Hadron Symmetry and Quarks

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I. Introduction

In these two lectures I am concerned with the recent development of hadron symmetry and quarks. The first lecture is intended to be an introductory review on the subject. In the second lecture I want to introduce to you the very recent progress in this field. Rather than making a complete review I intend to give you an idea of how new developments have been achieved step by step after recognizing the old difficulties. This progress in turn has helped us to make a further insight into the problem. It is naturally beyond the scope of these two lectures to cover in detail many interesting applications which have been done in this field every time a new idea has been proposed.

II. Approximate symmetry algebra

2-A. Definition

The approximate symmetry algebra (ASA) can be defined as a symmetry which is approximately valid in the physical world. We may formulate ASA as follows. Consider a physical system which is described by the hamiltonian H. Suppose we have some conserved quantum numbers O_a (a = 1, 2, ..., k):

$$[H, O_a] = 0, \quad a = 1, 2, \ldots, k \tag{2-1}$$

Further suppose that with some additional components O_a ($a = k + 1, \ldots, \ell$) we can form a closed algebra

$$[O_a, O_b] = f_{abc}O_c, \quad a,b,c = 1, 2, \ldots, \ell \tag{2-2}$$

If we can divide the hamiltonian such that

$$H = H_{inv} + \varepsilon H_{break}, \tag{2-3}$$

where

$$[H_{inv}, O_a] = 0, \ a = 1, 2 \ldots, \ell \tag{2-4}$$

and that we may consistently consider the exact symmetry limit of $\varepsilon \to 0$ as a good approximation to describe the system, then we call the symmetry algebra formed by O_a ($a = 1, 2, \ldots, \ell$) as ASA for the system. In this case it is useful to start with the exact symmetry limit and classify the eigenstates of H_{inv} ($\simeq$ H) under the irreducible representations of the algebra.

Let us consider, for example, the isotopic spin operators. As far as the strong interactions are concerned, we can think of them as constants of motion

$$[H_{st}, \underset{\sim}{I}] = 0, \tag{2-5}$$

obeying the commutation rules

$$[I_i, I_j] = i\varepsilon_{ijk} I_k. \tag{2-6}$$

Therefore we have the SU(2) invariance of H_{st} whose breaking is very small since it is only due to the electromagnetic and weak interactions. Thus we are led to classifying the hadron states under the isospin multiplets.

In the strong interactions we have another conserved quantum number called hypercharge Y or strangeness S. The two are related each other through the Nishijima-Gell-Mann rule Y = B + S where B is the baryon number which is always conserved,

$$[H_{st}, Y] = 0 \ . \tag{2-7}$$

Gell-Mann[1] proposed that we can form a simple closed algebra SU(3) by considering another four components in addition to the isospin and strangeness. Denoting the eight components by F_i ($i = 1, 2, \ldots, 8$) we obtain

$$[F_i, F_j] = if_{ijk}F_k \quad , \tag{2-8}$$

where the f_{ijk}'s are the structure constants of the SU(3) algebra. The isospin and hypercharge are related to these components as

$$I_i = F_i, \ i = 1,2,3 \text{ and } Y = \frac{2}{\sqrt{3}} F_8 \quad . \tag{2-9}$$

We then assume that the hamiltonian H_{st} can be divided such that

$$H_{st} = H^0_{st} + H'_{st} \quad , \tag{2-10}$$

where H^0_{st} is invariant under the SU(3) algebra

$$[H^0_{st}, F_i] = 0 \tag{2-11}$$

and H'_{st} denotes the symmetry breaking term (which is, for example, the quark mass term in the quark model of hadrons to be introduced later) of the SU(3) multiplets. In nature we have approximate realizations of the SU(3) symmetry: the pseudoscalar and vector mesons are nicely classified into the singlet and octet representations of the algebra, whereas the baryons are fit into the singlet, octet and decuplet representations.

2-B. Quarks

The fundamental representations of the SU(3) algebra are given by the triplet $\underline{3}$ and its conjugate $\underline{3}^*$. The quantum numbers of the states in the multiplets are shown in Fig. 1.

for developing a new study in particle physics. Some of the contents of these lectures discuss the results which are only obtained in the quark model, while most of the others are the abstractions based on quark concept.

2-C. $SU(6)_W$

It may be a natural temptation to attempt to incorporate spin S into the hadron symmetry scheme. This can be done by noting the spin structure of the quarks. The spin obeys the $SU(2)_S$ algebra

$$[S_i, S_j] = i\varepsilon_{ijk}S_k \tag{2-12}$$

Let us consider the product $SU(3) \times SU(2)_S$ of the SU(3) symmetry and the $SU(2)_S$ spin algebra. Then we are led to the $SU(6)_S$ symmetry as the minimal algebra including the subgroup $SU(3) \times SU(2)_S$. The quarks $q = (u\uparrow, d\uparrow, s\uparrow, u\downarrow, d\downarrow, s\downarrow)$ with up and down spins transform irreducibly as the basic 6 dimensional representation ($\underline{6}$) of $SU(6)_S$, whereas the anti-quarks $\bar{q}$ correspond to the conjugate representation $\underline{6}^*$. In $SU(6)_S$ the lowest mesons are classified into the single irreducible representations $\underline{35} = (\underline{8} + \underline{1}) \times \underline{3} + \underline{8} \times \underline{1}$ where the vector mesons with spin up for example are constructed by $(q\uparrow\bar{q}\uparrow)$ and the pseudoscalar mesons are made out of $((q\uparrow\bar{q}\downarrow) - (q\downarrow\bar{q}\uparrow))/\sqrt{2}$. The lowest baryons are classified into $\underline{56} = \underline{10} \times \underline{4} + \underline{8} \times \underline{2}$ where again the baryon spins 1/2 and 3/2 are made out of the quark spins. $SU(6)_S$ is essentially the static symmetry, so the orbital motion of particles involved are not treated properly. If one applies the symmetry to hadronic vertices, one immediately faces the difficulty. For example, let us consider the $\rho\pi\pi$ and $N^*N\pi$ couplings. If we require the $SU(6)_S$ symmetry, the quark spin must be conserved through these vertices. Therefore the above couplings are forbidden under $SU(6)_S$: $\rho(S = 1) \not\to \pi(S = 0) + \pi(S = 0)$, $N^*(S = 3/2) \not\to N\ (S = 1/2) + \pi(S = 0)$.

In order to remedy the above mentioned difficulty which is mainly due to the static treatment of quarks, Lipkin and Meshkov[3] proposed to consider the "W spin" independence instead of the "quark spin" independence. The W spin can be difined for quarks in motion. Let us consider a quark (or anti-quark) which moves in the y-z plane. Its angular momentum must then point along the x axis ($|L_x| = L$). We shall define the symmetry operation R_x which is a reflection in the y-z plane. Acting on the quark states it becomes

$$R_x = P\, e^{i\pi J_x} = (-1)^L P_{int} e^{i\pi(L_x + S_x)} = P_{int} e^{i\pi S_x} \qquad (2\text{-}13)$$

where P_{int} denotes the intrinsic parity (+1 for the quarks and -1 for the anti-quarks) and the relation J = L + S was used. Because the quark has spin 1/2 one can easily show that

$$R_x = P_{int} e^{i\pi\sigma_x/2} = e^{i\pi P_{int}\ \sigma_x/2} = iP_{int}\ \sigma_x. \qquad (2\text{-}14)$$

Thus we see that the R_x operation is commutable with the motion in the y-z plane and can be expressed in terms of the intrinsic quantum numbers of the quark states. Let us define

$$W_x = P_{int}\ \sigma_x/2 \qquad (2\text{-}15)$$

Then we see from eq. (2-14) that W_x is a good quantum number for the quark states in motion and further that R_x is generated by W_x. If we limit ourselves to a collinear motion (in the z direction) then we can define in an analogous way

$$W_y = P_{int}\sigma_y/2 \qquad (2\text{-}16)$$

which will also provide a good quantum number. One can now add a third conserved quantum number

$$W_z = \sigma_z/2 \qquad (2\text{-}17)$$

It is conserved because $J_z = S_z$ for any particle moving in the z direction.

Let us define the W spin in spinor notation which is given by

$$W_x = \beta\sigma_x/2, \quad W_y = \beta\sigma_y/2, \quad W_z = \sigma_z/2. \tag{2-18}$$

Then the $SU(6)_W$ is defined as the minimal algebra including the subgroup of $SU(3) \times SU(2)_W$ and its operators are given by

$$SU(6)_W \sim (\lambda_i \otimes W_a), \; i = 0, 1, 2, \ldots, 8 \text{ and } a = 0, x, y, z \tag{2-19}$$

where λ_i are the matrix representations of F_i first introduced by Gell-Mann, and λ_0 and $2W_0$ are the unit operators in each subgroup.

By definition the interrelation between the W spin and the S spin is obvious. The spin of the particle is naturally defined as the sum of the total spin of its quark and anti-quark constituents:

$$\underset{\sim}{S} = \underset{\sim}{S}_q + \underset{\sim}{S}_{\bar{q}} \tag{2-20}$$

Similarly we define

$$\underset{\sim}{W} = \underset{\sim}{W}_q + \underset{\sim}{W}_{\bar{q}} \tag{2-21}$$

Then we find that

$$W_{x,y} = (S_q)_{x,y} - (S_{\bar{q}})_{x,y}$$

$$W_z = (S_q)_z + (S_{\bar{q}})_z = S_z \tag{2-22}$$

Let us treat as an example the triplet and singlet $q\bar{q}$ eigenstates of S:

$$(\text{triplet})_s = \begin{bmatrix} V_{+1} \\ V_0 \\ V_{-1} \end{bmatrix} = \begin{bmatrix} q\uparrow\bar{q}\uparrow \\ \frac{1}{\sqrt{2}}(q\uparrow\bar{q}\downarrow + q\downarrow\bar{q}\uparrow) \\ q\downarrow\bar{q}\downarrow \end{bmatrix}, \quad (\text{singlet})_S = P = \frac{1}{\sqrt{2}}(q\uparrow\bar{q}\downarrow - q\downarrow\bar{q}\uparrow) \tag{2-23}$$

Correspondingly we find the eigenstates of W-spin:

$$(\text{triplet})_W = \begin{pmatrix} V_{+1} \\ -P \\ V_{-1} \end{pmatrix} = \begin{pmatrix} q\uparrow\bar{q}\uparrow \\ \frac{1}{\sqrt{2}}(-q\uparrow\bar{q}\downarrow + q\downarrow\bar{q}\uparrow) \\ -q\downarrow\bar{q}\downarrow \end{pmatrix}, \quad (\text{singlet})_W = -V_0 = -\frac{1}{\sqrt{2}}(q\uparrow\bar{q}\downarrow + q\downarrow\bar{q}\uparrow) \tag{2-24}$$

We note the interesting fact that the $S_z = W_z = 0$ eigenstates interchanged their places, i.e. the triplet S state became a singlet W states and vice versa. This peculiar property is called W-S flip. For the baryons there is no essential change in the quark configuration. Under the W-S transition the only effect is phase changes in assigning the anti-baryon to $\underline{56}^*$.

By definition the $SU(6)_W$ symmetry can only be applied to collinear processes such as three-point vertex functions and forward scattering amplitudes. Under $SU(6)_W$ the collinear couplings $\rho\pi\pi$ and $N^*N\pi$ are now allowed since the W spins are conserved through these vertices:

	$\rho \to$	π +	π		$N^* \to$	N +	π
$W =$	0	1	1	$W =$	3/2	1/2	1
$W_z =$	0	0	0	$W_z =$	1/2	1/2	0

$SU(6)_W$ preserves the good predictions of $SU(6)_S$ such as the F/D(=2/3) ratio of the couplings of pseudoscalars to baryon pairs, the (-2/3) ratio of the neutron and proton magnetic moments. Further it predicts the dominant M1 electro- and photo-production of N^* which are experimentally verified. It also gives the Johnson-Treiman relation

$$\frac{1}{2}[\sigma(K^-p)-\sigma(K^+p)] = \sigma(\pi^-p)-\sigma(\pi^+p)=\sigma(K^-n)-\sigma(K^+n) \tag{2-25}$$

There are some wrong predictions in the orbital excitation scheme of $SU(6)_W$ such as $SU(6)_W \times O(3)$ or $SU(6)_W \times O(2)_{L_z}$. However, as far as the lowest mesons and baryons are concerned, the $SU(6)_W$ symmetry seems to be working as a good ASA for hadron symmetry.

III. Algebra of currents

3-A Background

Instead of seaking for the invariance of the hamiltonian under a certain ASA we can start with the weak currents to form a closed algebra. Here the principle of the conserved vector current(CVC)[4] and the concept of universality[5] of weak interactions between leptons and hadrons play an important guiding role to construct a theory for the algebra of currents.

ASA tells you about the approximate invariance of the hamiltonian, thus becomes very powerful in classifying systematically the particle states. On the other hand the algebra of currents are based on the measurable current components irrespective of the invariance of the system. It has a predictive power for evaluating the matrix elements of the observed quantities.

Let us consider the hadronic weak current which is coupled to the lepton pairs

$$J_\alpha^{hadronic}=\cos\theta(V_\alpha^{(\Delta Y=0)}+A_\alpha^{(\Delta Y=0)})+\sin\theta(V_\alpha^{\Delta Y=1)}+A_\alpha^{(\Delta Y=1)}) \qquad (3\text{-}1)$$

where the relative strength between the strangeness preserving and changing currents are represented by the Cabibbo angle θ[6] and each current is split into a vector and axial vector part. Now the idea of CVC is that the first term is equal to the isotopic spin raising current and we can define

$$I_1 + iI_2 = \int V_0^{(\ Y\ =\ 0)}d^3x \qquad (3\text{-}2)$$

In the same manner we have for the electromagnetic current J_α^{em}= $J_{3\alpha}$ + isoscalar, and we can define I_3 to be the integral over space of the isovector time component of J_α^{em}

$$I_3 = \int J_{30}\ \ d^3x \qquad (3\text{-}3)$$

We see then that we can define the isospin operators in terms of quantities which, at least in the lowest order of electricity and weak interactions, are measurable, as an alternative to defining them as a set of good quantum numbers of strong interactions, obeying the commutation relation eq.(2-6)

This suggests that perhaps also the other portions of the weak vector current (and even axial current) have charge operators, i.e., space integrals of their time components, obeying some simple set of commutation rules. However, as we know very well, these charges are not conserved, so that they are not independent of time: we can talk only about equal time commutation relations. (To discuss different time commutation rules requires some knowledge of dynamics which is not at our disposal yet.) More than ten years ago, guided by the CVC hypothesis and the universality of weak interactions, Gell-Mann[1] were thus led to the following simplest possibility of the equal-time commutation relations between the weak vector and axial vector SU(3) currents:

$$
\begin{aligned}
&[F_i, F_j] = if_{ijk}F_k \\
&[F_i, F_j^5] = if_{ijk}F_k^5 \qquad\qquad (3\text{-}4)\\
&[F_i^5, F_j^5] = if_{ijk}F_k \quad , \quad i, j, k = 1, 2, \ldots, 8.
\end{aligned}
$$

where the operators $(F_i \pm F_i^5)$ generate two commuting SU(3) algebras, the so-called chiral SU(3) x SU(3). In the above $F_4 + iF_5 = \int V_0^{(\Delta Y = 1)} d^3x$, $F_1^5 + iF_2^5 = \int A_0^{(\Delta Y = 0)} d^3x$, and so on. Let us note that the vector charges can be taken to be conserved in the approximate SU(3) symmetry limit which implies the approximate invariance of the total hamiltonian under this algebra. Therefore the algebra of F_i given by the first commutators in eq.(3-4) can be identified with the ASA of SU(3) in the previous chapter.

3-B. Quark model for the algebra of currents

We have a model for the algebra of currents where the Gell-Mann's hypothesis about the current commutators is valid. We shall consider a lagrangian of the quarks*

$$\mathcal{L} = \bar{q}\ (\gamma\cdot\partial + m_0)q + \text{interaction} \tag{3-5}$$

from which we can deduce the currents

$$V^i_\alpha = i\ \bar{q}\ \frac{\lambda i}{2}\ \gamma_\alpha q, \tag{3-6}$$

$$A^i_\alpha = i\ \bar{q}\ \frac{\lambda i}{2}\ \gamma_\alpha \gamma_5 q$$

By using canonical anti-commutation rules for the q fields, we can compute, at least formally, the commutation relations of these currents which confirm the hypothesis eq. (3-4).

It is important to stress here that these commutation relations hold true, irrespective of how badly broken the symmetry is (for instance, by mass terms): SU(3) symmetry and the validity of equal time commutation relations are two quite independent things. This evidently tells us the important difference between the two algebras of F_i in the previous chapter and in the present section. In actual applications we do not distinguish them assuming SU(3) symmetry to be good.

One can try to extend the chiral SU(3) x SU(3) algebra by including the space integrals of all the vector and axial vector components and if one evaluates the commutators following the formal quark model we introduced above, one gets the chiral U(6) x U(6) algebra. One may go even further by introducing additional currents, like scalar, pseudoscalar and tensor currents, which have never been seen but which might be there, and one gets the compact U(12) algebra[7]. We denote its 144 compoennts

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* Our convention here is $\gamma_k = -i\beta\alpha_k$, $\gamma_4 = \beta$, $\gamma_5 = \gamma_1\ \gamma_2\ \gamma_3\ \gamma_4$.

by $(V^i_\alpha, A^i_\alpha, T^i_{\alpha\beta}, S^i, P^i)$, $(\alpha, \beta) = (0,x,y,z)$, $(i)=(0,1,2,\ldots, 8)$.

In the following we select those currents whose matrix elements between single particle states survive in the infinite momentum frame.[8) There are several advantages in considering the current commutators in the infinite momentum frame. Some of them are just kinematical, but more importantly, the others are deeply related to the physics which we are working on. For the details I suggest you to refer to the appropriate literature[8) 9)]. I only mention here that in the infinite momentum frame the estimate of intermediate contributions in the commutators can be done more systematically and transparently which allows us to approximate the sum over the intermediate states with a few resonance states, thus the algebraic properties of the commutators and the particle states are more enhanced.

In the infinite momentum frame half of the 144 currents give vanishing matrix elements between single particle states and these are therefore called "bad"[10)]. Thus we are left with only the following 72 "good"[10)] operators whose matrix elements do not vanish:

$$V^i_0 \sim V^i_z, \quad A^i_z \sim A^i_0, \quad T^i_{yz} \sim T^i_{0x}, \quad T^i_{zx} \sim T^i_{0y} \qquad (3\text{-}7)$$

where the notation $\sim$ means that between single particle states the two currents give identical matrix elements.

Out of these 72 operators we choose $(V^i_0, A^i_z, T^i_{yz}, T^i_{zx})$ and call it "$SU(6)_{W,}$ currents".[11)] In the quark model the integrals over space of these currents in fact form the closed algebra which is isomorphic to the $SU(6)_W$ which has been discussed in Sect. 2-C. Let us write down the quark model expression for the currents have components only in those SU(2) x SU(2) representations which satisfy $\left|I_A - I_B\right| = 1/2$. If the $J_z = 1/2$ component of the nucleon is purely in the representation specified by $[(I_A, I_B)_{1/2}, L_z = 0]$ where the isodoublet $\left|I_A - I_B\right| = 1/2$ corresponds to the nucleon N whereas the iso-multiplet $\left(\left|I_A - I_B\right| + 1\right) = 3/2 \leq (I_A + I_B)$ to $N^*(1236)$, we immediately find

$$G_A = \frac{2}{3}(k + 1) \tag{4-2}$$

$$G^* = \frac{1}{3}\sqrt{(2k - 1)(2k + 5)} \tag{4-3}$$

$$G_i^{**} = 0 \tag{4-4}$$

where $k = I_A + I_B$, G_A is the axial vector coupling constant in nucleon decay, G^* is the matrix element of the axial charge between N and $N^*(1236)$ normalized so that the Adler-Weisberger sum rule reads

$$1 = G_A^2 - (G^*)^2 + \text{higher contributions.} \tag{4-5}$$

G_i^{**} is the matrix element of F^5 between N and any I = 1/2 resonance N_i^*.

Now let us consider the electric dipole operator

$$D_i = \int x\, V_0^i(x)\, d^3x \tag{4-6}$$

whose matrix element between two nucleons at $p = \infty$ is equal to the anomalous magnetic moment. Since $D_i^{\pm} = \int \frac{1}{2}(x \pm iy)\, V_0^i\, d^3x$ transform as $[(3, 1)_0 + (1, 3)_0, \pm 1]$ we immediately find

$$\mu_A(p) = \mu_A(n) = 0, \quad \mu^* = 0 \tag{4-7}$$

where $\mu_A(N)$ is the anomalous magnetic moment of the nucleon N and μ^* is the matrix element for the magnetic M1 transition between N and $N^*(1236)$.

In Sect. 3-C we have seen that the particle states cannot be classified into an irreducible representation of the algebra of currents. From the results just obtained we find that even at the phenomenological level no irreducible representation can give a satisfactory description of the

$$
\begin{aligned}
F_i &= \int V_0^i \, d^3x = \int q^+(x)\frac{\lambda_i}{2} q(x) \, d^3x, \\
F_i^1 &= \int T_{yz}^i \, d^3x = \int q^+(x) \frac{\lambda_i}{2}\beta\sigma_x \, q(x) \, d^3x \\
F_i^2 &= \int T_{zx}^i \, d^3x = \int q^+(x) \frac{\lambda_i}{2}\beta\sigma_y \, q(x) \, d^3x, \qquad (3\text{-}8) \\
F_i^3 &= \int A_z^i \, d^3x = \int q^+(x) \frac{\lambda_i}{2} \sigma_z \, q(x) \, d^3x.
\end{aligned}
$$

As was emphasized above, the algebra of currents has nothing to do with the symmetry of the hamiltonian and the operators need not be conserved even approximately. On the other hand the ASA which we discussed before can only be introduced into particle physics when we have the approximate invariance of the hamiltonian under the algebra and the algebraic operators are given by approximately conserved quantum numbers. The $SU(6)_W$ as ASA is called "$SU(6)_{W,\ \text{strong}}$".[11)]

Still it is a nontrivial question to ask "can we think of a symmetric world under the algebra of currents ?". This is the topic we shall discuss next.

3-C. Intrinsically broken algebra of currents

Here we start to remind ourselves of the Coleman's theorems[12)] about the axiomatic consequences of assuming the algebra of local currents as a good symmetry of the hamiltonian.

Theorem I. If we have $\int_{x_0 = 0} j_0(x) \, d^3x \, |0\rangle = 0$, then the current must be conserved: $\partial_\alpha j_\alpha(x) = 0$.

Theorem II. If we have $\int_{x_0 = 0} j_\alpha(x) \, d^3x \, |0\rangle = 0$ for a certain $\alpha \neq 0$. then the current $j_\alpha(x)$ is rotation-free, $\partial_\alpha j_\beta(x) = \partial_\beta j_\alpha(x)$ and thus $\int_{x_0 = 0} j_\alpha(x) \, d^3x = 0$ for all $\alpha \neq 0$.

Theorem III. If we have $\int_{x_0 = 0} j_\alpha(x) \, d^3x \, | \, 0\rangle = 0$ for all α, then $j_\alpha(x)$ must vanish identically: $j_\alpha(x) = 0$.

For the detailed proof the original papers by S. Coleman[12] and S. Okubo[13] should be referred to, where it was shown by Coleman that if the algebra of local currents is a good symmetry a la Dashen and Gell -Mann[7], then it must actually be an exact symmetry. Okubo further proved that the exact symmetry under the algebra of currents is in reality impossible and hence that the group must be regarded as a broken symmetry at best. We only quote here the main assumptions involved in the proof: (i) $j_\alpha(x)$ is a local operator, (ii) there is no zero-mass particle in the Hilbert-space, which implies that the vacuum is non-degenerate, and (iii) the FederbushJohnson theorem[14] is valid. The last theorem tells you that under general assumptions of axiomatic field theory it can be shown that if the one-body Green's function equals its free-field value, the theory is that of a free field. From this theorem one can conclude that, if one has a local source of a field satisfying $j(x)\,|0\rangle = 0$, then $j(x) = 0$ identically.

Suppose that $SU(6)_{W,\ \text{currents}}$ is the exact symmetry of the hamiltonian. Then the particle states (i.e., the eigenstates) of the hamiltonian transform irreducibly under the algebra and the vacuum must be invariant. (Note that the vacuum is non-degenerate.) Thus we have in particular $\int A_z^i(x)\, d^3x \mid 0 > = 0$. Applying Theorem II to the axial current we get $\int A_z^i(x)\, d^3x = 0$. Therefore we are led to internal inconsistencies of the symmetry. Hence it must be intrinsically broken and the particle states can only be classified under mixed representations of the algebra. Similarly any other kind of algebras of currents including at least a space component of the vector or axial vector current must be intrinsically broken.

IV. Representation of chiral SU(3) x SU(3)

In this chapter we shall look for a representation of the chiral SU(3) x SU(3)[15]. Since in the infinite moemntum frame we have $F_i^3 \sim F_i^5$ for the matrix elements between single particle states, we may identify the chiral algebra with the subgroups SU(3) x SU(3) of $SU(6)_W$, currents which is composed of the elements F_i and F_i^3. The irreducible representations of SU(3) x SU(3) can be characterized by pairs of SU(3) multiplets $(A, B)_{S_z}$ where A is the SU(3) representation under $F_i + F_i^5$ (or $F_i + F_i^3$), B the representation under $F_i - F_i^5$ (or $F_i - F_i^3$) and S_z the eigenvalue of F_0^5, the singlet axial vector charge, which is naively the current quark spin component along the z direction since $F_0^5 \sim F_0^3 = \int q^+(x) \frac{\sigma_z}{2} q(x)\, d^3x$.

In general $S_z \neq J_z$, where J_z is the ordinary total angular momentum (spin) of the state. We therefore define an additional quantum number, the z component of the "internal orbital angular momentum" L_z satisfying[16)]

$$L_z = J_z - S_z. \tag{4-1}$$

In the quark model S_z and L_z can be regarded as the quantities explained above. We emphasize, however, that S_z, L_z and J_z are perfectly well defined regardless of the existence or relevance of quarks. Both S_z and L_z are separately conserved in all matrix elements and the states in any given SU(3) x SU(3) representation may have any common integral value of L_z.

We shall first work with the chiral SU(2) x SU(2) algebra which is simple but sufficient to treat the nonstrange baryons. In SU(2) x SU(2) the characterization of its representations is given by isospin multiplets $(I_A, I_B)_{S_z}$. Since the nucleon is an I = 1/2 state, it can experimental situation, particularly in view of the predicted vanishing of all the magnetic transitions. To improve the situation we have to have a mixed representation for the nucleon. For the existence of non-vanishing matrix elements for the dipole operator D_i and for predicting non-zero anomalous magnetic moments $\mu_A = 0$ as well as $\mu^* = 0$ the nucleon must have components in representations with both $L_z = 0$ and $L_z = \pm 1$. A possible candidate for the mixed representation is given by[17)]

$$| N^*(1236), J_z = \frac{1}{2} > = | (1, \frac{1}{2})_{1/2}, 0 >, \tag{4-8}$$

$$| N, J_z = \frac{1}{2} = \cos\theta | (1, \frac{1}{2})_{1/2}, 0 > + \sin\theta \{ \sin\phi | (0, \frac{1}{2})_{1/2}, 0> + \cos\phi [\cos\psi \; |(\frac{1}{2}, 0)\; 3/2, -1> + \sin\psi \; |(\frac{1}{2}, 0)_{1/2}, 0] \} \tag{4-9}$$

Then the experimental quantities are calculated to be

$$G^* = \frac{4}{3}\cos\theta \tag{4-10}$$

$$G_A = \frac{5}{3}\cos^2\theta + \sin^2\theta \cos 2\phi \tag{4-11}$$

$$\mu^* = M_A^1 \sin\theta \cos\phi \sin\psi \tag{4-12}$$

$$\mu_A(p) + \mu_A(n) = M_B^0 \sin^2\theta \;\; \sin 2\phi \;\; \sin\psi \tag{4-13}$$

$$\mu^* - \frac{\mu_A(p) - \mu_A(n)}{\sqrt{2}\cos\theta} = M_B^1 \sin^2\theta \;\; \sin 2\phi \;\; \sin\psi \; . \tag{4-14}$$

M_A^1 and $M_B^{1(0)}$ are the iso-vector (-scalar) reduced matrix elements for the transitions

$$[(1, \frac{1}{2})_{1/2}, 0] \xleftrightarrow{M_A} [(0, \frac{1}{2})_{1/2}, -1] \; , \tag{4-15}$$

$$[(0, \frac{1}{2})_{1/2}, 0] \xleftrightarrow{M_B} [(0, \frac{1}{2})_{1/2}, -1] \tag{4-16}$$

As we see from the results, we have five unknowns for the five observables through the five relations. We only have two consistency conditions

$$G^* \leq \frac{4}{3}, \tag{4-17}$$

$$\frac{8}{3}(G_A - 1) \leq G^{*2} \leq \frac{2}{3}(G_A + 1). \tag{4-18}$$

Thus we have much less predictive power once we make representations mixed, unless we find some systematics of how to mix various representations.

The above scheme can be generalized to the SU(3) baryon octet:

$$|\underline{10}, J_z = \frac{1}{2}> = |(6, 3)_{1/2}, 0> , \tag{4-19}$$

$$|\underline{8}, J_z = \frac{1}{2}> = \cos\theta\, |(6,3)_{1/2}, 0> + \sin\theta\{\sin\phi\, |(3^*, 3)_{1/2}, 0>$$

$$+ \cos\phi\, [\cos\psi\, |(8, 1)_{3/2}, -1> + \sin\psi |(3,3^*)_{1/2}, 1>]\} \tag{4-20}$$

In addition to the results obtained above in the SU(2) x SU(2) analysis we now have the expression for the F/D ratio of the axial vector transitions between members of the baryon octet:

$$a = \frac{D}{D+F} = \frac{1 + \tan^2\theta\ (\cos^2\phi\, \cos^2\psi - \sin^2\phi)}{\frac{5}{3} + \tan^2\theta\ (\cos^2\phi - \sin^2\phi)} \tag{4-21}$$

Again we get another consistency condition

$$\frac{1}{2G_A}(G_A + \frac{3}{4}G^{*2} - 1) \leq \alpha \leq 1 - \frac{3G^{*2}}{8G_A} \tag{4-22}$$

Using $G_A = 1.18$ we obtain from eq. (4-18)

$$0.7 \leq G^* \leq 1.2 \tag{4-23}$$

The experimental values for G^* are around 0.8 - 1.05 with some uncertainties. For $G_A = 1.18$, $G^* = 0.8$ eq. (4-22) predicts

$$0.28 \leq \alpha \leq 0.79 \tag{4-24}$$

and for $G^* = 1.05$ it gives

$$0.43 \leq \alpha \leq 0.66 \quad . \tag{4-25}$$

This is consistent with the phenomenological value $\alpha = 0.665$.

So far we have reviewed very briefly on the theoretical development in the study of hadron symmetry and quarks up to about 1967. For the last several years people[18] have been trying to apply the method to various cases without much progress from theoretical side, particularly in its systematic treatment of representations of the algebra. However, quite recently there has been some very rapid and important progress which seems to give us an idea of what the theory for hadron symmetry and quarks must be, something which is simple in its algebraic properties, systematics in treating all mesons and baryons in a unified way and definite in that the theory has a clear origin and structure. The main break-through for the progress was given by Melosh[19] who worked on the problem in the free quark model and formed a unitary transformation which relates the ASA of $SU(6)_{W,\ strong}$ with the $SU(6)_{W,\ current}$ algebra, thus gave a theoretical basis for studying the problem of representation mixing in a systematic way.

V. Transformation from current to constituent quarks

5-A. Current and constituent quarks

When we talk about $SU(6)_{W,\ strong}$, we use a simple model to obtain elementary results about the low-lying bound and resonant states of mesons and baryons and certain crude symmetry properties of these states, by saying that the hadrons act as if they were made up of subunits, the "constituent quarks"[11]. The quark model we have discussed in Chapter II is based on the constituent quarks.

There is another kind of basic fields which appear as bilinear products in the quark currents of $SU(6)_{W,\ current}$. These fundamental fields are called the "current quark fields"[11].

From what we have discussed in the previous chapters you may have understood why the two algebras, $SU(6)_{W,\ strong}$ and $SU(6)_{W,\ currents}$, cannot be directly identified with each other. If we do so, we are immediately led to poor predictions and some internal inconsistencies. We note, however, that the two symmetry algebras are complementary; $SU(6)_{W,\ strong}$ seems to be useful in classifying the hadrons into its irreducible representations, while the $SU(6)_{W,\ currents}$ algebra is composed partly of directly measurable components (vector and axial vector currents). The first algebra tells us about the symmetry of the hadron spectrum in nature, whereas the second provides us information about the vertex symmetry of hadrons in decay and scattering processes.

5-B. The Melosh transformation.

Even if the two algebras cannot be directly identified with each other, there still might be a unitary transformation V, which relates them:

$$[SU(6)_{W,\ strong}] = V[SU(6)_{W,\ currents}]\ V^{-1} \tag{5-1}$$

In the last year Melosh[19)] found in the free quark model that there is in fact a unitary transformation which connects the two algebras exactly as given by eq.(5-1)

Let us review briefly on the Melosh transformation. Consider a hamiltonian for the free quark model:

$$H_{free} = \int q^{+}(x)\ (-i\underset{\sim}{\alpha}\cdot\underset{\sim}{\partial} + m_o\beta)\ q(x)\ d^3x \tag{5-2}$$

where $q^i(x)$ (= (u(x), d(x), s(x))) is the local quark field obeying the free Dirac equation and the canonical commutation relations

$$\{q^i(x),\ q^j_\beta(y)^+\}_{x_0=y_0} = \delta_{ij}\ \delta_{\alpha\beta}\delta^3(\underset{\sim}{x}-\underset{\sim}{y}) \tag{5-3}$$

$SU(6)_{W,\ \text{currents}}$ is composed of the following local currents:

$$F_i = \int q^+(x)\,\frac{\lambda_i}{2}\,q(x)\,d^3x,$$

$$F_i^1 = \int q^+(x)\,\frac{\lambda_i}{2}\,\beta\sigma_x\,q(x)\,d^3x,$$

$$F_i^2 = \int q^+(x)\,\frac{\lambda_i}{2}\,\beta\sigma_y\,q(x)\,d^3x,$$

$$F_i^3 = \int q^+(x)\,\frac{\lambda_i}{2}\,\sigma_z\,q(x)\,d^3x. \tag{5-4}$$

We notice that the $SU(6)_{W,\ \text{currents}}$ algebra is not necessarily a good symmetry of the hamiltonian:

$$[H_{\text{free}}, F_i] = 0 \tag{5-5}$$

but

$$[H_{\text{free}}, F_i^k] = 0,\ k = 1, 2, 3. \tag{5-6}$$

Our problem here is to find a good symmetry of the hamiltonian which is isomorphic to $SU(6)_{W,\ \text{currents}}$, i.e. $SU(6)_{W,\ \text{strong}}$ as ASA which is composed of the operators W_i and W_i^k such that

$$W_i = VF_iV^{-1},\quad W_i^k = VF_i^kV^{-1},\quad k = 1, 2, 3$$

with

$$[H_{\text{free}}, W_i] = 0 \tag{5-7}$$

and*

$$[H_{\text{free}}, W_i^k] = 0. \tag{5-8}$$

There are some properties required for V. The SU(3) symmetry is generated by F_i and we want to keep the same SU(3) symmetry properties through the transformation. Thus V must be an SU(3) singlet: $[F_i, V] = 0$, i.e. $W_i = F_i$. Further we want to preserve the space-time and charge conjugation properties of the current operators, so V must have P = C = +1. Also V must be compatible with the collinearity along the z axis: $[J^3, V] = 0$.

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* It turns out that the $SU(6)_{W,\ \text{strong}}$ symmetry becomes exact rather than approximate in the free quark model. So we actually have $[H_{\text{free}}, W_i^k] = 0$.

We do not want to destroy the "good" properties of the current operators through the V transformation either. And, of course, V must be unitary.

Let us write a unitary operator V_{free} as

$$V_{free} = \exp(iY_{free}) \tag{5-9}$$

where Y_{free} is a hermitian operator. We shall choose

$$Y_{free} = \frac{1}{2} \int q^{+}(x) \tan^{-1}\left(\frac{i\underset{\sim}{\gamma}_{\perp} \underset{\sim}{\partial}_{\perp}}{m_0} \right) q(x)\, d^3x \tag{5-10}$$

Since this transformation is nonlocal due to the derivatives of infinite order, operators transformed by V_{free} are also nonlocal. Let us define

$$q'(x) = V_{free}\, q(x)\, V_{free}^{-1} = \exp(-iA/2)\, q(x) \tag{5-11}$$

where

$$\tan A = \frac{i\underset{\sim}{\gamma}_{\perp} \underset{\sim}{\partial}_{\perp}}{m_0} \tag{5-12}$$

Then we can rewrite H_{free} in terms of $q'(x)$:

$$H_{free} = \int q'(x)^{+} \left(-i\alpha_z \partial_z + m_0 \sqrt{1 + \left(\frac{i\underset{\sim}{\gamma}_{\perp} \underset{\sim}{\partial}_{\perp}}{m_0}\right)^2} \beta \right) q'(x)\, d^3x \tag{5-13}$$

where $(i\underset{\sim}{\gamma}_{\perp} \underset{\sim}{\partial}_{\perp})^2 = -\underset{\sim}{\partial}_{\perp}^2 = p_{\perp}^2$. From this expression of H_{free} it is apparent that the following W_i, W_i^k defined by

$$W_i = V_{free} F_i\, V_{free}^{-1} = \int q'(x) \frac{\lambda_i}{2} q'(x)\, d^3x = F_i \tag{5-14}$$

$$W_i^k = V_{free} F_i^k\, V_{free}^{-1} = \int q'(x) \frac{\lambda_i}{2} w^k\, q'(x)\, d^3x$$

with

$$w^{1,\,2} = \beta\sigma_{x,\,y}/2, \quad w^3 = \sigma_z/2 \tag{5-15}$$

are commutable with H_{free}, since $q'(x)$ obeys the same canonical commutation relations as $q(x)$. So we have

$$[H_{free}, W_i] = H_{free}, W_i^k = 0. \tag{5-16}$$

In terms of the $q(x)$ fields W_i and W_i^k are written as

$$W_i = F_i \tag{5-17}$$

$$W_i^1 = F_i^1 - \int q^+(x)\frac{1}{\kappa}\left(\frac{1}{1+\kappa}\frac{i\gamma_\perp\partial_\perp}{m_0} - i\right)\gamma_5\frac{\partial^1}{m_0}\frac{\lambda_i}{2}q(x)\, d^3x \tag{5-18}$$

$$W_i^2 = F_i^2 - \int q^+(x)\frac{1}{\kappa}\left(\frac{1}{1+\kappa}\frac{i\gamma_\perp\partial_\perp}{m_0} - i\right)\gamma_5\frac{\partial^2}{m_0}\frac{\lambda_i}{2}q(x)\, d^3x \tag{5-19}$$

$$W_i^3 = F_i^3 + \int q^+(x)\frac{1}{\kappa}\left(\frac{1}{1+\kappa}\frac{i\gamma_\perp\partial_\perp}{m_0} - i\right)\sigma_z\frac{i\gamma_\perp\partial_\perp}{m_0}\frac{\lambda_i}{2}q(x)\, d^3x \tag{5-20}$$

where

$$\kappa = \sqrt{1+\left(\frac{i\gamma_\perp\partial_\perp}{m_0}\right)^2} = \sqrt{1-\left(\frac{\partial_\perp}{m_0}\right)^2} \tag{5-21}$$

Annihilation of the vacuum: Since $SU(6)_{W,\ strong}$ is exact in the present model, we must have

$$W_i \mid 0 > = 0, \quad W_i^k \mid 0 > = 0. \tag{5-22}$$

Proof: The vacuum is the non-degenerate ground state of H_{free}: $H_{free} \mid 0 > = 0$. Since $[H_{free}, W_i] = [H_{free}, W_i^k] = 0$, we immediately obtain $H_{free} W_i \mid 0 > = H_{free} W_i^k \mid 0 > = 0$. From the non-degeneracy of the vacuum eq. (5-22) follows. The identities can be directly confirmed if the operators W_i and W_i^k are written in terms of the Fourier transformed creation and annihilation operators of $q(x)$.

Nonlocality: To study the nonlocality of the constituent quark fields $q'(x)$, we rewrite eq. (5-11) in terms of the kernel $K_{free}(x)$:

$$'q(x) = V_{free}\, q(x) V_{free}^{-1} = \int K_{free}(\underset{\sim}{x} - \underset{\sim}{y})\, q(x)\, d^3y \tag{5-23}$$

where

$$K_{free}(x) = \frac{\delta(z)}{(2\pi)^3}\int e^{i\, \underset{\sim}{p}_\perp \underset{\sim}{x}_\perp}\left(\frac{\omega + m_0 + i\, \underset{\sim}{p}_\perp \underset{\sim}{\partial}_\perp}{\sqrt{2\omega\,(\omega + m_0)}}\right) \tag{5-24}$$

and $\omega = \sqrt{p_\perp^2 + m_0^2}$. Since the integrand approaches unity only for $|p_\perp| \ll m_0$, we expect $K_{free}(x)$ to receive contributions from the region $|\underset{\sim}{x}_\perp| \lesssim 1/m_0$, i.e. from a distance comparable to the Compton wavelength of a quark. Although $q'(x)$ is nonlocal in the transverse directions, the microcausality is of course preserved.

The Melosh transformation is only explicit in the free quark model. It is not guaranteed that we may also be able to find a similar transformation in a more realistic interacting quark model. We could be optimistic, however, saying that the abstracted results from what we can work out explicitly in the free quark model may still turn out to be close to the true solution.

From this point of view we shall discuss a little more about the usage of the transformation and some of its applications.

5-C. Usage of the unitary transformation.

In this section we review the general usage of the unitary transformation V in evaluating the matrix elements of observables and also present some illustrative examples based on the Melosh transformation V_{free}.

In the approximate symmetric world physical single particle states are classified in irreducible representations of $SU(6)_{W,\ strong}$ in good approximation. Consider a physical vacuum $|0>$ and a physical single particle state $|A_\alpha> = a_s^+(A_\alpha)|\ 0>$. Then we have in good approximation

$$[W_i^k\ (\text{or } W_i),\ a_s(A_\alpha)] = \sum_\beta C_k^k(A)_{\alpha\beta}(\text{or } C_i(A)_{\alpha\beta})\ a_s(A_\beta) \tag{5-25}$$

and

$$W_i|\ 0> = W_i^k\ |0> = 0. \tag{5-26}$$

The second equalities imply also

$$F_i\ V^{-1}\ 0\ = F_i^k\ V^{-1}\ 0\ = 0. \tag{5-27}$$

From the first algebraic equality we see that $a_c(A_\alpha) = V^{-1}a_s(A_\alpha)\ V$

is in the same representation under $SU(6)_{W,\ currents}$. From the third equalities above we immediately find that the transformed state $|A_\alpha\rangle_c = a_c^+(A_\alpha)\ V^{-1}\ |0\rangle = V^{-1}\ a_s^+(A_\alpha)\ |\ 0\rangle$ transforms irreducibly under $SU(6)_{W,\ currents}$. These observations allow us to consistently formulate a systematic method for evaluating the matrix elements of observables in the current quark picture.

Suppose we want to evaluate the matrix element of an observable between physical single particle states $|A_\alpha\rangle_s$ and $|B_\beta\rangle_s$ which transform irreducibly under $SU(6)_{W,\ strong}$. Then we have

$$ {}_s\langle A_\alpha\ |\mathcal{O}\ |B_\beta\ \rangle_s = {}_c\langle\ A_\alpha\ |\ V^{-1}\mathcal{O}\ V\ |\ B_\beta\rangle_c \tag{5-29}$$

where $|A_\alpha\ \rangle_c$ and $|\ B_\beta\ \rangle_c$ transform irreducibly under $SU(6)_{W\ currents}$. If the transformed operator $V^{-1}\mathcal{O}V$ is given in terms of the current quarks or its properties are well known in the current quark picture, then the above matrix element can be calculated consistently only in the current quark picture.

As an example let us evaluate the matrix element of the axial vector current between the nucleon states based on the Melosh transformation. The matrix element concerned here is

$$ {}_s\langle\ \text{nucleon}\ |\ F_i^3\ |\ \text{nucleon}\ \rangle_s = {}_c\langle\ \text{nucleon}\ |\ V_{free}^{-1} F_i^3\ V_{free}|\ \text{nucleon}\ \rangle_c $$

$V_{free}^{-1}\ F_i^3\ V_{free}$ is evaluated to be

$$ V_{free}^{-1}\ F_i^3\ V_{free} = \int\ q^+(x)\ \frac{1}{k}(1+\ \frac{\gamma_\perp\partial_\perp}{m_0})\ \frac{\sigma_z}{2}\ \frac{\lambda_i}{2}\ q(x)\ d^3x \tag{5-30}$$

From this we note that $V_{free}^{-1}\ F_i^3\ V_{free}$ transforms as a sum of an $(\underline{8},\ \underline{1})_0 - (\underline{1},\ \underline{8})_0$ term and a $(\underline{3},\ \underline{3}^*)_1 - (\underline{3}^*,\ \underline{3})_{-1}$ term under the algebra of currents. In particular it should be mentioned that the first term has the orbital helicity $L_z = 0$ whereas the second term $L_z = \pm 1$. Of course the total helicity J_z must be zero, $J_z = 0$. Now the nucleon N and its excited state $N^*(1236)$ are classified under

$$|N \text{ or } N^*(1236),\ J_z = \frac{1}{2}\rangle_c = |(6,3)_{1/2},\ L_z = 0\rangle_c \qquad (5\text{-}31)$$

Because of the selection rule with respect to L_z the second term in $V^{-1}_{free} F^3_i V_{free}$ does not contribute to the matrix element concerned here. So we easily obtain

$$G_A = \eta\,\frac{5}{3}, \quad G^* = \eta\frac{4}{3}, \quad (F/D)_{axial} = \frac{2}{3} \qquad (5\text{-}32)$$

where

$$\eta = \left\langle \frac{1}{K} \right\rangle = \left\langle \frac{1}{\sqrt{1 + \frac{p_\perp^2}{m_0^2}}} \right\rangle \qquad (5\text{-}33)$$

which is a dynamical correction factor due to the relative motion between the quarks in the transverse directions. Taking $\langle p_\perp^2 / m_0^2 \rangle \sim 1$ the experimental number for G_A is well reproduced: $G_A = 5/3\sqrt{2} \sim 1.18$.

Melosh generalized the formalism onto the light-like plane and evaluated also the magnetic moment ratio between the proton and neutron which remarkably gives

$$\frac{\mu_T(p)}{\mu_T(n)} = -\frac{3}{2} \ . \qquad (5\text{-}34)$$

The M 1 transition moment for $N \to N^*(1236)$ was also calculated to be

$$\mu^* = \frac{2\sqrt{2}}{3}\,\mu_T(p) \qquad (5\text{-}35)$$

The interested reader is recommended to refer to the original literature[19) 20)] as to the light-like plane formalism and its advantage.

VI. Summary on recent works

In this chapter let us give a brief summary on recent works based on the Melosh transformation. Some of them are the direct applications of the transformation and the others are based on its abstracted properties.

De Alvis[20] applied the transformation to the bilocal operators which appear in light cone commutators in order to obtain the implications of the $SU(6)_W$ for the electro and neutrino production structure functions in the scaling region. For this purpose he casted the problem into the light plane language and obtained the following results. For the spin independent structure functions $F_2^{ep,n}$, $F_2^{p,n}$ on protons and neutrons

$$F_2^{ep}(\xi) = \frac{1}{6}(F_2^{\nu p}(\xi) + F_2^{\nu n}(\xi)) . \tag{6-1}$$

This is a consequence of pure f coupling for the vector bilocal. For the spin dependent functions $G_1^{ep,n}$ for electroproduction

$$G_1^{ep}(\xi) = \frac{7}{2} G_1^{en}(\xi) . \tag{6-2}$$

The above results are based on the transformation of the free quark model. If one allows for the curl free neutral vector gluon binding of the quarks, then neither eq. (6-1) nor eq. (6-2) is valid. The only result which survives is the integrated version of eq. (6-2), namely the Bjorken's sum rule.

$$\int_0^1 G_1^{ep}(\xi)\, d\xi = \eta \frac{7}{9} , \tag{6-3}$$

$$\int_0^1 G_1^{en}(\xi)\, d\xi = \eta \frac{2}{9}$$

where $\eta = G_A/(5/3)$ as defined by Melosh.

Gilman and Kugler[21] considered the pionic transitions of mesons and baryons using PCAC and the stronger forms of the algebraic structure for the vector and axial vector charges than those suggested by Melosh. They assumed that the transformed axial charge $V^{-1} F_i^5 V$ is a linear combination of F_i^5 itself and an operator K_i which transforms as $(1/2, 1/2)_1 - (1/2, 1/2)_{-1}$ under SU(2) x S(2): $V^{-1}F_i^5 V = \cos\alpha\ F_i^5 + \sin\alpha\ K_i$. The commutator $[K_i, F_i^5] = i\delta_{kj} S$ gives an additional scalar operator S. Then it can be found by the Jacobi identity that the algebra of F_i, F_i^5, K_i and S closes on that of Sp(4) or O(5). Based on this closed algebra they evaluated the pionic transition amplitudes. In particular they successfully obtained the results of a pure transverse decay for $B \to \omega\pi$ and a dominant longitudinal decay for $A_1 \to \rho\pi$. These decays are the famous difficulties of the usual $SU(6)_W \times O(3)$ scheme where the allowed transition is purely longitudinal (transverse) for $B \to \omega\pi$ ($A_1 \to \rho\pi$). As for baryons, the results are satisfactory except that any classification of the Roper resonance, $P_{11}(1470)$, as a radial excitation in the quark model results in its πN and πN^* (1236) decay modes being forbidden.

Hey and Weyers[22], on the other hand, only assumed the transformation properties (under the algebra of currents) of $V^{-1} F_i^5$ suggested by the Melosh transformation: $V^{-1}F_i^5 V$ transforms as a sum of an $(8, 1)_0 - (1, 8)_0$ term and a $(3, 3^*)_1 - (3^*, 3)_{-1}$ term under SU(3) x SU(3). Using PCAC they obtained predictions for pionic decays. They attempted to classify the Roper resonance together with a decimet partner N_R^* in a pure $[(6, 3)_{1/2}, 0]$ representation of $SU(3) \times SU(3)_{strong}$, which leads to allowed decays into $N\pi$ of the Roper multiplet as is trivially expected.

Gilman, Kugler and Meshkov[23] again based their argument on the above-mentioned transformation properties of $V^{-1} F_i^5 V$. It is not assumed that the $(8, 1)_0 - (1, 8)_0$ piece of $V^{-1} F_i^5 V$ is proportional to F_i^5 as in the work of Gilman and Kugler. However, unlike Refs. 21 and 22, they made a stronger assumption by employing $SU(6)_W$ to relate states with different values of the quark spin. They presented a systematic study of pionic transitions of mesons and baryons.

Hey, Rosner and Weyers[24] investigated in detail the interrelation between the present approach based on the Melosh transformation and the previous approach, the phenomenological "$^3P_0$" model of Micu, Colglazier, Petersen and Rosner[25], and found that for any hadronic pion decay the two are equivalent. On the other hand, the phenomenological prescription known as "ℓ broken $SU(6)_W$"[26] was found to be equivalent to the Melosh approach for many cases of physical interests.

There is a work by Oehme[27] which attempts to explain the suppression of $|\Delta S| = 1$ amplitudes in weak processes within the present framework of collinear $SU(6)_W$ algebras. This suppression originates in the mass breaking term in the quark hamiltonian: $H' = -u_0 - cu_8$[1]. It is claimed that the Cabibbo angle can be expresses as $\tan \theta = \cos (\pi c/2\sqrt{2})$. I am not quite sure whether this explains the whole mystery of the successful Cabibbo angle.

A paper by Fuchs[28] tried to anlyze the SU(3) symmetry breaking due to the quark mass differences. But I did not understand the content well. (I actually suspect that there exists no transformation of the type claimed in the paper unless you introduce uncontrolable singular behaviours in the transverse directions into the generalized unitary transformation.)

For details the reader should refer to the original papers as well as those other works which I did not happen to mention.

VII. Conclusion

We have reviewed quickly the developments on hadron symmetry and quarks for the last ten years or so. We have focused our arguments only on those topics associated with the currents. There has been developed another approach[29] toward the problem: the S matrix approach for hadron symmetry based on the concepts of boostrap and duality. It is quite possible that in the theory for hadron symmetry the quark model and the bootstrap will play a complementary role with each other.

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