Strong Interaction at High Energy

H. Sugawara

Institute for Nuclear Study, University of Tokyo

Tanashi, Tokyo, Japan

I. Introduction

I would like to talk about the hadronic interaction at high energy as I see it at the present stage. We already have much experimental data in this field and phenomenologists have clarified some of the basic properties of the strong interaction. Yet there are still many crucial experiments to be done and so many unsolved phenomenological problems left. Let me list some examples:

1. Under certain convention (Coulomb amplitude being real for example) what is the over all phase of an amplitude?[1]
2. How important is the diffraction dissociation?
   I read some of the phenomenological papers[2] but I could not get the clear idea.
3. There seems to be no established good explanation of the 'absorption'. Neither the strong nor the weak cut model can explain the real part of the amplitude. We do not understand why the helicity flip amplitudes seem to be explained by a simple Regge pole model.[3]
4. What is the value of $\alpha_P'(o)$?[4]
   possible values are
   1. 0 , fixed pole ,
   2. 1/2 , dual model ,
   3. ∞ , colliding pole .

   And who can exclude the values in-between?

All these are of course big problems but there are many easier (?) ones which will definitely contribute to clarify the hadron dynamics. Examples are:

5. Are the inelastic amplitudes dual? I want to see , for example, some polarization measurement in K^+p or pp inelastic collision to check the reality of the amplitude.

6. Although there may exist the 'pionization',

   I do not quite understand what it is: why the pionization product cannot carry away conserved quantities like the charge, the baryon number etc ?

7. Is scaling good at the energy of 100 TeV? Air shower experiment (μ^+/μ^- ratio for example) can provide us some information.

I would like to arrange my lecture in the following way. In the first lecture I will talk about the general picture of the hadron collision (Section III). In the second and the last lecture I will discuss the inelastic collision (Section IV).

II. Fragile particles

The hadrons are an extended object with the radius of $\sim$ 1 fm. So far there seems to be no evidence against this fact. Moreover this extended object seems to be 'fragile': When two particles collide face to face at rather high energy they are easily broken to pieces. Therefore the collision which produces a few particles occurs only when the particles collide peripherally[(5)] except for the diffraction for the elastic scattering.

There has long been a discussion whether there is a hard core or not inside an elementary particle. The indication so far is negative for its existence. This would show up in a large P_t event in the hadronic reactions.

Experimentally there is a backward peak in almost any two body scattering. This is rather hard to understand if the scattering is really 'backward'. The real situation, however, is: these peaks come from baryon number exchanged peripheral scattering in case of meson-baryon scattering. Backward collision will produce many secondary particles and no more leads to two body scattering.

As long as the two body scattering occurs peripherally (whether in the forward or in the backward region), we must be able to describe its angular dependence with one simple function for all range. Obviousely $J_\lambda(b\sqrt{-t})$ cannot do this. B function of Veneziano may be what we want but the justification for this is no longer clear. Our hope is that the direct channel Regge model will work.[6]

How do we see the very heavy resonances in these peripheral two body processes? We can identify the resonance if the two particles come to trap and go around each other for the period determined by the life time of the resonance. But in our case the particles collide only peripherally. The life time of the resonance we observe may be proportional to $\sim \sqrt{s}$ because the time necessary to pass each other in the C. M. system is proportional to $\sim 1/\sqrt{s}$. To measure the true life time we may have to consider only the resonance which is produced as a 'fragment'. Large angle supression of the amplitude at high energy may be connected with this fact. Suppose the life time of the resonances is proportional to log s instead of $\sqrt{s}$ this means that the binary resonance state stays longer and we expect less suppression in larger angle (fig.1).

We suggest[6]

$$\Gamma \propto \sqrt{s} \qquad \frac{\text{Large angle}}{\text{Forward}} \sim e^{-\sqrt{s}} ,$$

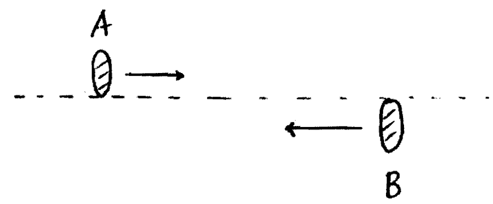
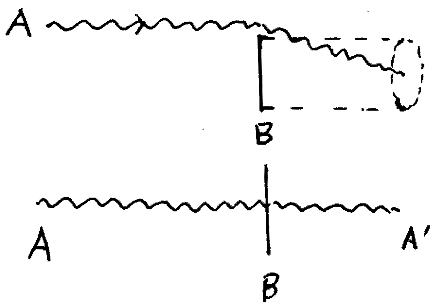
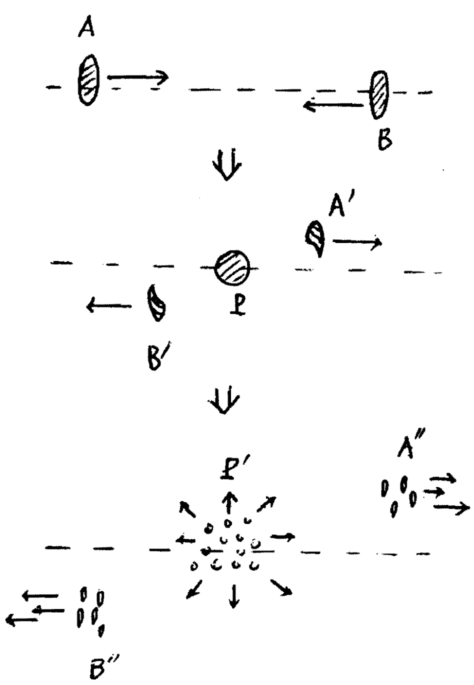
$$\Gamma \propto a \log s \qquad \frac{\text{Large angle}}{\text{Forward}} \sim s^{-a} .$$

Now when the impact parameter gets smaller and the substantial overlapping is observed the collision gets very inelastic. Overlapping area will tend to rest in the C. M. system and the others (fragments) rest in either the target or the projectile frame.[7] Uncertainty principle tells us that the excitation energy of the fragments is $\sim C/r_o$, where r_o is a measure of the extention of the particle. In the target rest system we expect that the target fragments have some finite energy and the number of the fragemnts is also finite. The energy dependent (increasing with the energy) multiplicity seems to come from the overlapping area (pionization).

The intuitive classical picture is sometimes dangerous and missleading. In our case it does not explain how the quantized conserved quantity is distributed after the collision. Suppose we have a proton as a target. The charge of the proton cannot be divided into the overlapping area and the fragmentation area because the proton has the unit charge. The fragmentation hypothesis tells us that the entire charge remains in the fragmentation region: particle and antiparticle are equally produced in the pionization region.

We have another mechanism in the high energy scattering of elementary particles: diffraction. We will have only one type of diffraction if the particles can be regarded as a complete black body i.e. shadow scattering. Yet the particles seem to have non zero transparancy in which case we have the diffraction dissociation[2] of a projectile when it goes through a target like the dissociation of a polarized light through a polarizer. Although some analyses[2] imply the existence of the diffraction dissociation at high energy, we still have no definite answer about how important it is. In this lecture I skip all about diffraction phenomena partly because I don't have enough time and partly because it is the least understood subject in the high energy scattering.

Summarizing this section let me make the following picture table to classify the high energy collisions:

I		peripheral collision	○ two body reaction ○ a part of 3∿4 body inelastic
II		shadow scattering diffraction dissociation	○ elastic ○ elastic ○ zero quantum number exchange ($\pi \to 3\pi$ etc.) reaction
III		non peripheral collision { pionization { fragmentation	○ most inelastic collisions

III. Peripheral collision (non diffractive two body scattering)

I will talk about two body collisions in this section. They are the most extensively studied subject in the high energy phenomena and it is through the investigation of these two body reactions that most of the ideas of high energy collisions were brought in.

Energy dependence of the peripheral cross section

To simplify the situation let us first consider the elastic scattering of a scalar particle. The partial wave expansion of the Feynman amplitude F(s, t) reads;

$$F(s, t) = 16\pi \sum_{\ell=0}^{\infty} (2\ell+1) a_\ell(s) P_\ell(\cos\theta) \quad . \tag{1}$$

Now suppose the scattering occurs only peripherally, we have;

$$\left.\begin{array}{lll} a_\ell \neq 0 & \text{for} & \frac{b\sqrt{s}}{2} + A \leq \ell \leq \frac{b\sqrt{s}}{2} + B \quad , \\ a_\ell = 0 & & \text{otherwise} \quad , \end{array}\right\} \tag{2}$$

where b is the impact parameter and A and B are constants. In general A and B can depend on the energy $\sqrt{s}$. If we assume they are independent of $\sqrt{s}$ we get

$$\begin{aligned} |F(s, o)| &\leq 16\pi \sum_{\ell=b\sqrt{s}+A}^{b\sqrt{s}+B} (2\ell+1)|a_\ell(s)| \quad , \\ &\leq c\sqrt{s} \quad , \end{aligned} \tag{3}$$

where we have used the unitarify condition

$$|a_\ell(s)| \leq 1 \quad . \tag{4}$$

In the actual case we have to subtract out the contribution of the diffraction. This can be done in the following way. Let us consider the amplitude $F^{\pi^+\pi^0}(s, t) - F^{\pi^-\pi^0}(s, t)$. Then the contribution of the diffraction scattering (Pomeron exchange) is cancelled because we believe the isospin cannot be exchanged in the diffraction. We have only the contribution from the peripheral partial waves for this amplitude

$$|F^{\pi\ \pi}(s, 0) - F^{\pi\ \pi}(s, 0)| \leq \sum_{b\sqrt{s}+A}^{b\sqrt{s}+B} (2\ell+1)(|a_\ell^+| + |a_\ell^-|) \quad ,$$

$$\leq c'\sqrt{s} \quad . \tag{5}$$

If we go to the Regge pole description of the peripheral scattering we get,

$$\alpha_R(0) \leq 1/2 \quad , \tag{6}$$

for all the Regge trajectories. Fig. 2 and Fig. 3 show the validity of the equation (6). From Fig. 3, we see that the baryon trajectories seem to satisfy,

$$\alpha_B(0) \leq 0 \quad . \tag{6'}$$

Presumably this occurs because of the cancellation between the odd and the even partial waves ($\cos\theta = -1$). In case of boson trajectories we have $\alpha_{\rho,A_2}(0) \sim \alpha_{\omega,f}(0) \sim 1/2$ from Fig. 2. Other trajectories stay lower. This may imply either that the K^* exchange processes for example deviate from the pure peripherality or that some other contributions like the absorption cut takes them back to peripherality. Actually even for the ρ-exchange processes it is known that the pure Regge pole model does not work at least for the helicity non-flip amplitude. To see this situation let us go to the next subsection.

t-dependence of the peripheral collision

The peripheral model has been studied mostly in connection with the angular dependence of the scattering.[8] As I stated in the section (II) the peripherality covers the whole range ($-1 \leq \cos\theta \leq 1$) of the scattering angle. I believe, therefore, that a simple function can describe the whole angular dependence of the peripheral collision.[6] But here I restrict myself to the narrow forward region and look for the evidence for the peripherality.[5],[8] We restrict ourselves for the time being to the discussions of πN scattering.

Let us first look at the $\pi^- p \to \pi^0 n$ process. This is a nice process to consider since we have no contamination from the diffraction and we expect the helicity non flip amplitude to be small because of the forward dip in the cross section (Fig. 4).

We have

$$\frac{d\sigma}{dt} = \frac{1}{64\pi s q^2} \sum_{\lambda,\lambda'} |F_{\lambda,\lambda'}(s, t)|^2 \quad , \qquad (7)$$

where q is the center of mass momentum and Σ means the spin summation and the average and

$$F_{\lambda,\lambda'}(s, t) = 16\pi \sum_{J=M}^{\infty} (2J+1) A^J_{\lambda,\lambda'}(t) d^J_{\lambda\lambda'}(\cos\theta) \quad , \qquad (8)$$

where λ = initial proton helicity,

λ' = final neutron helicity

and $M = \max\{|\lambda|, |\lambda'|\}$.

Suppose we consider only the helicity flip amplitude $(\lambda, \lambda') = (+, -)$ and restrict the summation over J to the range $\frac{b\sqrt{s}}{2} + A \leq J \leq \frac{b\sqrt{s}}{2} + B$ and take $s \to$ large fixing t, we get[5]

$$\frac{d\sigma}{dt} \propto J_1^2(b\sqrt{-t}) + \text{nonflip} \quad , \qquad (9)$$

where J_1 is the usual Bessel function. For the value of $b \sim 1$ Fermi^{-1} we conclude that $\frac{d\sigma}{dt}$ should have a dip at $t \sim -0.5$. The experiment is consistant with this prediction. Some other processes which have a similar dip are,

$\pi^+ p \to \pi^0 \Delta^{++}$, (fig. 5)

$\pi^- p \to K^0 \Lambda$,

$$\pi^+ p \rightarrow K^+ \Sigma^+ \quad ,$$

$$\pi^- p \rightarrow K^0 \Sigma^0 \qquad\qquad \text{etc.}$$

Regge pole model also predicts this dip since we have

$$F_{+-} \propto \alpha_p(t) \frac{1 - e^{-i\pi\alpha_p(t)}}{\sin \pi\alpha_p(t)} s^{\alpha_p(t)} \quad , \tag{10}$$

where the $\alpha_p(t)$ in front is the non-sense factor.[9] At $\alpha_p(t) = 0$ $(t \sim -0.5)$ both ReF_{+-} and ImF_{+-} vanish. Thus the peripheral model and the Regge pole model seems to be consistent here. But if we look more carefully we immediately notice that we are not so fortunate.

1. If we take $J_1(b\sqrt{-t})$ both for real and the imaginary parts of F_{+-}, ReF_{+-} has a simple zero at $t \sim -0.5$, whereas ReF_{+-} in the Regge pole model has the double pole at this point as is clear from the equation (10). The famous mirror symmetry of the $\pi^{\pm}p$ polarization tells us that (10) is better fig.6 e.s. the Regge model is good here:

$$P^{\pi\pm p} \propto \frac{1}{\frac{d\sigma^{\pi\pm p}}{d\Omega}} \text{Im} \left(F_{++}^{I_t=0} \pm F_{++}^{I_t=1}\right)^* \left(F_{+-}^{I_t=0} \pm F_{+-}^{I_t=1}\right)$$

$$\propto \; ReF_{+-}^{I_t=1} \tag{11}$$

where I_t is the isospin in the t-channel. The equation (11) comes because F_{++} is dominated by the Pomeron and F_{+-} is dominated by $I_t = 1$ part. This fact clearly tells us the simple Bessel function model does not work. Yet this does not imply the failure of the entire peripheral model.

2. Although for the helicity flip amplitude in the forward region the simple Regge pole model seems better than the naive peripheral model, this is no longer true for the helicity non flip πN scattering. This is demonstrated in the famous cross over of the $\pi^- p$ and $\pi^+ p$ differential cross section at

$t \simeq -0.2$;

$$\frac{d\sigma^+}{dt} - \frac{d\sigma^-}{dt} \propto \left|F_{++}^{I=0} + F_{++}^{I=1}\right|^2 - \left|F_{++}^{I=0} - F_{++}^{I=1}\right|^2$$

$$+ \text{ (helicity flip part)}$$

$$= 2\mathrm{Re}F_{++}^{I=0} F_{++}^{I=1\,*} \qquad . \qquad (12)$$

This follows because of the large pomeron contribution to $F_{++}^{I=0}$. If we take the simple Regge pole model (ρ-exchange) $\mathrm{Im}F_{++}^{I=1}$ vanishes at $t = -0.5$ instead of $t = -0.2$ assuming the non-sense choosing mechanism (the same expression equation (10) for $F_{++}^{I=0}$). On the other hand the simple peripheral model gives zero at $t \sim -0.2$ since $F_{++}^{I=0}$ is proportional to $J_0(b\sqrt{-t})$.

There have been many attempts within the Regge exchange model to save this situation. A possible phenomenological approach will of course be to assume the sense choosing for ρ at $\alpha_\rho = 0$ and give the accidental 0 at $t \simeq -0.2$.(10) Another approach was to introduce cuts or the absorption effect to modify the Regge pole model. There were or still are two popular ideas on the cut model.

(A) Weak cut model(11)

Here the non-sense zero is shifted to $t \sim -0.2$ by the cut.

(B) Strong cut model(12)

Cross over zero is caused because of the cancellation of a pole and a cut.

All these models were more or less successful if we consider only the cross over zero.

3. For the real part as well as for the imaginary part the peripheral model claims the form $F_{\Delta\lambda} \propto J_{\Delta\lambda}(b\sqrt{-t})$. It is not very nice for $\Delta\lambda = 1$ as we saw above. What is the situation for $\Delta\lambda = 0$? And what is the

prediction of the above cut models?

Both of the cut models essentially correspond to the absorption diagram of fig. 7 . We have, therefore,

$$F_{cut} \propto i\int F_p(s, t')F_R(s, t'')d\phi(t', t'') \quad (13)$$

where F_p is the Pomeron amplitude and F_R is the Regge amplitude $d\phi$ is an appropriate integration in the intermediate state. You can understand 'i' in front of the integration in the following way.

$$S = 1 + iT$$

$$T = S_pT_RS_p^+$$

$$= (1 + iT_p)T_R(1 - iT_p)$$

$$= T_R + i(T_pT_R - T_RT_p) \quad (14)$$

T_R ↓ Regge pole; $i(T_pT_R - T_RT_p)$ → absorptive cut

The essential point of the euqatuon (13) is that the zero point in the real part and the imaginary part is shifted in the same manner because the F_p is almost pure imaginary near the forward direction. Since we must have zero at smaller value of $|t|$ than in the simple Regge model for the imaginary part we see that the real part also has zero at more forward position than the simple Regge pole model. The detailed analysis shows that the zero of the real part comes even more forward than that of the imaginary part. Actually the strong cut model claims $F_{\Delta\lambda} \propto J_{\Delta\lambda}(b\sqrt{-t})$ just as in the peripheral model. Let us look at the experiment. The best place is the πN charge exchange polarization. (fig. 8)

$$P^{C.E.} \propto \mathrm{Im}\, F_{++}^{I_t=1} F_{+-}^{I_t=1\,*} \quad (14)$$

The Bessel function model says the polarization should vanish both at $t = -0.2$ and $t = -0.5$. Although the experiment is not yet conclusive it seems unlikely that it has two zeros in the forward region. Weak cut model gives somewhat different value and it still has a hope. If we look at the amplitude at 6 GeV/c constructed by Halzen and Michel (13) or $\bar{K}N$ amplitude of Fukugita and Inami (14) there seems to be no zero in the $F_{++}^{I=1}$ more forward than that of $\mathrm{Im}\, F_{++}^{I=1}$.

Thus the prospect does not seem to be bright for any of the above models. Yet if you allow me to tell my own prejudice I feel that the most important is the peripherality. First of all it is very intuitive. Secondly the angular momentum plane seems to be rather complicated with poles and cuts colliding and merging in all the places. Thirdly in the peripheral model there is a hope to describe the amplitude by a simple function for the whole range of the scattering angle[(6)] not just the forward or the backward region separately.

Let me skip the discussion of the backward scattering which is rather poorly described in terms of poles and cuts and so far has not been much discussed from the peripheral point of view.

exoticity and exchange degeneracy

One of the most important dynamical property of the hadronic reaction is the correlation between the resonances in different channels.[(15)] Let me start with the Froissart-Gribov formula for the scattering of a scalar particle:

$$a_J^{\pm}(t) = \int Q_J(z)\,[A_s(t,\,z) \pm A_u(t,\,z)]dz \quad , \qquad (15)$$

where A_s and A_t represent the s-channel and u-channel forces respectively. In the absence of the exchange force ($A_u \equiv 0$) we have $a_J^+ = a_J^-$ and this is called the exchange degeneracy. In this case we have even and odd trajectories degenerate: $\alpha^+(t) = \alpha^-(t)$. Strictly speaking for certain trajectories $\alpha^+ \equiv \alpha^-$ can be realized without having the regorous equality $a_J^+ \equiv a_J^-$. All we need is the absence of the exchange forces which are relevant to make the bound states.

In a relativistic theory this situation is not realised in general. There should be some deep reason for this if it happens. Actually we have $\alpha_\rho = \alpha_{A_2}$, $\alpha_\omega = \alpha_f$, $\alpha_{K^*} = \alpha_{K^{**}}$ etc. Simple explanation was given in terms of the exoticity and the pole-pole duality: Although the exoticity is not well defined yet, at least for meson-meson channels and for meson-baryon channels we can say that all the channels other than those made from $q\bar{q}$ (meson-meson case) and qqq (meson-baryon case) are exotic i.e. have no resonances. The pole-pole duality says the forces A_s and A_u are dominated by the resonances in S and u channels respectively. If we consider K^-p scattering for example $A_u \equiv 0$ and we have the degeneracy of ρ and A_2 trajectories.

At the present stage it is not clear how we should formulate the pole-pole duality. The trouble comes from the fact that most of the scattering amplitudes are not dominated by Regge poles at high energy. The crossed channel angular momentum plane seems to have a rather complicated structure except for a few cases. The finite energy sum rules or the dual resonance model in its simplest form cannot be directly used without a serious modification due to cuts. More over the pole-pole duality itself may not be a good approximation for some cases. For example K^* and K^{**} trajectories seem to be rather different from each other (fig. 2).

Following line reversed reactions are frequently discussed in connection with the exchange degeneracy:

$$\sigma(K^-p \to \bar{K}^0 n) = \sum_i \left| \beta_i \frac{e^{-i\pi\alpha_\rho}}{\sin \pi\alpha_\rho} S^{\alpha_\rho} \right|^2 , \tag{15a}$$

and

$$\sigma(K^+n \to K^0 p) = \sum_i \left| \beta_i \frac{1}{\sin \pi\alpha_\rho} S^{\alpha_\rho} \right|^2 . \tag{15b}$$

These two cross sections should be equal at least approximately because we neglected the possible contribution of the cuts which may be large in the helicity non-flip part. Another example which probably shows the violation of the exchange degeneracy is: (fig.10)

$$\sigma(\pi^+p \to K^+\Sigma^+) = \sum_i \left| \beta_{1i} \frac{1-e^{-i\pi\alpha_{K*}}}{\sin \pi\alpha_{K*}} + \beta_{2i} \frac{1+e^{-i\pi\alpha_{K**}}}{\sin \pi\alpha_{K**}} \right|^2 , \qquad (16a)$$

$$\sigma(K^-p \to \pi^-\Sigma^+) = \sum_i \left| -\beta_{1i} \frac{1-e^{-i\pi\alpha_{K*}}}{\sin \pi\alpha_{K*}} + \beta_{2i} \frac{1+e^{-i\pi\alpha_{K**}}}{\sin \pi\alpha_{K**}} \right|^2 . \qquad (16b)$$

These two are equal if $\alpha_{K*} \equiv \alpha_{K**}$.

The most pesimistic view point of the whole discussion concerning the exchange degeneracy will be that it means not more than the relative smallness of A_u (absorptive part of the amplitude in the u channel which is exotic) compared with the other absorptive parts. If that is the case it would not be very promising to take it so seriousely and regard it as a dynamical principle.

So much for the two body peripheral reactions and let us now turn to the inelastic processes.

IV. Inelastic collision.

In the previous section we discussed the peripheral collision of two particles. The smaller the impact parameter of the collision the less likely the chance of the particles stay in its original form since they are fragile or they are strongly interacting. In other words the process will become inelastic.

Scaling or limiting fragmentation(7)

When several particles are produced in the collision it is possible that the dominant process still is the peripheral production of resonances.

But if the energy gets higher and higher and the multiplicity also becomes large enough the peripheral resonance production will be no longer the dominant process. The collision will look something like the third picture of the table of section II. In this picture let us go to the rest frame of the particle A. The assumption of the limiting fragmentation says that the production cross section of a particle with the finite energy in this system remains finite in the infinite incident energy limit. For these particles (fragments of A) we have the following energy momentum conservation law:

$$E_B + M_A = \sum E_C^A + \sum E_C^R \quad , \tag{17a}$$

and

$$P_B = \sum P_{C\parallel}^A + \sum P_{C\parallel}^R \quad , \tag{17b}$$

where the first sum is taken over the fragments of A and the second sum is over the rest. In the high energy limit E_C^A remains finite and E_C^R becomes large. Subtracting (17b) from (17a) and using the fact the $E_B - P_B \to 0(\frac{1}{s})$, $E_C^R - P_C^R \to 0(\frac{1}{s})$ and the number of terms in the second term is restricted by $c\sqrt{s}$ we obtain,

$$\left.\begin{aligned} &\sum x_C^A = 1 \ , \\ \text{where} \qquad &x_C^A = \frac{E_C^A - P_{C\parallel}^A}{M_A} \ . \end{aligned}\right\} \tag{18}$$

The transeverse momentum of a produced particle will be finite because of the Heisenberg's uncertainty principle combined with the fragility of the particle and/or because of the finite temprature in the collision. The pionization products come from a fire ball-like object with a large mass ($\sim \sqrt{s}$). The level density of this object will be very large and we may describe it in terms of a statistical model. But the fragmentation part does not seem to allow the statistical description. In this case the small $P_\perp$ will be attributed to the uncertain principle and to the fact that the particle has no hard core in it.

In two body collisions the mere fact that they are peripheral will probably be enough to determine the angular distribution of the scattered particle at least qualitatively. While in the inelastic scattering the distribution functions will reflect the structure of the initial particles. At high energy the distribution of the fragments is independent of what particle hit the original particle of the fragments. All that matters is the fact that the particle was hit and fragmented i.e. excited in a definite manner.

Elementary kinematics of inclusive reactions

The difinition of the inclusive reaction is:[16]

$$a + b \to c + \text{'anything'} \quad ,$$

where 'anything' means that no measurement is done for those particles in it. 'C' can contain any number of definite particles. We have,

C	experiment
null	total cross section
1	single particle distribution
2	two particle distribution
.	.
.	.
.	.

For the single particle distribution function we have,

$$\frac{d\sigma^C}{d^3K^C} = \frac{1}{VTv_r} \sum_n |\langle n|a^C_{out}(K^C)|i\rangle|^2 \quad , \tag{19}$$

where K^C is the momentum of C, v_r is the relative velocity of a and b (initial particles) and we have taken the normalization

$$\langle P|P'\rangle = (2\pi)^3\delta(P-P') \quad . \tag{20}$$

If we integrate over the momentum K^C we get

$$\sigma^C \equiv \int \frac{d\sigma^C}{d^3K^C} d^3K^C = \sum n_C\sigma(n_C) \equiv \langle n_C\rangle \sigma_T \quad , \tag{21}$$

where n_c is the multiplicity of c and $\sigma(n_c)$ is the inclusive cross section where c particles are produced with the definite number n_c. The invariant distribution function is defined:

$$\frac{d\sigma^c}{d^3K^c} = \frac{1}{K_0^c} f\,(S,\, K^c) \quad , \tag{22}$$

where S is the center of mass energy squared. Suppose c is a fragment of 'a' for convenience. The assumption of limiting fragmentation says:

$$\ell_{S\to\infty}\ f(S,\, K^c) = f(x^c,\, p^c) \tag{23}$$

where x^c is defined in (18). In the center of mass system the variable x^c becomes

$$x^c = \frac{2K^*_{c/\!/}}{\sqrt{S}} \quad (16) \quad , \tag{24}$$

since

$$K_{c/\!/} = \gamma(K^*_{c/\!/} - \beta K^*_{c0})$$

and

$$K_{c0} = \gamma(K^*_{c0} - \beta K^*_{c/\!/}) \quad , \tag{25}$$

where the star symbol is put to denote the center of mass quantity and $\gamma = 2M_a/\sqrt{S}$. In terms of the variables x^c and p^c we have

$$\frac{d\sigma^c}{dxdp^2} \overset{S\to\infty}{\cong} \pi \frac{f(x^c,\, p^c)}{\sqrt{x^2 + \dfrac{4(m_c^2 + p_c^2)}{S}}} \quad . \tag{26}$$

From the equation (21) we get for the multiplicity,

$$\langle n_c \rangle = \frac{\pi}{\sigma_T^{ab}} \left(\int dK_\perp^{c2} f(K_\perp^c,\, 0)\right) \log S + \text{const} \quad , \tag{27}$$

where we have assumed that $f(K^c,\, 0)$ is finite. From the fig.11 we cannot exclude $S^{\frac{1}{4}} \sim S^{\frac{1}{2}}$ dependence of $\langle n_c \rangle$. Regge pole theory of Mueller (17) predicts at high energy

$$\langle n_c \rangle = \langle n_{\bar{c}} \rangle \quad , \qquad (28)$$

where $\bar{c}$ is the antiparticle of c. Fig. 12 shows that this is not yet reached at 16 GeV/c where

$$\frac{n_{\pi^+}}{n_{\pi^-}} \simeq 2 \quad .$$

At I.S.R. energy (28) looks good even for the proton and the anti-proton (fig. 13 & 14)At this energy we may say that the limiting fragmentation has been realized for the whole range of x. For larger values of x the scaling seems to be reached even at lower energy. Moreover the independence of the target fragmentation on the projectile particles is also evident (fig. 17). In the fig. 17 only the reaction $\pi^- p \to \pi^- +$ 'anything' seems to be defferent from the rest. This was explained by Chan et al.:[(23)] Scaling is reached quickly when the $ab\bar{c}$ channel is 'exotic'. And it was claimed that all the channels in fig. 17 except for $\pi^- p\pi^+$ are exotic. Yet it is by no means a clearly understood matter which three body channel is exotic and why the scaling is reach quickly when $ab\bar{c}$ is exotic. There are some other proposals for the criterion of the fast reaching to the scaling limit.[(23)]

Let us discuss next the bumps in the x dependence of the distribution (fig. 12). In the reaction

$$\pi + p \to \pi + 'x' \quad ,$$

suppose x is a resonance. The pion emitted with it has the x value of

$$x = 1 - \frac{M_R^2}{S} \xrightarrow{S\to\infty} 1 \quad , \qquad \text{since}$$

$$\begin{aligned} M^2 = (p_\pi + p_p - K_\pi)^2 &= S + m_\pi^2 - 2\sqrt{S}\, K^*_{\pi 0} \quad , \\ &= (1 - x)S \quad , \qquad (29) \end{aligned}$$

where p_π and p_k are initial π and p momenta and K_π is the emitted momentum. This shows that the bump moves away to the edge as the energy increases as is supported experimentally. This is a peripheral process which is not essentially inelastic according to our classification.

Another interesting kinematical phenomenon related to the resonance production was found by Yen and Barger.[18] They explained the sharp peak in the $p_\perp$ distribution of pion (fig. 15) in terms of the resonance production. For the detail see the reference 18.

To discuss the two and more particle distribution functions it is convenient to introduce the following functional:

$$F(f) = \frac{1}{VTv_r} \; {}_{in}\langle i| \, T^{+} e^{\sum_c \int f_c(K^c) n_c^{in}(K^c) d^3K^c} \, T|i\rangle_{in} \quad , \tag{30}$$

where $n_c^{in}(K) = a_c^{+in}(K) a_c^{in}(K)$ and $S = 1 + iT$. The summation $\sum_c$ is taken over all the hadrons. Clearly we have

$$\frac{d\sigma}{d^3K^c} = \frac{\delta F}{\delta f_c(K^c)} \Bigg|_{\text{all } f \equiv 0} \quad . \tag{31}$$

And

$$\frac{\delta^2 F}{\delta f_0(K^c)\delta f_d(K^d)} \Bigg|_{f\equiv 0} = \frac{1}{VTv_r} \langle i|T^{+} n_c(K^c) n_d(K^d) \pi i\rangle$$

$$= \frac{d\sigma}{d^3K^c d^3K^d} - \delta_{cd}(2\pi)^3 \delta(K^c - K^d) \frac{d\sigma}{d^3K^c} \quad . \tag{32}$$

The energy conservation law is obtained using the fact $[T, H] = 0$;

$$\langle i| \; T^{+} e^{\sum_c \int f_c n_c^{in}} [T, H]|i\rangle_{in} = 0 \quad . \tag{32}$$

From this we get

$$E_i F(f) = \sum_c \int \frac{\delta F}{\delta f_c(K^c)} E_c d^3K^c \quad , \tag{33}$$

where E_i is the initial energy and E_c is the energy of the emitted particle c. For any other conserved quantity Q we have the similar form,

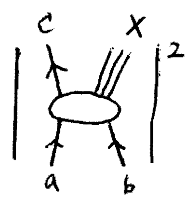
$$Q_1F(f) = \sum_c \int \frac{\delta F}{\delta f_c(K^c)} Q_c d^3K^c \quad . \tag{34}$$

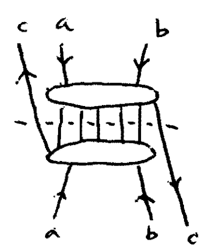
From (33) and (34) we get restrictions for the single particle distributions, two particle correlations, etc. but we will not go into the detail any further.

<u>Regge pole theory of inclusive reactions</u>

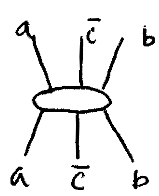
In the high energy two body collision the Regge pole description was sometimes successful and sometimes not. In many cases a simple intuitive peripheral model looks better. Likewise we cannot expect the Regge theory to be almighty in the analysis of inclusive reactions. As was claimed previousely the distribution functions should reflect some sort of structure of the particle: cloud inside the particle. Then cuts in the angular momentum plane seem to play a very inportant role (fig. 16) and I believe we should be able to find a simpler approach to the problem. Here we sketch very roughly works initiated by Mueller.[17] The equation (19) in terms of Feynman diagram looks;

$$\frac{d}{d^3k^c} \equiv \frac{F}{2k^c_0 2p_{a0} 2p_{b0}} = \frac{1}{2k^c_0 VTv} \sum_x \left| \quad \right|^2 \propto$$





$$= \text{Disc}_c \tag{35}$$

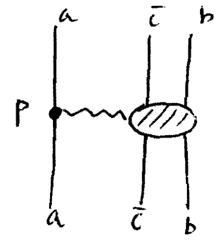


where v is the relative velocity of a and b and $VT \sim (2\pi)^4\delta(0)$ and Disc_c means the dicontinuity of all the connected diagrams. This is because T defined through $S = 1 + iT$ contains the partly connected diagrams.

In the fragmentation region of b where $M^2 = (p_a + p_b - k_c)^2 \to \infty$ and $t = (p_b - k_c)^2$ stays constant we have

$$F = \beta\left(t, \frac{M^2}{s} \right)\left(\frac{M^2}{M_0^2} \right)^{\alpha_P(0)} \quad , \tag{36}$$

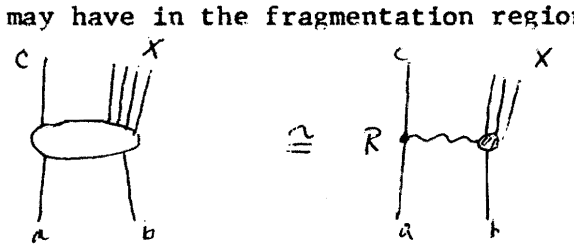
which corresponds to the following diagram



Unfortunately all the information we can get out of this formula is that the pomeron exchange between a and $b\bar{c}$ gives a scaled amplitude at high energy. All the physics which is typical of the inelastic processes is contained in $\beta(t, \frac{M^2}{s})$. We cannot determine this function without certain dynamical consideration.

In a fortunate case (since even in the two body scattering a simple Regge pole model is not necessarily successful) the following possibility exists. In the $M^2 \to \infty$ limit let $\frac{M^2}{s} \to 0$ ($x \to 1$ because of (29)

Then we may have in the fragmentation region of b,

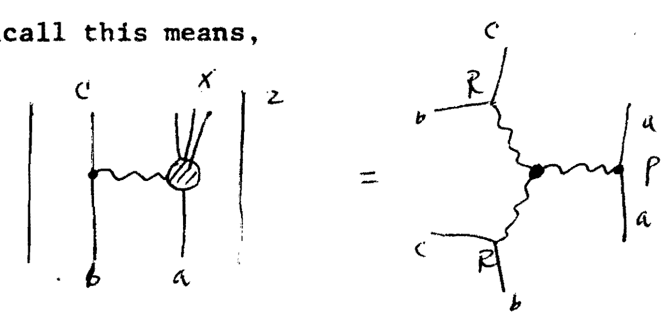


where R is an appropriate Regge pole. Then we get[19]

$$F = \beta(+) \left(\frac{M^2}{M_0^2} \right)^{\alpha_P(0)} \left(\frac{s}{M^2} \right)^{2\alpha_R(t)} , \tag{37}$$

where M^2 in the denominater comes from a group theoretical consideration.

Diagramaticall this means,

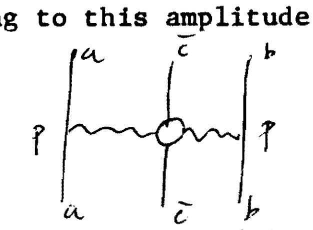


There are many theoretical investigations of this triple Regge vertex which[20] I do not think is suitable to discuss in this lecture.

Finally in the pionization region where $t = (p_b - k_c)^2 \to \infty$, $u = (p_a - k_c)^2 \to \infty$ and $\frac{tu}{M^2} \to$ const we have,

$$F = \beta\left(\frac{tu}{M^2}\right) t^{\alpha_p(0)} u^{\alpha_p(0)} , \tag{38}$$

where it is easy to check that $\frac{tu}{M^2}$ depends only on k_c . Feynman diagram corresponding to this amplitude is



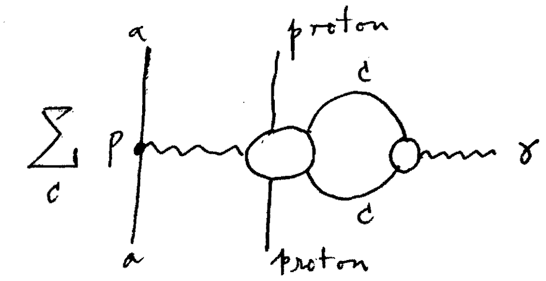
Here again the physics is contained in $\beta\left(\frac{tu}{M^2}\right)$.

IV. Conclusion

I would like to make two remarks in this section. First is concerned with the distribution function in inclusive reactions. Suppose we consider a pion as a fragment of a proton for example. The distribution function will depend on how the pions (necessarily off mass shell) are distributed in the proton before the collision. It will of course depend on the way the collision takes place but in the high energy limit we have seen that it is independent of what projectile is used to fragment the proton. This means that the mechanism of the interaction can be taken into account in a fairely universal manner. As the zeroth approximation we may consider only the energy-momentum transfer. Since the pion distribution inside the proton is not a measurable quantity let us consider instead the charge distribution function;

$$\frac{dQ}{d^3k} = \sum_c Q^c \frac{d\sigma^c}{d^3k^c} , \tag{39}$$

where $\frac{d\sigma^c}{d^3k^c}$ is the inclusive cross section $(p \to c)$. Diagram for (39) is:



where 'a' can be any particle. It is natural to suspect some relationship between this function and the electro-magnetic form factor of the proton. In the above diagram c particle is on the mass shell and γ also is on the mass shell unlike in the case of the electro-magnetic form factor. Therefore the connection is not straight forward. We speulate in the following way. Before the collision the charge is distributed according to the form factor $F(q^2)$. After the collision a particle with momentum q will go out with the momentum,

$$q_{\perp}^{L} = q_{\perp} \quad , \tag{40}$$

and

$$q_{11}^{L} = q_{11} + q_0 \quad , \tag{41}$$

where q_0 is the momentum transfer from the projectile. $q_{\perp}^{L}$ and q_{11}^{L} are the transverse and longitudinal momenta of the emitted particle respectively. We have not yet satisfied the energy conservation law,

$$\frac{E^L - q_{11}^L}{M_p} \leq 1 \quad . \tag{42}$$

Those particles left backward after the collision may have come from the core part of the proton which corresponds to $q^2 \to \infty$. Taking into this and (42) we modify (40) and (41) in the following way.

$$q_{\perp}^{L} = q_{\perp} \left\{ 1 - \left(\frac{E^L - q_{11}^L}{M_p} \right)^{\kappa} \right\} \tag{40'}$$

and

$$q_{11}^{L} = (q_{11} + q_0) \left\{ 1 - \left(\frac{E^L - q_{11}^L}{M_p} \right)^{\kappa} \right\} \quad , \tag{41'}$$

where κ is a free parameter. By making κ very large we can make the effect of the factor $(1 - x^\kappa)$ as small as we want unless $x \simeq 1$. Then we have,

$$\frac{dQ}{d^3k} = F(q^2) \quad , \tag{43}$$

where

$$q_\perp = k_\perp /(1 - x^\kappa) \quad , \tag{44a}$$

$$q_\parallel = k_\parallel /(1 - x^\kappa) - q_0 \quad , \tag{44b}$$

and $x = \frac{\sqrt{M^2 + k^2} - k}{M_p}$, M^2 being an appropriate mass.

For the function $F(q^2)$ we may use

$$F(q^2) = \frac{m^4}{(q^2 + m^2)^2} \quad . \tag{45}$$

In this case we have

$$\frac{dQ}{d^3k} \propto \frac{(1 - x^\kappa)^4}{[k_\perp^2 + \{k_\parallel - (1 - x^\kappa)q_0\}^2 + (1 - x^\kappa)^2 m^2]^2} \quad . \tag{46}$$

There are three free parameters κ q_0 and M^2 to be fixed. We have no way of determining these parameters unambiguously. But $q_0 \sim \frac{1}{\tau_{int}} \sim \frac{1}{\gamma_0}$ and $M^2 \approx M_p^2$ will be the reasonable estimation.

Finally let me make a speculation on the possibility of physics in the high energy limit. In the high energy limit both physics and mathematics are expected to be simpler.

I. In the high energy limit only difraction phenomena and inclusive reactions survive and we can forget about the peripheral collisions.

II. In the finite energy we have a very complicated function for measurable quantities. Fox example we will have something like

$$\sigma(S) \sim \sum_{n<S} \frac{(a\log S)^n}{n!} \sqrt{\frac{S - n}{S}} \; S^{-a} \quad . \tag{47}$$

But in the high energy limit we have

$$\underset{S\to\infty}{\ell} \quad \sigma(S) = \text{const} \quad . \tag{48}$$

Of course this is a tremendous simplification.

In a theory where the dimensioned quantity are $\hbar$, c and ℓ like that of Heisenberg's[21] we have

$$\sigma(S) = \ell^2 f\left(\frac{\ell\sqrt{S}}{\hbar c} \right) + \frac{\hbar^2 c^2}{S} g \left(\frac{\ell\sqrt{S}}{\hbar c} \right) \tag{49}$$

Similar expression can be written for $\frac{d\sigma}{d(\cos\,\theta)}$ and other inclusive cross sections. The essential point is that the dimensionless quantity is

$$d = \frac{\ell\sqrt{S}}{\hbar c} \tag{50}$$

In the high energy limit we have

$$\sigma = \ell^2 f(\infty) \quad .$$

Notice that

$$\lim_{S\to\infty} d = \lim_{\hbar\to 0} d \quad . \tag{51}$$

It looks as if we can calculate cross sections in the classical limit. Actually the situation is not that simple. Since in such a theory all the masses of hadrons should be determined in terms of $\frac{\hbar}{\ell c}$ these vanish in the classical limit. We may encounter the infrared divergence in passing to $\hbar = 0$. If may happen that all we need is the most singular infrared divergent diagrams. Another point is that our limit is different from that of W. K. B. limit of Nambu.[22] In the case of Nambu the wave vector instead of the energy momentum was kept fixed when passing to $\hbar = 0$. Thus he reached the low energy limit which is consistent with current algebra. In our case we should keep the momentum fixed. We will be in a completely different world which I do not know how one can reach.

Acknowledgment

I would like to thank J. Arafune, M. Fukugita, Y. Shimizu and K. Takahashi for valuable discussions.

References

1. Near the forward direction the interference with the Coulomb Scattering gives the phase.

   G. Bellettini etal. Phys. Letters 14, 164 (1965)

   This is a very important experiment since this can distinguish various Pomeron models.

   Suppose ReF/ImF → const in the high energy limit $\alpha'_p(0) = 0$
   See also G. G. Beznogikh etal. Phys. Letters 39B, 4?1 (1972)

2. The original paper is

   M. L. Good and W. D. Walker, Phys. Rev. 126, 1857 (1960)

   G. Cohen - Tannoudji, G. L. Kane and C. Quigg
   Nuclear Physics B37, 77 (1972)

   Aachen - Berlin - CERN - London - Vienna
   Collaboration, CERN/D.Ph.II - Phys. 71 - 30 (1971)

3. See the discussions in section III.

4. The serpukov experiment is consistent with $\alpha'_p(0) \sim 0.4$. But at the I.S.R. energy $\alpha'_p(0)$ looks smaller.

   G. G. Beznogikh etal. Phys. Letters, 30B. 274 (1969)

   G. Barbiellini etal. Phys. Letters 39B, 663 (1972)

   V. Amaldi etal. Phys. Letters 36B, 504 (1971)

5. H. Harari Phys. Rev. Letters 26, 1400 (1971)

   H. Harari Ann. Phys. 63, 432 (1971)

   M. Imati, S. Otsuki and F. Toyoda Prog. Theor. Phys. 43, 1105 (1970)

6. T. Kondo, Y. Shimizu and H. Sugawara (To be published)

7. J. Benecke, T. T. Chou, C. N. Yang and E. Yen
   Phys. Rev. 188, 2159 (1969)

8. M. Davier and H. Harari Phys. Letters 35B, 239 (1971)

9. See for example

   P. D. B. Collins Phys. Report, 1c (1971)

10. Y. Barger and R. J. N. Phillips Phys. Rev. 187, 2210 (1969)

11. R. C. Arnold Phys. Rev. 153, 1523 (1967)

R. C. Arnold and M. L. Blackmon Phys. Rev. 176, 2082 (1968)

M. L. Blackmon and G. R. Coldstein Phys. Rev. 179, 1480 (1969)

C. Lovelace Nuclear Phys. B12, 253 (1969)

12. F. Henyey, G. L. Kane, J. Pumplin and M. Ross Phys. Rev. Letters 21, 946 (1968)

F. Henyey, G. L. Kane, J. Pumplin and M. Ross Phys. Rev. 182, 1579 (1969)

M. Ross, F. Henyey and G. L. Kane Nuclear Phys. B23, 269 (1970)

R. L. Kelly, G. L. Kane and F. Henyey Phys. Rev. Letters 24, 1511 (1970)

13. F. Halzen and C. Michael Phys. Letters 36B, 367 (1971)

14. M. Fukugita and T. Inami, University of Tokyo Preprint, UT-140 (1972)

15. R. Dolen, D. Horn and C. Schmidt Phys. Rev. Letters 19, 402 (1967)

16. R. P. Feynman Phys. Rev. Letters 23, 1415 (1969)

17. A. H. Muller Phys. Rev. D2, 2963 (1970)

18. E. Yen and E. L. Barger, Phys. Rev. Letters 24, 695 (1970)

19. C. E. Detar, C. E. Jones, F. E. Low, J. H. Weis and J. E. Young Phys. Rev. Letters 26, 675 (1971)

20. Fox example for Pomeron-Regge- Regge vertex,

H. D. I. Abarbanel, G. F. Chew, M. L. Goldberger and L. M. Saunders, Phys. Rev. Letters 26, 937 (1971)

In the dual model

C. E. Detar, Kyungsik Kang, Chung-I Tan and J. H. Weis, Phys. Rev. D4, 425 (1971)

For more complete references see for example J. H. Weis, 'Singularities in complex angular momentum and helicity' M.I.T. preprint, April (1972)

21. W. Heisenberg 'Introduction to the Unified Field Theory of Elementary Particles' Interscience Publishers, N. Y. (1966)

22. Y. Nambu Phys. Letters 26B, 626 (1968)

23. Chang Hong-Mo, C. S. Hsue, C. Quigg and Jiunn-Ming Wang
Phys. Rev. Letters 26, 672, (1971)

S. H. H. Tye and G. Veneziano, Phys. Letters 38B, 30 (1972)

T. Matsuoka, Prog. Theor. Phys. 47, 1643 (1972)

For more complete references, following review articles are recommended.

Two body collisions :

P. D. B. Collins Physics Report 1C (1971)

C. Lovelace Invited talk at the Cal. Tech. Conference, March, 1971.

D. R. O. Morrison
Repporteur's talk at the Kiev Conference, September, 1970

A. Yokosawa KEK, 72-6, Current two body final states hadronic physics at intermediate energy region Tsukuba, Japan

Inclusive reactions;

E. L. Barger, ANL/HEP 7148 October, 1971.

D. R. O. Morrison CERN/D.Ph. II/PHYS 72-19 June, 1972

W. Kittel CERN/D.Ph. II/PHYS 72-11 March, 1972

O. Czyzewski, Proceedings of the colloquium on Multiparticle Dynamics Helsinki, 1971

W. R. Frazer, L. Ingber, C. H. Mehta

C. H. Poon, D. Silverman, K. Stowe
P. D. Ting and H. J. Yesian Rev. Mod. Phys. 44, 284 (1972)

figure captions

fig. 1. The Suppresion of large angle scattering due to the smaller interaction time.

fig. 2. Chew-Frautschi plot for meson Trajectories;

P. O. B. Collins, Phys. Report 1 (1971)

fig. 3. Chew-Frautschi plot for baryon Trajectories ; Ibid.

fig. 4. $\pi^- p \to \pi^\circ n$ differential cross section; taken from the review article by A.Yokosawa, KEK, 72-6, Current two body final states hadronic physics at intermediate energy region Tsukuba, Japan

fig. 5. differential cross section and spin density matrix for $\pi^+ p \to \pi^\circ \Delta^{++}$: A. Yokosawa, Ibid.

fig. 6. $\pi^\pm p$ polarization :A.Yokosawa, Ibid.

fig. 7. absorption diagram

fig. 8. πN charge exchange polarization : A. Yokosawa, Ibid.

fig. 9. Comparison between $K^+ n \to K^\circ p$ and $K^- p \to \bar{K}^\circ n$; A. Yokosawa, Ibid.

fig.10. Comparison between $\pi^+ p \to K^+ \Sigma^+$ and $K^- p \to \pi^- \Sigma^+$

fig.11. the energy dependence of multiplicity

fig.12. the distribution function for $\pi^+ p \to \pi^\pm$ + 'x' at 8 and 16 GeV/c

Aachen-Berlin-Born-CERN-Cracow-Heidelberg-Warsaw Collaboration

CERN Report CERN/D.Ph. II Phys/71 (27, 6 1971)

fig.13. the distribution of π^+, p and K^+ in p + p

L. G. Ratner etal. Phys. Rev. Letters 27, 68 (1971)

fig.14. the distribution of p from p + p

A. Bertin etal. Phys. Letters 38B, 260 (1972)

fig.15. p distribution of π compared with that of p.

L. G. Ratner etal. Phys. Rev. Letters 27, 68 (1971)

fig.16. emission of a could particle 'C' from 'b' as its fragment.

Final state interaction is expressed as a Regge exchange.

fig. 17. Exoticity and the scaling

taken from SLAC-Berkeley-Tufts Collaboration, K. C. Foffeit etal., SLAC-PUB-1004 and LBL-572

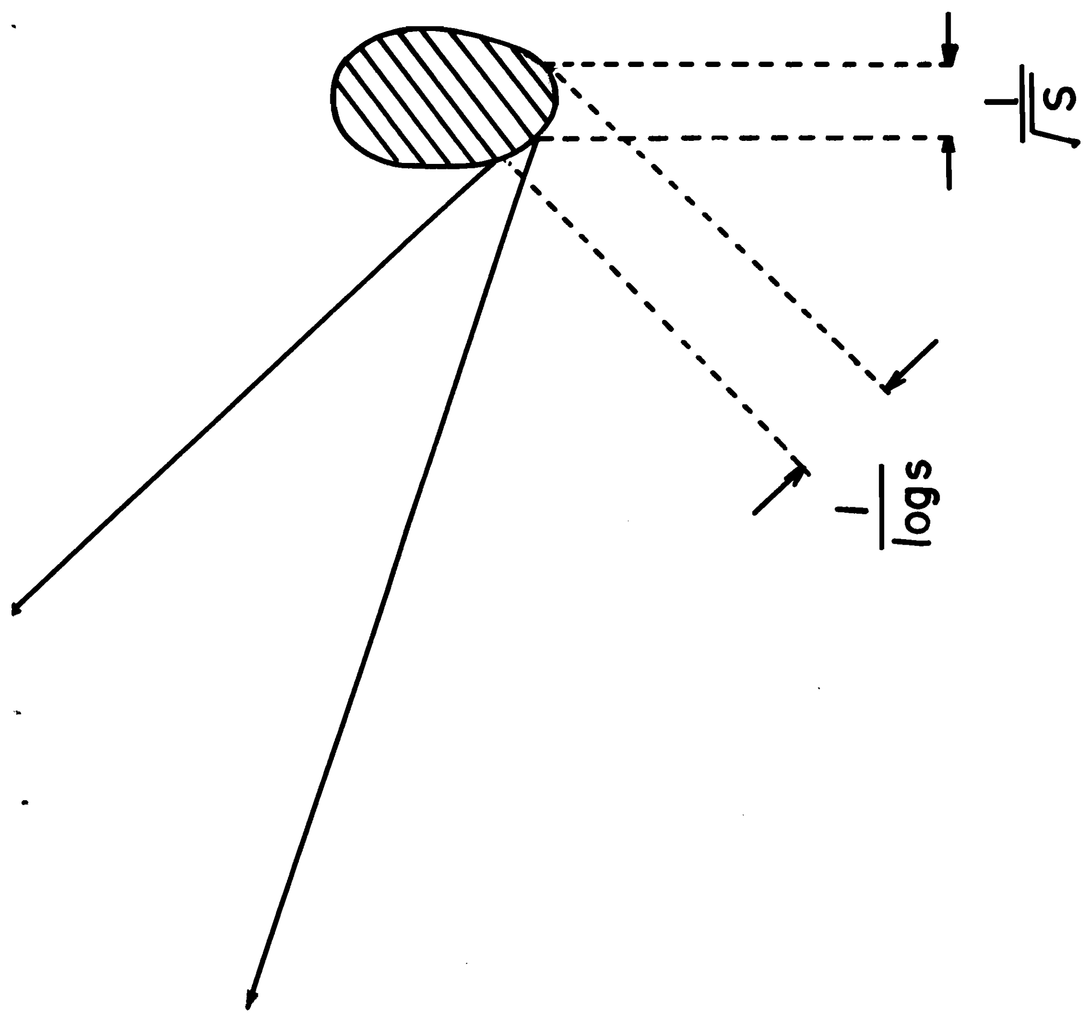


fig. 1

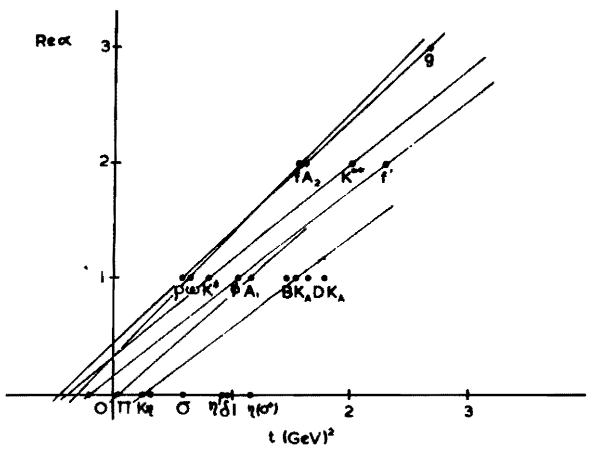
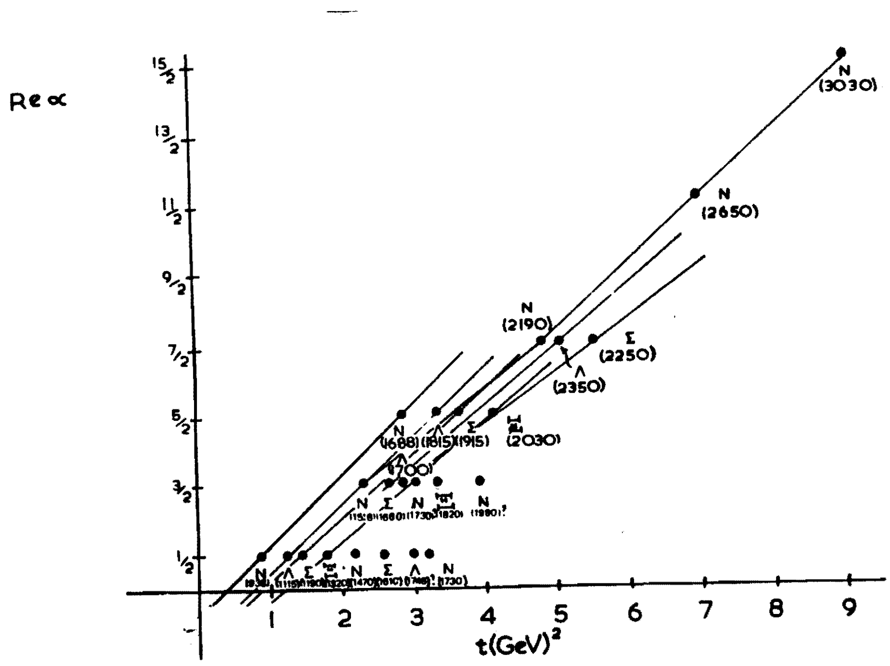
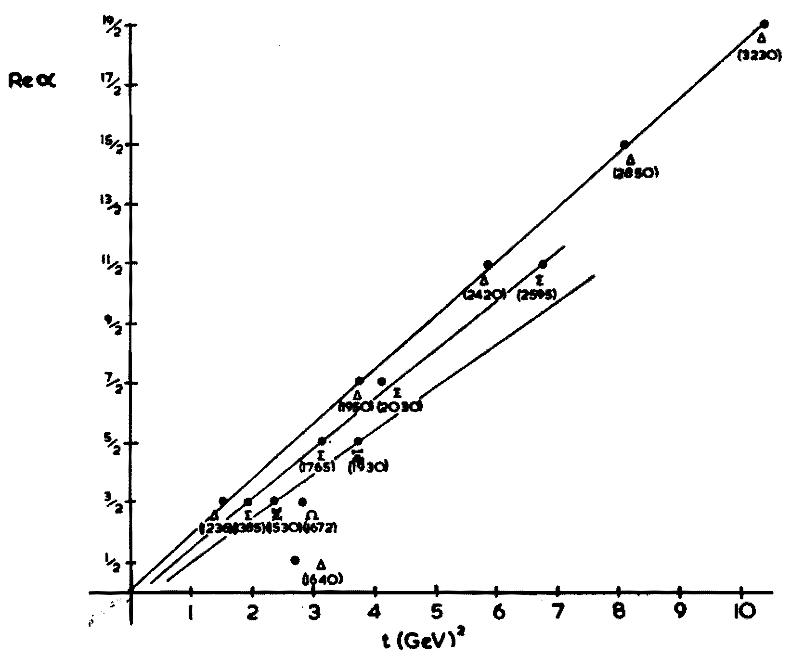


fig. 2



Re α
15/2
13/2
11/2
9/2
7/2
5/2
3/2
1/2
N (3030)
N (2650)
N (2190)
Σ (2250)
Λ (2350)
N (1688)
(1815)
(1915)
(2030)
(1700)
1 2 3 4 5 6 7 8 9
t(GeV)²



Re α
19/2
17/2
15/2
13/2
11/2
9/2
7/2
5/2
3/2
1/2
Δ (3230)
Δ (2850)
Δ (2420)
Σ (2595)
Δ (1950)
Σ (2030)
Σ (1765)
(1930)
Δ (1236)
Σ (1385)
(1530)
Ω (1672)
Δ (1640)
1 2 3 4 5 6 7 8 9 10
t (GeV)²

fig. 3

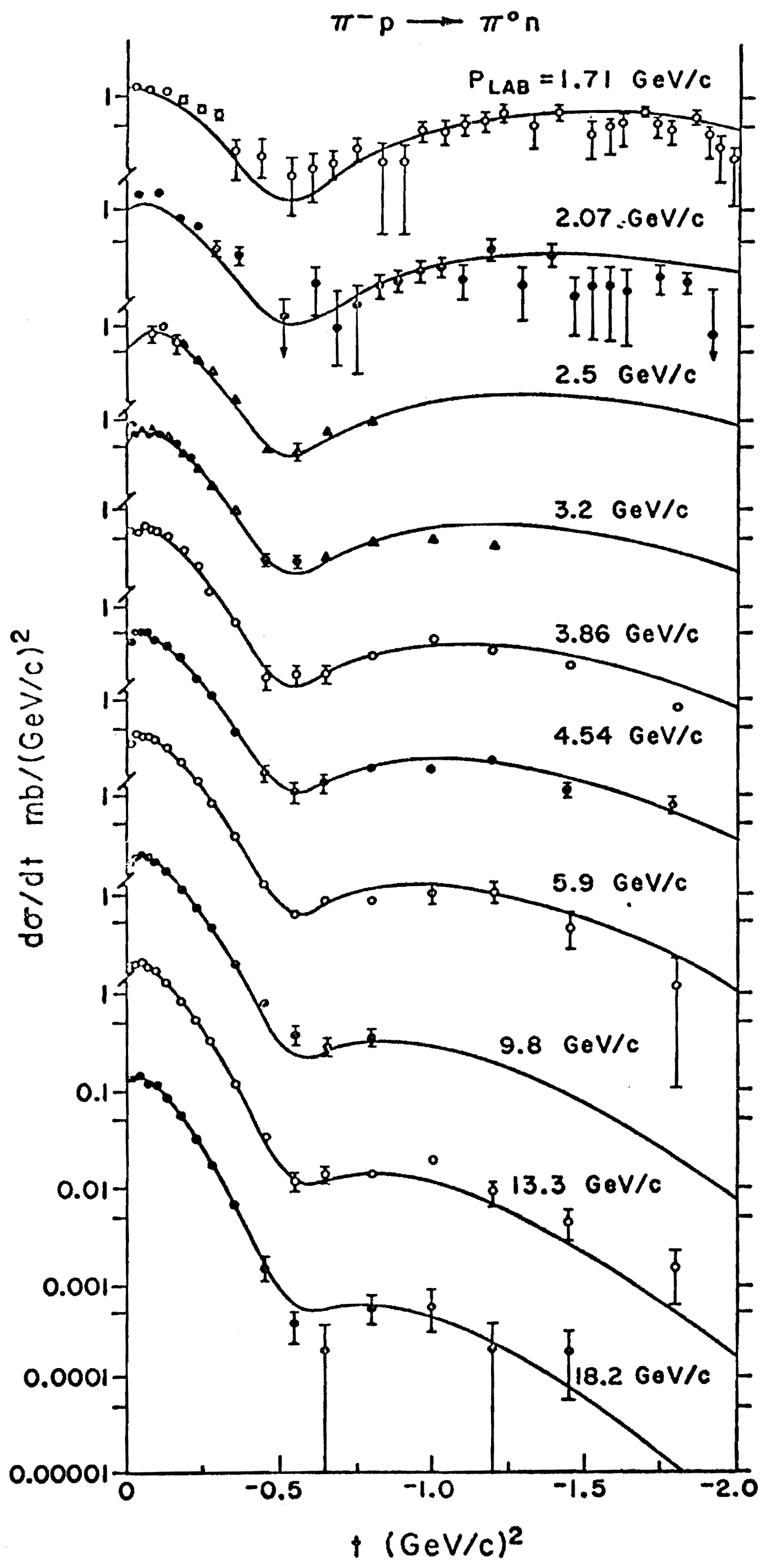
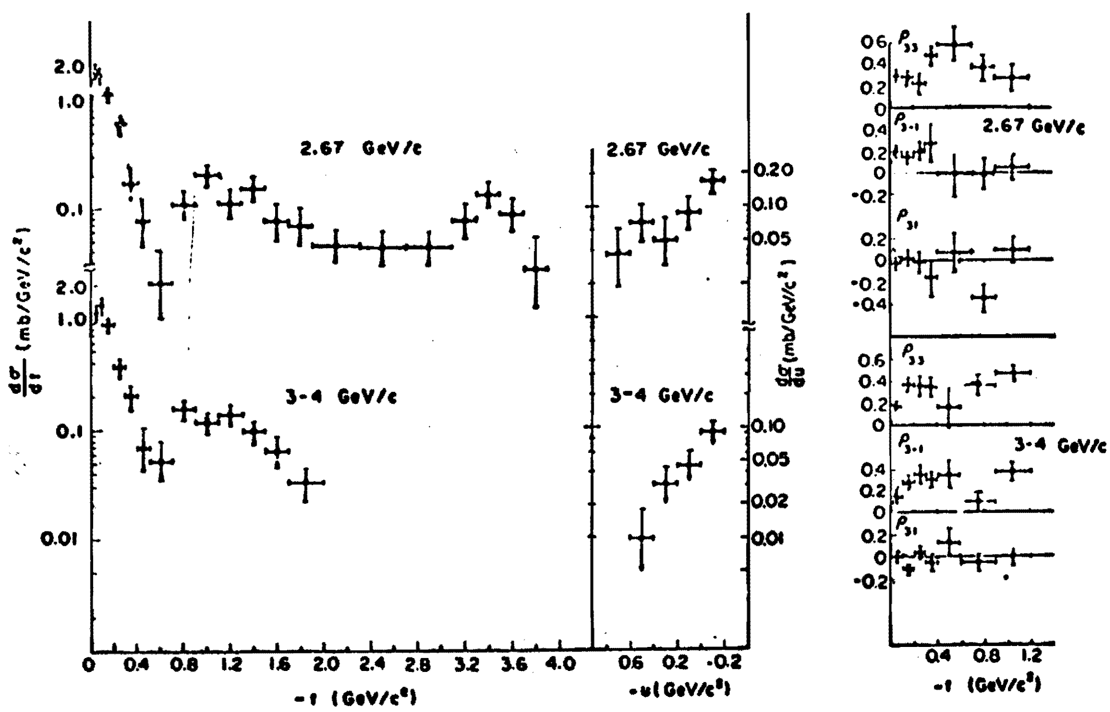
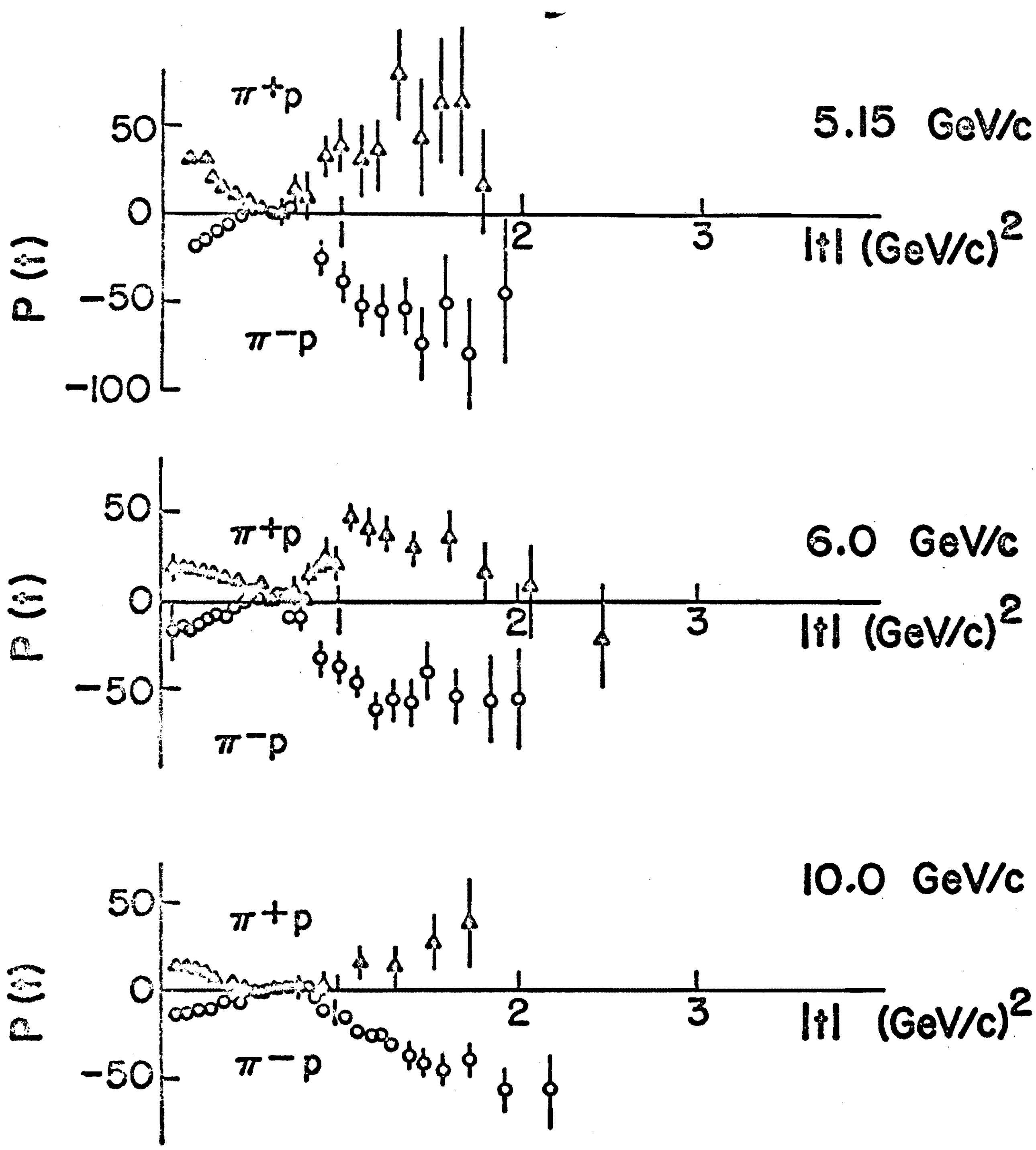

π⁻p ⟶ π°n
P_LAB = 1.71 GeV/c
2.07 GeV/c
2.5 GeV/c
3.2 GeV/c
3.86 GeV/c
4.54 GeV/c
5.9 GeV/c
9.8 GeV/c
13.3 GeV/c
18.2 GeV/c
dσ/dt mb/(GeV/c)²
1
0.1
0.01
0.001
0.0001
0.00001
0
-0.5
-1.0
-1.5
-2.0
t (GeV/c)²

fig. 4



2.67 GeV/c
3-4 GeV/c
$\frac{d\sigma}{dt}$ (mb/GeV/c²)
$\frac{d\sigma}{du}$ (mb/GeV/c²)
-t (GeV/c²)
-u (GeV/c²)
ρ33
ρ3-1
ρ31

fig. 5



50
0
-50
-100
P (↑)
π⁺p
π⁻p
5.15 GeV/c
1
2
3
|t| (GeV/c)²
50
0
-50
P (↑)
π⁺p
π⁻p
6.0 GeV/c
1
2
3
|t| (GeV/c)²
50
0
-50
P (↑)
π⁺p
π⁻p
10.0 GeV/c
2
3
|t| (GeV/c)²

fig. 6

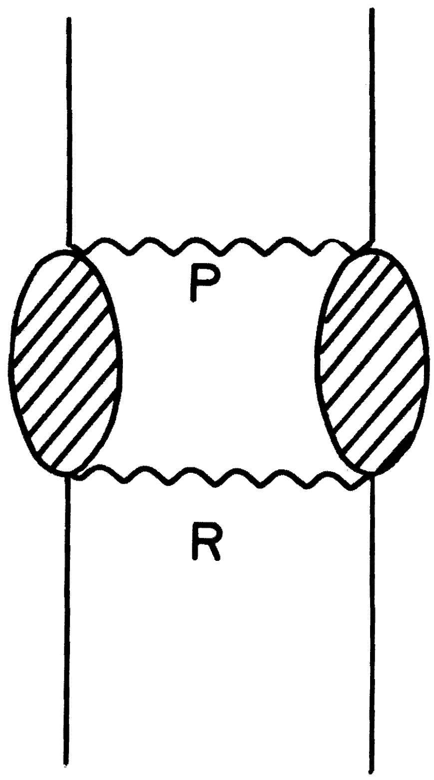


fig.7

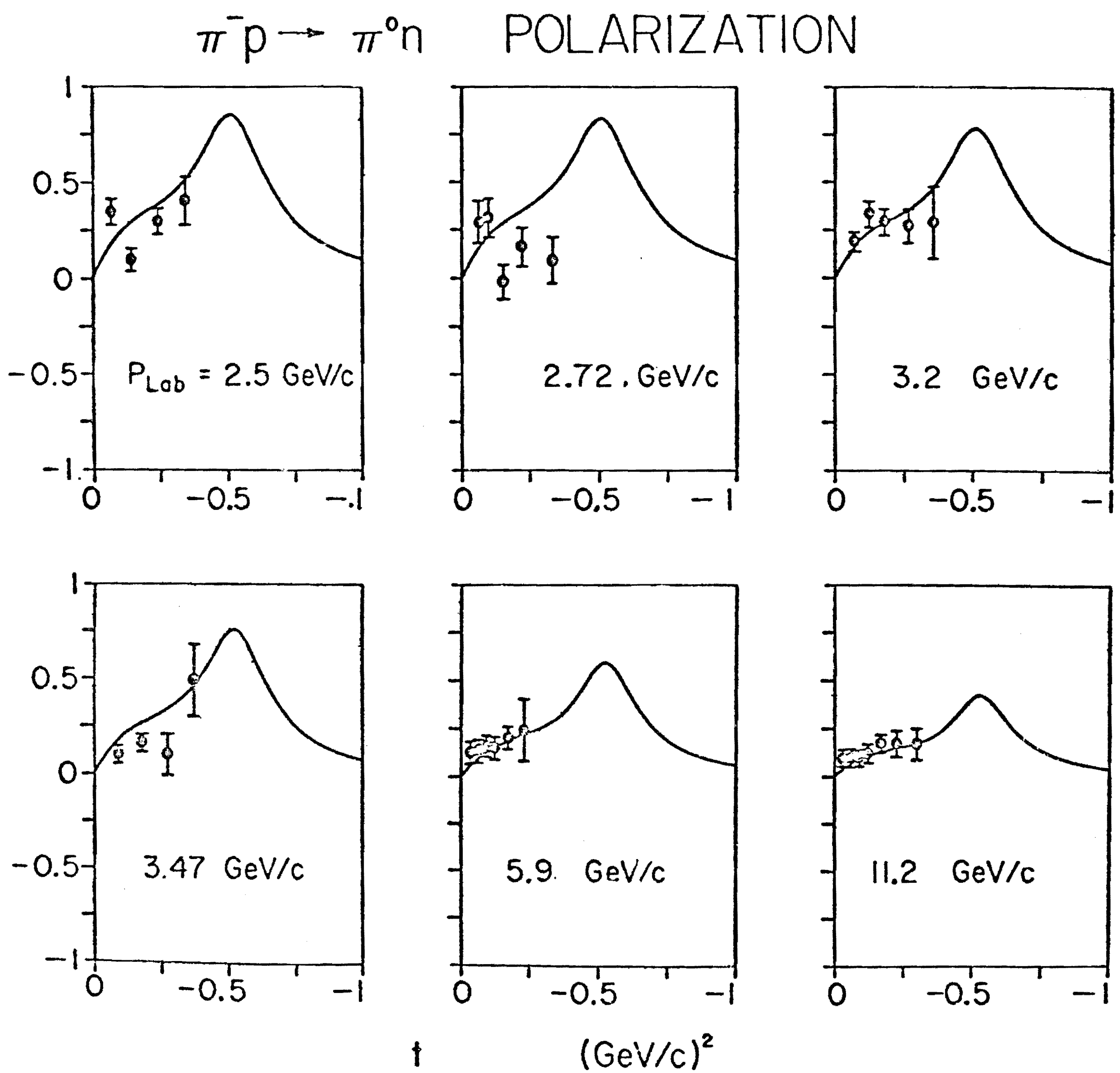
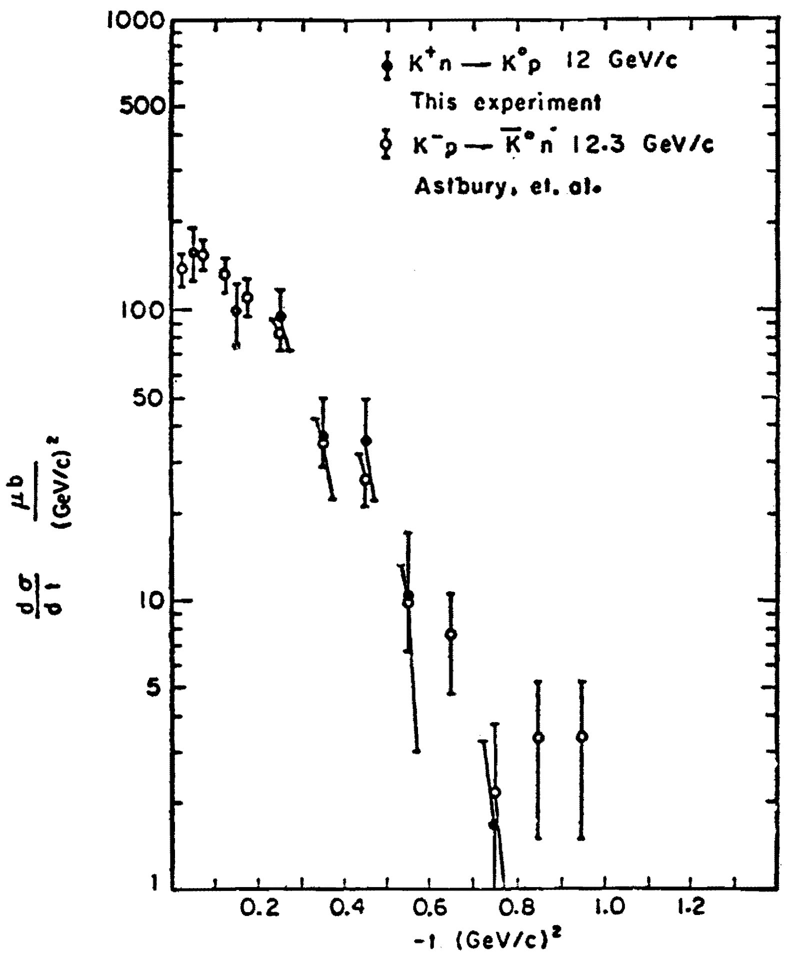
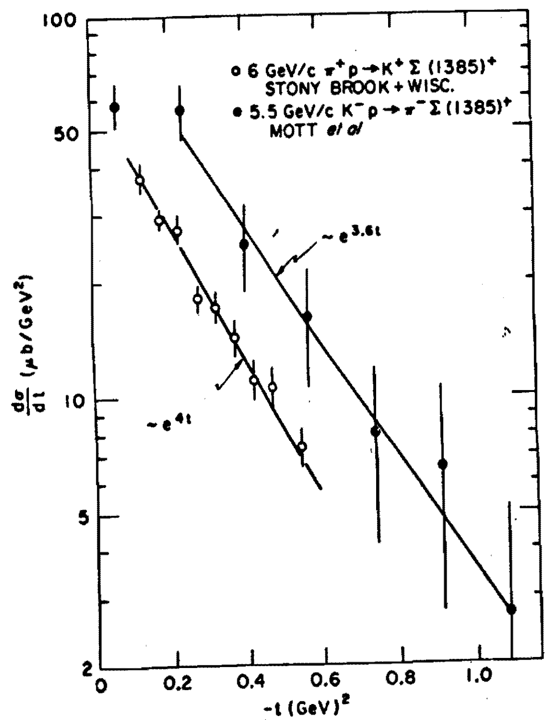

π⁻p → π°n POLARIZATION
1
0.5
0
-0.5
-1
0
-0.5
-1
P_Lab = 2.5 GeV/c
2.72 GeV/c
3.2 GeV/c
3.47 GeV/c
5.9 GeV/c
11.2 GeV/c
t
(GeV/c)²

fig. 8

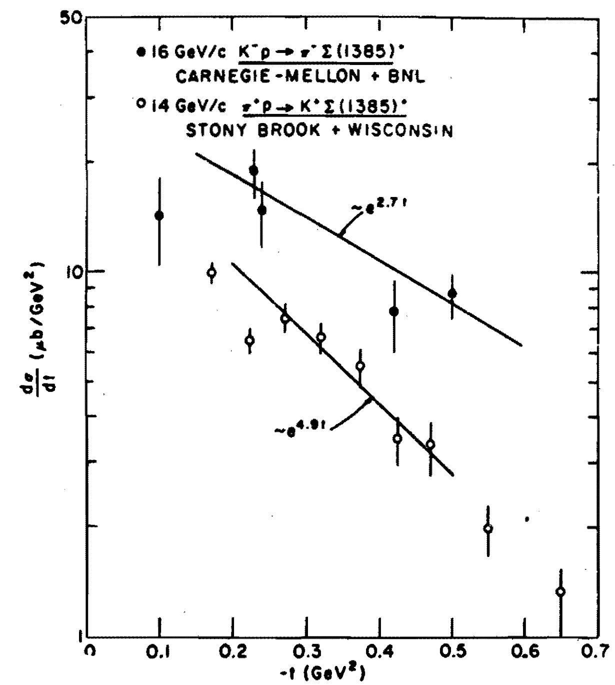


K⁺n — K°p 12 GeV/c
This experiment
K⁻p — K̄°n 12.3 GeV/c
Astbury, et. al.
1000
500
100
50
10
5
1
0.2
0.4
0.6
0.8
1.0
1.2
-t (GeV/c)²
dσ/dt μb/(GeV/c)²

fig. 9



Differential cross sections for the reactions $\pi^+ p \to K^+ \Sigma(1385)^+$ at 6 GeV/c and $K^- p \to \pi^- \Sigma(1385)^+$ at 5.5 GeV/c.



Differential cross sections for the reactions $\pi^+ p \to K^+ \Sigma(1385)^+$ at 14 GeV/c and $K^- p \to \pi^- \Sigma(1385)^+$ at 16 GeV/c.

fig. 10 from K. W. Laiand J. Louie N. P. B19 (1970) 205

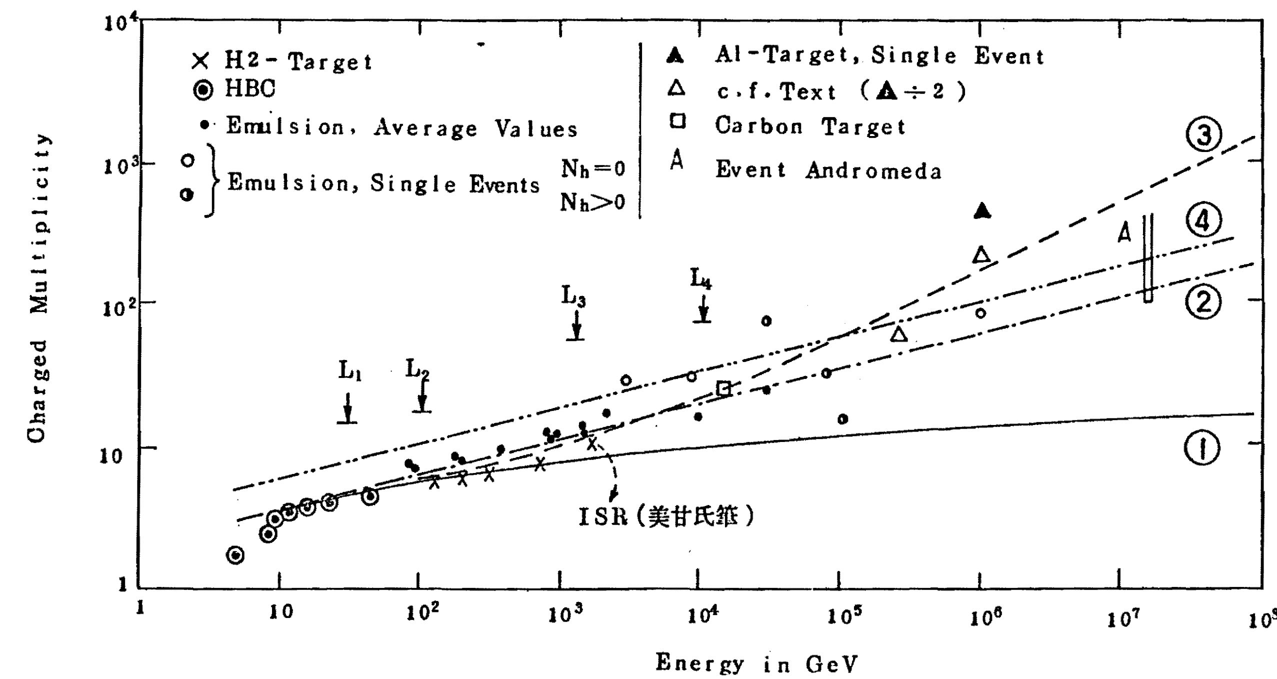
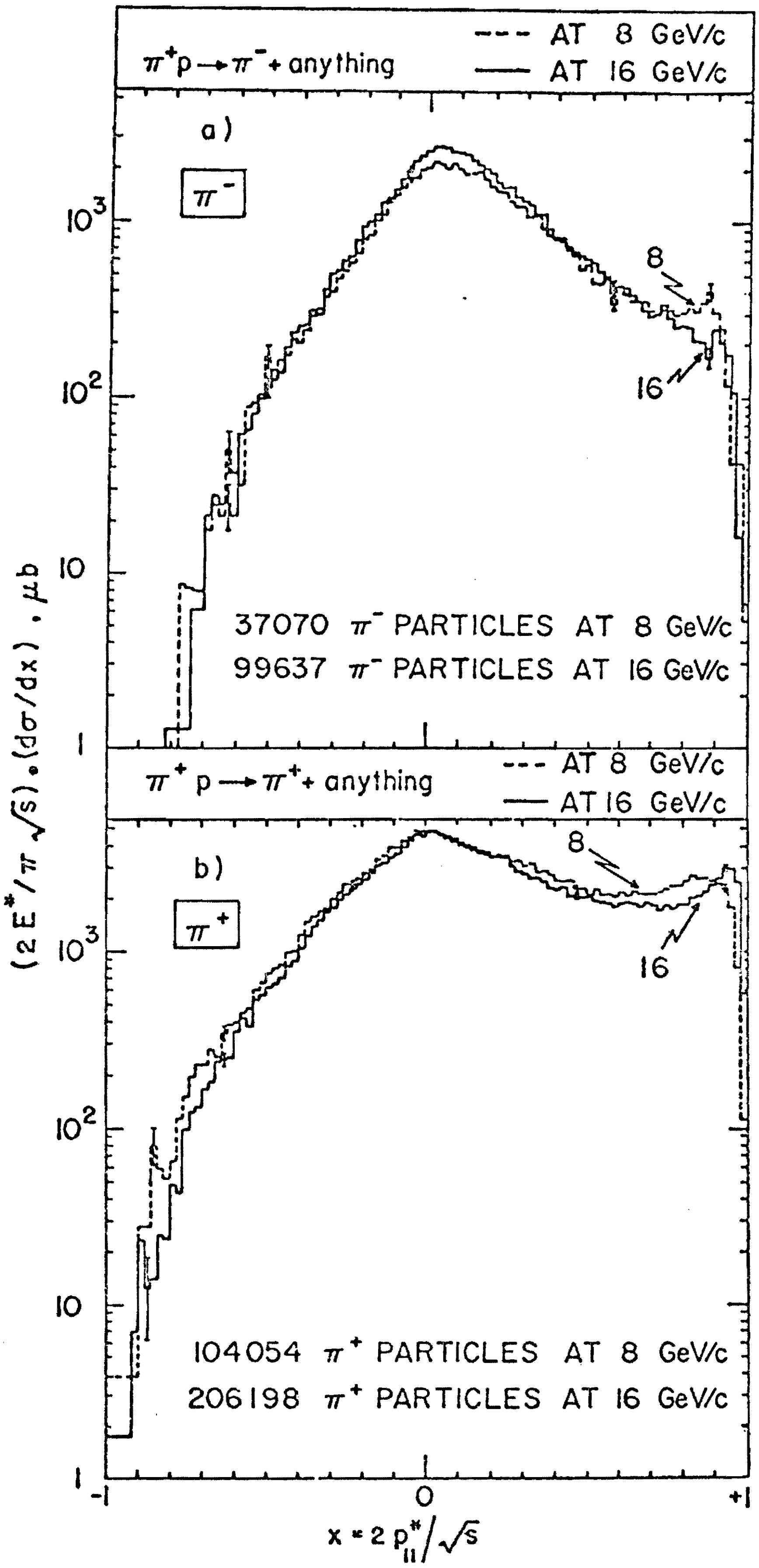


Fig. 11 Multiplicity of charged parjeles versus laboratory energy. (P. K. F. Grieder, TASMANIA international conference on cosmic ray physics (1971))



$\pi^+ p \to \pi^- +$ anything
--- AT 8 GeV/c
— AT 16 GeV/c
a)
π^-
37070 π^- PARTICLES AT 8 GeV/c
99637 π^- PARTICLES AT 16 GeV/c
$\pi^+ p \to \pi^+ +$ anything
--- AT 8 GeV/c
— AT 16 GeV/c
b)
π^+
104054 π^+ PARTICLES AT 8 GeV/c
206198 π^+ PARTICLES AT 16 GeV/c
$(2E^*/\pi\sqrt{s}) \cdot (d\sigma/dx)$, μb
$x = 2p^*_{\parallel}/\sqrt{s}$

Fig. 12

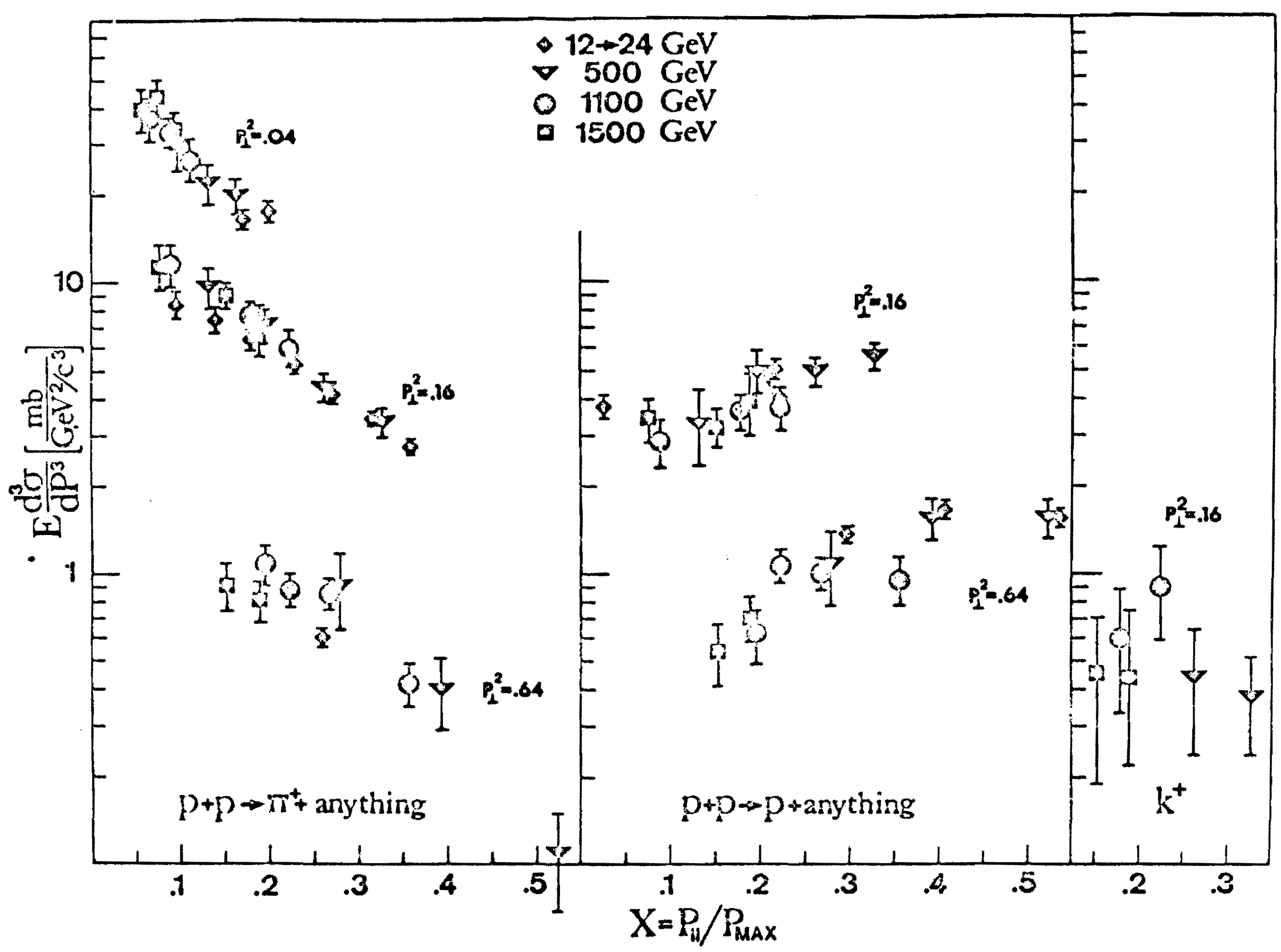
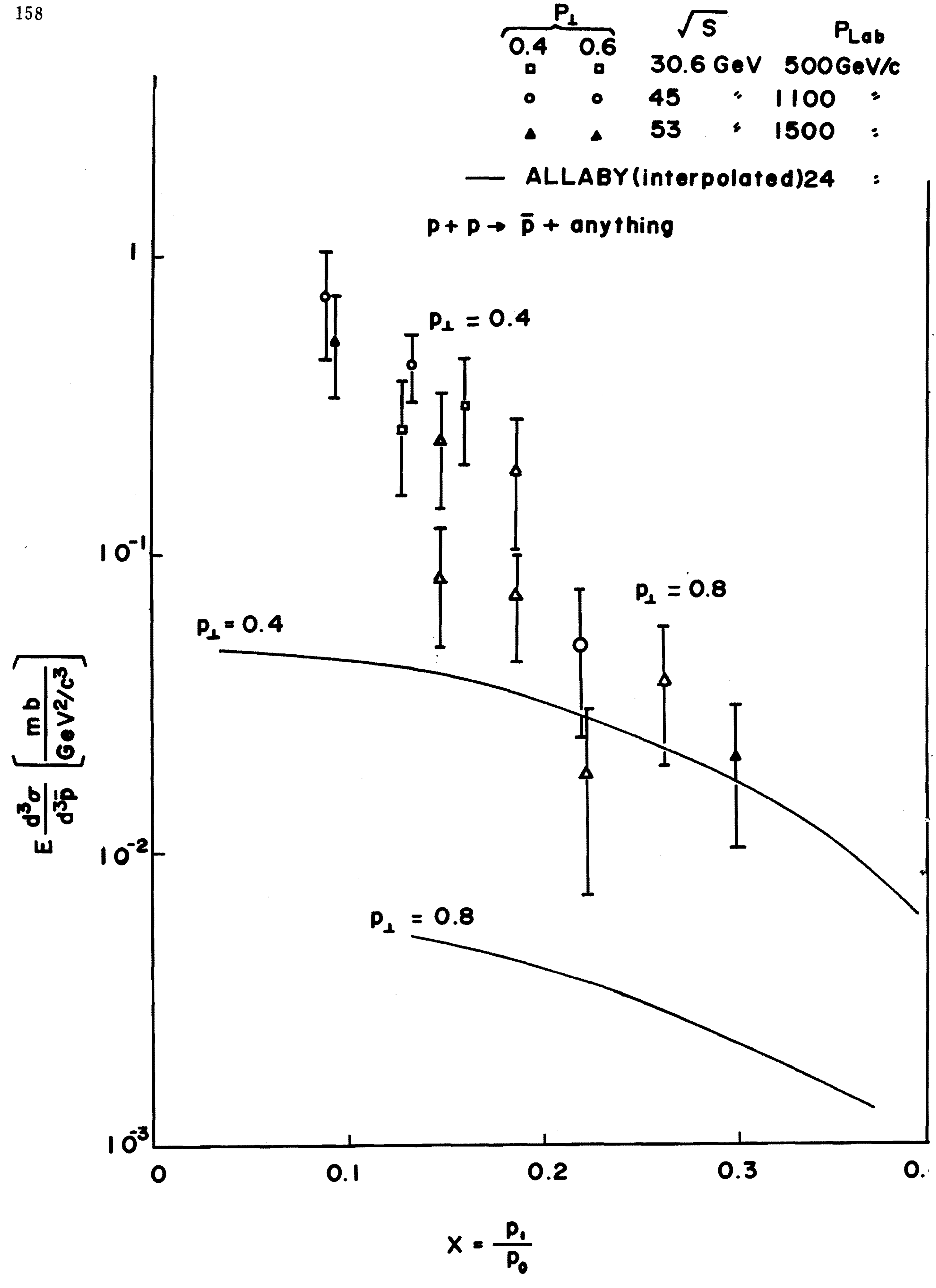

◇ 12→24 GeV
▽ 500 GeV
○ 1100 GeV
□ 1500 GeV
$P_\perp^2$=.04
$P_\perp^2$=.16
$P_\perp^2$=.64
$E\frac{d^3\sigma}{dP^3}\left[\frac{mb}{GeV^2/c^3}\right]$
10
1
p+p→π⁺ anything
p+p→p+anything
k⁺
.1
.2
.3
.4
.5
X=P∥/P_MAX

fig. 13



P⊥
0.4 0.6
√S
PLab
30.6 GeV 500GeV/c
45 1100
53 1500
ALLABY(interpolated)24
p + p → p̄ + anything
p⊥ = 0.4
p⊥ = 0.8
p⊥ = 0.4
p⊥ = 0.8
1
10⁻¹
10⁻²
10⁻³
E d³σ/d³p̄ [mb/GeV²/c³]
0
0.1
0.2
0.3
X = p∥/p0

fig.14

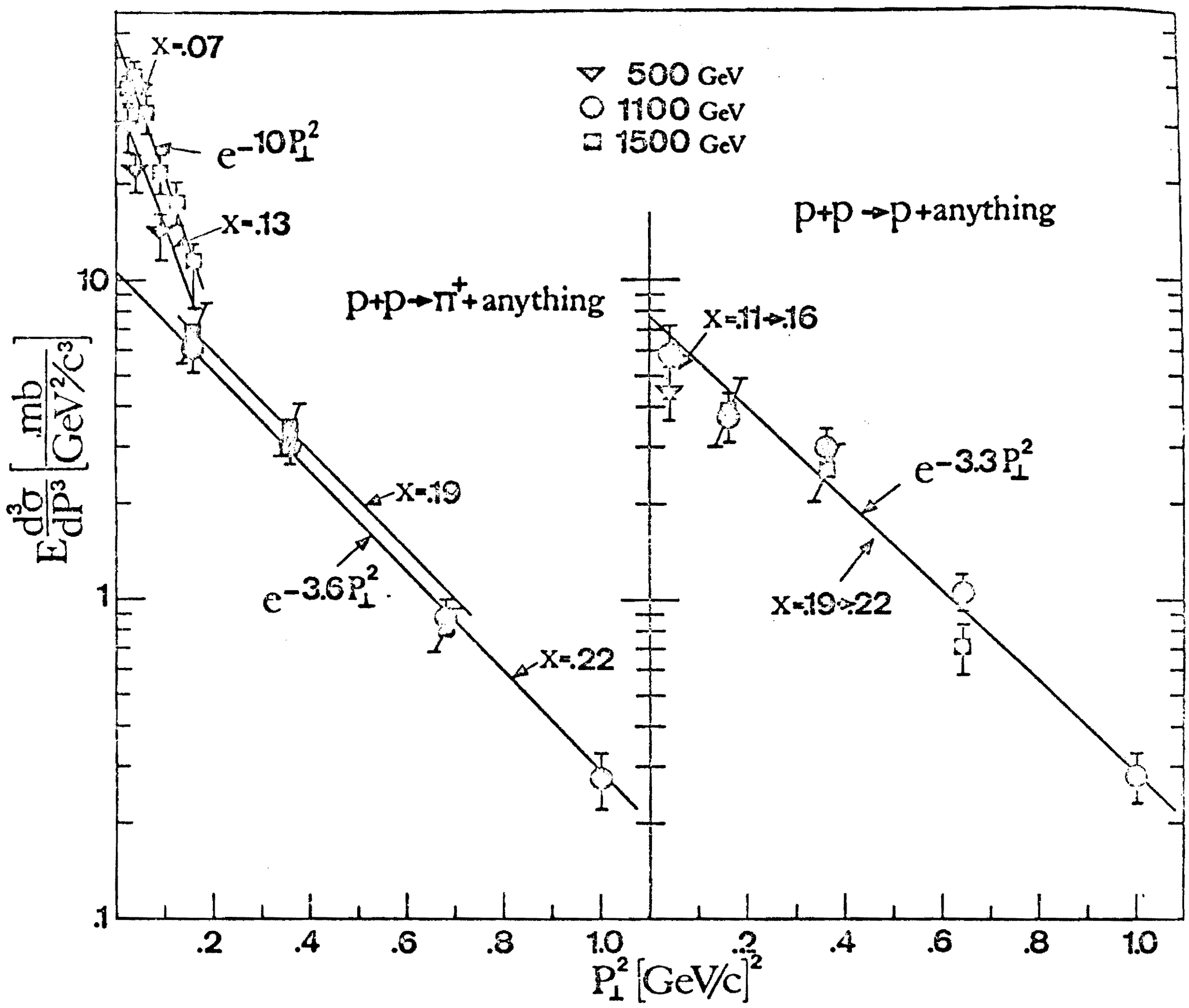

x=.07
500 GeV
1100 GeV
1500 GeV
$e^{-10P_\perp^2}$
x=.13
p+p→p+anything
10
p+p→π⁺+anything
x=.11→.16
$E\frac{d^3\sigma}{dP^3}\left[\frac{mb}{GeV^2/c^3}\right]$
x=.19
$e^{-3.3P_\perp^2}$
$e^{-3.6P_\perp^2}$
1
x=.19→.22
x=.22
.1
.2
.4
.6
.8
1.0
.2
.4
.6
.8
1.0
$P_\perp^2\ [GeV/c]^2$

Fig. 15

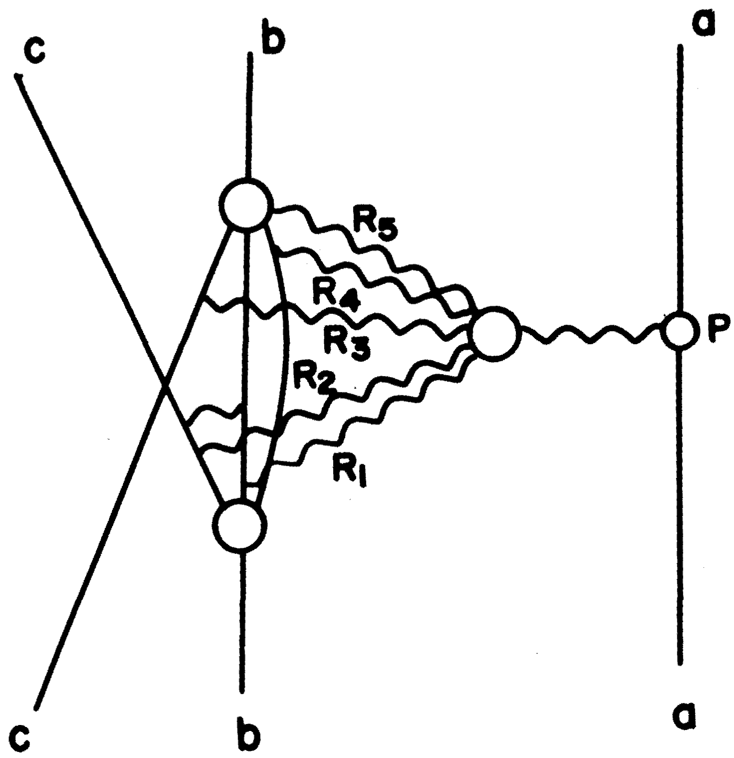


fig.16

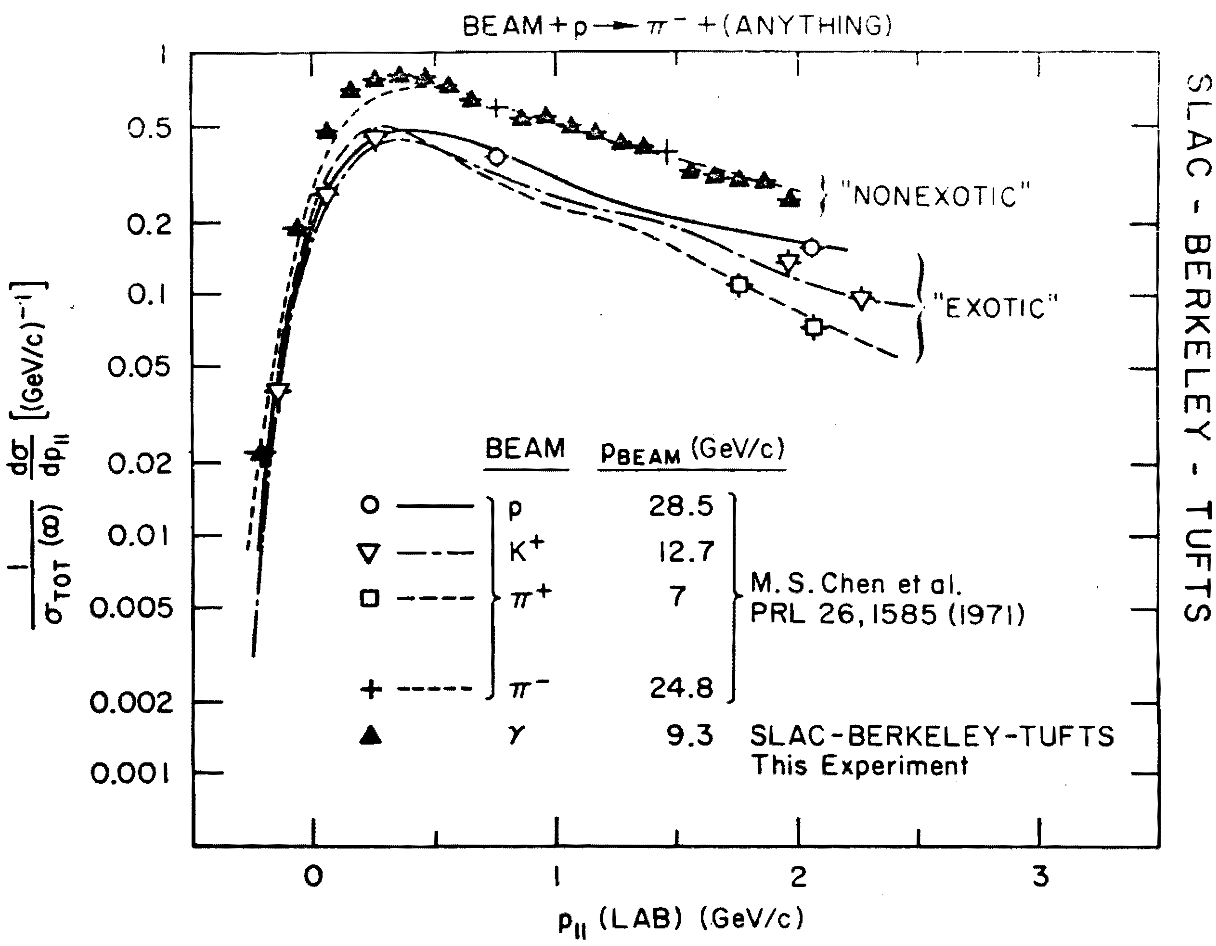

BEAM + p → π⁻ + (ANYTHING)
SLAC - BERKELEY - TUFTS
"NONEXOTIC"
"EXOTIC"
BEAM
PBEAM (GeV/c)
p 28.5
K+ 12.7
π+ 7
π- 24.8
M. S. Chen et al.
PRL 26, 1585 (1971)
γ 9.3
SLAC-BERKELEY-TUFTS
This Experiment
1/σTOT(∞) dσ/dp∥ [(GeV/c)⁻¹]
p∥ (LAB) (GeV/c)
1
0.5
0.2
0.1
0.05
0.02
0.01
0.005
0.002
0.001
0
1
2
3

fig. 17