HYBRID MAGNETS

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Abstract

Neither those magnets employing superconductors alone nor those employing resistive conductors alone are capable of the most economical generation of continuous magnetic fields above approximately 20 T. A so-called "hybrid system," which consists of a large-bore superconducting magnet surrounding a small-bore, high-current-density resistive magnet, however, can generate fields above those possible with a purely superconducting magnet, and at a substantial saving in conductor volume and/or power over that required by a purely resistive magnet.

This power consumption has been computed by an extraordinarily accurate, yet remarkably simple, equation which acknowledges all the major causes that make a high-field magnet less efficient than a low-field one: decreased space factor to preserve adequate cooling, increased magnet temperature, and increased conductor resistivity associated with the stronger conductors needed to withstand the greater magnetic stresses. The formula computes the field generated by a coil of specified bore consuming a specified amount of power, assuming that the fraction of conductor removed to provide cooling and the temperature rise in the coil each are proportional to the square root of the power to be consumed, and that the 20°C resistivity of the conductor is either that of copper or else proportional to the maximum stress in the coil, approximated by the product of the inner radius, the current density there, and the total field there, including the contribution of the coil itself.

The wire cost of the superconducting magnet required to bring the total field up to its prespecified value is computed assuming tape-like conductors carrying 1,000 A and cryostatically stabilized at a heat flux of 0.5 W/cm^2. The cost per meter of this conductor has been estimated as $\$/m = \$_1 V_{Cu} + \$_2 V_{SC} + \$_3$, where V_{Cu} and V_{SC} are the volumes of copper and superconductor in a meter of conductor, and the coefficients are calibrated from existing conductors.

The paper concludes with a review of the three hybrid systems now being tested at Oxford University (16 T, 2 MW), M. I. T. (22 T, 5 MW), and McGill University (25 T, cryogenic insert), and a brief study of the proposed Nijmegen system (25 T, 6 MW). The fact that this system should require only $50,000 or so of wire, whereas a purely resistive magnet, even of massive dimensions, would require an additional 5 MW of power, attests to the value of the hybrid concept.

Introduction

Hybrid magnets--that is, magnets which combine superconductors and resistive conductors--are the very-high-continuous-field magnets of the future. This is a bold prediction, considering that there are at this moment only three hybrid systems in the entire world, and even these are only in the testing stage. Yet, barring any major breakthrough in very-high-field superconductor technology, the prediction should be a safe one. The following graphs, which indicate the difficulty of generating a really intense field with superconducting magnets alone or with resistive magnets alone, should explain why.

High-Field, Small-Bore Superconducting Magnets

With superconducting magnets, the difficulty in generating a very intense magnetic field arises from the limited current density which a superconductor can carry at fields approaching its upper critical field, as seen in Fig. 1 for the three commercial materials niobium-titanium, niobium-tin, and the best samples of vanadium-gallium manufactured by Dr. Tachikawa and tested by Iwasa[1] at the Francis Bitter National Magnet Laboratory.

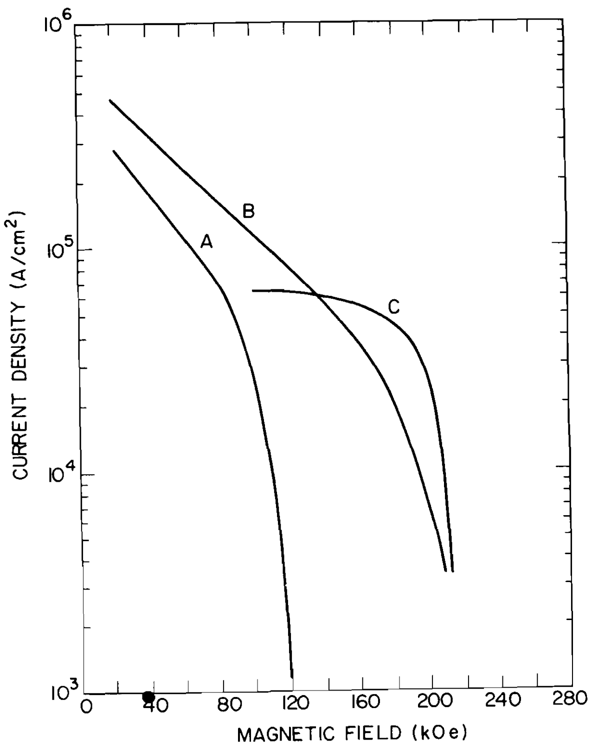


Fig. 1: Typical current-density-vs.-magnetic-field curves for small-diameter Nb-Ti wire, Nb_3Sn tape, and the best Va_3Ga tapes, as compiled by Y. Iwasa.

Note that Nb-Ti can carry only one-fifth the current density at 10 T as at 5 T, and above approximately 12 T can carry no current at all. Similarly, Nb_3Sn becomes useless at fields above approximately 22 T. In fact, at fields much above 15 T its current-carrying capacity already has become so low that a magnet of this material might be prohibitively expensive. Va_3Ga, despite an upper critical field actually slightly less than that of Nb_3Sn, then may become the more economical--depending crucially on its cost.

Of course the current density as limited by the maximum field seen by any part of the magnet need not be retained throughout. Instead the magnet may be subdivided into regions, in each one of which the current density may be the maximum permitted by the field seen by the region itself. Even so, however, the cost of a superconducting magnet rises very rapidly as the desired field approaches the upper critical field of the highest-field conductor used, as seen in Fig. 2.

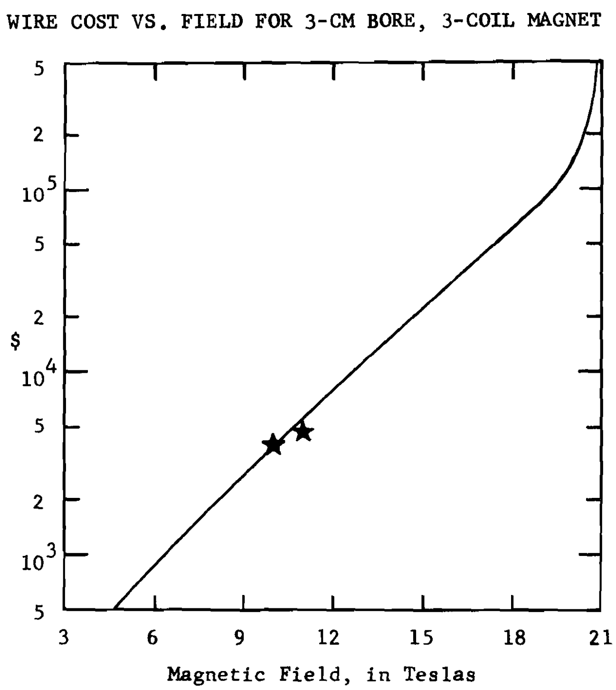


Fig. 2: Wire cost vs. field for magnets of 4 cm winding i.d. and 20 cm length wound with tape-like conductors of 1.27 cm width and carrying 1,000 A. The magnets consist of three concentric sections generating respectively 1/6 (innermost coil), 1/3, and 1/2 of the total field. Nb-Ti is used to 9 T, Nb_3Sn between 9 T and 15 T, and Va_3Ga above 15 T. The conductors are spaced axially by 1 mm and radially by 0.1 mm, and are partially stabilized (heat flux with copper carrying current = 1 W/cm^2). The wire cost per meter has been estimated by means of a formula calibrated from existing conductors. The stars indicate magnets available commercially from CSCC, the actual price being converted to wire cost by dividing by 2½.

Note that the wire cost for a magnet to generate 15 T is over five times that for a magnet to generate 10 T, and a magnet for 20 T is another factor of five over this. Thus we conclude that present-day superconductors are incapable of the economical generation of fields over 20 T.

<u>High-Field Bitter-Type Magnets</u>

If, on the other hand, we try to generate an intense field with resistive magnets alone, we discover a similar, though somewhat less severe, phenomenon of diminishing returns. Figure 3 graphs the power required to generate a specified magnetic field with a two-coil conventional-conductor magnet of modest size and 3 cm working bore, a typical diameter for high-field magnets.

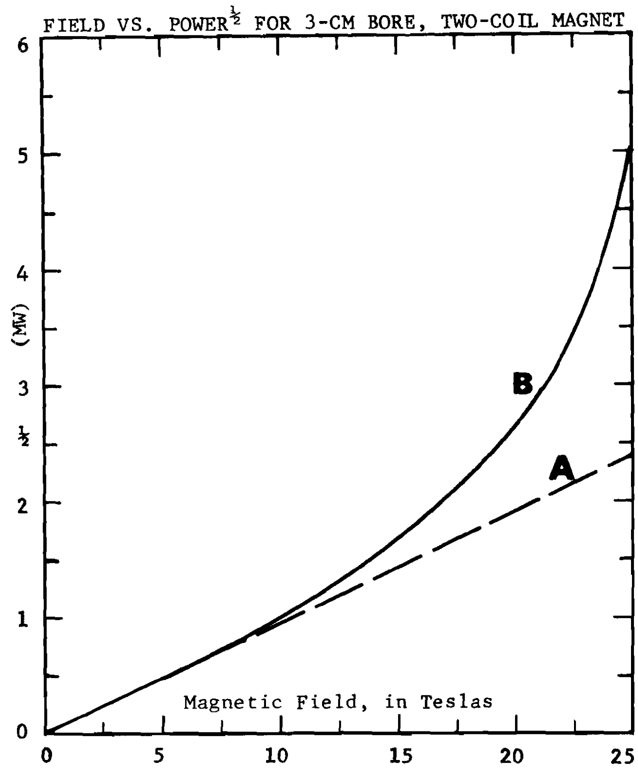


Fig. 3: Power consumption vs. field for 4-cm i.d. magnets consisting of two concentric Bitter coils of identical shape, each with an o.d./i.d. ratio of 3, the outer coil being three times the size of the inner. The inner and outer coils are modeled on those of the highest-field magnet at M. I. T., with the efficiency upgraded about 5% in order to achieve 10 T at 1 MW. With the maximum feasible amount of cooling (1/3 of conductor volume sacrificed to this end), the coils can dissipate 4 MW and 36 MW respectively at a temperature rise of 100° C. Curve A: power assuming high-field magnets can retain the efficiency of low-field magnets. Curve B: actual power consumption.

Curve A indicates the power that would be required if high-field magnets could retain the same efficiency as low-field magnets: the power then would be proportional to the square of the desired field.

At a field as low as 10 T, however, this assumption of constant efficiency already is somewhat unrealistic, and rapidly becoming much more so. First--and very important at all fields--in a magnet of fixed size, the higher the power, the higher the power density, and the greater must be the sacrifice of conductor volume to preserve adequate cooling. Second, the higher the power, the higher the magnet temperature--and therefore resistivity--because one can not afford to add sufficient cooling channels to limit the heat flux to the value present at low power. Third--and quite important at very high fields--the high-conductivity conductor--copper--which is adequate in strength for moderate-field magnets, would rupture under the much higher stresses induced by the Lorentz interaction between a high-field magnet's greater field and current density. Consequently either the copper must be replaced by a conductor strong enough to withstand these higher stresses itself, or else it must be reinforced with an even sturdier material, such as stainless steel or tungsten. A fourth source (related to the first two) of unavoidable inefficiency in the highest-field magnets arises because of the "burn-out heat flux" limit to the heat flux permissible across cooling surfaces, coupled with the impossibility of increasing indefinitely the available cooling surface area, while at the same time maintaining adequate structural strength. The consequence is that much of the power added to a magnet to raise its field must go into inefficient regions not yet carrying so much current as to be subject to this cooling limitation. When all of these factors have been taken into account, the result is curve B of Fig. 3. Note that if to generate 10 T requires 1 MW, then to generate 20 T requires not 4 MW but nearly 7 MW; and to generate 25 T with the limited conductor envelope assumed by Fig. 3 requires not 6 1/4 MW but approximately 25 MW.

Fortunately this forbidding amount of power can be reduced significantly by expanding the conductor envelope to truly massive proportions, for example by surrounding the two-coil magnet of Fig. 3 with a third coil, again identical in shape, but now nine times the size of the innermost coil. This coil can have little more than one-third the efficiency of the coil immediately inside it, however, and thus may require considerable power to raise by very much the field from the system.

The Hybrid Concept

Combining superconductors with conventional conductors, however, can circumvent the limitations on each component used separately, and thus has great potential, as recognized by Wood and Montgomery[2,3,4] as early as 1965. By restricting the use of the superconductor to regions in which the ambient field does not drastically limit its current density, one need employ only a modest amount to replace a resistive coil which because of its large bore generates relatively little field, considering the power that it consumes. We will show that this new type of system--the hybrid system--is able to generate very intense fields with significantly less power than with a purely resistive magnet, and significantly less superconductor than with a purely superconducting magnet--if indeed a superconducting magnet would be capable of the field at all. For example, a total field of 20 T can be generated with only 2 1/4 MW, a factor of three less than for a purely resistive magnet, and only approximately $30,000 of wire, a savings of a factor of four compared to a purely superconducting magnet. 25 T can be generated with 4 MW and $70,000 of wire, or 6 1/4 MW and $30,000 of wire. Were one to have generated this very high field with the resistive conductor magnet of Fig. 3 augmented by the huge third coil mentioned earlier, the power consumption would have been over 10 MW. With only the superconductors of Fig. 1, this field could not have been generated at any price.

Topics to be Covered

Let us shortly take a closer look at the theory governing the design of high-field resistive magnets, large-bore superconducting magnets, and hybrid systems. In other words, let us calculate: 1) the field which can be generated in a specified bore with a specified amount of power, using a resistive-conductor magnet of known conductor envelope, with inner windings subject to a specified maximum field; 2) the wire cost of superconducting magnets to generate a specified field in a specified bore; and 3) near-optimum parameters for hybrid systems. The optimization procedure will focus on the trade-off between the power consumption of the resistive insert and the wire cost of the superconductor, and the diameter of the insert that requires the least amount of superconductor to complete the system. We will apply some of the procedures derived to the design of a particular system to generate 25 T in a 3-cm working bore with 6 MW of power. Finally, let us review the three hybrid systems now being tested. These are the 16 T, 2 MW system of Oxford University; the 22 T, 5 MW system of the Francis Bitter National Magnet Laboratory at M. I. T.; and the 25 T system of McGill University, which operates not near room temperature, but at cryogenic temperatures. We will discover that for each system the parameters more or less specified at the outset were available power and desired field in a given bore; minimization of the total system cost--essentially proportional to the superconductor cost--was the criterion for selection. Therefore the design procedure derived below likewise assumes power, central field, and bore to be prespecified.

High-Field Resistive Magnet Design

First let us calculate the field increment ΔH which can be generated in a specified bore of $2a_1$ by a given amount of power P, given that the innermost windings are subject to a certain maximum field H. The product of this field, inner radius, and current density j there determines the stress in the windings and therefore the strength s required of the conductor:

$$s = c_s H j a_1 . \qquad (1)$$

But the current density is proportional to the field increment generated by the coil, with the constant of proportionality involving only known geometric parameters of the coil:

$$j = c_j \Delta H / \lambda_r \quad ; \qquad (2)$$

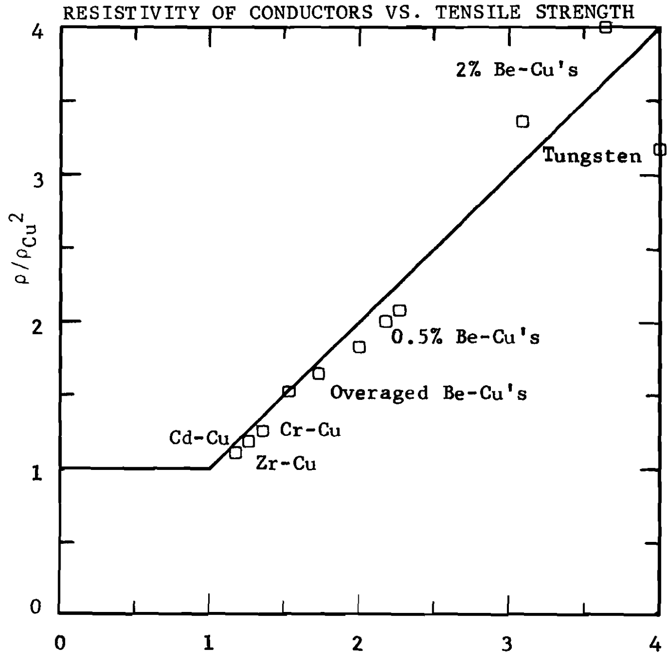
the "apparent radial space factor" λ_r has been left separate from the constant of proportionality in order that it may be adjusted as a function of the power which the coil is to consume. When j in Eq. (1) is replaced by its value as given by Eq. (2), the result is:

$$s = c' H \Delta H / \lambda_r \quad , \qquad (3)$$

where c' is the constant of proportionality relating needed conductor tensile strength to the product of total field and coil-field increment, divided by its radial space factor. c', as with c_s, must take account of stress and current density concentrations due to slits and cooling holes, as well as adjusting for the factor by which the actual stress in the coil may differ from that in an unsupported loop.

Now, how does conductor resistivity depend on this needed strength? If one tabulates conducting alloys, including as candidates for use in magnets only those materials which are most conductive for a given strength, one finds conveniently that for alloys which are stronger and more resistive than hard copper, the resistivity at 20 °C is approximately proportional to tensile strength, as seen in Fig. 4.

RESISTIVITY OF CONDUCTORS VS. TENSILE STRENGTH



Tensile Strength Normalized to that of Copper

Fig. 4: Electrical resistivity vs. ultimate tensile strength, both normalized to that of hard copper, for various copper alloys and other conductors.

This piecewise-linear relationship between 20 °C resistivity and required strength is expressed by:

$$\rho_o = \begin{cases} \rho_{Cu} = 1.7241 \times 10^{-8} \text{ ohm m} \quad (s \le s_{Cu}) & (4a) \\ c_o\, s = c_o\, c' H \Delta H / \lambda_r \quad (s \ge s_{Cu}) & (4b) \end{cases}$$

where: $c_o = \rho_{Cu} / s_{Cu}$

$$s_{Cu} = 3.8 \times 10^9 \text{ n/m}^2 = 55{,}000 \text{ psi}$$

and Eq. (3) has been used to obtain the rightmost expression of Eq. (4b). (For laminated composites of two materials, conductivity is linear with tensile strength. The Appendix presents a detailed derivation of a very accurate and very versatile set of equations based on this relationship, the work of C. F. Weggel.)

If we now choose a family of coils for which increases in power consumption are accompanied by a decrease in radial space factor and an increase in coil operating temperature which parallel one another, then each must be proportional to the square root of the power, in order that their product be proportional to the power consumed, which should be true if the overall heat transfer coefficient--bulk, boundary layer, and thermal gradient--remains constant:

$$\lambda_r = 1 - c_1 P^{\frac{1}{2}} \quad , \qquad (5)$$

$$\rho = \rho_o + c_2 P^{\frac{1}{2}} \quad . \qquad (6)$$

In writing Eq. (6), use has been made of the fact that all high-conductivity copper alloys experience approximately the same increase in resistance (absolute, not %) for the same temperature rise. One may now replace ρ_o in Eq. (6) by its value from Eq. (4), and use the resulting expression for ρ in the magnetic-efficiency equation:

$$H = G\,(P \lambda_x \lambda_e / \rho a_1)^{\frac{1}{2}} \qquad (7)$$

presented on page 88 of Montgomery.[5] The equation was derived by Weggel based on the experimental studies by Fournier and Rapport,[6] who showed that the actual "effective radial space factor" λ_e to be used in Fabry's efficiency equation should be $\lambda_e = \lambda_r / (2 - \lambda_r)$. The resulting equation is:

$$\Delta H = G \left[\frac{P \lambda_x \lambda_e}{(\rho_{Cu} + c_2 P^{\frac{1}{2}})\, a_1} \right]^{\frac{1}{2}} \qquad (8a)$$

$$\Delta H = G \left[\frac{P \lambda_x \lambda_e}{(c_o c' H \Delta H / \lambda_r + c_2 P^{\frac{1}{2}})\, a_1} \right]^{\frac{1}{2}} \qquad (8b)$$

Equation (8a) gives explicitly the field increment generated by a non-stress-limited coil consuming a specified amount of power. Equation (8b) may be made similarly explicit as the solution to the cubic equation:

$$\Delta H^3 + A_2 \Delta H^2 - A_o = 0 \quad , \qquad (9)$$

where:

$$A_o = G^2 \lambda_x \lambda_r \lambda_e P / c_o c' H a_1 \, , \qquad (10a)$$

$$A_2 = c_2 \lambda_r P^{\frac{1}{2}} / c_o c' H \, . \qquad (10b)$$

Define:

$$q = - A_2^{\,2} / 9 \qquad (11a)$$

$$r = + A_o / 2 - A_2^{\,3} / 27 \qquad (11b)$$

$$R = (q^3 + r^2)^{\frac{1}{2}} . \qquad (11c)$$

Then the solution is:

$$\Delta H = (r + R)^{1/3} + (r - R)^{1/3} - A_2 / 3 . \qquad (12)$$

This equation for the field increment ΔH generated by a coil consuming P megawatts and operating at a temperature implied by Eq. (6) when cooled according to the space factor of Eq. (5), and subject to a field H at its inner radius, applies whenever the stress level precludes the use of copper. To my knowledge this equation and the one in the Appendix by C. F. Weggel have no rival in the literature when it comes to the number of factors considered which govern the efficiency of a high-field Bitter-type magnet.

Results computed using Eqs. (8a) and (12) may be seen in Fig. 5.

FIELD INCREMENT VS. POWER$^{\frac{1}{2}}$ FOR 3-CM BORE INSERTS STRESSED BY VARIOUS MAXIMUM FIELDS

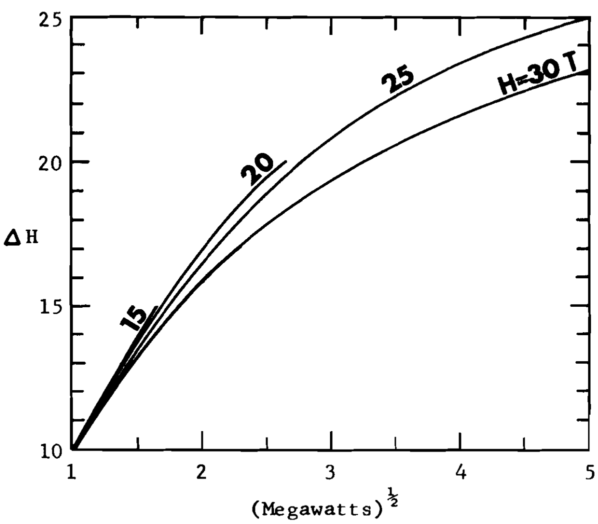


Fig. 5: Field increment generated by the two-coil Bitter magnet of Fig. 3, as a function of power, for various values of central magnetic field H.

Note that at any given level of power the field is somewhat less for high values of central field, provided that the coil is stressed beyond the limit for copper, and that at high levels of power the field rises dramatically less rapidly than the square root of power. In fact, even had we neglected the decrease in space factor and the increase in temperature rise with increasing power ($c_1 = c_2 = 0$), and considered only the increase in conductor resistivity due to the increase in stress level, already the field increment would be proportional only to the cube root of the power. Thus the field from a highly-stressed high-power coil is far less than would have been predicted from a naive use of the square-root-of-the-power law applicable to low-field coils.

Large-Bore Superconducting Magnet Design

The additional field increment necessary to bring the field shown in Fig. 5 up to the central value intended for the system is shown in Fig. 6.

REQUIRED SUPERCONDUCTOR FIELD VS. POWER$^{\frac{1}{2}}$ OF INSERT

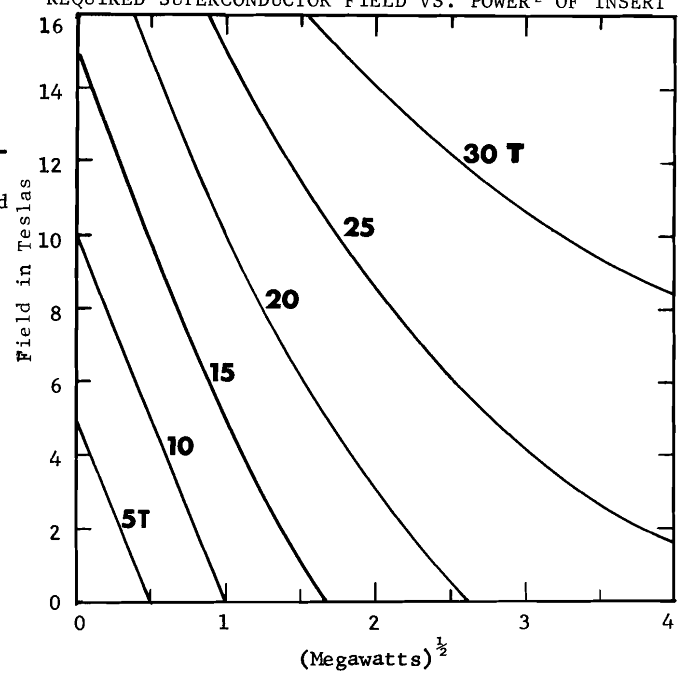


Fig. 6: Additional field increment necessary to bring the field increment of the magnets of Fig. 5 up to the central field intended for the complete system, as a function of power consumption, for various values of central field.

This additional field can be generated either by an additional resistive coil, as mentioned in the Introduction, or by a superconducting magnet. The power required by the inefficient, large-bore resistive coil has been shown to be large, however. Therefore let us use a superconducting magnet. We know the field it must generate. We also presume that its bore should be larger than the o. d. of the resistive magnet by just enough to guarantee adequate cryogenic insulation between the two. We therefore can calculate its cost. The magnet may be made of only one type of wire if the field is sufficiently low, or of several types, each selected to minimize the cost of the total system.

Estimation of Wire Cost per Meter

The dimensions of each wire used have been selected and its cost per meter estimated in the following way. Since large-bore coils usually are assembled from pancakes, the conductor shape has been assumed to be tape-like. Its width has been set at 1.27 cm, because this is standard for high-current Nb_3Sn conductor. A current of 1,000 A was chosen as typical for magnets of this size, and sufficient cross section of superconductor (and of substrate and reinforcement, if they accompany the superconductor, as was considered the case with Nb_3Sn and Va_3Ga) included to carry this current at the maximum field which it sees. (Note: in order to agree with values for existing large-cross-section Nb-Ti and highly-reinforced Nb_3Sn, the current densities of Fig. 1 had to be reduced by a factor of 2½.) The maximum field was approximated by:[7]

$$H_{max} = H\,(1 + 0.64 / 4^{(\text{length/i. d.})}) \qquad (13)$$

Sufficient copper cladding, taking into account its magnetoresistance, as given by[8]

$$\rho\,(H_{max}) = (1 + H_{max} / 24) \times 10^{-10} \text{ ohm m} \qquad (14)$$

was then added for stabilization at a heat flux over all surfaces--faces and edges--of 0.5 W/cm^2, the value given by Wilson[9] for the recovery of nucleate boiling from film boiling. (Jackson and Fruin[10] give a value of 0.4 W/cm^2, while Montgomery[11] gives values of 0.31 and 0.25 W/cm^2, respectively, for passages which are open or vapor-locked.) The overall current density has then been calculated assuming axial and radial spacing of 2 mm and 0.2 mm, respectively (1 mm and 0.1 mm, respectively, and a heat flux of 1 W/cm^2 in the case of small-bore coils.) These assumptions lead to an overall current density as shown in Fig. 7.

1.27-CM-TAPE OVERALL CURRENT DENSITY VS. FIELD

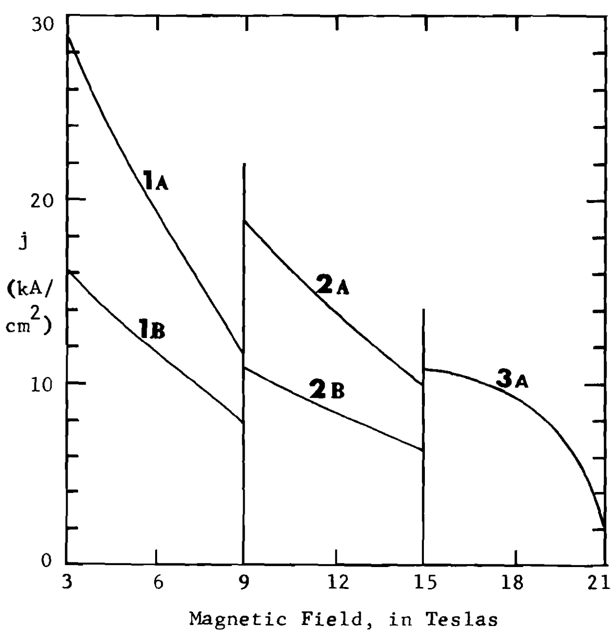


Fig. 7: Overall current density of 1,000 A, 1.27-cm wide conductors of (1) Nb-Ti, (2) Nb_3Sn, and (3) Va_3Ga stabilized to (A) 1 W/cm^2 for use in small coils and to (B) 0.5 W/cm^2 for use in large coils.

These current densities are much smaller than those shown in Fig. 1, but stabilization of at least some degree seems highly advisable for coils of this size.[12]

Required Outer Radius of Superconducting Coil(s)

The outer radius of coil necessary to generate a specified increment in field has been calculated by inverting the standard field equation for a uniform-current-density solenoid to obtain:

$$a_2 = 0.5\,(c^2 - b^2) / c \ , \qquad (15a)$$

where b is the half-length of the coil, and

$$c = (a_1 + (a_1^2 + b^2)^{\frac{1}{2}}) \exp\,(\Delta H / 0.4\pi j\, b).$$

Wire Cost of Superconducting Coils

Finally, the cost of coils has been computed from a cost per meter estimated as:

$$\$/m = \$_1\, V_{Cu} + \$_2\, V_{SC} + \$_3 \ , \qquad (16)$$

where V_{Cu} and V_{SC} are the cm^3 of copper and superconductor, respectively, in a meter of conductor, and the coefficients have been calibrated from commercially-available wires. For Nb-Ti $\$_1$ = \$0.40, $\$_2$ = \$1.60, and $\$_3$ = \$1.00, based on four high-current conductors manufactured by Supercon Corporation;[13] for Nb_3Sn the corresponding values, based on data supplied by Intermagnetics General Corporation[14] were: $\$_1$ = \$0.40, $\$_2$ = \$7.20, and $\$_3$ = \$1.20, subject to somewhat greater error than for Nb-Ti for conductors different from those used in the curve-fitting. (Coefficients for CSCC Nb_3Sn tape may be about 1/4 lower.) Coefficients for Va_3Ga were taken identical to those for Nb_3Sn.

WIRE COST VS. FIELD OF 40-CM I.D., 2-COIL MAGNETS

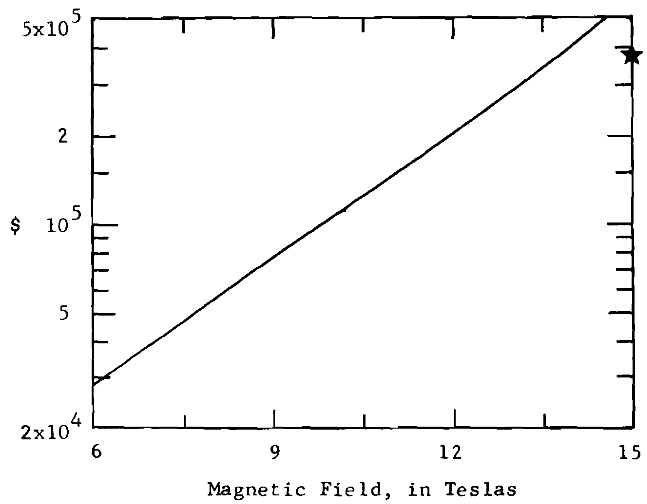


Fig. 8: Wire cost vs. field for magnets of 40 cm i. d. and 80 cm length. Line: two coils generating 1/3 and 2/3, respectively, of the field. Star: three coils (1/6 + 1/3 + 1/2). j as in Fig. 7 (B).

Optimization of Coil Length

The wire costs shown in Fig. 8 are for coils with a length-to-i.d. ratio of 2.0, which is near the optimum for the highest-field cases, for which optimization is most critical. For lower-field cases deviation from the optimum length by as much as - 1/3 to + 1/2 results in a penalty of less than 10%, as demonstrated by Fig. 9.

NORMALIZED WIRE COST VS. LENGTH OF 10 T COIL

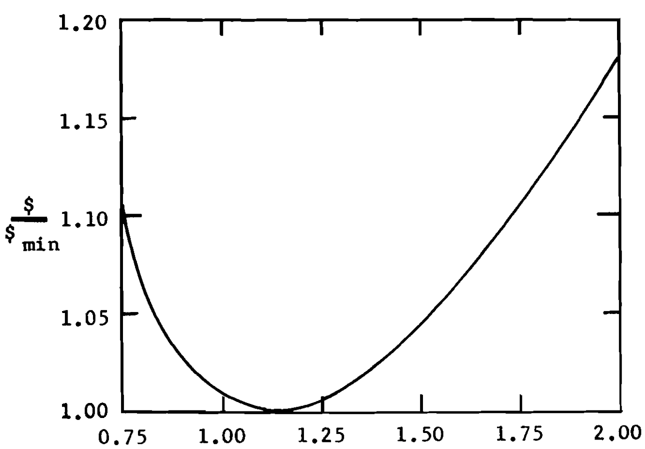


Fig. 9: Relative wire cost vs. coil length for a 10 T magnet identical, in all respects but length, to the magnets of Fig. 8.

Radius-Optimization of Hybrid Systems

The field which a resistive magnet can generate tends to increase with any increase in the conductor envelope allotted to it. The wire cost of a superconducting coil increases with increasing bore (presumably a constant amount larger than the resistive conductor envelope), but decreases with the decreasing field now demanded of it. Consequently there is an optimum radius at which to terminate the resistive coil. Calculating the wire cost of various superconducting coils, each larger in bore than its predecessor but having to generate a smaller field, since the insert has generated more, we find a curve very similar to that of Fig. 9. Again one can afford to deviate appreciably from the optimum before suffering an unacceptable penalty. One discovers that the greater the central field and the greater the power consumed by the resistive insert, the greater is the optimum radius. This is as we might expect, because the higher the field, the more essential it is that the resistive insert generate as much field as it can, and the higher the power, the more essential it is that adequate space be provided for its dissipation, lest cooling channels otherwise occupy an unreasonably large fraction of the limited conductor volume available. We can observe this trend when we review the various hybrid systems in existence or in the design stage.

Graphs Relating Field, Power, and Wire Cost

Combining the concepts and equations leading to the preceding Figures, we arrive at Figs.10 and 11, relating the central field to be generated, the total power available, and the wire cost of the superconductor required.

WIRE COST VS. FIELD FOR 3-CM BORE HYBRID SYSTEMS

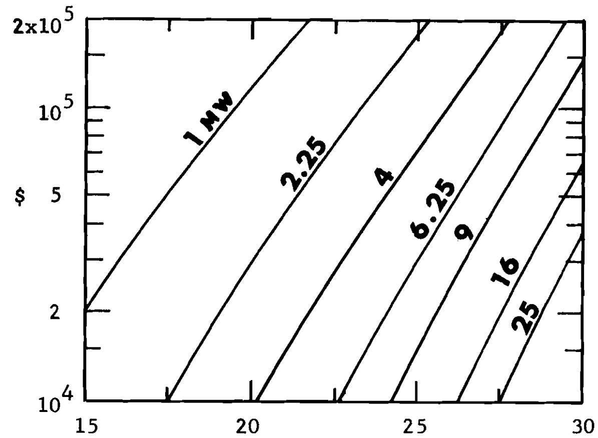


Fig. 10: Superconducting wire cost vs. central magnetic field for 4-cm i. d. hybrid systems consuming various amounts of power. Resistive coils as in Figs. 3, 5, and 6; superconducting coils as in Fig. 8.

WIRE COST VS. POWER$^{\frac{1}{2}}$ FOR 3-CM BORE HYBRID SYSTEMS

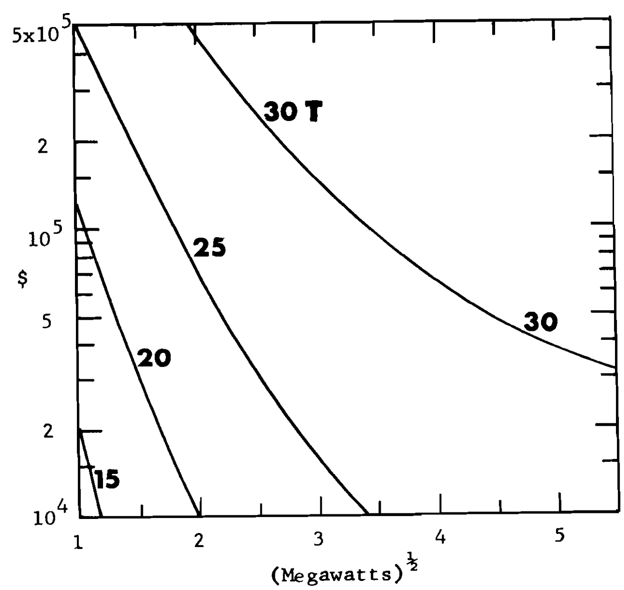


Fig. 11: Superconducting wire cost vs. power$^{\frac{1}{2}}$ for 4-cm i. d. hybrid systems generating various central magnetic fields. Resistive coils as in Figs. 3, 5, and 6; superconducting coils as in Fig. 8.

These two graphs are restatements of the same relationship. However, each is useful in its own right in displaying explicitly how wire cost varies as a function of either the field to be generated or the power which is available. Note that if power is constrained, then the wire cost rises rapidly with the field to be generated. Similarly, if wire cost is constrained, then the power consumption rises rapidly with field. Therefore, for any particular field that is to be generated, a judicious compromise should be made between the amount of power which is to be made available to the resistive magnet and the amount of money which is to be invested in the superconductor. Any undue limit on one of the two parameters results in an excessive value for the other. In particular, a value of zero for either can result in an astronomical value for the other. This confirms our opening assertion that any very high field can be more economically generated with both resistive conductors and superconductors together than by either alone.

Existing Hybrid Systems

Now for a review of the three hybrid systems presently being tested.

Oxford University System

The 16 T, 2 MW Oxford University system[15] employs a 9.4 T solenoid of 5.32 cm i. d., 21.2 cm o. d., and 10 cm length. It is composed of nested machined helices, in the manner of the 30 T, 30 MW Canberra magnet inner stack.[16] The field is sufficiently modest that copper is adequate in strength, and the coil can be designed to emphasize efficiency, rather than stress or heat flux minimization. The heat flux is 600 W/cm^2.

The superconductor is a twisted bundle of "fine filaments of Nb-Ti in a matrix of high purity copper formed into rectangular sectioned wire. The wire is wound into double pancakes and bonded with epoxy and glass. The windings are said to be both intrinsically and cryostatically stable."[17]

M. I. T. System[18]

The system at the Francis Bitter National Magnet Laboratory consumes 5 MW to generate a total of 22 T. The system includes two resistive coils that nearly duplicate the inner two coils of a three-coil resistive system which generates a similar field--but at double the power. The inner coil is of 1½" i.d., 4 3/8" o.d., and 4" length, and consumes 1.4 MW to generate 7 T. The heat flux is over 1,200 W/cm^2. Because of the high central field, 0.5% Be-Cu of 110,000 psi strength has been used, despite its conductivity of only 55% IACS. According to the calculations of this paper, however, a more conductive (weaker) conductor should suffice.

The outer resistive stack is of copper, with an i.d. of 4 3/4", and o.d. of 13 1/8", and a length of 8½". The heat flux is approximately 550 W/cm^2, and as with the Oxford stack is of significant but not paramount concern.

The superconducting coil has been described by Leupold, Iwasa, and Montgomery.[19] In brief, the coil generates 5.8 T in a 40 cm i. d. using 24 double pancakes of 66 cm o. d. wound of 180 m each of Nb-Ti copper composite of 2 mm by 10 mm cross section. The winding length is 58 cm. This coil is well into the regime of large size in which instabilities can be a problem, especially with untwisted wire. The wire purchase antedates the recognition of the advantages of twisting, and the coil did indeed experience flux jumping during energizing. Nevertheless the coil is so conservatively designed with regard to heat flux (w_s = 0.25 W/cm^2) and recovery current that the design value of current of 1,500 A was essentially achieved.

McGill University System

The 25 T system at McGill University[20] employs an innermost stack of ultrapure aluminum cooled by supercritical helium initially at 8 $^\circ$K but which warms during operation to 18 $^\circ$K, the pressure rising from 10 atm to 40 atm. Circulation through the closed system is ensured by a piston-type pump. The resistivity is reduced about 5,000 times below its room-temperature value. Unfortunately, the conductor is very weak, and substantial support was essential, as was true also with the cryogenic magnet at the NASA Lewis Research Center.[21]

The superconducting magnet is built of two sections, the outer of Nb-Ti and generating 7.5 T, the inner of Nb_3Sn and bringing the field to 15 T. Considering the novelty of the cryogenic coil and the high field of the various components, the system is an ambitious undertaking.

Proposed Nijmegen Hybrid System

Examining finally the proposed 25 T, 6 MW Nijmegen system, we discover that the consequences of the high central field are even more pronounced than for the M. I. T. system. The innermost conductor need not change, but the extra megawatt in the middle stack may mean that copper no longer is adequately strong. The resistive coils together will generate approximately 16.5 T for a relatively conservative and inefficient design, or as much as 18 T for a design more nearly approaching the efficiency assumed possible elsewhere in this paper. The wire cost for a 7.5 T magnet built with two wire grades is about $50,000.

Thus, this hybrid system is not really inexpensive; but then, no system to generate such a field would be. If the field were to have been generated exclusively with resistive conductors, not 6 MW but 11 MW would have been required, even with a large, efficient three-coil magnet.[22] Assuming a minimum cost of $50,000 per MW, multiplied by two for regulation and delivery, this additional 5 MW represents an investment of $500,000. Instead, less than $150,000, including a factor of three for design and assembly expenses, has been required. Such is the potential for hybrid magnets.

APPENDIX

DERIVATION OF THE HIGH-ACCURACY FORMULAS FOR THE MAGNETIC FIELD OF A BITTER MAGNET AS A FUNCTION OF ITS POWER CONSUMPTION

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As explained in the body of the paper, if the central field is greater than approximately ten teslas, the simple Fabry G factor is no longer very useful for predicting the magnetic field which can be generated with a given power consumption--in fact, it very often can be downright misleading. Not only does it grossly overestimate the field which can be generated, it also erroneously predicts optimum coil geometries which are considerably smaller than the true optimum coils. Therefore, new, more accurate formulas are imperative if one wishes to predict accurately the performance of any magnet system incorporating resistive coils if the central field is greater than approximately ten teslas. To be accurate, such formulas must take into account the following factors:

1. The true "effective" radial space factor resulting from the sacrifice of conductor volume for cooling passages. This effective radial space factor λ_e should also include the effect due to the distorted current paths around the cooling passages as determined experimentally by Fournier and Rapport to be

$$\lambda_e = \frac{\lambda_r}{2 - \lambda_r} \tag{1}$$

2. The increase in the average operating temperature of the magnet and the consequent increase in the conductor resistivity. Since the efficiency of a coil can be improved by lowering its average operating temperature, and since this can be accomplished by reducing the power density in the coil, the dimensions of truly optimum coils will be larger than those predicted by the simple Fabry G factor.

3. The reduction in the axial space factor λ_x due to the necessary sacrifice of conductor volume for reinforcing sheets to enable the coil to withstand the enormous stresses produced by the interaction of the magnet current with the ambient magnetic field. Alternatively (or simultaneously), a stronger but (unavoidably) more resistive material than copper may be used for the conducting turns of the coil; this results in an improved axial space factor, but a higher value of the conductor resistivity.

The eight basic equations required to derive such high-accuracy formulas are as follows:

The Field-Generation Equation:

$$\Delta H = J_o a_1 \lambda_x F \tag{2}$$

where

ΔH = the magnetic field generated by the coil, in teslas;

J_o = the current density in the conductor at the i. d. of the coil, in amperes per square meter;

a_1 = the inner radius of the coil, in meters;

λ_x = the dimensionless axial space factor as defined in Equation (8); and

F = the dimensionless "field factor" F, a function only of the geometry of the coil and its current distribution.

The Power-Consumption Equation:

$$\Delta P = J_o^2 \rho \lambda_x a_1^3 W \tag{3}$$

where

ΔP = the power consumption of the coil, in watts;

ρ = the conductor resistivity of the coil at its operating temperature, in ohm-meters; and

W = the dimensionless "power factor" W, a function only of the geometry of the coil and its current distribution.

Note that since the effective radial space factor is not a constant, it cannot be factored out of the field-generating equation or the power-consumption equation; therefore, both F and W have been herein redefined to include the effect of λ_e.

The Conductor Resistivity Equation:

$$\rho = \rho_o + \alpha \Delta T \tag{4}$$

where

ρ_o = the conductor resistivity of the coil at zero power consumption, in ohm-meters;

α = $d\rho/dT$ = approximately 6.81×10^{-11} ohm-meters per degree centigrade for copper <u>and</u> for most high-conductivity copper alloys; and

ΔT = the average temperature rise in the coil, in degrees centigrade.

The Temperature-Rise Equation:

$$\Delta T = C_T J_o^2 \rho \tag{5}$$

where

C_T = the calculated constant of proportionality between the average temperature rise in the coil and the power density in the conductor at the i. d. of the coil. It includes the effects of the average coolant temperature rise, the boundary-layer temperature rise, and the average temperature rise in the conductor. The assumption that C_T is constant is highly accurate providing that the heat-transfer coefficient (h) remains constant as the power density in the coil is increased.

The temperature-rise equation actually applies only at the i. d. of the coil, but since the cooling passages in a coil are generally located so as to yield a coil whose temperature rise is constant with respect to radius, in such cases the same formula will apply at any point in the coil.

The Stress Equation:

$$\sigma = C_s H J_o a_1 \tag{6}$$

where

σ = the stress in the conductor at the i. d. of the coil (where the stress in a Bitter coil is greatest), in newtons per square meter; and

C_s = the dimensionless "stress-concentration" factor. It includes the effects of stress concentration around the slits and the cooling passages and the effect of the thick build of the coil turns.

H = the magnetic field at the i. d. of the coil, in teslas.

The Mechanical Strength Equation:

$$\sigma t_c = S (Y_c t_c + Y_s t_s) \tag{7}$$

where

t = the thickness of the various components of each turn, in meters;

the subscript "c" applies to the conductor material;

the subscript "s" applies to the reinforcing material; and

the subscript "i" applies to the insulator material;

Y = the yield strength of the conductor or reinforcing material, in newtons per square meter; and

S = the dimensionless safety factor.

Note that the mechanical strength of the turn must be calculated at its weakest point--underneath one of the slits in the conductor sheets or the reinforcing sheets. Therefore, either Y_c or Y_s must be multiplied by the factor $(1 - 1/N_p)$ where N_p is the number of parallel conductor plates per turn or reinforcing plates per turn, the correct choice being that which predicts the lower mechanical strength for the turn.

The Axial Space Factor Equation:

$$\lambda_x = \frac{t_c}{t_c + t_s + t_i} \tag{8}$$

The final equation is the definition

$$\lambda_i = 1 - \frac{t_i}{t_c + t_s + t_i} \tag{9}$$

These eight equations are all that are required to calculate the power consumption of any resistive coil of a magnet system as a function of the field generated by that coil. Despite the emphasis on accuracy rather than simplicity, the resulting equations are still remarkably simple--cubic or quartic equations result at the very worst.

The derivation proceeds as follows:

Use Equation (5) to eliminate ΔT from Equation (4), and solve the resulting equation for ρ to yield

$$\rho = \frac{\rho_o}{1 - \alpha C_T J_o^2} \tag{10}$$

Substitute this equation for ρ into Equation (3) for the power consumption of the coil to yield

$$\Delta P = \frac{J_o^2 \rho_o \lambda_x a_1^3 W}{1 - \alpha C_T J_o^2} \tag{11}$$

Use Equation (2) to eliminate J_o wherever it occurs in Equations (11) and (6) to yield

$$\Delta P = \frac{C_1 \Delta H^2 \lambda_x}{\lambda_x^2 - C_2 C_T \Delta H^2} \tag{12}$$

and

$$\sigma = \frac{C_s H \Delta H}{\lambda_x F} \tag{13}$$

where

$$C_1 = \frac{\rho_o a_1 W}{F^2} \tag{14}$$

and

$$C_2 = \frac{\alpha}{a_1^2 F^2} \tag{15}$$

To eliminate the unknowns λ_x and σ from these equations, make use of Equations (7), (8), and (9) as follows:

Divide Equation (9) by Equation (8) and solve the result for t_s/t_c to yield

$$t_s/t_c = \lambda_i/\lambda_x - 1 \tag{16}$$

Use this result to eliminate t_s from Equation (7) to yield

$$\sigma = S (Y_c + (\lambda_i/\lambda_x - 1) Y_s) \tag{17}$$

Eliminate the unknown σ between this equation and Equation (13) and solve the resulting equation for λ_x to yield

$$\lambda_x = C_3 - C_4 H \Delta H \tag{18}$$

where

$$\Delta Y = Y_s - Y_c \tag{19}$$

$$C_3 = \frac{\lambda_i Y_s}{\Delta Y} \tag{20}$$

and

$$C_4 = \frac{C_s}{S F \Delta Y} \tag{21}$$

Of course, λ_x can never be greater than λ_i; λ_x will equal λ_i whenever the stress in the coil is low enough so that no reinforcing material is required, the conductor being able to withstand the entire stress by itself.

Equations (18) and (12) together enable one to calculate the power consumption of any resistive coil as a function of the field which it generates: First use Equation (18) to solve for the axial space factor. Then use the smaller of the two quantities λ_x and λ_i in Equation (12) to calculate the power consumption ΔP. (Actually, if these equations are being used to design a coil rather than to analyze an existing coil, some minor iteration will typically be required since F, W, and C_T are slowly-varying functions of λ_e and, therefore, of ΔP.)

On the other hand, the solution to the inverse problem of determining the field generated by the coil as a function of its power consumption is given by a remarkably simple implicit equation for ΔH as a function of ΔP. It is derived by substituting the expression for λ_x given by Equation (18) into Equation (12); clearing the result of fractions; and grouping terms with the same power of ΔH to yield

$$P_0 - P_1 H \Delta H - (P_{21} + C_2 C_T - P_{22} H^2) \Delta H^2 + P_3 H \Delta H^3 = 0 \quad (21)$$

where

$$P_0 = C_3^2 \quad (22)$$

$$P_1 = 2 C_3 C_4 \quad (23)$$

$$P_{21} = C_1 C_3 / \Delta P \quad (24)$$

$$P_{22} = C_4^2 \quad (25)$$

$$P_3 = C_1 C_4 / \Delta P \quad (26)$$

Note that if the field outside the coil is specified rather than the field inside the coil,

$$H = H_{outer} + \Delta H \quad (27)$$

and Equation (21) becomes a simple quartic equation. If, furthermore, the field outside of the coil should happen to be zero, the equation reduces to an especially simple form--a quadratic equation in ΔH^2 which, of course, can be solved explicitly for ΔH as a function of ΔP.

Before accepting the results of Equation (21), however, care should be exercised to ensure that no constraints have been violated by any of the variables; these constraints are as follows:

1. The axial space factor can never be greater than λ_i.
2. The average temperature of the coil should not greatly exceed the boiling point of the coolant (usually, water), or disasterous film boiling and consequent burnout may ensue.
3. The effective radial space factor should not be permitted to become unrealistically small; $\lambda_e = 0.5$ represents about the lowest practical limit. From Equation (1), this implies that λ_r should not become smaller than approximately 2/3.

To ensure that no constraints have been violated in the proposed design, the calculated value of ΔH should be substituted back into the earlier equations as a check. For example, if Equation (18) were to predict a value of λ_x that is greater than λ_i, this would imply that the stresses in the coil are low enough so that no reinforcing sheets are required in the coil. Therefore, $\lambda = \lambda_i$, and the correct value for ΔH is obtained from Equation (12) to yield

$$H = \frac{\lambda_i}{(C_2 C_T + C_1 \lambda_i / \Delta P)^{1/2}} \quad (28)$$

which is the applicable formula to use whenever the coil is not "stress-limited".

These formulas should permit much more accurate predictions of the performance of high-field resistive magnets and hybrid magnet systems. The correspondingly accurate formulas for the F factor and the W factor will be presented in the near future.

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