Invalid IR limit of the Hořava gravity

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Attempts at quantum theory of gravitation

- Einstein's GR: relativistically (diffeomorphism) invariant classical theory of gravitation, extremely successfully describes all available experimental data

- GR is inconsistent (non-renormalizable,…) quantum mechanically, and thus can not be treated as an ultimate theory of gravitation
- Higher-derivative extension of GR is a renormalizable theory ! (Stelle 79)
- DoF : 2 (massless graviton) + 5 (massive (~M_P) spin-2) + 1 (massive (~M_P) scalar)

  spin-2 quantum states have negative norm – unitarity is compromised !

  Alternative quantizations

  - Analogy with PT symmetric QM – rigorous proof for QFT is still absent (Bander, Mannheim)
  - Spin-2 states are "shadow states", i.e. they do not realized as asymptotic on-shell states and so do not spoil S-matrix unitarity. Analiticity of S-matrix, and hence causality is compromised ! (Lee-Wick, Sudarshan, …)
- Currently, string theory is the only known consistent UV completetion of quantum theory of gravitation.

Hořava's proposal

- P. Hořava (**0901.3775**) has proposed a new way to remove the ghost states in higher-derivative gravity at a price of breaking full diffeomorphism invariance down to the "foliation-preserving" diffeomorphism symmetry. Hořava's gravity is claimed to be a renormalizable theory of gravitation with unitarity/causality maintained.
- A lot of excitement – more than 100 works on the subject in first 6 months.
- Does the Hořava theory reproduces Einstein's GR in the IR regime ?
- Diffeomorphism or local gauge invariance = redundancy of the description = statement about physical DoFs. Therefore, due to the reduced Diff invariance one expects extra DoF(s) in Hořava gravity.
- Do these DoFs affect physics in the IR ?
- Several authors have identified extra DoF in Hořava gravity using the linearized approximation of the theory both on flat and cosmological backgrounds. The results of these studies are controversial: some authors claim decoupling of the extra DoF in the IR, some found the extra DoF is problematic (strong coupling, instabilities…)
- Note: linearized approximation might be misleading (especially in LV theories). Dirac's constraint analysis provide more rigorous counting of DoFs in gauge theories.

Dirac's constraint analysis of Hořava gravity

- M. Li and Y. Pang (0905.2751) have studied non-projectable version of Hořava gravity from UV perspectives. They have demonstrated that the phase space contains odd number of fields, that is, no standard canonical structure is available (quantization using Nambu brackets ?)

- Projectable version of Hořava gravity in the IR regime:

$$S_{\text{Horava}} = \int dt \int_{\Sigma_t} d^3x \left(\pi^{ij}\dot{h}_{ij} - N\mathcal{H}_0 - N_i\mathcal{H}^i\right) ,$$

$$\mathcal{H}_0 = -\sqrt{h}\ {}^{(3)}R + h^{-1/2}\pi^{ij}\pi_{ij} - \frac{(3\lambda - 1)}{4\lambda}h^{-1/2}\pi^2 ,$$

$$\mathcal{H}^i = -2\nabla_i(\pi^{ij}) ,$$

- Lapse function depends only on time coordinate: $N \equiv N(t)$ (consistent with foliation-preserving diffs, $\xi^0(t)$, $\xi^i(t,x)$,).

- We also set **λ = 1** in the IR. In the UV, λ = 1/3 (conformal symmetry with asymmetric scaling, t -> s^3t, x -> sx).

λ (UV) --> λ (IR) due to the RG flow (???)

Dirac's constraint analysis of Hořava gravity

- Canonical variables are defined on a const. time spatial hyper-surface Σ_t:

$$(N, \pi_N) \ , \quad \left(N_i(\vec{x}), \pi_N^i(\vec{x})\right) \ , \quad \left(h_{ij}(\vec{x}), \pi^{ij}(\vec{x})\right)$$

- Primary constraints:

$$\pi_N \equiv \frac{\partial \mathcal{L}_{\rm Horava}}{\partial \dot{N}} \approx 0 \ ,$$
$$\pi_N^i \equiv \frac{\partial \mathcal{L}_{\rm Horava}}{\partial \dot{N}_i} \approx 0$$

- Canonical Poisson algebra:

$$\{N, \pi_N\}_{\rm PB} = 1 \ ,$$
$$\left\{N_i(x), \pi_N^j(y)\right\}_{\rm PB} = \delta_i^j \delta^3(x-y) \ ,$$
$$\{h_{ij}(x), \pi^{mn}(y)\}_{\rm PB} = \frac{1}{2}\left(\delta_i^m \delta_j^n + \delta_i^n \delta_j^m\right) \delta^3(x-y) \ .$$

Dirac's constraint analysis of Hořava gravity

- Secondary constraints:

$$\dot{\pi}_N = \{\pi_N, H\}_{\rm PB} = \int_{\Sigma_t} d^3y \mathcal{H}_0(y)$$

$$\Downarrow$$

$$H_0 \equiv \int_{\Sigma_t} d^3x \mathcal{H}_0 \approx 0$$

Thus we have only global Hamiltonian is constrained !

The secondary momentum constraint must be satisfied at each spatial point x :

$\mathcal{H}^i(x) \approx 0$, just like in GR

Dirac's constraint analysis of Hořava gravity

- Next we check that the algebra of secondary constraints is closed:

$$\{H_0, H_0\}_{\rm PB} = \int\int d^3x d^3y \left(2\mathcal{H}^i(x)\partial_i^{(x)}\delta^3(x-y) + \partial_i^{(x)}\mathcal{H}^i(x)\delta^3(x-y)\right)$$

$$= \int d^3x \partial_i^{(x)}\mathcal{H}^i(x) = 0 ;$$

$$\{\mathcal{H}_i(x), H_0\}_{\rm PB} = \mathcal{H}_0(x)\partial_i^{(x)}\int d^3y \delta^3(x-y) = 0 ;$$

$$\{\mathcal{H}_i(x), \mathcal{H}_j(y)\}_{\rm PB} = \mathcal{H}_i(y)\partial_j^{(y)}\delta^3(x-y) - \mathcal{H}_j(x)\partial_i^{(x)}\delta^3(y-x) \approx 0 .$$

- No further constraints are generated:

$$\dot{H}_0 = \{H_0, H\}_{\rm PB} = 0 , \quad \dot{\mathcal{H}}_i = \{\mathcal{H}_i, H\}_{\rm PB} \approx 0 .$$

Thus, the constraints are the first-class constraints

- # of physical DoF = ½ (# of canonical var.) – (# of 1st class constr.)

Dirac's constraint analysis of Hořava gravity

- Zero (spatially homogeneous) modes: ½ ·20 (can. var.) – 8(constraints) = 2
- Propagating modes: ½ ·18 (can. var.) – 6(constraints) = 3

Thus, there is 1 (per each point x) extra propagating DoF in Hořava gravity

- Compare with unimodular gravity:

  Reduced Diffs: 4-volume preserving diffs, $\nabla_\mu \xi^\mu = 0$;

  Lapse function is fixed – no secondary Hamiltonian constraint ;

  However, the algebra of constraints is not closed, tertiary 1st class constraint is generated: $\partial_i (h^{-1/2}\mathcal{H}_0) = 0 \quad \Longrightarrow \quad \mathcal{H}_0' \equiv \mathcal{H}_0 + h^{1/2}\Lambda(t) \approx 0$

  $$\dot{\mathcal{H}}_0' \approx 0 \quad \Longrightarrow \quad \Lambda = const$$

  1 extra global ("vacuum" field) DoF – the cosmological constant which is canonically conjugated with cosmic time.

Diff-invariant action

- Promote *N(t)* to full-fledged field *N(t,x)* and introduce 2nd class constraints. The constraint is introduced through the auxiliary spatial two-form field $A_{ij}(t,x)$ with the tree-form field strength $F_{ijk} = \partial_{[i} A_{jk]}$. Dual field strength transform as the spatial density:

$$\tilde{\mathcal{F}} \equiv h^{1/2} \frac{1}{3!} \epsilon^{ijk} \mathcal{F}_{ijk} = \partial_i \tilde{\mathcal{A}}^i$$

- Then, we write the action,

$$S_{\text{equiv.}} = S_{\text{GR}} + \int d^4 x N \partial_i \tilde{\mathcal{A}}^i \ ,$$

which is invariant under the full Diffs (by construction)

- The constraint eq.: $\partial_i N = 0 \quad \Rightarrow \quad N = N(t)$
- Global Hamiltonian constraint is also reproduced:

Integrating the local Hamiltonian constraint $\mathcal{H}_0 + \partial_i \tilde{\mathcal{A}}^i \approx 0$ we obtain:

$$\int d^3 x \left(\mathcal{H}_0 + \partial_i \tilde{\mathcal{A}}^i \right) = \int d^3 x \mathcal{H}_0 + \tilde{\mathcal{A}}^i_{|x| \to \infty} = \int d^3 x \mathcal{H}_0 \approx 0$$

Modified Einstein's equations

- Define another auxiliary scalar as: $F = 2\tilde{\mathcal{F}}/h^{1/2}$
- Then the Einstein equations can be written as:

$$^{(4)}R_{\mu\nu} - 1/2^{(4)}g_{\mu\nu}R = -F n_\mu n_\nu \ , \quad n_\mu = (1,0,0,0).$$

- The free (vacuum) gravitational eqs in Horava theory look like the GR equations sourced by the dust-like fluid (Mukohyama [0905.3563] – "dark matter as an integration constant")
- Not an ordinary dust, since: $\int d^3x h^{1/2} F = \int d^3x \tilde{\mathcal{F}} = 0$.
- Let us solve for the auxiliary field F, $F = \frac{1}{g^{00}} R$, and put back into Einstein's eqs:

$$R_{\mu\nu} - \left(\frac{1}{2}g_{\mu\nu} - \frac{1}{g^{00}} n_\mu n_\nu\right) R = \left(T_{\mu\nu} + n_\mu n_\nu T^\alpha_\alpha\right) \ ,$$

- Plus the constraint eq.: $\partial_i \left((-g^{00})^{-1/2}\right) = 0$

Weak field approximation

- Expand around flat metric: ${}^{(4)}g_{\mu\nu} \approx \eta_{\mu\nu} + \gamma_{\mu\nu}$.
- Take temporal gauge: $n^{\mu}\gamma_{\mu\nu} = -\gamma_{0\nu} = 0$, the constraint eq. is automatically satisfied.
- Linearized equations:

$$00: \quad 2\ddot{\gamma} + \left(\partial^k\partial^l\gamma_{kl} - \triangle\gamma\right) = -2(T_{00} + T^{\alpha}_{\alpha}) \ ,$$

$$0i: \quad \partial^k\dot{\gamma}_{ki} - \partial_i\dot{\gamma} = 2T_{0i} \ ,$$

$$ij: \quad \partial_\mu\partial^\mu\gamma_{ij} - \partial_i\partial^k\gamma_{kj} - \partial_j\partial^k\gamma_{ik} + \partial_i\partial_j\gamma + \delta_{ij}\left(\partial^k\partial^l\gamma_{kl} - \partial_\mu\partial^\mu\gamma\right) = -2T_{ij} \ .$$

- Note: 00 equation is not the independent one – there is no local Hamiltonian constraint

Weak field approximation

- Next, expand the graviton field in Poincare-irreducible components:

$$\gamma_{ij} = \gamma_{ij}^{\mathrm{TT}} + \partial_i V_j^{\mathrm{T}} + \partial_j V_i^{\mathrm{T}} + \left(\delta_{ij} - \frac{\partial_i \partial_j}{\triangle}\right)\phi + \frac{\partial_i \partial_j}{\triangle}\psi \ .$$

- $\gamma_{\mu\nu}^{\mathrm{TT}}, \quad V_i^{\mathrm{T}}$ have the same solutions as in GR;
- Scalar modes:

$$\phi = -\frac{1}{\triangle}T_{00} + h(\vec{x})$$

$$\ddot{\psi} = f^{\mathrm{GR}} + \triangle h$$

- $h(\vec{x}) \neq 0, \quad \triangle h \neq 0, \quad \int d^3x \triangle h = 0.$ In the free limit, this field cannot be gauged away! It contributes to the gravitational propagator, and hence to S-matrix.

Conclusion

- We have established through the constraint analysis that the projectable version of the Horava gravity in the IR regime contains extra propagating DoF (even if $\lambda = 1$ strictly). Therefore, it does not reproduces GR.

- More work is required on the phenomenological validity of the theory.