\rho - g'A_\rho][gZ^0_\sigma - g'A_\sigma](\delta_{\mu\nu}\delta_{\rho\sigma} \\ & -\delta_{\mu\rho}\delta_{\nu\sigma}) + \frac{g^2}{2}[|W_\mu W_\mu|^2 - (W_\mu W_\mu{}^\dagger)^2]. \end{aligned} \tag{4·12}$$

This interaction contains electromagnetic coupling of W boson in a specific manner, from which g-factor of the bare magnetic moment of W can be determined, giving a value of the gyromagnetic ratio $g = 2$, i.e., $m = (e/M_W)S$. Note that this is not the value predicted by the "minimum" electromagnetic coupling[54], which predicts $g = 1$. As pointed out by Weinberg[55], the value of $g = 2$ is just the one which is required for the Compton scattering off a W target to satisfy the Gerasimov-Drell-Hearn sum rule (GDH sum rule)[56]. In fact, for a target of spin S, magnetic moment m and mass M, the GDH sum rule reads[57]

$$S_Z\left(\frac{m}{S} - \frac{e}{M}\right)^2 = \frac{1}{\pi}\int_0^\infty \frac{\sigma_+(\nu)-\sigma_-(\nu)}{\nu}\,d\nu \tag{4·13}$$

where S_Z is the Z component of the spin S of the target and $\sigma_+(\nu)\,[\sigma_-(\nu)]$ is the forward total cross-section due to the helicity +1[-1] photons. Obviously, the right-hand side of (4·13), if expanded in power series of α, will start with the order of α^2, so that for the GDH sum rule to be valid, the magnetic moment m which appears in the left-hand side of (4·13) must be[57]

$$m = \frac{e}{M}(1 + O(\alpha))S, \tag{4·14}$$

that is, the bare g factor is equal to 2.

As is known, the GDH sum rule is derived from low energy theorems and unsubtracted dispersion relations for the helicity flip Compton scattering amplitude[58]. The above result would then imply that for arbitrary target, helicity flip Compton scattering amplitude will have a gentle behaviour at higher energies, only when the g factor of the target is equal to 2.

§5. Hadrons, Gauge Symmetries and Approximate Symmetries in Strong Interactions

(5-1) Hadrons in unified gauge models

So far we have not discussed hadrons within the scheme of unified theories of weak and electromagnetic interactions. But, it is very important to apply the gauge theories to processes involving hadrons, since our experimental informations about weak interactions mainly come from semi-leptonic processes. A trouble is, however, that once strong interactions come in, it is not easy to make a quantitative estimations of hadronic effects on weak processes.

Perhaps, to perform such calculations, it will be necessary to appeal to "current algebra" technique or dual models with currents[59).

Now, ignoring such problems for the moment, let us first consider the simple quark model and assume a doublet

$$\psi_L = \frac{1+\gamma_5}{2}\begin{pmatrix} p \\ n' \end{pmatrix} \qquad (Y = +\tfrac{1}{3}) \qquad (5\cdot 1)$$

and four singlets

$$\lambda'_L = \frac{1+\gamma_5}{2}\lambda' \quad (Y = -\tfrac{2}{3}), \qquad \lambda'_R = \frac{1-\gamma_5}{2}\lambda' \quad (Y = -\tfrac{2}{3}),$$
$$p_R = \frac{1-\gamma_5}{2}p \quad (Y = +\tfrac{4}{3}), \qquad n'_R = \frac{1-\gamma_5}{2}n' \quad (Y = -\tfrac{2}{3}), \tag{5·2}$$

with $n' = \cos\theta\ n + \sin\theta\ \lambda$ and $\lambda' = -\sin\theta\ n + \cos\theta\ \lambda$.

Proceeding then in a manner similar to which (4·5) was obtained, we can easily incorporate hadrons in the W-S model. For charged currents to which W couple, we obtain

$$j_\mu{}^h = i[\bar{p}\gamma_\mu(1+\gamma_5)n\cos\theta + \bar{p}\gamma_\mu(1+\gamma_5)\lambda\sin\theta] \tag{5·3}$$

The neutral current, on the other hand, is given by

$$j_\mu{}^{(Z)} = j_\mu{}^{(3)} - \sin^2\phi\ j_\mu{}^{e.m.} \tag{5·4}$$

with

$$\begin{aligned} j_\mu{}^{(3)} = \tfrac{i}{2}[&\bar{p}\gamma_\mu(1+\gamma_5)p - \cos^2\theta\ \bar{n}\gamma_\mu(1+\gamma_5)n \\ &-\sin\theta\cos\theta(\bar{\lambda}\gamma_\mu(1+\gamma_5)n + \bar{n}\gamma_\mu(1+\gamma_5)\lambda) \\ &-\sin^2\theta\ \bar{\lambda}\gamma_\mu(1+\gamma_5)\lambda] \end{aligned} \tag{5·5}$$

and

$$j_\mu{}^{e.m.} = i[\tfrac{2}{3}\bar{p}\gamma_\mu p - \tfrac{1}{3}\bar{n}\gamma_\mu n - \tfrac{1}{3}\bar{\lambda}\gamma_\mu\lambda]. \tag{5·6}$$

As is easily seen, $\Delta s = 1$ neutral current which appeared in (5·4) and (5·5) is not suppressed at all in this model.

To eliminate such strangeness changing current, one must extend the simple quark model to, for example, $SU_L(4)\times SU_R(4)$ model[60) or three triplets model[61). It is then possible to cancel $\bar{\lambda}n$ term in (5·5) completely, and at the same time, to suppress a process of $\bar{\lambda}+n \to W^+ + W^-$, which, through $\bar{\lambda}+n \to W^+ + W^- \to \mu^+ + \mu^-$, contributes to the induced $\Delta s = 1$ neutral current[62).

Although, in this way, one can achieve a consistent unified model of weak and electromagnetic interactions for both leptons and hadrons, we still feel that models involving hadrons with non-zero Cabibbo angle is not satisfactory in the sense that elimination of $\Delta s = 1$ neutral current is rather ad hoc. Perhaps, more detailed informations on the nature of $\Delta s = 0$ and $\Delta s = 1$ neutral currents and/or heavy leptons will be necessary before we can construct a more realistic model of weak interactions.

(5-2) Approximate Symmetries

Strong interactions obey a group of approximate symmetries, for which it is convenient to make the following distinction; the one is algebraic and the other is dynamical[63). Algebraic symmetries are those in which invariance of the Lagrangian is manifested in algebraic condition on the S-matrix, such as SU(2) invariance and isotopic spin conservation law. In this case, particles are usually classified into multiplets of simple representations of the invariance groups. Dynamical symmetries, on the other hand, are not symmetries of multiplets. Instead, like chiral $SU(2)\otimes SU(2)$[64) or $SU(3)\otimes SU(3)$, they allow us to derive various low energy theorems. Presence of massless particles is a common feature in the latter symmetries. And in both cases, it is usually thought that intrinsic symmetry breaking can be put in by hand afterwards.

The non-Abelian gauge symmetries we have been discussing clearly belong to the latter category. One difference is that the symmetries are further broken spontaneously in such a way the except photons, there remains no massless gauge particles associated with them. Since these gauge particles are supposed to be very massive, there is no low energy theorems, either. As a result, we can expect only very indirect evidences in nature for such symmetires, even if they exist. We may call the symmetires of this sort the dynamical gauge symmetries and distinguish them from those in which there exist exact or approximate low energy theorems[65].

Now, in order to incorporate hadrons into the unified gauge models, it is necessary to couple $W^{\pm}$, Z and A to the currents of an exact $SU_L(2)\times U(1)$ symmetry of the strong interaction. Otherwise, the renormalizability or the unitarity bound will be violated. This implies among others that divergences of hadronic vector and axial vector currents must be independent of strong interactions. A question then arises. How do we understand the successful hypothesis of PCAC? The only answer will be that pion mass should be generated by the same spontaneous symmetry breaking mechanism responsible for $W^{\pm}$ and Z masses; that is, weak interactions with Higgs scalars generate pion masses and thus play a role of producing intrinsic symmetry breaking of strong interactions. As emphasized by Weinberg[66] and Weinstein[67], from such point of view of gauge theories, it may be possible to obtain deeper understanding of current algebra in conformity of universality of weak interactions.

Recently, origins of the approximate symmetries of strong interactions have also been discussed from a point of view of renormalizable gauge theories broken spontaneously[68]. It was shown that in some cases there occurs a certain symmetry relation among masses after spontaneous symmetry breaking, which may or may not a consequence of the gauge symmetries assumed. It is called "the zeroth order symmetry" or "natural symmetry", which can be broken by higher order effects and thus can account for observed approximate symmetries.

Although such a view point is extremely interesting, at the time of preparing this lecture note, it appears that no realistic model of such approximate symmetries has been constructed yet.

§6. Conclusion

We have discussed some of the ideas of combining weak and electromagnetic interactions on a unified basis. As a conclusion of my lecture, I would like to emphasize that from a purely theoretical point of view, this is indeed a very natural approach. Consider the process (3·1) again. Without weak interactions, it proceeds, to the order of α, with one photon annihilation and, as discussed previously, violates the unitarity bound at higher energies, if W boson has anomalous magnetic moment. This would then imply that the electromagnetic interactions of massive charged W with anomalous magnetic moment are not renormalizable. On the other hand, weak interaction (3·2) also violates the unitary bound [see (3·3)] and these difficulties will remain as long as we treat two interactions separately. It is therefore very appealing and gratifying that we can in fact eliminate such difficulties by unifying them in a gauge invariant manner. The price we have to pay for this was, of course, the necessity of introducing of additional neutral currents and/or heavy new leptons. Also, we must assume massive Higgs scalar particles, which are a consequence of spontaneously broken gauge symmetries. Thus, experimental verifications for or against those evidences will be one of the central issues of weak interactions of early seventies.

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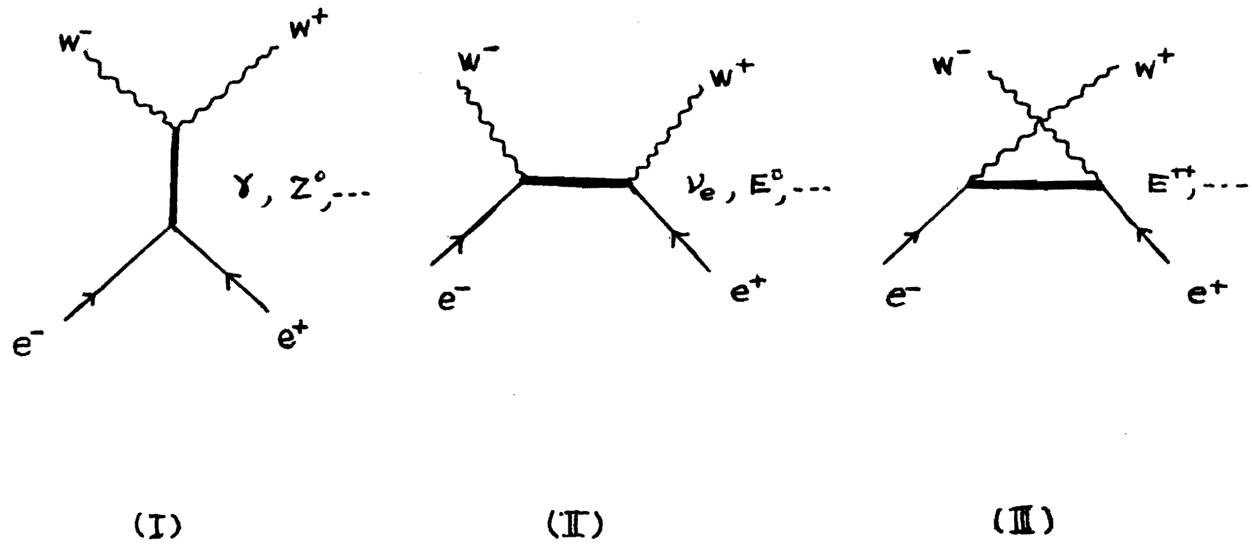

Fig. 1 Feynman Diagrams for $e^{-}+e^{+} \rightarrow W^{-}+W^{+}$

The diagram (I) represents a class of s channel exchange processes, while the diagrams (II) and (III) are those of t and u exchanges, respectively.