Lorentz Invariance Violation: theory and phenomenology

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Outline

- Motivation
- Theories: SME, A new theory SMS
- Phenomena: neutrinos, GZK cutoff, time-lag of photons
- Summary

LV-Window of Quantum Gravity

- Quantum Gravity

  String theory and canonical quantum gravity (QG):
  Both expect the typical quantum gravity scale is $\mathrm{M}_{\mathrm{Planck}}$,
  which is far unattainable from accelerators。

- Relic probe to QG

  ▷ Quantum decoherence and state collapse
  ▷ Cosmological variations of couplings
  ▷ Extra-dimension, TeV-Black Holes
  ▷ QG imprint on Cosmological perturbations
  ▷ Violation of some sacred Symmetries, e.g. Lorentz and CPT symmetry

Space-time Symmetry

Space-time symmetry: Lorentz symmetry vs Galileo symmetry

- Poincare algebra

$$i[J^{\mu\nu}, J^{\rho\sigma}] = \eta^{\nu\rho} J^{\mu\sigma} - \eta^{\mu\rho} J^{\nu\sigma} + \eta^{\mu\sigma} J^{\nu\rho} - \eta^{\nu\sigma} J^{\mu\rho}$$
$$i[P^{\mu}, J^{\rho\sigma}] = \eta^{\mu\rho} P^{\sigma} - \eta^{\mu\sigma} P^{\rho}$$
$$[P^{\mu}, P^{\nu}] = 0, \quad (1)$$

i.e.

$$[J^i, J^j] = i\epsilon^{ijk} J^k \quad [J^i, K^j] = i\epsilon^{ijk} K^k \quad [K^i, K^j] = -i\epsilon^{ijk} J^k$$
$$[J^i, P^j] = i\epsilon^{ijk} P^k \quad [P^i, K^j] = i\delta^{ij} H \quad [H, K^i] = iP^i$$
$$\text{other commutators} = 0 \quad (2)$$

- Galileo algebra

$$[J^i, J^j] = i\epsilon^{ijk} J^k \quad [J^i, K^j] = i\epsilon^{ijk} K^k \quad [K^i, K^j] = 0$$
$$[J^i, P^j] = i\epsilon^{ijk} P^k \quad [P^i, K^j] = i\delta^{ij} M \quad [W, K^i] = iP^i$$
$$\text{other commutators} = 0, \quad (3)$$

Where $H = M + W, W = \frac{1}{2} M \vec{v}^2$

Pioneers' study of Lorentz symmetry violation

The early discussion on the effects of Lorentz violation

- Dirac's æther and nonlinear electrodynamics
  P.A.M. Dirac, Nature **168**, 906 (1951).
- Goldstone boson associated to Spontaneous Lorentz symmetry breaking(SLSB)
  ▷Bjorken's earlier attempts: Photon as Goldstone boson associated to SLSB. J.D. Bjorken, Ann.Phys. **24**, 174 (1963).
  ▷Is Graviton also a Goldstone boson?
  P.R. Phillips, Phys. Rev. **146**, 966 (1966)....
- An universal length scale
  T.G. Pavlopoulos, Phys. Rev. **159**, 1106 (1967)
- Nielsen's renomalization group calculation of the beta-function for a non-covariant pure Yang-Mills theory
  H.B. Nielsen and M. Ninomiya, Nucl. Phys. B **141**, 153 (1978). ...

Why Lorentz violation?

- Quantum Gravity (QG)?
  - spacetime foam [Ellis et al.'08, PLB]
  - loop gravity [Alfaro et al.'00, PRL]
  - torsion in general gravity [Yan'83, TP]
  - vacuum condensate of antisymmetric tensor fields in string theory [Kostelecky & Samuel'89 & '91, PRL]
  - double special relativity [Amelino-Camelia'02, Nature & '02 IJMPD]

Various theories on Lorenz violation

Effective Field Theory

Standard Model Extension

an explicitly introduction of condensation of background tensor field

$\mathcal{L}_{LV} \sim \frac{\lambda}{M_{\mathrm{Planck}}^k} < T > \overline{\psi}\Gamma(i\partial)^k\psi$

D. Colladay and V.A. Kostelecký, Phys. Rev. D **58**, 116002 (1998).

Dynamical critical exponent of space and time scaling

$t \to \lambda^z t, \quad \vec{r} \to \lambda \vec{r}$

Lorentz symmetry emergent at low energies as $z \to 1$

P. Horava, Phys. Rev. D **79**, 084008 (2009); JHEP **020**, 0903 (2009).

Non EFT

Double Special Relativity with two universal invariants:

photon limiting velocity c,

Planck length scale $l_{\mathrm{Planck}} = 1.616 * 10^{-33}\mathrm{cm}$

Stringy space time foam model

Effective Field Theory

SME

- The total Lagrangian

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \delta\mathcal{L}, \tag{4}$$

where $\delta\mathcal{L}$ denotes tiny LV parts.

- take QED as example

$$\delta\mathcal{L}_{\rm QED} = \delta\mathcal{L}_{\rm photon} + \delta\mathcal{L}_{\rm electron}, \tag{5}$$

where

$$\delta\mathcal{L}_{\rm photon} \supset -\frac{1}{4}(k_F)_{\kappa\lambda\mu\nu}F^{\kappa\lambda}F^{\mu\nu} + \frac{1}{2}(k_{AF})_\kappa\epsilon^{\kappa\lambda\mu\nu}A_\lambda F_{\mu\nu}, \tag{6}$$

$$\begin{aligned}\delta\mathcal{L}_{\rm electron} \supset\ & \frac{1}{2}i\overline{\psi}(\widetilde{c}^{(\nu\mu)}\gamma_\nu + \widetilde{d}^{\nu\mu}\gamma_5\gamma_\nu + \frac{1}{2}\widetilde{g}^{\lambda\nu\mu}\sigma_{\lambda\nu})\overleftrightarrow{D}_\mu\,\psi \\ & -\overline{\psi}(\widetilde{b}_\mu\gamma_5\gamma^\mu + \frac{1}{2}\widetilde{H}_{\mu\nu}\sigma^{\mu\nu})\psi.\end{aligned} \tag{7}$$

Effective Field Theory

SME

- Lorentz violation-conflict with covariance?

$$\overline{\psi}(x)(a^\nu\gamma_\nu)\psi(x) \to [U\overline{\psi}(x)U^{-1}]([Ua^\nu U^{-1}][U\gamma_\nu U^{-1}])[U\psi(x)U^{-1}]$$
$$= [\overline{\psi}(\Lambda x)S^{-1}](a^\nu[\Lambda^\rho{}_\nu\gamma_\rho])[S\psi(\Lambda x)] = \overline{\psi}(\Lambda x)(a^\nu\Lambda^\rho{}_\nu\gamma_\rho)\psi(\Lambda x)$$

So Lorentz violation can be incorporated in a covariant form

Lorentz invariance breaks;
But Lorentz covariance works.
So we call it Lorentz invariance violation (LV or LIV)

A new theory of Lorentz violation

- a replacement of the common derivative operators by covariant co-derivative ones

$$\partial^\alpha \to M^{\alpha\beta}\partial_\beta, \quad D^\alpha \to M^{\alpha\beta}D_\beta,$$

- The effective minimal Standard Model

$$\mathcal{L}_{SM} = \mathcal{L}_G + \mathcal{L}_F + \mathcal{L}_{HG} + \mathcal{L}_{HF},$$
$$\mathcal{L}_G = -\frac{1}{4}F^{a\alpha\beta}F^a_{\alpha\beta},$$
$$\mathcal{L}_F = i\bar{\psi}\gamma^\alpha D_\alpha\psi,$$
$$\mathcal{L}_{HG} = (D^\alpha\phi)^\dagger D_\alpha\phi + V(\phi),$$

- A new standard model with supplement terms

$$\mathcal{L}_{SMS} = \mathcal{L}_{SM} + \mathcal{L}_{LV},$$
$$\mathcal{L}_{LV} = \mathcal{L}_{GV} + \mathcal{L}_{FV} + \mathcal{L}_{HFV}$$

Zhou Lingli and B.-Q. Ma, Preprint

Consequences of Lorentz violation

- **Could provide explanation of neutrino oscillation without neutrino mass**

  S.Yang and B.-Q.Ma, IJMPA (09), arXiv:0910.0897

  Z.Xiao and B.-Q.Ma, IJMPA24(09)1539

- **Modified dispersion relation could increase threshold energy of photo-induced meson production of the proton: an increase of GZK cutoff energy**

  Z.Xiao and B.-Q.Ma, IJMPA24(09)1539

- **Modified dispersion relation may cause time lag of photons with different energies when they propagate in space from far-away astro-objects**

  Z.Xiao and B.-Q.Ma, arXiv:0909.4927

  L.Shao, Z.Xiao and B.-Q.Ma, arXiv:0911.2276

Lorentz violation in three-family neutrino oscillations

The general equation for neutrino oscillation probabilities

- The Lagrangian density for neutrino sector

$$L = \frac{1}{2} i \overline{\nu}_A \gamma^\mu \overleftrightarrow{\partial}_\mu \nu_B \delta_{AB} + \frac{1}{2} i c_{AB}^{\mu\nu} \overline{\nu}_A \gamma^\mu \overleftrightarrow{\partial}^\nu \nu_B - a_{AB}^\mu \overline{\nu}_A \gamma^\mu \nu_B \tag{3.1}$$

- From eq.(3.1), we figure out the Hamiltonian density

$$H = (-i \overline{\nu}_A \vec{\gamma} \bullet \nabla \delta_{AB} - i c_{AB}^{\mu i} \overline{\nu}_A \gamma^\mu \bullet \nabla_i) \nu_B + a_{AB}^\mu \overline{\nu}_A \gamma_\mu \nu_B \tag{3.2}$$

- Transforming the description into quantum mechanics

$$\hat{H} = -i \gamma^0 \vec{\gamma} \bullet \nabla \delta_{AB} - i c_{AB}^{\mu i} \gamma^0 \gamma^\mu \bullet \nabla_i + a_{AB}^\mu \gamma^0 \gamma_\mu \tag{3.3}$$

The general equation for neutrino oscillation probabilities

- Up to now, we have not detected right-handed neutrinos or left-handed anti-neutrinos, so we choose the basis vector as

$$\begin{pmatrix} u_L(p) \\ v_R(p) \end{pmatrix} \tag{3.4}$$

- We could get the dynaminal equation for neutrinos

$$\left(i\frac{\partial}{\partial t} - H_{AB} \right) \begin{pmatrix} a_B \\ b_B \end{pmatrix} = 0 \tag{3.5}$$

- The Hamiltonian matrix for neutrino given

$$H_{AB} = \begin{pmatrix} |p|\delta_{AB} + c_{AB}^{\mu j}\frac{p_\mu p_j}{|p|} + \frac{a_{AB}^{\mu} p_\mu}{|p|} & 0 \\ 0 & |p|\delta_{AB} + c_{AB}^{\mu j}\frac{p_\mu p_j}{|p|} + \frac{a_{AB}^{\mu} p_\mu}{|p|} \end{pmatrix} \tag{3.6}$$

The general equation for neutrino oscillation probabilities

- Diagonalizing H_{AB} to get the energy spectrum for neutrinos

$$\mathrm{E} = U^{+} H U \tag{3.7}$$

- The relationship between energy eignstates ν_i and flavor eigenstates ν_α

$$\left|\nu_\alpha\right\rangle = \sum_i U^{*}_{\alpha i}\left|\nu_i\right\rangle \tag{3.8}$$

- The general equation for neutrino oscillation

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{i>j} \mathrm{Re}[(U^{+})^{*}_{i\alpha}(U^{+})_{i\beta}(U^{+})^{*}_{j\alpha}(U^{+})_{j\beta}]\sin^2[\frac{\Delta E_{ij}}{2}t]$$
$$+2\sum_{i>j} \mathrm{Im}[(U^{+})^{*}_{i\alpha}(U^{+})_{i\beta}(U^{+})^{*}_{j\alpha}(U^{+})_{j\beta}]\sin[\Delta E_{ij}t]$$

(3.9)

Neutrino oscillation for Lorentz violation:

- We carried out Lorentz violation contribution to neutrino oscillation by the effective field theory for LV and give out the equations of neutrino oscillation probabilities.
- In our model, neutrino oscillations do not have drastic oscillation at low energy and oscillations still exist at high energy.
- Neutrinos may have small mass and both LV and the conventional oscillation mechanisms contribute to neutrino oscillation.

S.Yang and B.-Q.Ma, IJMPA (09), arXiv:0910.0897
Z.Xiao and B.-Q.Ma, IJMPA24(09)1539

LV from cosmological VHE photon emissions

Z.Xiao and B.-Q.Ma, arXiv:0909.4927

L.Shao, Z.Xiao and B.-Q.Ma, arXiv:0911.2276

Modified photon dispersion relation from LV

$$v(E) = c_0 \left(1 - \xi \frac{E}{M_{\rm P} c^2} - \zeta \frac{E^2}{M_{\rm P}^2 c^4}\right)$$

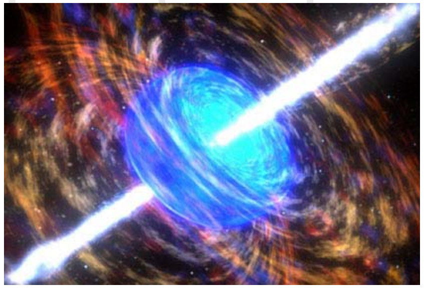
$$\sqrt{\hbar c/G} \simeq 1.22 \times 10^{19}\ \mathrm{GeV/c^2}$$

For reviews, see, e.g.,

Jacobson et al.'06, Ann. Phys.
Kostelecky & Mewes'09, PRD
Mattingly'05, Living Rev. Rel.
Amelino-Camelia & Smonlin'09, PRD

Gammy-ray Bursts (GRBs)

- the most energetic astrophysical process except the Big Bang
- 2 types [Piran'05, Rev. Mod. Phys.]
  - long GRBs: duration > 2 s; collapses of massive rapidly rotating stars
  - short GRBs: duration < 2 s; coalescence of two neutron stars or a neutron star and a black hole
- use GRBs to test LV [Amelino-Camelia et al.'98, Nature]



Time lag by LV effect

- expansion universe [Jacob & Piran'08, JCAP]

$$\Delta t_{\rm LV} = \frac{1+n}{2H_0}\left(\frac{E_{\rm h}^n - E_{\rm l}^n}{M_{\rm QG}^n c^{2n}}\right)\int_0^z \frac{(1+z')^n {\rm d}z'}{h(z')}$$

$$M_{\rm QG,L} = |\xi|^{-1} M_{\rm P} \;\; \text{and} \;\; M_{\rm QG,Q} = |\zeta|^{-1/2} M_{\rm P}$$

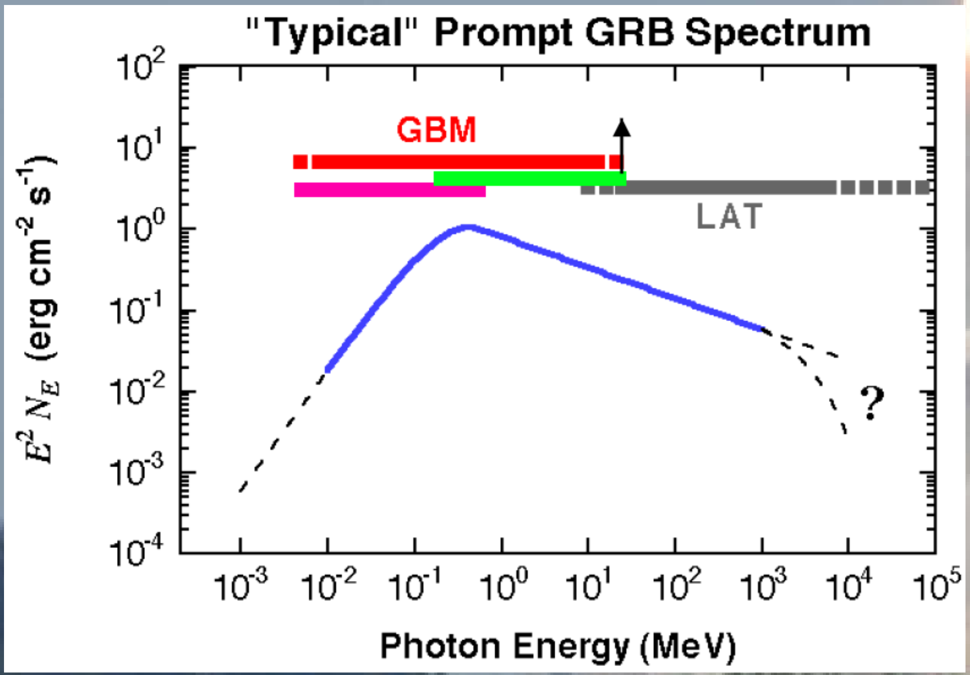
$$h(z) = \sqrt{\Omega_\Lambda + \Omega_{\rm M}(1+z)^3}$$

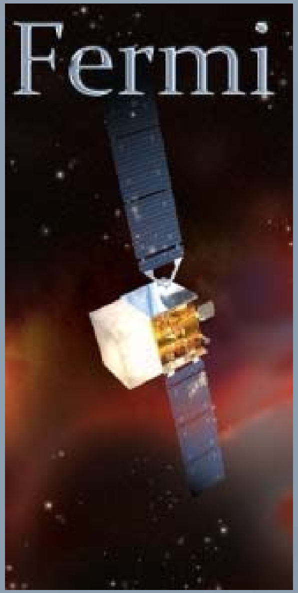
$$H_0 \simeq 71 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

$$\Omega_\Lambda \simeq 0.73 \;\; \Omega_{\rm M} \simeq 0.27$$

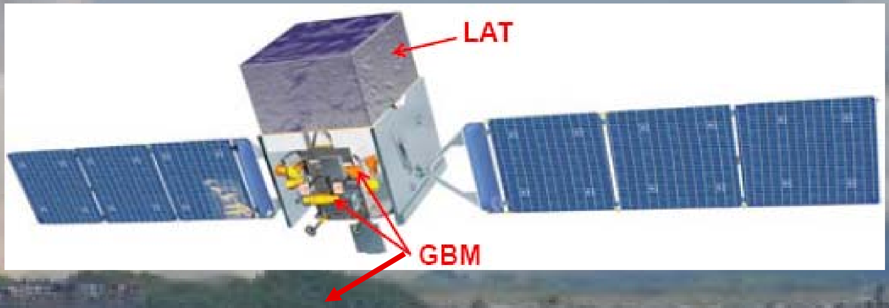
June 11, 2008

Fermi instruments

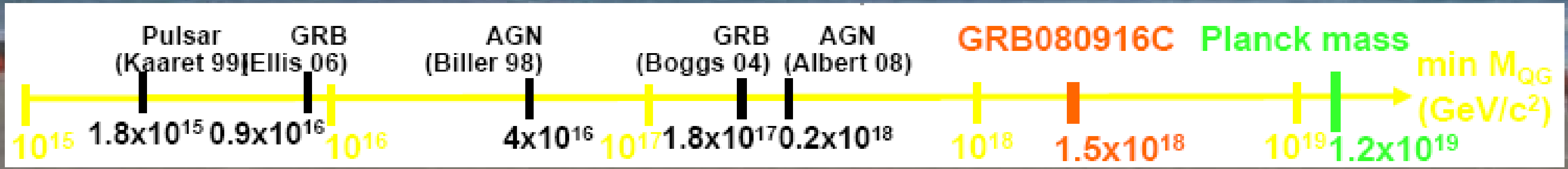




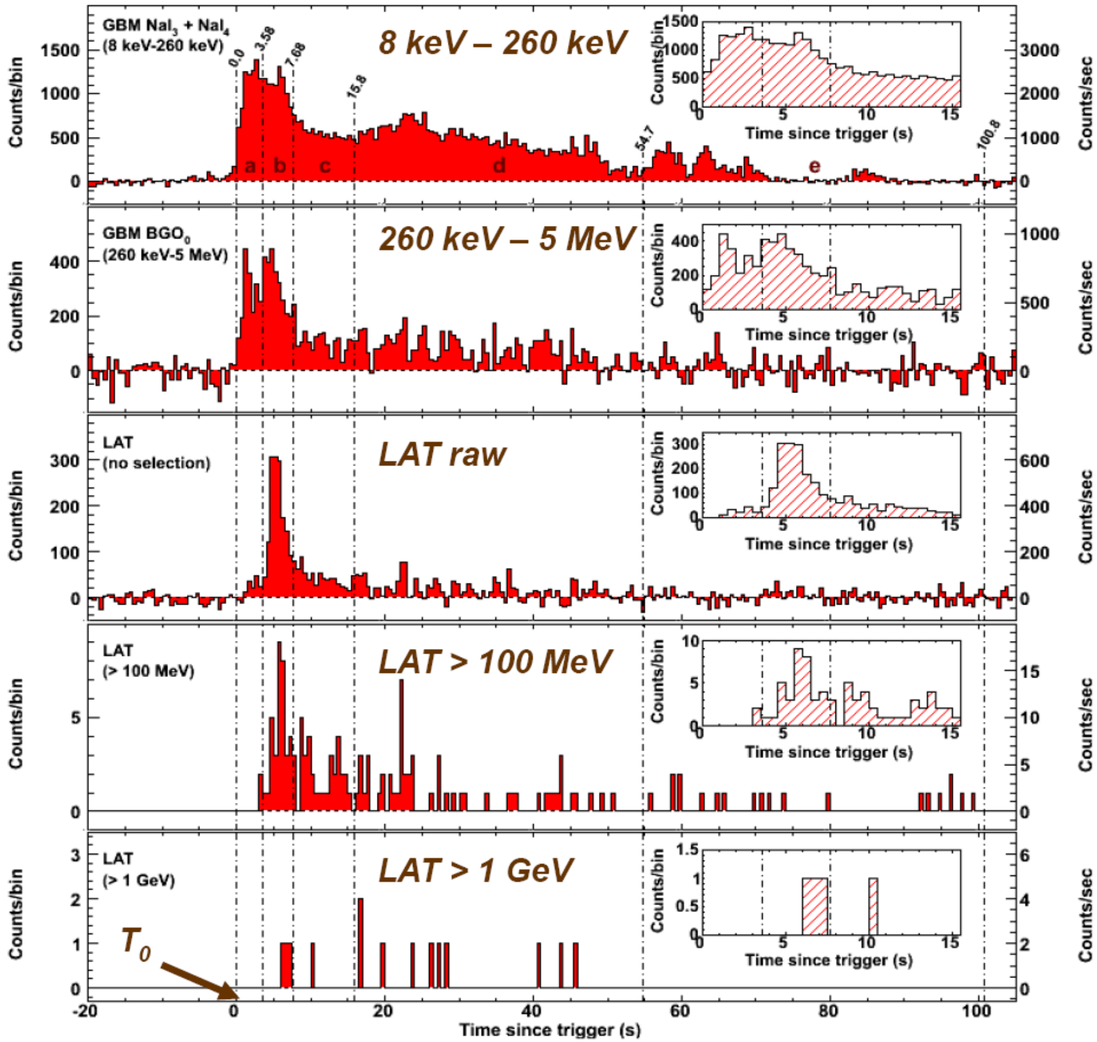
~ 300 GeV



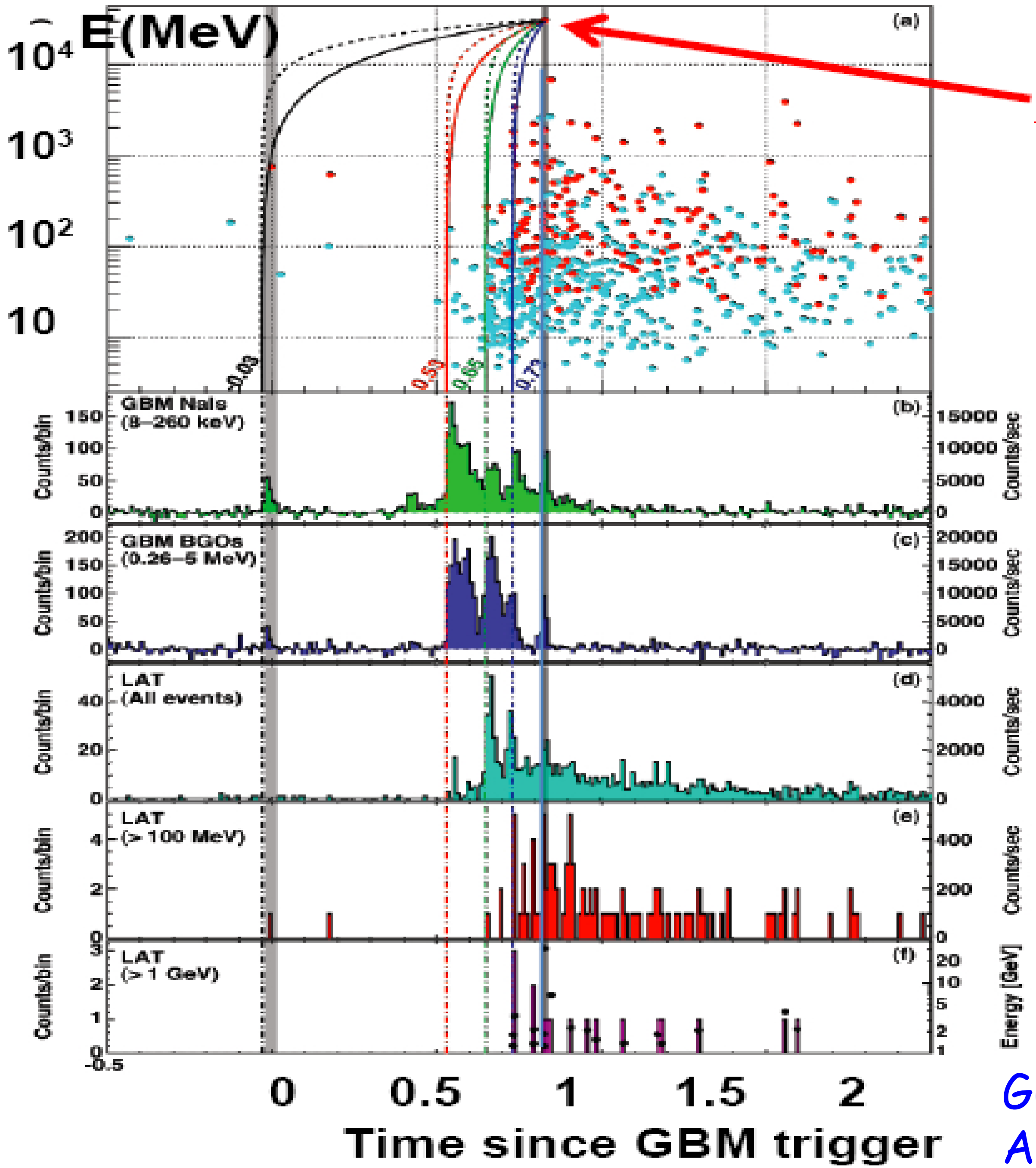
trigger photons ~ 0.1 MeV



Lag determinations



GRB080916C -- Abdo et al.'09, Science



Time lags are affected both artificially and instrumentally

GRB090510
Abdo et al.'09, Nature

Four Fermi observations

the arrival of the highest energy photon to GBM trigger

GRBs	z	E (GeV)	$\Delta t_{\rm obs}$ (s)	$M_{\rm QG,L}$ (GeV/c^2)	$M_{\rm QG,Q}$ (GeV/c^2)
080916C [19]	4.35 [21]	13.22	16.54	1.5×10^{18}	9.7×10^{9}
090510 [20]	0.903 [22]	31	0.829	1.7×10^{19}	3.4×10^{10}
090902B [23]	1.822 [24]	33.4	82	3.7×10^{17}	5.9×10^{9}
090926A [25]	2.1062 [26]	19.6	26	7.8×10^{17}	6.8×10^{9}

$$\Delta t_{\rm obs} = \Delta t_{\rm LV}$$

$$M_{\rm QG,L} \sim (4.9 \pm 8.1) \times 10^{18}\ \mathrm{GeV}$$

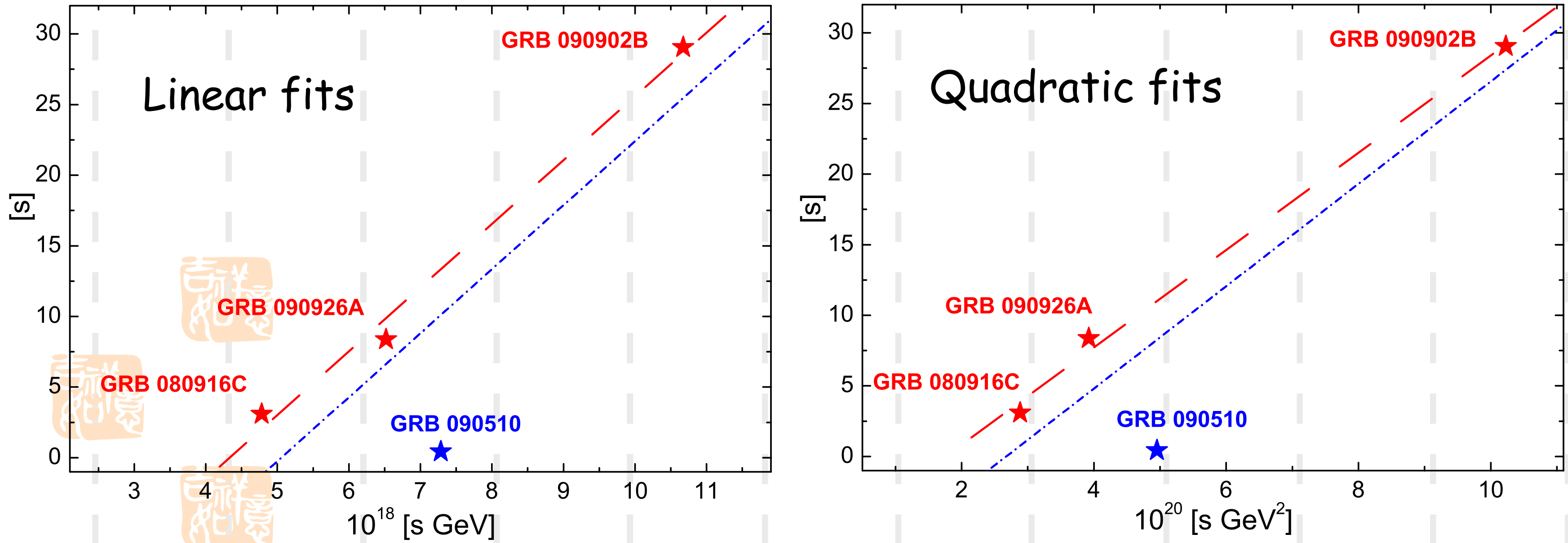
$$M_{\rm QG,Q} \sim (1.4 \pm 1.3) \times 10^{10}\ \mathrm{GeV}$$

Separation of astrophysical time lags from LV delay

- imperfect knowledge of radiation mechanism of GRBs
- a survey of GRBs at different redshifts
  - the time lag induced by LV accumulates with propagation distance
  - the **intrinsic source** induced time lag is likely to be a distance independent quantity
- A robust survey [Ellis et al.'06 & 08, Astropart. Phys.]

$$\Delta t_{\rm LV} = \frac{1+n}{2H_0}\left(\frac{E_{\rm h}^n - E_{\rm l}^n}{M_{\rm QG}^n c^{2n}}\right)\int_0^z \frac{(1+z')^n {\rm d}z'}{h(z')}$$

$$\Delta t_{\rm obs} = \Delta t_{\rm LV} + \Delta t_{\rm in}(1+z)$$

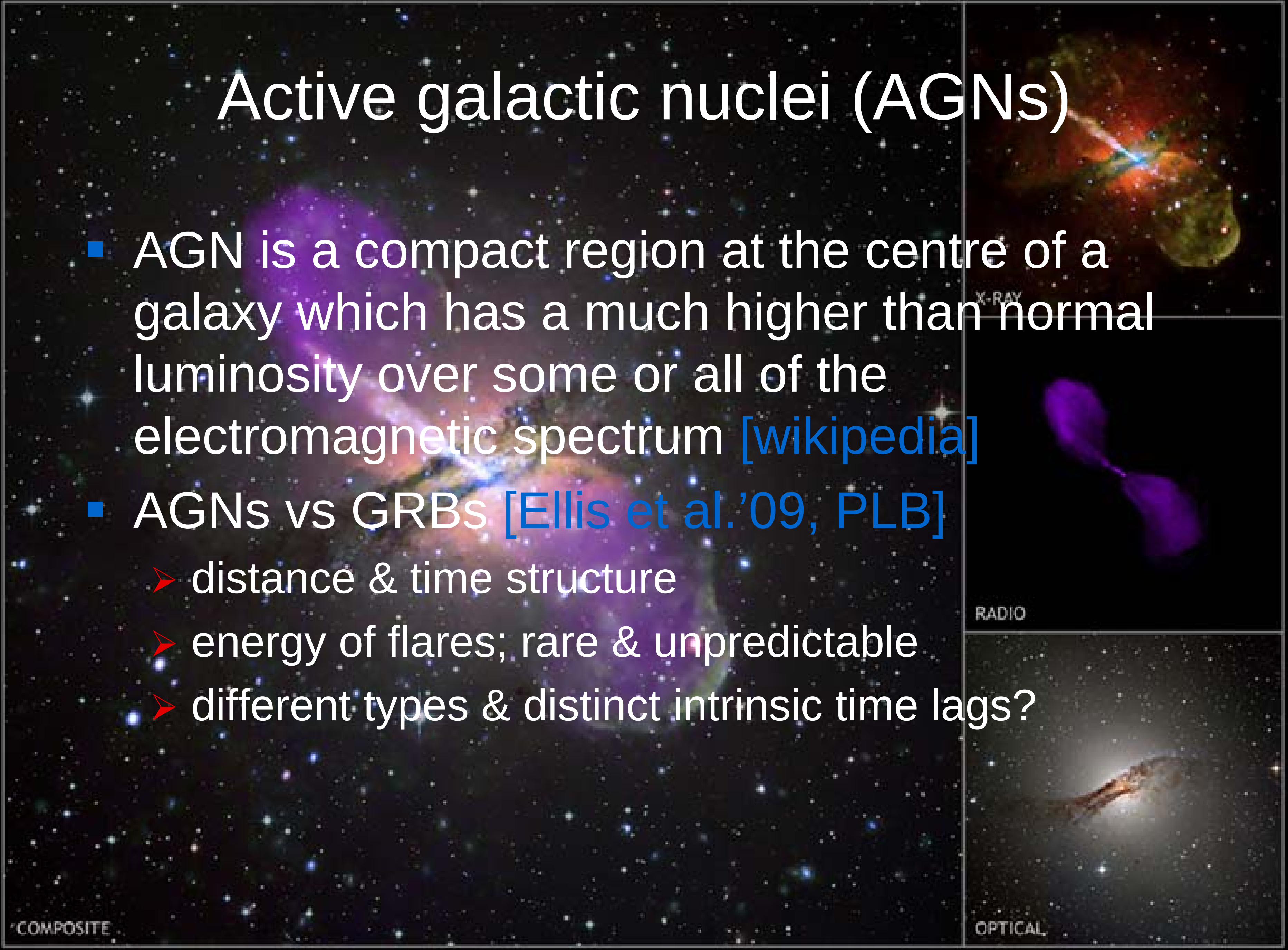


$$M_{\rm QG,L} = (2.2 \pm 0.2) \times 10^{17}\ {\rm GeV/c^2}\ \text{and}\ M_{\rm QG,Q} = (5.4 \pm 0.2) \times 10^{9}\ {\rm GeV/c^2}$$

$$M_{\rm QG,L} = (2.2 \pm 0.9) \times 10^{17}\ {\rm GeV/c^2}\ \text{and}\ M_{\rm QG,Q} = (5.3 \pm 0.8) \times 10^{9}\ {\rm GeV/c^2}$$

Active galactic nuclei (AGNs)

- AGN is a compact region at the centre of a galaxy which has a much higher than normal luminosity over some or all of the electromagnetic spectrum [wikipedia]
- AGNs vs GRBs [Ellis et al.'09, PLB]
  - distance & time structure
  - energy of flares; rare & unpredictable
  - different types & distinct intrinsic time lags?



A brief review on LV AGNs

- **Markarian 421** – no time lag > 280 s between energy bands < 1 TeV and > 2 TeV [Biller et al.'99, PRL]

$$M_{\rm QG,L} > 4.9 \times 10^{16}\ {\rm GeV/c^2}\ \text{and}\ M_{\rm QG,Q} > 1.5 \times 10^{10}\ {\rm GeV/c^2}$$

- **Markarian 501** – 4 min lag for ΔE ~ 2 TeV [Albert et al.'08, PLB]

$$M_{\rm QG,L} \sim 1.2 \times 10^{17}\ {\rm GeV/c^2}$$

- **PKS 2155-304** – ~20 s lag for ΔE ~ 1.0 TeV & ΔE^2 ~ 2.0 TeV^2 [Aharonian et al.'08, PRL]

$$M_{\rm QG,L} \sim 2.6 \times 10^{18}\ {\rm GeV/c^2} \quad M_{\rm QG,Q} \sim 9.1 \times 10^{10}\ {\rm GeV/c^2}$$

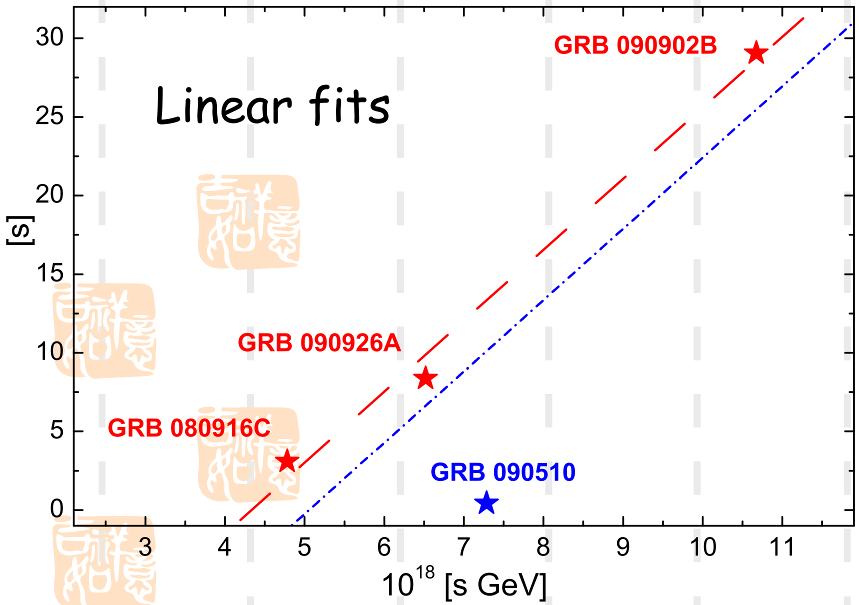
$$\Delta t_{\rm in} = 0$$

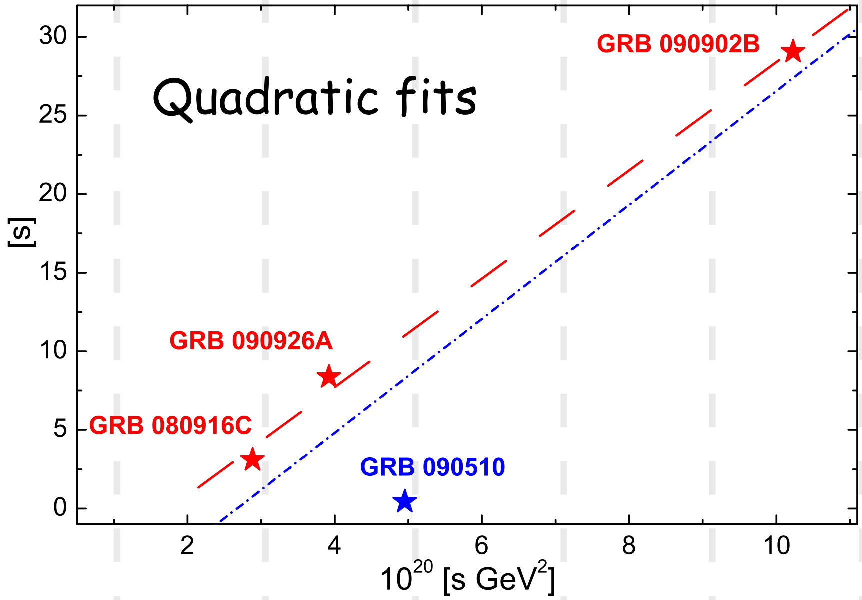
Discussions

- GRBs vs AGNs
  - AGNs data are inadequate to carry out a robust analysis
  - a complementary probe: different observational method and distinct origins
  - Fermi LAT: GRB 090323, GRB 090328, & GRB 091003
- a set of VHE celestial events for LV hints or quantum gravity mass boundaries?
- disentangle astrophysical origins from LV effects?

Discussions

- negative intercepts





~ -20 s for linear dependence

-6 s ~ -10 s for quadratic dependence

A brief summary:

- QG -> LV -> modified dispersion relation -> time lag -> high energy & long distance -> astrophysics -> GRBs & AGNs etc.
- We (re)analyse 4 Fermi LAT GRBs and review 3 AGNs -> surprising consistency
- A robust survey on GRBs of different redshifts to separate source effects -> how about AGNs?

$$\sim 2 \times 10^{17}\ \mathrm{GeV}/c^2$$
$$\sim 5 \times 10^{9}\ \mathrm{GeV}/c^2$$

Thanks for your attentions!

Summary

- **New theory for Lorentz violation can be introduced.**
- **Lorentz violation can provide an alternative explanation for neutrino oscillation.**
- **Photon time-lag effect from Lorentz violation can be observed from GRBs, AGN, and Pulsars.**
- **Lorentz violation is being an active frontier both theoretically and experimentally.**