Dependence of $\ddot{a}(z)$ on different models of dark energy

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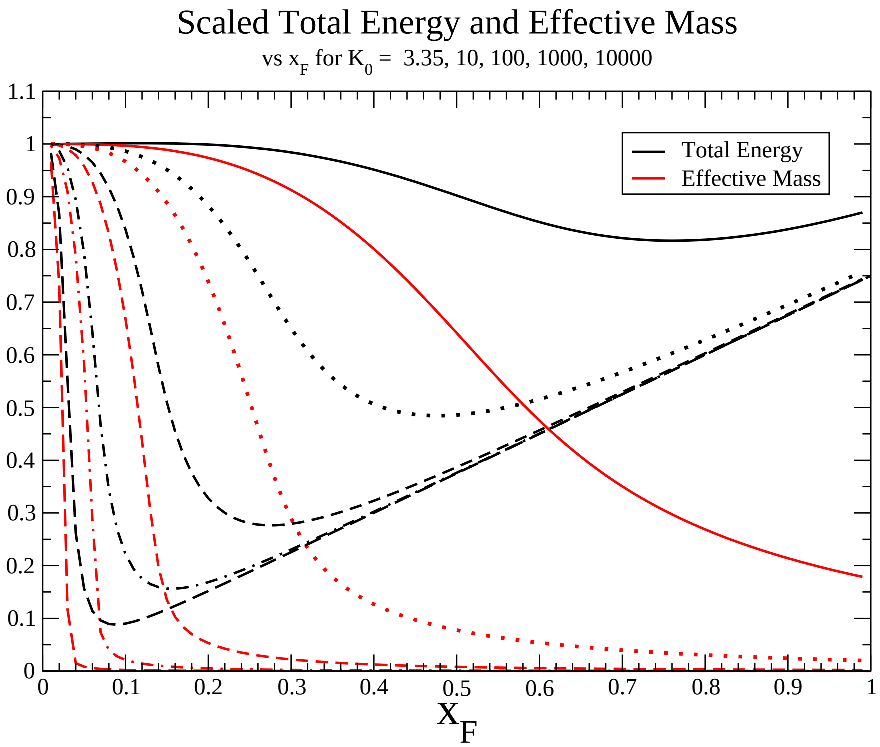
Any fermion, Majorana or Dirac, with vacuum mass m_0, interacting with coupling g with a scalar field with mass m_s, develops an effective mass

$$m^* = m_0 - \frac{g^2}{m_s^2}\langle\bar{\psi}\psi\rangle \tag{1}$$

Using scaled variables $y = \frac{m^*}{m_0}$, $x_F = \frac{k_F}{m_0}$, $e_F = \sqrt{x_F^2 + y^2}$ and $K_0 = \frac{g^2 m_0^2}{\pi^2 m_s^2}$, this becomes

$$y = 1 - \frac{y K_0}{2} \left[e_F x_F - y^2 \ln \left(\frac{e_F + x_F}{y} \right) \right] \quad (2)$$

Define $< e >$ as the total energy per fermion, including scalar field energy.


Scaled Total Energy and Effective Mass
vs x_F for K_0 = 3.35, 10, 100, 1000, 10000
Total Energy
Effective Mass
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
x_F

y and $< e >$ vs x_F

The acceleration of the time evolution of the scale parameter is given by the Friedmann Equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) + \frac{\Lambda}{3} \tag{3}$$

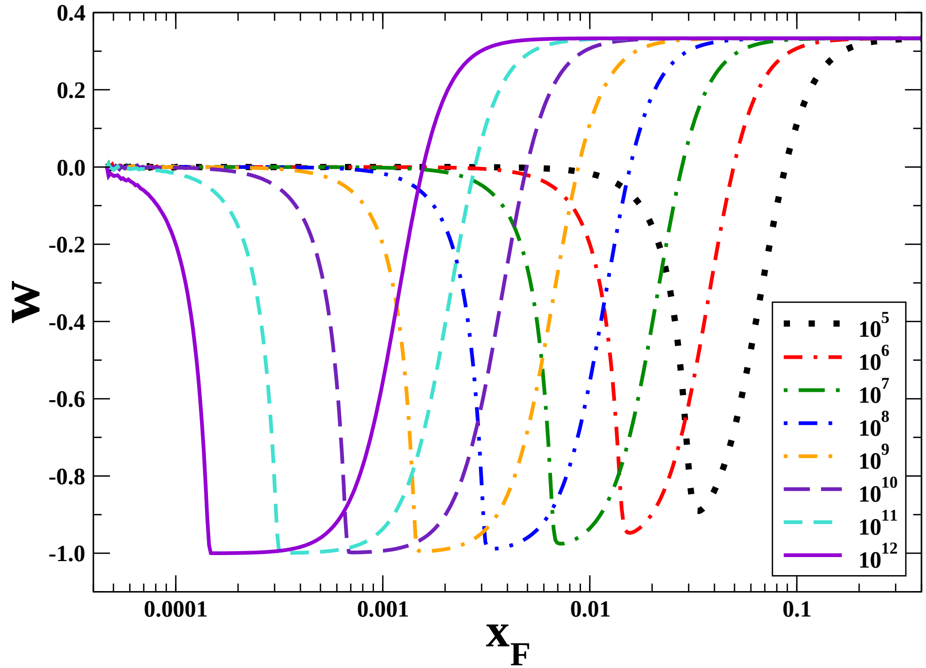
where the energy density ρ and pressure P refer to the matter being considered and Λ is the cosmological constant. The Equation of State relates ρ and P through

$$P = w\rho \tag{4}$$

For the system under consideration,

$$w = \frac{1}{3}\frac{\partial < e >}{\partial x_F}$$

$$= \frac{1}{3}\frac{e_F K_0 x_F^3 - 3(2-y)(1-y)}{e_F K_0 x_F^3 + (2-y)(1-y)} \quad (5)$$



w vs $\log(x_F)$ for 8 values of K_0

The figure suggests scaling for large enough K_0 and this can be shown with the use of analytic expansions. w becomes, to a very good approximation, $w(\xi)$ where

$$\xi = K_0 x_F^3 \tag{6}$$

Empirically, the location of the minimum of w, x_{Fmin} is given by

$$x_{Fmin} = (3.83/K_0)^{\frac{1}{3}} \tag{7}$$

and the minimum of w is

$$w \approx -1 + 2x_{Fmin} \tag{8}$$

To connect with reality, we need to know the density required at some value of z. Without a separate fitting of the model to data we have taken the energy density to be equal to that deduced for dark energy in the ΛCDM model, $\rho_{DE} = (2.4meV)^4$. A good approximation in this model is

$$\rho_{DE} = \frac{m_0^4}{2\pi^2 K_0} \tag{9}$$

Look at two examples, $m_0 = 160 meV, 160 GeV$.

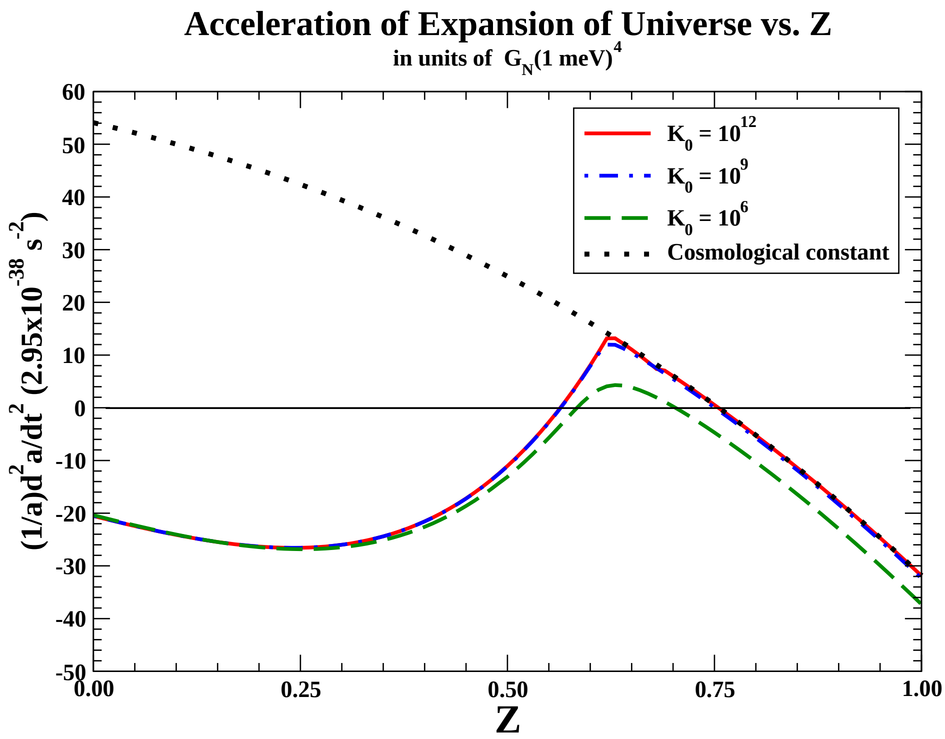
$$\begin{aligned} m_0 &= 160 meV \\ K_0 &= 10^6 \\ \rho &= 33 \times 10^3 (cm)^{-3} \end{aligned} \tag{10}$$

$$\begin{aligned} m_0 &= 160 GeV \\ K_0 &= 10^{54} \\ \rho &= 33 \times 10^{-9} (cm)^{-3} \end{aligned} \tag{11}$$

where the densities hold at $z \approx .75$.

$x_F \propto (1+z)$ implies that, as z becomes large for any m_0, $w \longrightarrow \frac{1}{3}$ appropriate to a relativistic gas of fermions. The number density of light, active neutrinos today is about 110 per $(cm)^3$ per spin per flavor. That becomes about 185 at $z \approx .75$. Therefore BBN effectively rules out that example. Larger masses, however, will have too small a number density to affect BBN and are not constrained.

For all m_0, w returns towards 0 as one approaches the present epoch leading to very different predictions for $\frac{\ddot{a}}{a}$ from that of ΛCDM.



$\frac{\ddot{a}}{a}$ vs z for several values of K_0

CONCLUSIONS

1) Neutral Majorana fermions, interacting with a very light scalar, could lead to $\frac{\ddot{a}}{a} > 0$ for a wide range of vacuum masses m_0 and strength parameter K_0

2) $\frac{\ddot{a}}{a}$ exhibits small differences as a function of these variables for $z > 1$. Such differences vanish as z increases and have no effect on Big Bang Nucleosynthesis.

3) For all m_0, this interacting model predicts a rapid return to $\frac{\ddot{a}}{a} < 0$ by the present epoch.