The Composite Structure of Hadrons and Two-Body Reactions

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§ 1. Introduction

Recently, the experimental studies of high energy two-body hadronic reactions have been progressing so that the structures of the amplitudes decomposed by the t-channel isospin and s-channel helicity states have been able to be investigated. Accordingly, it has became possible to judge fairly definitely which model is adequate for explaining the characteristic features of the scattering amplitudes. Particularly the Regge pole and cut models confront with some difficulties in the interpretation of t(squared four momentum transfer)-dependence of the forward amplitudes. In the context of the Regge model it is impossible to explain the zero of the imaginary parts of helicity nonflip amplitude for the πN and $\bar{K}N$ scattering, i.e. $t \simeq -0.2$. On the other hand, in the impact parameter (b) representation it has been found that the structures of the imaginary parts of the nonflip and flip amplitudes could be understood on the basis of common properties of the b-distribution, i.e. the b-distributions have a peak centered around $b \sim 1$ fm. Furthermore, it has been shown for $B\bar{B}$ scattering that the characteristic b-distributions share their properties in common except for a peak centered around $b \sim 1.2$ fm. In the present situations, it would be important to investigate wether the high energy amplitudes are characterized by the t-channel exchange objects (e.g. s-channel resonance) and how these circumstances are entangled in the structures of the amplitudes. Table I shows a comparison between the Regge model and the dual effective resonance model in the forward regions ($0 \leq |t| \lesssim 1$ $(GeV/c)^2$).

In addition to the above mentioned features, it is interesting to study the breaking of line reversal relations and D/F ratios of the amplitudes. In order to grasp the dynamical features of hadrons, it is useful to study systematically the prominent structures of the scattering amplitudes for two-body reactions.

§ 2. Breaking of line reversal relations.[1)]

In the context of the Regge pole model, we have the following amplitudes for the processes $K^-p \to \overline{K}^o n$ and $K^+n \to K^o p$,

$$T(K^-p \to \overline{K}^o n) = -\beta_{A_2}(t)[e^{-i\pi\alpha_{A_2}(t)} + 1]\, s^{\alpha_{A_2}(t)} - \beta_\rho(t)[e^{-i\pi\alpha_\rho(t)} - 1] s^{\alpha_\rho(t)}$$

$$T(K^+n \to K^o p) = -\beta_{A_2}(t)[e^{-i\pi\alpha_{A_2}(t)} + 1]\, s^{\alpha_{A_2}(t)} + \beta_\rho(t)[e^{-i\pi\alpha_\rho(t)} - 1] s^{\alpha_\rho(t)}$$

Then we get the equation of differential cross section between the processes provided that $\alpha_\rho(t) = \alpha_{A_2}(t)$, i.e.

$$\frac{d\sigma}{dt}(K^-p \to \overline{K}^o n) = \frac{d\sigma}{dt}(K^+n \to K^o p)$$

The same arguments lead to the following relations,

$$\frac{d\sigma}{dt}(\pi^+p \to K^+\Sigma^+) = \frac{d\sigma}{dt}(K^-p \to \pi^-\Sigma^+)$$

$$\frac{d\sigma}{dt}(\pi^-p \to K^o\Lambda) = 2\,\frac{d\sigma}{dt}(K^-p \to \pi^o\Lambda)$$

From the standpoint of the composite model of hadrons, the above relations mean that in the urbaryon rearrangement scheme the moduli of the H-type amplitudes are equal to those of the X-type ones.[2)] The KN and $\overline{K}N$ charge exchange data show that the relation holds good above P_L~5 GeV/c, but the modulus $|X|$ is larger than $|H|$ below P_L~5 GeV/c.[3)] In the processes the helicity flip amplitudes dominate.

For the strangeness exchange processes it has been found that the breaking of line reversal relations are rather conspicuous.[4)] All the present data show that the breaking is strong in the slope, while weak in the absolute values of the differential cross sections at $t \cong 0$, i.e.

$b_H > b_X$

$$|H| \sim |X| \qquad \text{at } t \cong 0$$

where b_H and b_X are the slope parameters in the differential cross sections. Since the slope b_H has a marked change near $t = -0.4(\text{GeV/c})^2$, the slope parameters are determined in the region $0 \lesssim |t| \lesssim 0.4\ (\text{GeV/c})^2$. Integrating the differential cross sections in the forward region, we have

$$\sigma_H < \sigma_X$$

In Fig. 3 we display the slope parameters and the integrated cross sections in the region $0 \leq |t| \leq 0.4\ (\text{GeV/c})^2$ versus s. Figure 3 shows that b_H increases with energy as $b_H \simeq 2 \ln s + 4$, while b_X increase at the rate of $b_X \simeq 4 \ln s - 4$. If we extrapolate the presently available data, both the slopes and the integrated cross sections converge about $P_L = 20$ GeV/c. Therefore it is expected that above P_L~20 GeV/c the line reversal relations hold good for the strangeness exchange processes. In these processes the helicity nonflip amplitudes dominate.

In summary for the processes in which the helicity flip and nonflip amplitudes dominate, the line reversal relations break down below P_L~5 GeV/c and P_L~20 GeV/c respectively. These features contradict the Regge pole model.

§3. D/F ratios[5)]

So far the accurate values of D/F ratios have not been obtained in the two-body reactions. The general tendency of the values of D/F ratios, however, can be studied. From data of the total cross section of πN and KN ($\bar{K}N$) scattering the nonflip D/F ratio $F_+ = \frac{f_+}{f_+ + d_+}$ is determined at $t = 0$. The total cross section of the processes can be parameterized approximately as follows,[6)]

$$\sigma_T\ (\pi^- p) = 21.3 + \frac{17.6}{\sqrt{\nu}}\ \text{mb}$$

$$\sigma_T\ (\pi^+ p) = 21.3 + \frac{11.2}{\sqrt{\nu}}\ \text{mb}$$

$$\sigma_T\ (K^- p) = 17.1 + \frac{17.1}{\sqrt{\nu}}\ \text{mb}$$

$$\sigma_T\ (K^- n) = 17.1 + \frac{11.45}{\sqrt{\nu}}\ \text{mb}$$

$$\sigma_T\ (K^+ p) = \sigma_T\ (K^+ n) = 17.1\ \text{mb}$$

where ν is the incident laboratory energy measured by GeV unit. The nonflip amplitudes of the processes can be expressed in terms of F_+ as

$$T(\pi^+ p) = D_\pi + (2F_+ - 1)\, H_{++} + 2\, F_+\, X_{++}$$

$$T(\pi^- p) = D_\pi + 2\, F_+\, H_{++} + (2F_+ - 1)\, X_{++}$$

$$T(K^- p) = D_K + 2\, F_+\, H_{++}$$

$$T(K^- n) = D_K + (2F_+ - 1)\, H_{++}$$

$$T(K^+ p) = D_K + 2F_+\, X_{++}$$

$$T(K^+ n) = D_K + (2F_+ - 1)\, X_{++}$$

where D_π, D_K are the diffraction amplitudes for πN and KN ($\overline{K}N$) scattering, respectively. Therefore, we obtain $F_+ \cong 1.5$. As is well known, in SU(6) symmetry model we have $F_+ = 1$ and $F_- = 2/5$.

Next consider the value of F_+ at $t \neq 0$. Writing the t-channel isovector and odd charge conjugation part of nonflip πN amplitudes and the t-channel isoscalar odd charge conjugation part of nonflip KN ($\overline{K}N$) amplitudes as $\rho_{\pi N}$ and ω_{KN} respectively, the following relation is obtained,

$$\omega_{KN} = \frac{4\,F_+ - 1}{2}\,\rho_{\pi N}$$

and then

$$I_m\omega_{KN} = \frac{4\,F_+ - 1}{2}\,I_m\rho_{\pi N}$$

Recognizing that in the forward region the diffraction amplitude dominates the imaginary part, $I_m\omega_{KN}$ and $I_m\rho_{\pi N}$ are extracted from the differential cross sections of the πN and KN ($\overline{K}N$) scattering as follows,

$$I_m\rho_{\pi N} \cong \frac{\frac{d\sigma}{dt}(\pi^- p) - \frac{d\sigma}{dt}(\pi^+ p)}{\sqrt{8[\frac{d\sigma}{dt}(\pi^- p) + \frac{d\sigma}{dt}(\pi^+ p)]}}$$

$$I_m\omega_{KN} \cong \frac{\frac{d\sigma}{dt}(K^- p) - \frac{d\sigma}{dt}(K^+ p)}{\sqrt{8[\frac{d\sigma}{dt}(K^- p) + \frac{d\sigma}{dt}(K^+ p)]}}$$

In Fig. 4 the r.h.s. and l.h.s. of the above relation are shown. It is found that the D/F ratio F_+ has a tendency to increase with $|t|$.

In the large momentum transfer regions ($1 \lesssim |t| \lesssim 2(\text{GeV/c})^2$), it is expected that the contribution of diffraction amplitude becomes very small.

As a first approximation, if we neglect the contribution of the diffraction amplitude, we have the following relation,

$$(P^-\sigma^- - P^+\sigma^+ + P^\circ\sigma^\circ) - \frac{1}{4F_+ -1}(P^-\sigma^- + P^+\sigma^+ - P^\circ\sigma^\circ) = 4\,F_+\,P^\circ\sigma^\circ,$$

where σ^-, σ^+, σ°, P^-, P^+, P° stand for the differential cross sections and polarizations for $\pi^- p \to \pi^- p$, $\pi^+ p \to \pi^+ p$ and $\pi^- p \to \pi^\circ n$ scattering respectively. In the region $1 \leq |t| \leq 2$ $(\mathrm{GeV/c})^2$ where $P^- \cong -P^+$ and $\sigma^- \cong \sigma^+ >> \sigma^\circ$ above $P_L = 5$ GeV/c, we have

$$(P^-\sigma^- - P^+\sigma^+ + P^\circ\sigma^\circ) >> (P^-\sigma^- + P^+\sigma^- - P^\circ\sigma^-).$$

Taking $F_+ > 1.5$ into account, the relation is simplified as

$$\begin{aligned}(4\,F_+ - 1)P^\circ\sigma^\circ &\cong P^-\sigma^- - P^+\sigma^+ \\ &\cong -2P^+\sigma^+.\end{aligned}$$

From the experimental data, the rough estimation of σ°/σ^+ and P°/P^+ can be made in the region $1 \leq |t| \leq 2$ $(\mathrm{GeV/c})^2$ and $P_L > 5$ GeV/c as

$$\sigma^\circ/\sigma^+ \simeq \frac{1}{5} \sim \frac{1}{10}$$

$$0 \lesssim -P^\circ/P^+ \lesssim 1/2.$$

Then we obtain $F_+ \gtrsim 5$ which is considerably large compared with $F_+ \cong 1.5$ at $t = o$.

On the assumption of the s-channel helicity conservation and the dominance of the imaginary part for the diffraction amplitude, we have the following relations,

$$\frac{P^-\sigma^-}{P^+\sigma^+} \cong \frac{2F - \mathrm{Re}H_{+-} + (2F_- - 1)\,X_{+-}}{(2F_- - 1)\mathrm{Re}H_{+-} + 2E\,X_{+-}}$$

The denominator and numerator become zero at $t = -0.6$ $(\mathrm{GeV/c})^2$ and around the value of t the mirror symmetry holds. This means that

$F_- = 1/4$. From the results of Regge pole analyses, it is known phenomenologically that

$$\frac{\mathrm{Re}H_{+-}}{X_{+-}} = \cos\pi\alpha\ (t)$$

and

$$\alpha(t) = 1/2 + 0.9\ t.$$

It follows that

$$\frac{P^-\sigma^-}{P^+\sigma^+} \cong \frac{2F_-\cos\pi\alpha(t) + 2(F_- - 1)}{(2F_- - 1)\ \cos\rho\alpha(t) + 2F_-}$$

In Fig. 5 the r.h.s. of the relation is compared with the l.h.s. calculated from experimental data. As is seen from Fig. 5, the ratio F_- is nearly equal to 1/3 at $t \cong -0.1$ and F_- approaches 0.25 ~ 0.26 with increasing $|t|$. If F_- is a constant lying between 2/5 and 0.26 in the region $0 \leq |t| \leq 0.5\ (\mathrm{GeV}/c)^2$, σ^-P^- is expected to have a zero in the region. The experimental data on $P^-\sigma^-$ do not show such a zero.

§ 4. Counting of urbaryon rearrangements[5)]

On the assumption of U(6) symmetric wave functions for hadron and s-channel helicity conservation for the spectator urbaryons, in the pseudoscalar meson-octet baryon scattering the baryon part of effective interaction caused by two urbaryon rearrangements can be given by

$$J^{\delta\ell}_{\alpha i} = \left[\epsilon_{jk} B_{\alpha\beta\gamma,i} + \epsilon_{ki} B_{\beta\gamma\alpha,j} + \epsilon_{ij} B_{\gamma\alpha\beta,k}\right] \times \left[\epsilon_{jk} B_{\delta\beta\gamma,\ell} + \epsilon_{k\ell} B_{\beta\gamma\delta,j} + \epsilon_{\ell j} B_{\gamma\delta\beta,k}\right]^*$$

where Greek and Rome suffices represent the unitary spin and the s-channel helicities of urbaryons respectively. Here we have reinterpreted the spin as the helicity. For baryon the unitary spin wave function

$B_{\alpha\beta\gamma}$ is antisymmetric under the substitution of the latter two suffices. In the above equation the spectator urbaryons have unitary spins and helicities labelled by (βj) and (γk) and the rearranged have (αi) and $(\delta 1)$. From the equation it follows that the D/F ratios of helicity non-flip and flip amplitudes F_+ and F_- are 1 and 2/5 respectively.

The urbaryon rearrangements are counted in equal weight irrespective of the unitary spin part of the urbaryon states being antisymmetric or not. The fact $F_+ = 1.5$ at $t = 0$ suggests that it is needed to introduce some distinctions between constituent urbaryons of a octet baryon. Taking into account that the U(6) symmetry is realized only approximately, we may consider that there are some differences between interactions or binding of the urbaryon in the unitary spin anti-symmetric states and in the non-antisymmetric ones. As the result of the differences, the rearrangements of the constituent urbaryons are not counted completely in equal weight. In this viewpoint, the D/F ratios will deviates from the values given by the U(6) symmetry and are more dynamical quantities which reflect the urbaryon rearrangement interactions and composite structures of baryon but not simple parameters in the U(3) symmetry. Therefore, it is not unnatural that the ratios depend on s and/or t. We introduce a weight parameter x which distinguishes the rearrangement of urbaryon in an antisymmetric unitary spin state from that of a non-antisymmetric state. Then the equation is modified as

$$J_{\alpha i}^{\delta l} = \left[x\, \epsilon_{jk} B_{\alpha\beta\gamma, i} + \epsilon_{ki} B_{\beta\gamma\alpha, j} + \epsilon_{ij} B_{\gamma\alpha\beta, k} \right]$$

$$\times \left[x\, \epsilon_{jk} B_{\delta\beta\gamma, l} + \epsilon_{kl} B_{\beta\gamma\delta, j} + \epsilon_{lj} B_{\gamma\delta\beta, k} \right]^*$$

Then we have

$$2x^2 + 2x - 1 = \frac{3}{2F_+ - 1}$$

$$2x^2 + 2x + 1 = \frac{1}{2F_- - 1}$$

Eleminating x, we obtain

$$(4F_+ + 1)\ (4F_- - 1) = 3$$

The ratios $F_\pm$ are not independent with each other. If x = 1, the set of ratios is $(F_+, F_-) = (1, 2/5)$. When $F_+ = 3/2$ which is approximately obtained from experiments on total cross sections, we have $(F_+, F_-) = (3/2, 5/14)$. If F_+ increases infinitely, F_- tends to 1/4. The set $(F_+, F_-) = (\infty, 1/4)$ leads to complete s-channel helicity conservation in the t-channel isoscalar amplitudes, and realized the mirror symmetry of polarizations of $\pi^\pm p$ elastic scattering. The mirror symmetry holds rather well also around $t \sim -1\ (GeV/c)^2$. If $E_+ \geq 1.5$, F_- is restricted to $1/4 \leq F_- < 5/14$ which is consistent with experimental data in the small t regions.

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Table I

		zeros of imaginary part of amplitudes		
		non flip	single flip	double flip
MB	Regge model	×	O	?
	dual effective resonance model	O	O	?
$B\bar{B}$	Regge Model	×	?	?
	dual effective resonance model	O	?	?

Figure Captions

Fig. 1. Examples of the urbaryon rearrangement scheme for the H-type amplitude and the X-type.

Fig. 2. Data for the KN and $\overline{K}N$ charge exchange reactions. Data from Ref. 3).

Fig. 3. (a) Slope-parameter, b as a function of s in the $\pi^{\pm}N$, the KN and the $\overline{K}N$ reactions. (b) shows the s-dependence of integrated cross section, σ of the $\pi^+p \to K^+\Sigma^+$ and the $K^-p \to \pi^-\Sigma^+$ reactions.

Fig. 4. t-dependences of the t-channel isovector, odd charge conjugation part of non-flip amplitude, $\rho_{\pi N}$ and the isoscalar one for non-flip $KN(\overline{K}N)$, ω_{KN}. (for details, see text)

Fig. 5. Ratio of the polarizations $(\underline{p}^-/\underline{p}^+)$ for the reactions of $\pi^-p \to \pi^-p$ and $\pi^+p \to \pi^+p$, as a function of t. Data are for the reactions at p_L = 14 GeV/c.

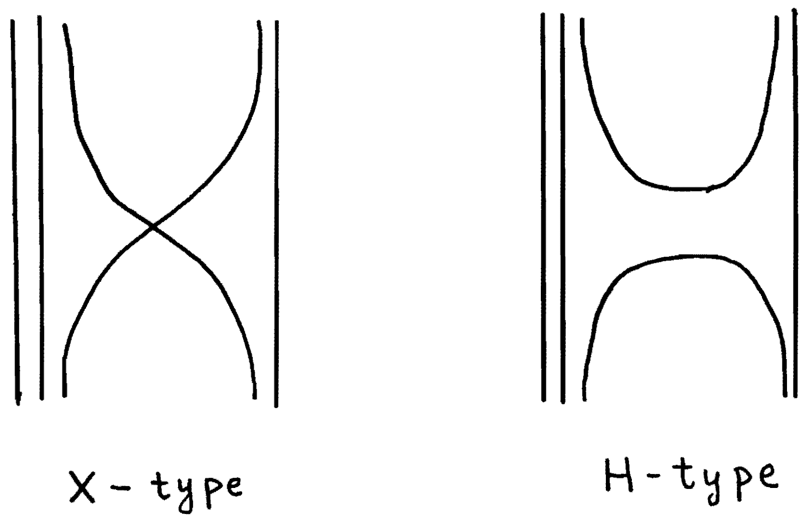

X - type
H - type

Fig. 1

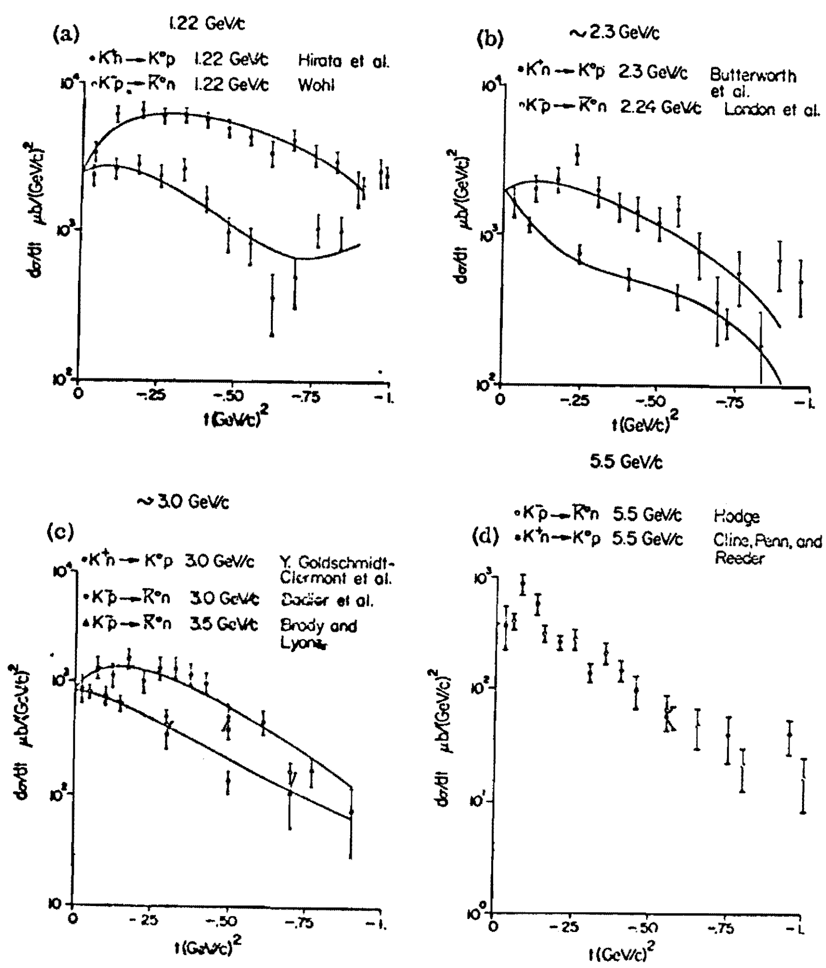

(a)
1.22 GeV/c
•K⁺n→K°p 1.22 GeV/c Hirata et al.
◦K⁻p→K̄°n 1.22 GeV/c Wohl
dσ/dt μb/(GeV/c)²
t (GeV/c)²
(b)
~2.3 GeV/c
•K⁺n→K°p 2.3 GeV/c Butterworth et al.
◦K⁻p→K̄°n 2.24 GeV/c London et al.
dσ/dt μb/(GeV/c)²
t (GeV/c)²
(c)
~3.0 GeV/c
•K⁺n→K°p 3.0 GeV/c Y. Goldschmidt-Clermont et al.
•K⁻p→K̄°n 3.0 GeV/c Dacier et al.
▲K⁻p→K̄°n 3.5 GeV/c Brody and Lyon
dσ/dt μb/(GeV/c)²
t (GeV/c)²
(d)
5.5 GeV/c
◦K⁻p→K̄°n 5.5 GeV/c Hodge
•K⁺n→K°p 5.5 GeV/c Cline, Penn, and Reeder
dσ/dt μb/(GeV/c)²
t (GeV/c)²

Fig. 2

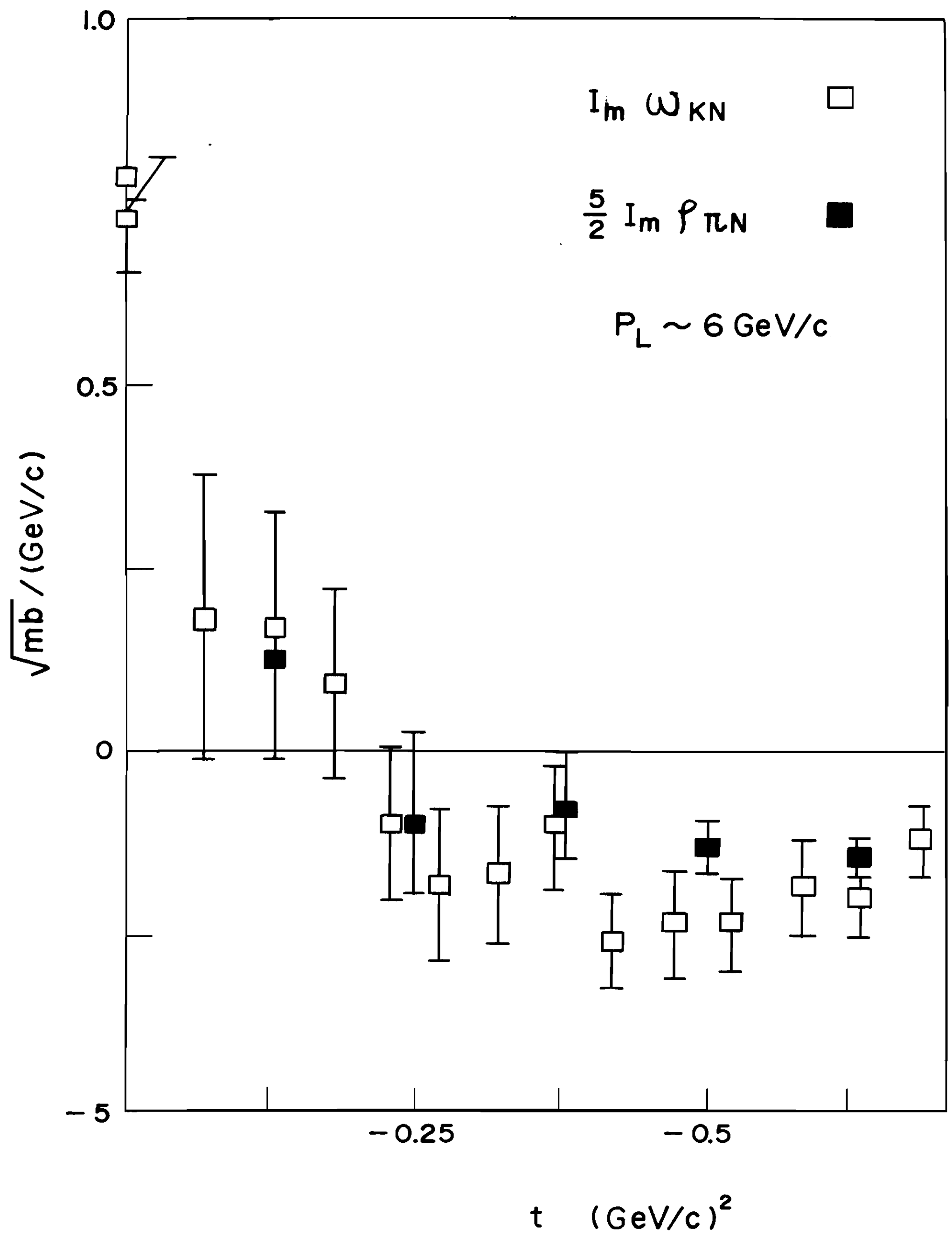
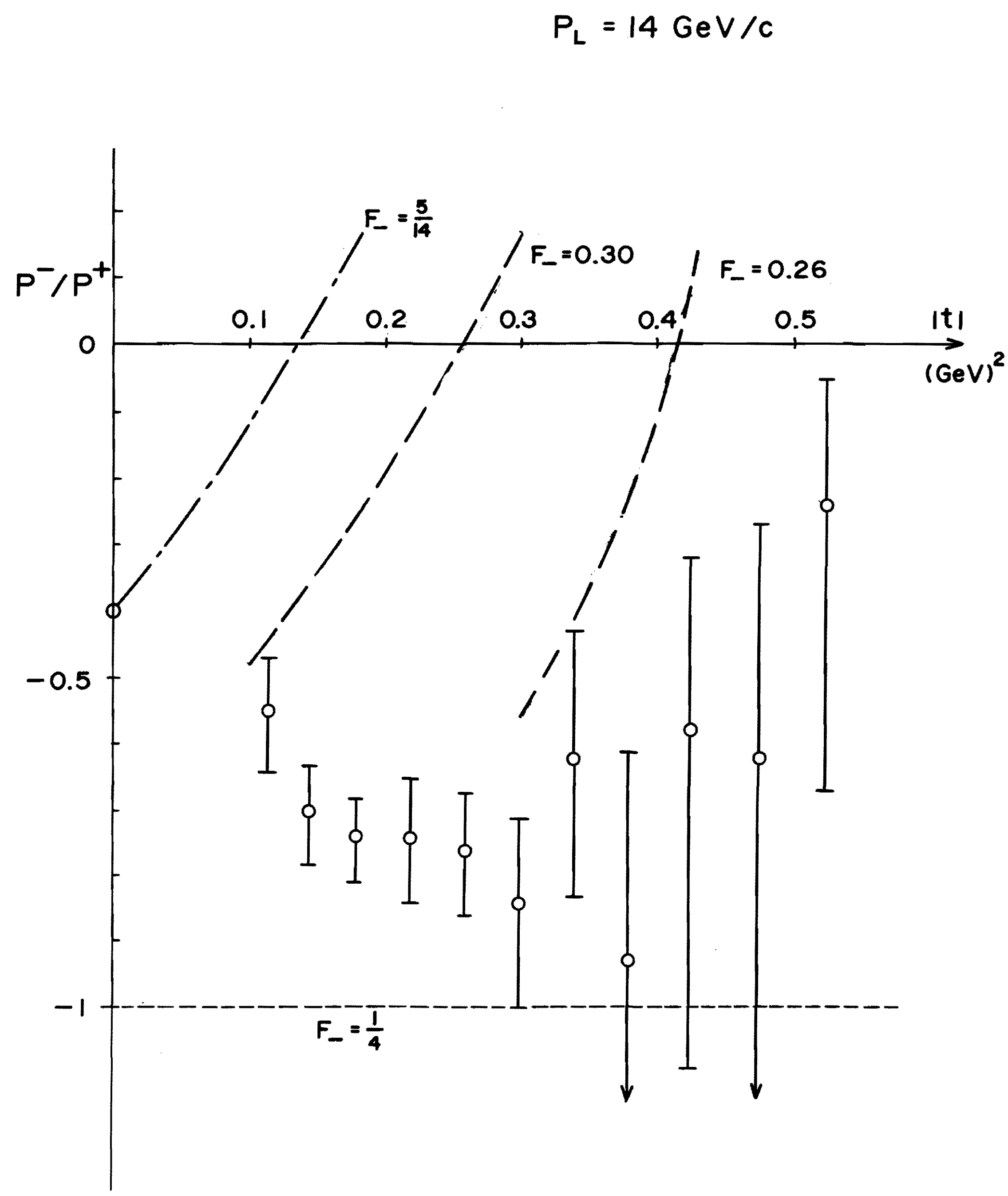

1.0
0.5
0
– 5
$\sqrt{mb}$ / (GeV/c)
Im ω_{KN}
$\frac{5}{2}$ Im $\rho_{\pi N}$
$P_L \sim 6$ GeV/c
– 0.25
– 0.5
t (GeV/c)^2

Fig. 4

Fig. 5



P_L = 14 GeV/c
P^-/P^+
0
−0.5
−1
0.1
0.2
0.3
0.4
0.5
|t|
(GeV)^2
$F_- = \frac{5}{14}$
F_-=0.30
F_-=0.26
$F_- = \frac{1}{4}$