TOPICS IN PARTICLE PHENOMENOLOGY*

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Abstract

Some recent developments are reviewed concerning those aspects of particle reactions which are entirely or almost entirely independent of dynamical models. Of the three sections, the first discusses the spin structure of particle reactions, the second deals with the application of conformal mapping to partial wave expansions, and the third outlines angular momentum decomposition at high energies.

I. THE SPIN STRUCTURE OF PARTICLE REACTIONS

The fact that some particles have spins gives us a more detailed insight into the interaction of quantum mechanical particles, without having to have knowledge about the nature of the forces acting between these particles. This opportunity has not been fully realized in the past, partly because the experimental requirements for measuring the spin dependence in reactions is in some cases rather severe, partly because often theorists were not oriented toward making predictions for anything but differential cross sections. Lately, however, theoretical interest in polarization phenomena has increased, and experimental techniques in terms of increased beam intensities, polarized beams, and polarized targets have made giant strides. It is therefore important to discuss what information can be obtained from the study of the spin structure of particles reactions (high energy, nuclear, or atomic) and how that information can be acquired through experiments.

There are a number of different ways of describing the force-independent, or non-dynamical structure of particle reactions. Which one we choose depends on what purpose we want to use it for. We can have the following requirements:

a) The description should be Lorentz invariant.

b) The description should cover all reactions regardless of the values of the spins of the particles.

c) The description should be simple, that is, should exhibit in a transparent way the relationship of experimental observables to scattering amplitudes.

d) The description should be able to take into account easily the constraints of conservation laws.

e) The description should be easy to compute and manipulate with.

f) The description should be easily compatible with actual experimental considerations.

g) The formalism should be compatible with past, present, and future theoretical dynamical models.

Not very surprizingly, it turns out that none of the existing formalisms are ideal for all these purposes. Especially the last one is difficult to fulfill. At the moment, it can be applied to S-matrix theory and thus, for example, one can demand that the formalism should be as free of kinematic singularities as possible.

In this Section I want to discuss one such non-dynamical formalism which stands quite high in terms of the above requirements, which has been worked out in great detail, and which can therefore illustrate the type of results one can obtain from these kinds of considerations. Other formalisms would probably yield similar results, though detailed studies are generally not available.

Since detailed results of the formalism are available in the published literature [1-28], the purpose of this review is to give an overall picture of the salient features. A summary up to that time was given in Reference 25. I will supplement it here with recent developments as well as some conceptual clarifications. In some of the references[3,4,6,8,10,11,12,13,16,17,18,23,25,26,28] a number of specific examples were worked out in great detail and these should be consulted for illustrations.

Our aim is to describe the reaction matrix (here called the M-matrix) in terms of its spin-momentum structure. For this purpose we will use[3] irreducible spin tensors $S_{[J]}(s, s')$ (for notation see below) as well as momentum tensors $T_{[J]}(\{p\})^r$. Though these tensors look formidable, their explicit role in the structure of the reactions can be eliminated by performing once and for all, the evaluation of the traces[3,10] involving them. The resulting

tabulated numbers[21] will then bear the only reminder that the S's and T's ever entered the problem. Thus I will spend no time on the properties of these tensors.

Let us first consider the four-particle reaction

$$a + b \rightarrow c + d \tag{1.1}$$

with arbitrary spins s_a, s_b, s_c and s_d, and let its M-matrix be M_1. From the point of view of all properties independent of the details of the dynamics, we can factorize this reaction in terms of the simpler reactions

$$a + 0 \rightarrow c + 0 \tag{1.2}$$

and

$$0 + b \rightarrow 0 + d \tag{1.3}$$

where 0 describes a particle with spin 0. If the M-matrices of these reactions are denoted by M_2 and M_3, we have the factorization

$$M_1 \ (=) \ M_2 \otimes M_3 \tag{1.4}$$

where (=) denotes equality for all dynamics-independent purposes, and $\otimes$ denotes the outer product in the spin space of the four particles. With this result, we now have to consider only the simpler reaction given by Eq.(1.2), since more complicated reactions can be composed from it.

The M-matrix for the reaction of Eq. (1.2) can be written as

$$M = \sum_{J,r} a_J^r \, S_{[J]}(s, s') : T_{[J]}^r(\{p\}) \tag{1.5}$$

where the a_J^r's are amplitudes containing all the dynamical information, the $S_{[J]}$'s are the spin tensors already mentioned, the $T_{[J]}^r$'s are the momentum tensors also already mentioned, s and s' are the spins of particles a and c respectively, and the : means total contraction over all tensorial indices.

The sum is over the rank of the S and T tensors (which is denoted by [J]) and over the index r distinguishing the various T tensors of the same rank.

The experimental observables, denoted by $L\big((S:\,T)_I,\,(S:\,T)_F\big)$, are obtained from the M-matrix by the operation

$$L[(S:\,T)_I,\,(S:\,T)_F] = \mathrm{Tr}\{\, M(S:\,T)_I\, M^{+}(S:\,T)_F \,\} \tag{1.6}$$

which, using Eq. (1.5), can be rewritten as

$$L[(S:\,T)_I,\,(S:\,T)_F] = \sum_{\substack{J_1 J_2 \\ r_1 r_2}} a_{J_1}^{r_1}\, a_{J_2}^{r_2}\, {}^{*}X_{J_1 J_I J_2 J_F}^{r_1 r_I r_2 r_F} \tag{1.7}$$

where

$$X_{J_1 J_I J_2 J_F}^{r_1 r_I r_2 r_F} = \mathrm{Tr}[(S:\,T)_{J_1}^{r_1}\,(S:\,T)_{J_I}^{r_I}\,(S:\,T)_{J_2}^{r_2\,\dagger}\,(S:\,T)_{J_F}^{r_F}\,] \tag{1.8}$$

is a dynamics-independent quantity that, as mentioned earlier, has been tabulated[21] for various values of s and s'.

In the above formulae $(S:\,T)_I$ and $(S:\,T)_F$ are the spin-momentum tensors describing the initial and final polarization states of the particles. Thus each experiment corresponds to a different L (or a linear combination of L's).

As an example, in pion-nucleon scattering ($s = \frac{1}{2}$, $s' = \frac{1}{2}$), with rotation invariance only (i.e. no parity conservation)

$$M = b_0 1 + b_1 \vec{\sigma}\cdot\vec{q}_1 + b_2 \vec{\sigma}\cdot\vec{q}_1\times\vec{q}_2 + b_3 \vec{\sigma}\cdot\vec{q}_2 \tag{1.9}$$

where $\vec{q}_1$ and $\vec{q}_2$ are two momenta characterizing the kinematics of the reaction. To establish the correspondence with Eq. (1.5), the a_J^r's there are the $b_0, b_1, \cdots, b_3$ here, the $S_{[J]}^r$'s there are the unit matrix 1 and the $\vec{\sigma}$ here, (that is, [J] here is 0 or 1), the $T_{[J]}^r$'s there are 1, $\vec{q}_1$, $\vec{q}_1\times\vec{q}_2$,

and $\vec{q}_2$ here (that is, r is 1 for [J] = 0 and r is 1, 2, 3 for [J] = 1), and the : there is the dot product here.

Observables for pion-nucleon scattering are, for example

$$L(1, 1) = \mathrm{Tr}(M\, 1\, M^{+}\, 1) \tag{1.10}$$

which would have 16 terms (in principle), such as $|\, b_0\, |^2 \mathrm{Tr}\, 1$, $b_0 b_1^{*} \mathrm{Tr}(\vec{\sigma} \cdot \vec{q}_1^{+})$, etc. In general, the X's here are the traces of 0, 1, 2, 3, or 4 Pauli matrices.

The momenta used to span the tensors T can be chosen in an infinite number of different ways. In principle, no significant results should depend on the choice of the basis. In partice, however, the transparency of the results and the simplicity of the relationship between amplitudes and observables will depend crucially on the choice of the basis.

It is not known what choice of the basis is optimal[22] from the point of view of simplicity or transparency, as well as from the other criteria enumerated at the beginning of the section. The one I will use ranks high but has not been proven optimal.

Denoting by $\vec{q}$ and $\vec{q}'$ the center of mass momenta of particles a and c respectively, I will use the three orthonormal unit vectors

$$\hat{\ell} \equiv \frac{\vec{q}' - \vec{q}}{|\vec{q}' - \vec{q}|} \qquad \hat{m} \equiv \frac{\vec{q}' \times \vec{q}}{|\vec{q}' \times \vec{q}|} \qquad \hat{n} \equiv \ell \times \hat{m} \tag{1.11}$$

to span the momentum space. Therefore, for example, the pion-nucleon M-matrix will be written as

$$M = b_0 + b_1 \vec{\sigma} \cdot \hat{\ell} + b_2 \vec{\sigma} \cdot \hat{m} + b_3 \vec{\sigma} \cdot \hat{n} \tag{1.12}$$

The a_J^r's in Eq. (1.5) are rank-zero tensors, which can depend on rank-zero tensors formed of the momenta characterizing the reaction. Note that in a four-particle reaction such rank-zero combinations of momentum tensors

(q_1^2, q_2^2, and $\vec{q}_1 \cdot \vec{q}_2$) are all scalar (and not pseudoscalar), while for a reaction containing more than four particles, where at least three momenta are needed to specify the kinematics, some of these combinations are scalar ($\vec{q}_1$, $\vec{q}_2 \cdot \vec{q}_3$ etc.) but one of them is pseudoscalar ($\vec{q}_1 \cdot \vec{q}_2 \times \vec{q}_3$). Thus for a four-particle reaction the amplitudes are always scalar, while for a reaction with more than four particles the amplitudes do not in general have a definite behavior under reflection. This has the consequence that a more-than-four-particle reaction in general cannot be used to determine the intrinsic parities of particles[1,2,20].

The number of terms in Eq. (1.5), if one assumes only rotation invariance, can be shown to be $x \equiv (2s + 1)(2s' + 1)$. In general, the number of amplitudes in a four-particle reaction is $X \equiv \prod_{i=1}^{4} (2s_i + 1)$, where s_i is the spin of particle i. The number of different bilinear combinations of amplitudes is therefore x^2. It turns out that the number of different observables L is also x^2. Thus there are x^4 different X's, which can be thought of as elements of a matrix connecting the x^2 observables and the x^2 bilinear products of amplitudes.

Note that (if we consider only rotation invariance), the number of observables is the same as the number of bilinear products, and hence the observables are all <u>linearly</u> independent.

Even though the values of the x^4 different X's can be computed once and for all, the existence of such large number of X's, connecting all of the observables to all of the bilinear products creates a clumsy situation. After all, even for one of the simplest reactions (pion-nucleon scatterint), x is 4, so x^4 is 256. For something more complicated, but still very realistic, such as rho-nucleon scattering, we have x = 36, and so x^4 = 1, 679, 616. Thus, unless this general structure can be simplified, a systematic discussion of the observable structure of reactions appears to be impractical.

Actually, however, the situation is vastly better, since on account of the judicious choice of basis vectors plus the properties of the irreducible spin tensors, a very large fraction of the X's are actually zero. For the pion-nucleon scattering case, only 40 of the 256 X's are non-zero. For a multiply factorizable reaction like rho-nucleon scattering, the ratio is even more favorable: over 99% of the X's in that case are zero.

Not only are most of the X's zero, but they vanish in such a way so as to decompose the problem into many small problems. An unfactorizable reaction breaks down into eight groups of observables, each of which is related to only one of eight groups of bilinear products of amplitudes. In a once factorizable reaction there are 32 such small groups, in a twice factorized one 128, and in a three-times factorized one 512. Furthermore, there is a very simple recipe for determining which observables go with which bilinear products[10, 25]. Thus the structure of the relationship between observables and amplitudes is actually enormously simpler than it appears at first sight.

This simplified structure is useful for a number of reasons. It points out immediately which experiments furnish information about which amplitudes, thus providing a dynamics-independent basis for planning new experiments to gain additional knowledge about the amplitudes. Unfortunately, the related general problem of what sets of experiments give a unique determination of the amplitudes has not been solved yet, though some partial results are available[19,24]

The simplified structure is also helpful in finding tests of conservation laws and in determining the intrinsic quantum numbers of particles participating in the reaction. I will therefore turn now to the discussion of conservation laws.

Up till now we have assumed only rotation invariance. For example, Eq. (1.5) was constructed by requiring only that M be rotation invariant. As an example, let us now consider reflection invariance[10] also and see how

the structure of observables and amplitudes changes as a result of this add additional assumption.

The changes are easy to follow[25]. In the M-matrix given by Eq. (1.5), all amplitudes, as I have said, are scalar. Hence, if we want to demand that the M-matrix be specifically scalar or specifically pseudoscalar, we must see if the (S: T)'s are scalar or pseudoscalar. It turns out that approximately half of the (S: T)'s are always scalar and half are pseudoscalar. Therefore, if parity is conserved and the product of the four particles participating in the reaction is scalar (pseudoscalar), then the M-matrix given by Eq. (1.5) must also be scalar (pseudoscalar), and hence the inappropriate terms in M must be zero (that is, the corresponding amplitudes must vanish indentically).

We then have only $x/2$ amplitudes, and so $x^2/4$ bilinear products of these. The number of observables is, of course, still x^2, since the number of ways one can set up polarization experiments certainly does not depend on whether certain conservation laws hold or not. In this case, therefore, we will have x^2 observables linearly depending on $x^2/4$ bilinear products of amplitudes and hence there must be $3x^2/4$ <u>linear</u> relationships among the x^2 observables. These relationshpis did not exist when we assumed only rotation invariance, and are the direct consequence of our additional imposition of parity conservation. Thus these linear relations among the observables are tests of parity conservation, and in fact one can prove that they represent <u>all</u> the parity tests which are dynamics-independent.

These tests can be divided in two groups. About $x^2/2$ of them test: whether parity is <u>partially</u> violated or not. In other words, they give one result if either parity is conserved or completely violated, and they give another result if parity is partially violated. These tests, however, cannot distinguish between parity conservation and complete parity violation. In the case of parity, complete parity violation in some cases can be

eliminated by a judicious assignment of intrinsic parities to the particles. In fact, this is the justification of the concept of intrinsic parities. Thus, we can also say that while these $x^2/2$ tests can indicate (partial) parity violation, they cannot serve to determine the intrinsic parities of the participating particles. In the case of some other symmetries (such as time reversal invariance[13]) the concept of a corresponding intrinsic quantum number for particles is not appropriate, and hence in these two cases only the first of the above two formulations is relevant.

In addition to these $x^2/2$ tests, we have about $x^2/4$ other types of tests which, in addition to testing partial parity violation, can also tell parity conservation from total parity violation, that is, they can determine the intrinsic parities of the participating particles.

Of these two types of tests, the first one is of a much simpler form. They consist either of certain observables vanishing identically (such as in the case of parity conservation) or of two observables (differing only in the order of the polarization tensors in their arguments) being either equal to each other or one being equal to the negative of the other. This latter type of relations is called mirror relation[17].

The tests of the second type (which are the more versatile ones) unfortunately are somewhat more elaborate in structure. They state that certain linear combinations of observables must vanish identically (i.e. at all angles and energies). Experimentally these are more difficult to carry out, since one has to measure a number of different observables, and in forming the linear combinations experimental errors might add up into a large uncertainty. For relatively low spin values, however, these linear combinations are relatively simple, and hence the tests are not unfeasible.

One can discuss this way not only parity conservation [5,7,10,15,(P)], or time reversal invariance[13] (T), but also P + T, PT[16], and CPT [16, 27].

Furthermore, one can treat in a formally similar fashion detailed balancing (B), which, though not a conservation law, is likely to hold in certain types of reactions and hence can be a useful tool. One can then also discuss BP, BT, BPT, CPTB, etc.

Furthermore, one can also discuss in a similar way the constraints of identical particles[23,27] and the case of collinear reactions[26], both of which are of practical importance.

In the discussion of all of these symmetries and constraints, one follows the same procedure. This procedure consists of two main parts.

First, one wants to determine the type of reaction that goes into itself under the particular symmetry or constraint under consideration. Such reaction is called a self-transforming reaction[25] under that particular symmetry or constraint. For example, under time reversal, the reaction $a + b \to c + d$ goes into itself if $a = c$ and $b = d$, that is, if we have elastic scattering.

The reason for concentrating on self-transforming reactions is two-fold. First, it is experimentally often easier if, when performing a test of a conservation law, one can concentrate on observables of <u>one</u> particular reaction instead of having to compare observables of two reactions, as in the case of non-self-transforming reactions. (For example, one can test time reversal invariance also on reactions other than elastic scattering, but such tests would involve comparing the reactions $a + b \to c + d$ with $c + d \to a + b$.)

The second reason for concentrating on self-transforming reactions is that they are the <u>only</u> ones capable of supplying the type of tests which can distinguish conservation from complete violation. It turns out that non-self-transforming reactions can provide <u>only</u> mirror relations[17] which, as I have said, cannot tell conservation from complete violation (and hence cannot determine intrinsic quantum numbers).

The second main part of the procedure to discuss symmetries or constraints is to determine what such constraints do to the spin vector $\vec{S}$ and to

the basic momentum vectors $\hat{\ell}$, $\hat{m}$, and $\hat{n}$. These four vectors transform differently under the various constraining transformations, and correspondingly different terms in the original M-matrix (Eq. 1.5) will have the 'wrong' behavior from the point of view of the additional constraint. It turns out that one can define a characteristic quantity[10,25] for each symmetry which quickly tells whether a term in Eq. (1.5) is eliminated or not by the imposition of that symmetry. For example, for a term to survive in Eq. (1.5) under parity conservation[10] (with the product of all intrinsic parities being positive) the number of ℓ's plus the number of n's in that term (i.e. in the argument of the momentum tensor $T^{r}_{[J]}$) must be even.

The same characteristic quantity can also be used to sort out the experimental observables. For example, if parity conservation is imposed, all observables L in the argument of which the total number of ℓ's plus the total number of n's is odd must be identically zero at all energies and angles. This requirement, in fact, supplies the $x^2/2$ relations discussed earlier.

This completes the very brief summary of the spin structure of particle reactions. We have seen that by an appropriate choice of a basis for the spins and the momenta the relationship between amplitudes and observables can be made fairly simple and quite transparent. As a consequence, the effects of symmetries and other constraints on the observables can be easily determined, and thus tests of the existence of such constraints can be devised. Also, the transparent relationship between observables and amplitudes allows one to plan experiments better, since it is relatively easy to ascertain what experiments will supply what type of information about the amplitudes. Since it is my guess that the greatest advances in particle physics in the next decade will be made by careful and detailed measurements of <u>various types</u> of observables in the medium energy range (up to 20 GeV), I also believe that the understanding of the spin structure of particle reactions is likely to play a significant role in the coming years.

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