Dark Energy/Cosmological Constant in Models with Warped Extra Dimensions

Ishwaree Neupane

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Observations fit 4d Einstein GR with a Cosmological Constant term (dark energy density) and dark matter: How to get this picture from a compactified 10d string theory or supergravity models?

General Motivation:

- **String theory - the best (only?) theory of quantum gravity**

- **Warped Extra dimensions - a generic prediction of string theory**

- **Implications:**

– Particle physics (very model dependent)

– Cosmology (has universal aspects through extra-dimensional gravity)

- Testing String theory on CMB /Cosmological Observations (inflation, dark energy, dark matter….)

Warped Compactification

- **Basic Ideas**
  - **Extra Dimensions $\Rightarrow$ Higher dim spacetime: "bulk"**
  - **Localization of matter to a subspace "3-brane"**

    (in contrast with Kaluza-Klein approach)

- **Motivations**
  - **Particle Physics: Mass Hierarchy (why $\boxed{M_{EW} \ll M_{Pl}}$?)**
  - **Brane Gravity: A New Compactification Scheme**
  - **Brane inflation / braneworld cosmology**

Randall-Sundrum single brane model

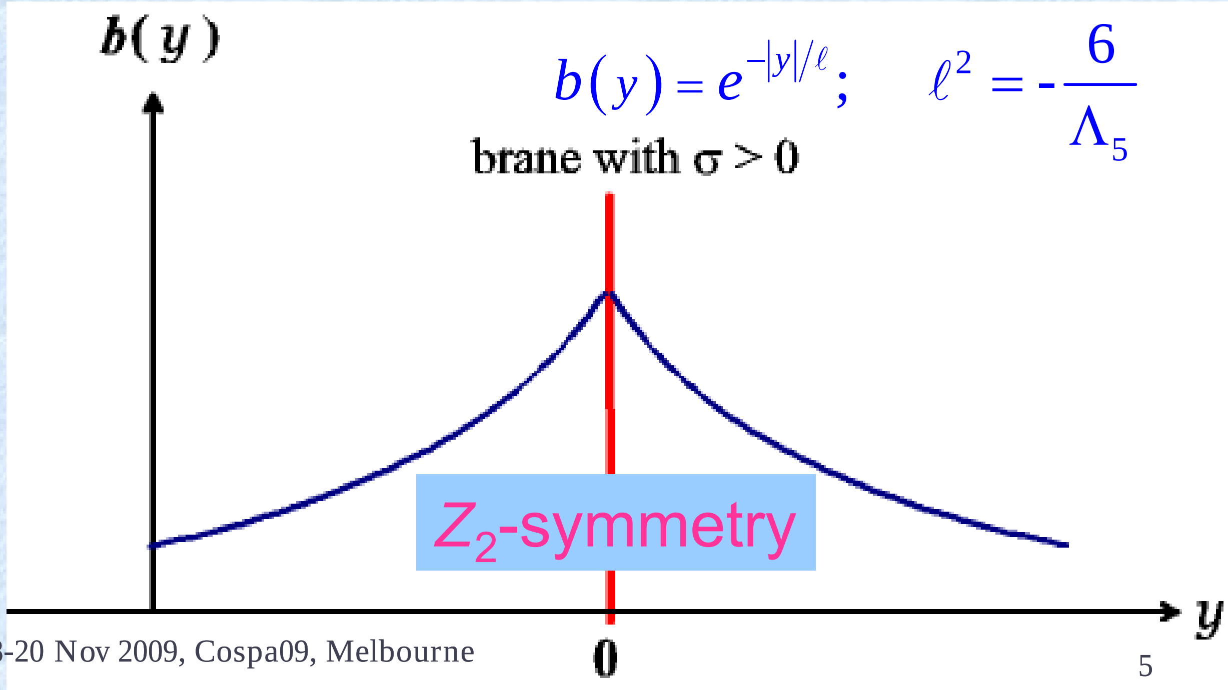
$$S = \frac{1}{2\kappa_5^2}\int_{M_5} d^5x\sqrt{-g}\left(R-2\Lambda_5\right)-\int_{\Sigma_4} d^4x\sqrt{-q}\,\sigma$$

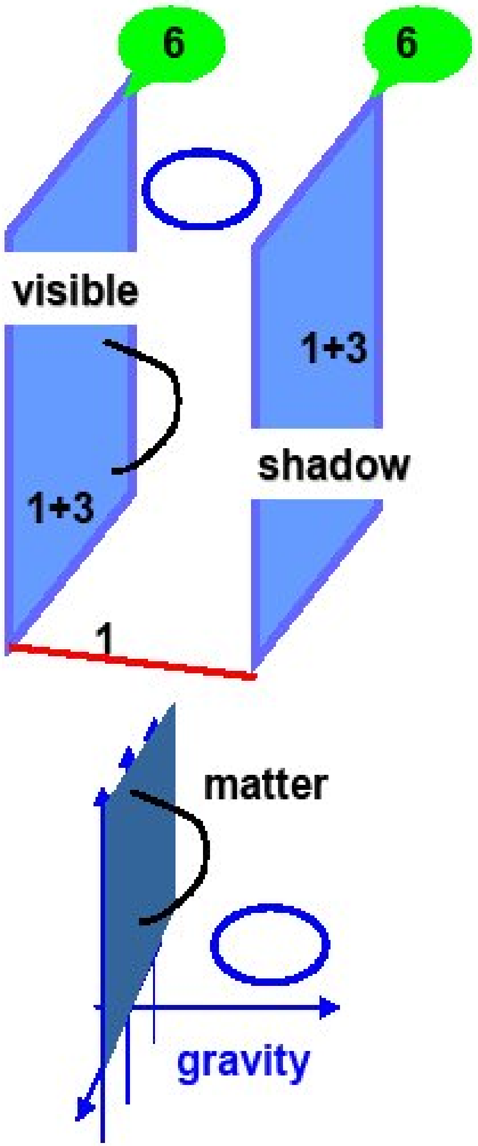
Must be $g_{\mu\nu}$ for a realistic cosmology

$$\Rightarrow \quad ds^2 = dy^2 + e^{-2|y|/\ell}\eta_{\mu\nu}dx^\mu dx^\nu \quad \text{for} \quad \kappa_5^2\sigma = \sqrt{-6\Lambda_5}$$

- Warp factor $A(y)=e^{-|y|/\lambda}$ decreases exponentially as $|y|\rightarrow$ infty

- Extra dim is effectively compact with size $\sim\lambda$





M theory

1 time + 10 space dimensions

1+10 → 1+3+1+(6)

braneworld

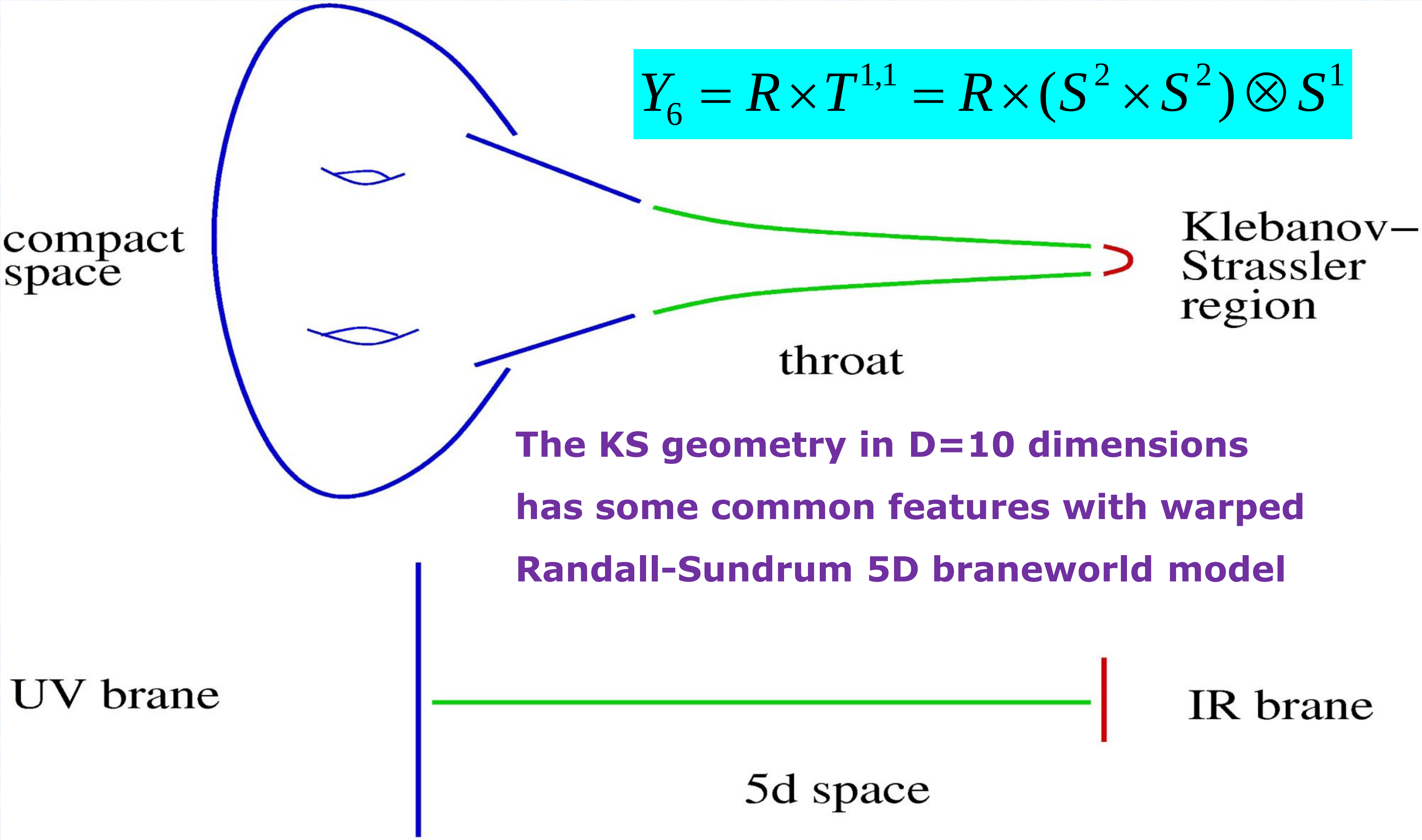
large extra dimension

→ effective 5D braneworld

$$M_5 \sim \left(M_4^2 / L\right)^{1/3} << M_4$$

RS type (effectively) 5d-regions can arise in warped compactifications of type IIB string theory

$$Y_6 = R \times T^{1,1} = R \times (S^2 \times S^2) \otimes S^1$$



The KS geometry in D=10 dimensions has some common features with warped Randall-Sundrum 5D braneworld model

Inflation in String Theory

A typical potential obtained by string compactification

$$V(\Phi,\psi,\sigma) \sim e^{\sqrt{2}\Phi-\sqrt{6}\Psi}\, V(\sigma)$$

Φ and Ψ are canonically normalized fields corresponding to the dilaton field and to the volume of the compactified space; σ is the field driving inflation

The potential with respect to Φ and Ψ is very steep: these fields rapidly run down, and the potential energy V vanishes. One needs to stabilize these fields.

Dilaton stabilization:

Polchinski et al 2001
Kachru et al 2003
Burgess et al 2003

Λ or dark energy in string theory

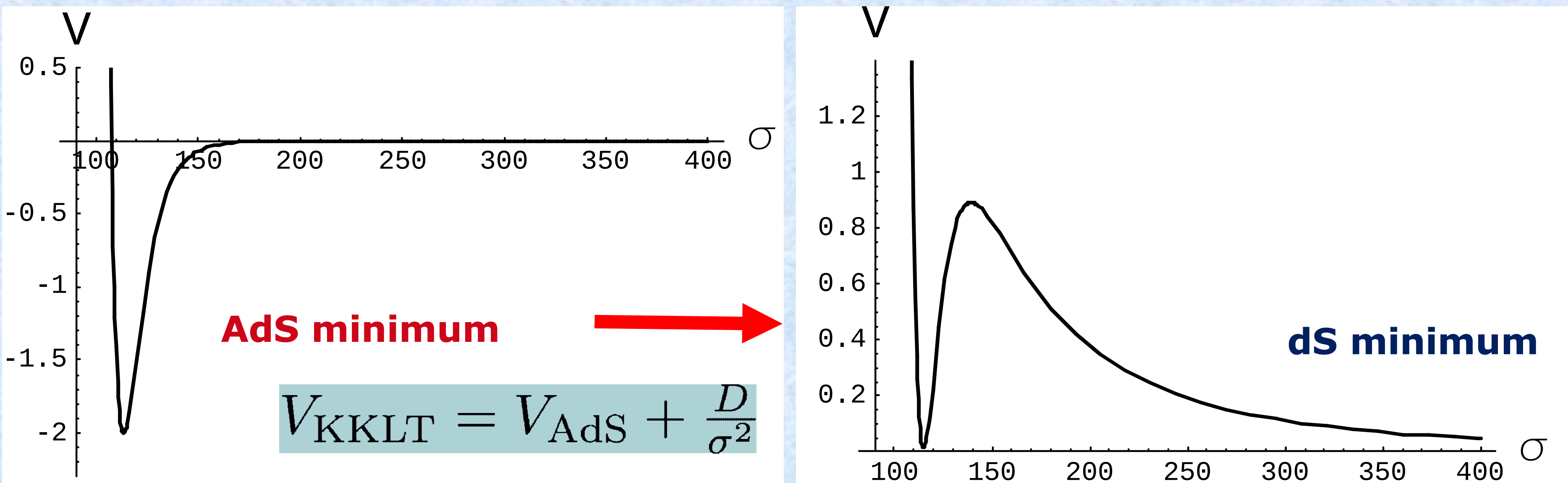
KKLT (Kachru, Kallosh, Linde, Trivedi) 2003

1) **Start with a theory with runaway potential (AdS minimum)**

2) **Bend this potential invoking some (non-perturbative) effects**

3) **Uplift the minimum to the state with positive vacuum energy by adding a positive energy of an anti-D3 brane in warped Calabi-Yau space**

This proposal suffers from fine-tuning issues associated with the necessary flatness of the potential and or the level of fine tuning required for Λ to be the present-day gravitational vacuum (dark energy) density



$$V_{\text{KKLT}} = V_{\text{AdS}} + \frac{D}{\sigma^2}$$

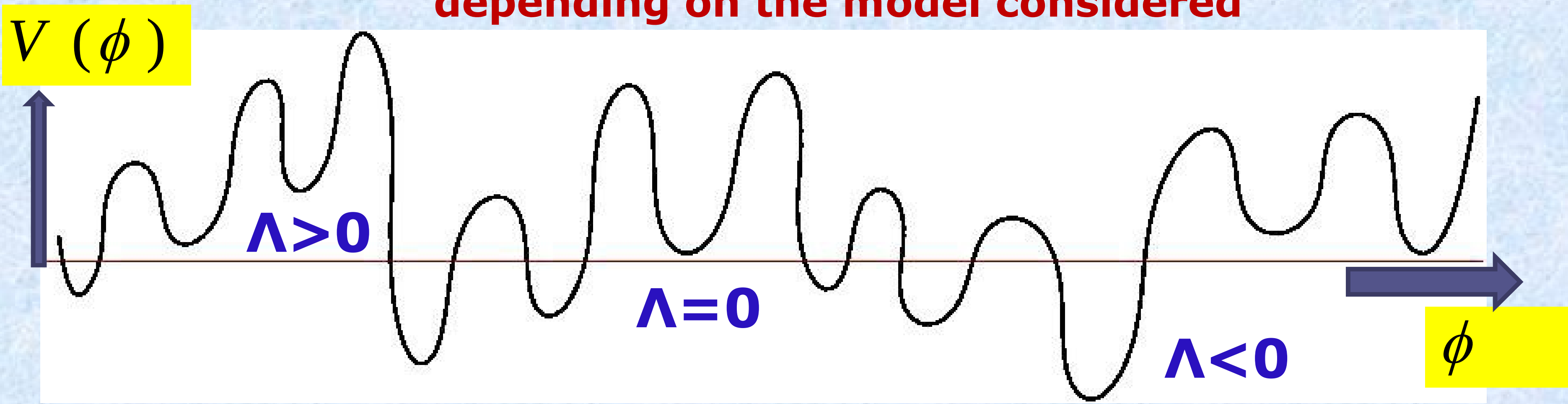
String Theory and the Dark Universe

• **Dark energy / cosmological constant Λ :**

• Quintessence – likely to be linked to moduli fields, e.g., size and shape of compact space

Λ – could depend on fundamental (UV) structure of theory and statistics of string theory vacua: internal space geometry, branes, external fluxes, etc

The runaway involves one or more scalars, depending on the model considered



Is there a room for
a ``Cosmological Constant-like term”
and/or “Quintessence”
in String or Supergravity theory?

What are the obstacles in finding
explicit (inflationary) de Sitter
solutions of 10D supergravity equations?

The celebrated answer:
Brane world “no-go” theorem!

“No-go” theorem forbids cosmic acceleration in cosmological solution arising from compactification of pure SUGRA where the internal space is time-dependent, non-singular compact manifold without boundary.

- Gibbon (1984)
- Maldacena-Nunez (2001)

Why?

Acceleration requires violation of 4D strong energy condition

$$R_{00}^{(4)} < 0 \quad \text{or} \quad T_{AB}\xi^A\xi^B < 0$$

If extra dimensions are warped and static, then in a compactified theory

$$R_{00}^{(4)} \geq 0$$

Provided that SEC holds for D=10 or 11D SUGRA

$$R_{00}^{(D)} \geq 0$$

Why should the SEC be violated?

To see this one considers a FRW metric

$$ds_4^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1-kr^2} + r^2 d\Omega_2^2 \right)$$

The time-time component of *4D* Ricci tensor gives

$$\boxed{R_{00}^{(4)} = -\frac{3\ddot{a}}{a}}$$

Acceleration requires $\dot{a}/a > 0$ and $\ddot{a}/a > 0$ hence

$$R_{00}^{(4)} < 0$$

The 4D Einstein field equations imply that

$$R_{00}^{(4)} = T_{00} + g^{ij} T_{ij} = \rho + 3p$$

An accelerated expansion is possible in a universe governed by Einstein's gravity only if the matter in it violates SEC.

A celebrated version of “no-go” theoreom

Consider a (4+m)-dimesional metric ansatz

$$\mathrm{ds}_{\mathrm{D}}^{2} = e^{2A(y)} ds_4^2(x) + g_{mn}(y) dy^m dy^n$$

This gives

$$\mathrm{R}_{00}^{(\mathrm{D})}(x,y) = R_{00}^{(4)} - \tfrac{1}{4} e^{-2A(y)} \nabla_y^2 e^{4A(y)}$$

$$\Rightarrow \left[e^{2A(y)}\right] R_{00}^{(4)} = \int e^{2A(y)} R_{00}^{(D)} + \frac{1}{4}\int \nabla^2 e^{4A(y)}$$

If the last term above vanishes, then

$$\mathrm{R}_{00}^{(D)} \geq 0 \ \text{ only if } \ \mathrm{R}_{00}^{(4)} \geq 0$$

Any time that one does not understand something, one can point to details that do not work.

This has been more or less the case with braneworld models and string theory compactifications with warped extra dimensions

It is always important to identity what is wrong qualitatively and give the best clue to possible future progress/success.

1. **Allow internal space to be time-dependent, analogue of time-dependent scalar fields – Lukas et al ‘00**

2. **Drop condition that internal space is flat or positive, it may be negatively curved (hyperbolic)**

Townsend-Wohlfarth, N. Ohta, IPN et al 2003

A compactified theory on hyperbolic spaces leads to cosmologies with transient accelerating phase.

SUGRA solutions describing accelerating cosmologies from twisted spaces: IPN&Wiltshire: hep-th/0502003 (PLB), hep-th/0504135 (PRD), hep-th/0609086 (PRL)

Recent Finding

The limitation with warped models studied previously have arisen from an over simplification of 10d metric ansatz.

IPN: arXiv:0901.2568 [hep-th]
IPN: arXiv:0903.4190 [hep-th]
IPN:arxiv:0911.xxxx

These papers give a few explicit examples of non-singular warped compactifications on de Sitter space dS4.

The explicit de Sitter solutions given in these papers serve as explicit models of accelerating universe (attributed to dark energy or cosmological constant-like term).

Start with

$$ds_{10}^2 = e^{2A(y)} ds_4^2(x) + e^{-\alpha A(y)} ds_6^2(y)$$

$$ds_4^2(x) = -dt^2 + a(t)^2\, dx_3^2$$

$$\begin{aligned} ds_6^2 &\equiv g_{mn}(y)\, dy^m dy^n \\ &= dy^2 + \alpha_1 y^2 ds_{X_5}^2 \end{aligned}$$

S^5 or $T^{1,1}$

A general 6d metric

$$R_6 = \frac{20(1-\alpha_1)}{\alpha_1 y^2}$$

The metric ansatz

$$\mathrm{ds}_{10}^2 = e^{2A(y)} ds_4^2(x) + e^{-\alpha A(y)} ds_6^2(y)$$

$$\mathrm{ds}_6^2 \equiv g_{mn}(y) dy^m dy^n = \lambda^2 (\mathrm{dy}^2 + \alpha_1 y^2 ds_{X_5}^2)$$

explicitly solves the 10d vacuum Einstein equations when

$$e^{(\alpha+2)A} = \frac{3(\alpha+2)^2}{32} \frac{y^2}{L^2} \qquad \alpha_1 = \frac{(\alpha+2)^2}{8}$$

$$a(t) \propto e^{Ht} \qquad H = \sqrt{\frac{1}{\lambda^2 L^2}}$$

The drawback of this solution is that the 6d metric and hence the warp factor is singular at y=0

It's more convenient to write our solution in z-coordinate:

$$\alpha = 2$$

$$ds_{10}^2 = e^{-k_0} e^{-k|z|} \left[\begin{array}{l} (-dt^2 + a(t)^2 dx_3^2) \\ + \frac{9}{2} r_c^2 k^2 dz^2 \\ + r_c^2 (e_\psi^2 + \frac{3}{2} \Sigma_{i=1}^4 e_i^2) \end{array} \right]$$

The warped volume of 6d space

$$V_6^{\mathrm{w}} \sim \frac{54|k|\pi^3 r_c^6 e^{-4k_0}}{\sqrt{2}} \int_{-\infty}^{+\infty} e^{-4k|z|} \approx \frac{27\pi^3 r_c^6 e^{-4k_0}}{\sqrt{2}} \qquad M_{Pl}^2 \equiv \frac{M_{10}^8 \times V_6^{\mathrm{w}}}{(2\pi)^6}$$

To get a finite 6d warped volume one introduces a Dp-brane (imposing Z2 symmetry) at z=0

Why did the previous authors -- including Gibbons, Maldacena-Nunez, Giddings et al and many others -- not realise (or somehow rule out) these explicit de Sitter solutions of 10D Einstein equations or background supergravity equations?

The two major assumptions that went into the earlier discussions of braneworld no-go theorems are

1.

$$V_6^{w} = \int d^6 y \sqrt{g_6}\, e^{(2-3\alpha)A} = const\ ,$$

$$M_{Pl}^2 \equiv \frac{M_{10}^8 \times V_6^{w}}{(2\pi)^6} = const\ !$$

Can the natural "constants" change in time and or as one moves away from the D3-brane?

2.

$$\int \nabla_y^2\, e^{nA(y)} = 0$$

for any n and A(y)

These constraints are 'strict' which are generally *not* satisfied by cosmological warped solutions, especially, in the presence of some localised (brane) sources at y=0 where A(y)=const

An alternative to a true cosmological "constant" ?

$$ds_{10}^2 = e^{2A(y)}\left(-dt^2 + a(t)^2 d\vec{x}_3^2\right) + e^{-\alpha A(y)} ds_6^2(y)$$

$$ds_6^2 \equiv g_{mn}(y) dy^m dy^n$$
$$= \rho^2 (\sinh^2(y - y_0) + \alpha_1 \cosh^2(y - y_0) ds_{X_5}^2)$$

This solves the 10D Einstein equations when

$$A(y) = \frac{2}{2+\alpha} \ln\left(\cosh(y - y_0) \times \sqrt{\frac{3(2+\alpha)^2 \mu^2}{32}}\right) \qquad \alpha_1 = \frac{(2+\alpha)^2}{8}$$

$$a(t) \propto e^{Ht} \qquad H = \sqrt{\frac{\mu^2}{\rho^2}}$$

$$^{(10)}R_{00}(x,y) = {}^{(4)}\hat{R}_{00}(x) + \frac{e^{(2+\alpha)A}}{\rho^2 \sinh^2(y+y_0)}\left(2(2-\alpha)A'^2 + \nabla_y^2 A\right)$$

$$= {}^{(4)}\hat{R}_{00}(x) + \frac{\mu^2}{\rho^2}\left(3 + \frac{3(2+\alpha)}{8}\coth(y-y_0)\delta(y)\right)$$

Here it is possible to get acceleration $\hat{R}_{00} < 0$ **without violating the 10D strong energy condition** $^{(10)}R_{00} \geq 0$

$$ds_{10}^2 = e^{-A_0}\left(\frac{3(2+\alpha)^2\mu^2\cosh^2 y}{32}\right)^{2/(2+\alpha)}$$

$$\times\left(ds_4^2 + \frac{32\rho^2}{3(2+\alpha)^2\mu^2}\tanh^2 y\left(dy^2 + \frac{(2+\alpha)^2}{8}\coth^2 y\, ds_{X_5}^2\right)\right)$$

$$V_6 \sim (\rho/\mu)^6 \ln\cosh y \sim y$$

Dimensional Reduction

$$\frac{6\mu^2}{\rho^2}$$

$$ds_{10}^2 \sim M_{10}^8 \int d^{10}x\sqrt{-g_{10}}\left(R_{(10)} + \ldots\ldots\right)$$

$$= \frac{e^{-4A_0} M_{10}^8 \times V_6}{\rho^6} \int d^4x \sqrt{-\hat{g}_4}(\hat{R}_{(4)} - 2\Lambda_4)$$

$$- \frac{e^{-4A_0} M_{10}^8 V_5}{\rho^6} \int (\cosh\ y)^{16/(2+\alpha)} \delta(y)\, dy$$

Requiring that $M_{10} \geq TeV$ **and** $\Lambda_4 \sim 10^{-120} M_{Pl}^2$

forces us to make the choice $A_0 \geq 136$

It may not be natural to just set Λ_4 **to zero!**

This is true also in lower dimensions: Take D=5

$$S = \frac{M_5^3}{2}\int d^5x\sqrt{-g_5}\,(R - 2\Lambda_5) + \frac{M_5^3}{2}\int d^4x\sqrt{-g_b}\,(-T_3)$$

$$ds_5^2 = e^{2A(z)}\left(-dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] + \rho^2 dz^2\right)$$

$$a(t) = \frac{1}{2}e^{\mu t/\rho} + \frac{k\rho^2}{2\mu^2}e^{-\mu t/\rho}$$

$$A(z) = \frac{24\mu^2}{24\mu^2 e^{\mu|z|} + \Lambda_5\rho^2 e^{-\mu|z|}}$$

$$S_{eff} \supset \frac{M_5^3\rho}{2}\int d^4x\sqrt{-\hat{g}_4}\int dz\, \mathrm{e}^{3\mathrm{A(z)}}\left(\hat{R}_4 - L_\Lambda - 2\Lambda_5\, e^{2A(z)}\right)$$

$$L_\Lambda = \frac{12\mu^2}{\rho^2}\left(1 - \frac{160\Lambda_5\mu^2\rho^2}{\left(24\mu^2 e^{\mu|z|} + \Lambda_5\rho^2 e^{-\mu|z|}\right)^2}\right) - \frac{16\mu}{\rho^2}\left(\frac{24\mu^2 e^{\mu|z|} - \Lambda_5\rho^2 e^{-\mu|z|}}{24\mu^2 e^{\mu|z|} + \Lambda_5\rho^2 e^{-\mu|z|}}\right)\delta(z)$$

Summary

- Warped models give a new picture of the universe as well as new opportunities for both particle physics and cosmology (inflation, dark energy, dark matter, ...)

- Cosmic acceleration (attributed dark energy) is intrinsically an extra-dimensional phenomenon in many compactifications of supergravity models.

- String theory can, of course, accommodate some null energy condition violating objects -- negative tension branes and O-planes -- but they may not be required just to get an accelerating universe from higher dimensional Einstein's theory