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Analytical Modeling of Turbomolecular Pumps and Roots Blowers

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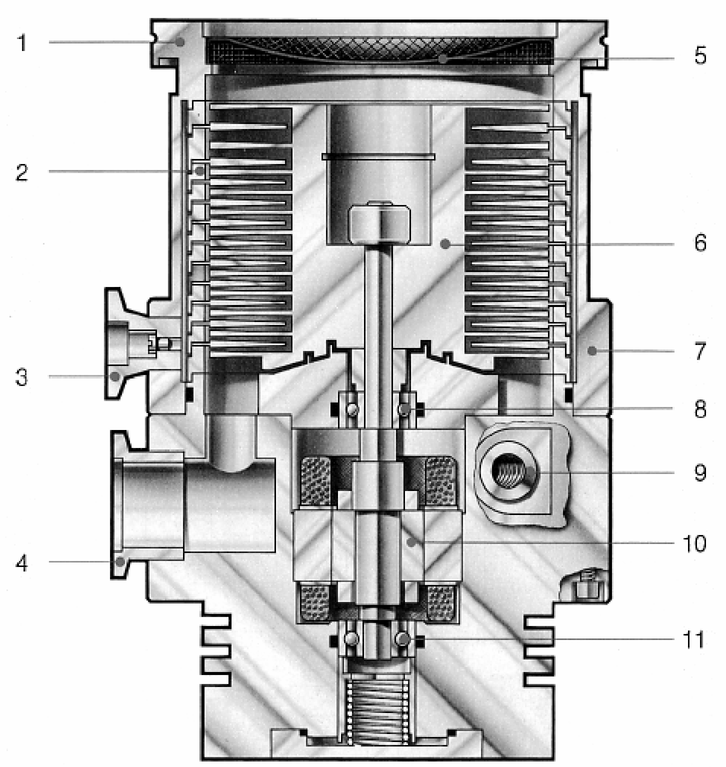
Classical Turbomolecular Pump



1	Hochvakuumanschlußflansch;	5	Splitterschutz;	9	Kühlwasseranschluß;
2	Stator-Paket;	6	Rotor;	10	3-Phasen-Motor;
3	Belüftungsanschlußflansch;	7	Pumpengehäuse;	11	Kugellager
4	Vorvakuumanschlußflansch;	8	Kugellager;		

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How to calculate the compression curve of a classical turbomolecular pump ?

Classical turbomolecular pump (TMP) = TMP without compound stage

Analytical model:

TMP in one dimension:

$x = 0$: High-vacuum flange (= inlet flange)

$x = L$: Foreline port; L is taken as the effective length of the TMP

Basic assumptions

Ansatz:

$$Q = Q_{max} - Q_{bs}$$

Net gas throughput = Maximum gas throughput in forward direction – Backstreaming

$$p_1 \cdot S_{max} = p \cdot S_{max} - r \cdot (1/h_v) \cdot (p + p_2) \cdot (dp/dx)$$

h_v : Dynamic viscosity in the range of viscous flow

p_2 : Empirical parameter (depends on the geometry of the TMP) { $p_2 = p_2(Q)$ }

Differential equation

Differential equation: $(p + p_2)\cdot(dp/dx) = (p - p_1)\cdot(C_2/L)$

Boundary condition: $p(x = L) = p_{FV}$ (= foreline pressure)

x : Position inside the TMP $\{ 0 \leq x \leq L \}$

p : Pressure inside the TMP $\{ p(x = 0) = p_{HV} \leq p = p(x) \leq p_{FV} = p(x = L) \}$

$\mathbf{p_1}$: Lowest high-vacuum pressure which can be attained with the TMP at the prescribed gas throughput Q $\{ p_1 = p_1(Q) \}$

$\mathbf{C_2}$: $\mathbf{C_2 = h_v \cdot S_{max} \cdot L/r}$

h_v : Dynamic viscosity in the range of viscous flow

$\mathbf{S_{max}}$: Maximum pumping speed at the inlet flange at the prescribed gas throughput Q $\{ S_{max} = Q/p_1 = S(p_1) \}$

L : Effective length of the TMP

r : Backstreaming coefficient $\{ r = r(Q) \}$

Solution of the differential equation

$$p = p_1 + (p_{FV} - p_1)\cdot \exp(b\cdot(p_{FV} - p - C_2\cdot[1 - x/L]))$$

$$b = 1/(p_1 + p_2) \qquad\qquad C_2 = h_v\cdot S_{max}\cdot L/r$$

p_2 and C_2 are the crucial parameters of the model.

$$p(x = 0) = p_{HV} = p_1 + (p_{FV} - p_1)\cdot \exp(b\cdot(p_{FV} - p_{HV} - C_2))$$

Þ solve for the high-vacuum pressure p_{HV} numerically or

Þ use an analytical approximation, if $p_{HV} << p_{FV}$:

$$p_{HV} = p_1 + (p_{FV} - p_1)\cdot \exp(b\cdot(p_{FV} - C_2))$$

The **compression curve $K(p_{FV})$ of the TMP** is given by

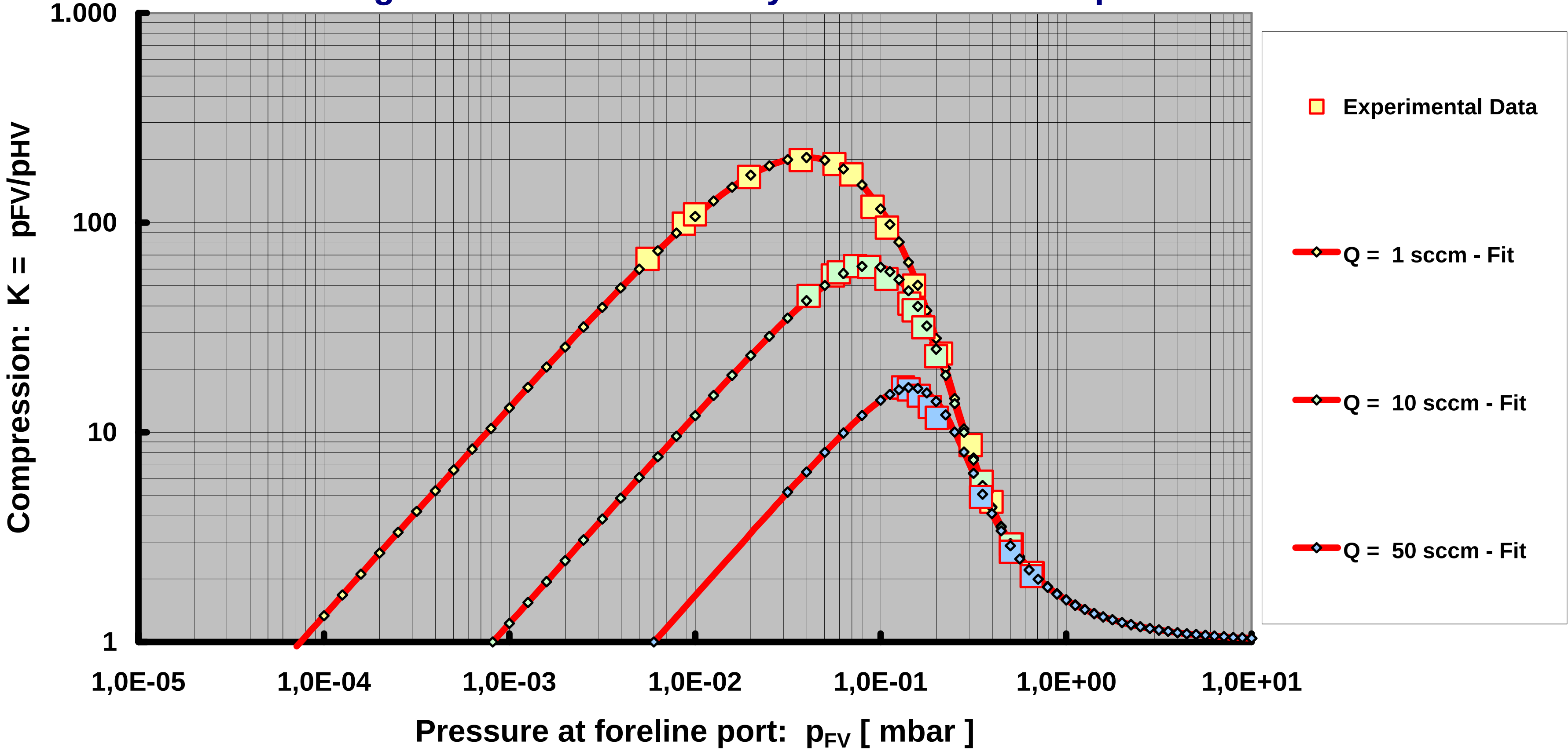
$$K(p_{FV}) = p_{FV} / p_{HV}$$

TMP without compound stage (100 ISO-K; Rotational speed: 750 Hz)

Compression - Hydrogen

(p_2 = 6,2 x 10^{-2} mbar, C_2 = 0,4 mbar)

Excellent agreement between analytical model and experimental data



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Intuition

The analysis of the compression curves leads to the following intuition:

The parameters p_2 and C_2 are constants and do not depend on throughput, provided the throughput Q is less than 100 sccm.

Consequently,

$$S_{max}/r = C_2/(\eta_v \cdot L)$$

does not depend on throughput and is a constant.

Zero throughput compression

$Q = 0$ Þ $p_1 = 0$

Þ $p_{HV} = p_{FV} \cdot \exp((p_{FV} - p_{HV} - C_2)/p_2)$

Þ $K_0(p_{FV}) = p_{FV}/p_{HV} = \exp(- (p_{FV} - p_{HV} - C_2)/p_2)$

In the limit $p_{HV} << p_{FV} << C_2$ we obtain for the zero throughput compression of the TMP:

Þ K_0 ® $k_0 = \exp(C_2/p_2)$

Hypothesis: The zero throughput compression curve of a TMP can be derived from a compression curve for finite throughput, e.g. Q = 1 sccm.

This can be achieved by using the parameters p_2 and C_2 determined from the Q = 1 sccm-compression curve to calculate the zero throughput compression curve.

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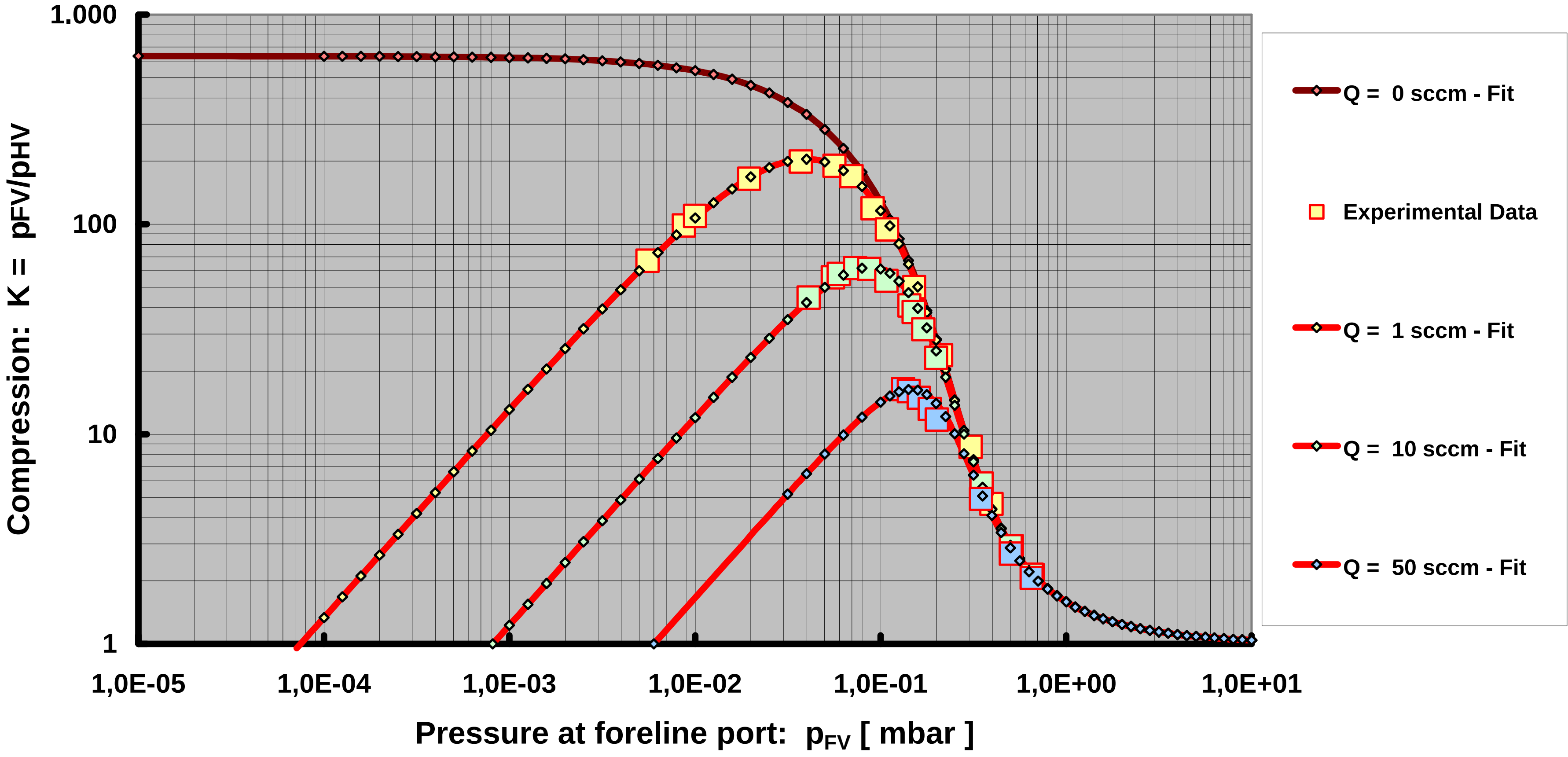
TMP without compound stage (100 ISO-K; Rotational speed: 750 Hz)

Compression - Hydrogen

($p_2 = 6{,}2 \times 10^{-2}$ mbar, $C_2 = 0{,}4$ mbar, $k_0 = \exp(C_2/p_2) = 634$)



TMP without compound stage (100 ISO-K; Rotational speed: 750 Hz)

Compression - Hydrogen

(p_2 = 6,2 x 10^{-2} mbar, C_2 = 0,4 mbar, k_0 = $\exp(C_2/p_2)$ = 634)

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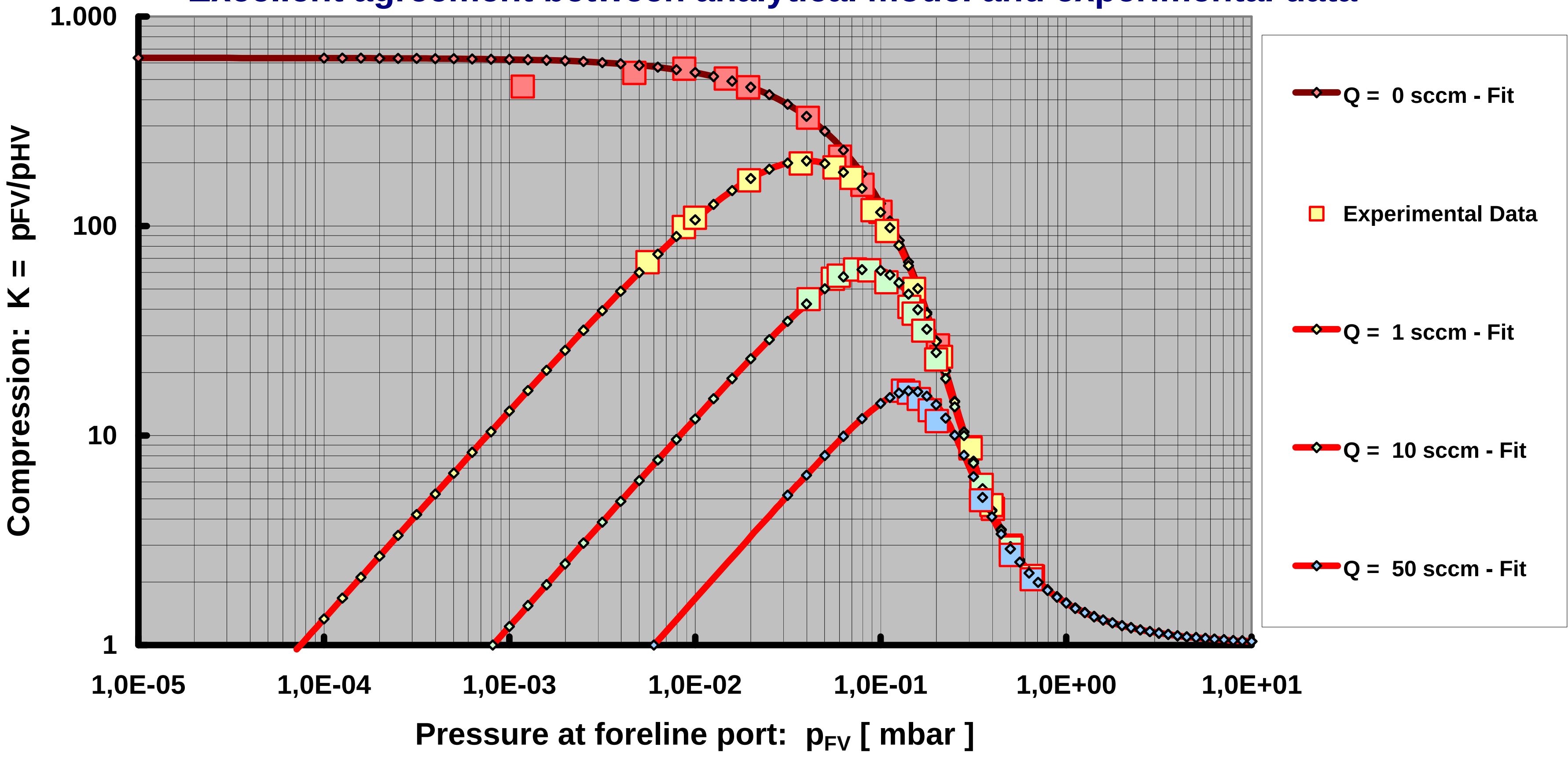

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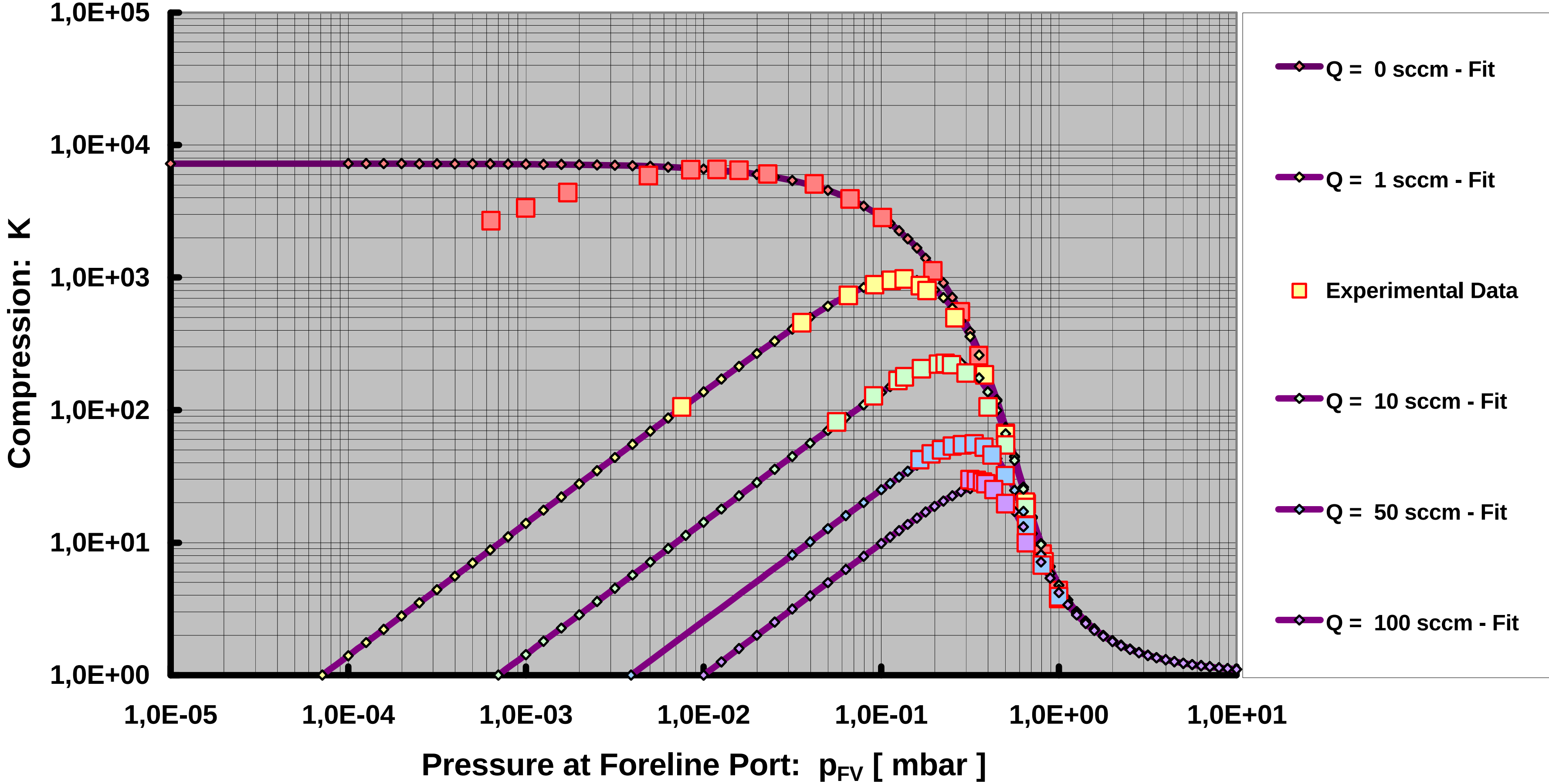
TMP w/o Compound Stage (100 ISO-K; Rotational Speed: 750 Hz)
Compression - Helium



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How to calculate the compression curve of a wide range turbomolecular pump ?

Wide range turbomolecular pump (TMP) =
Classical TMP plus compound stage

Apply the analytical model twice !

Turbo stage: compression from p_{HV} to intermediate pressure p_{int}

$K_1 = p_{int} / p_{HV}$

Compound stage: compression from intermediate pressure p_{int} to p_{FV}

$K_2 = p_{FV} / p_{int}$

Total compression: $K = K_1 \cdot K_2 = p_{FV} / p_{HV}$

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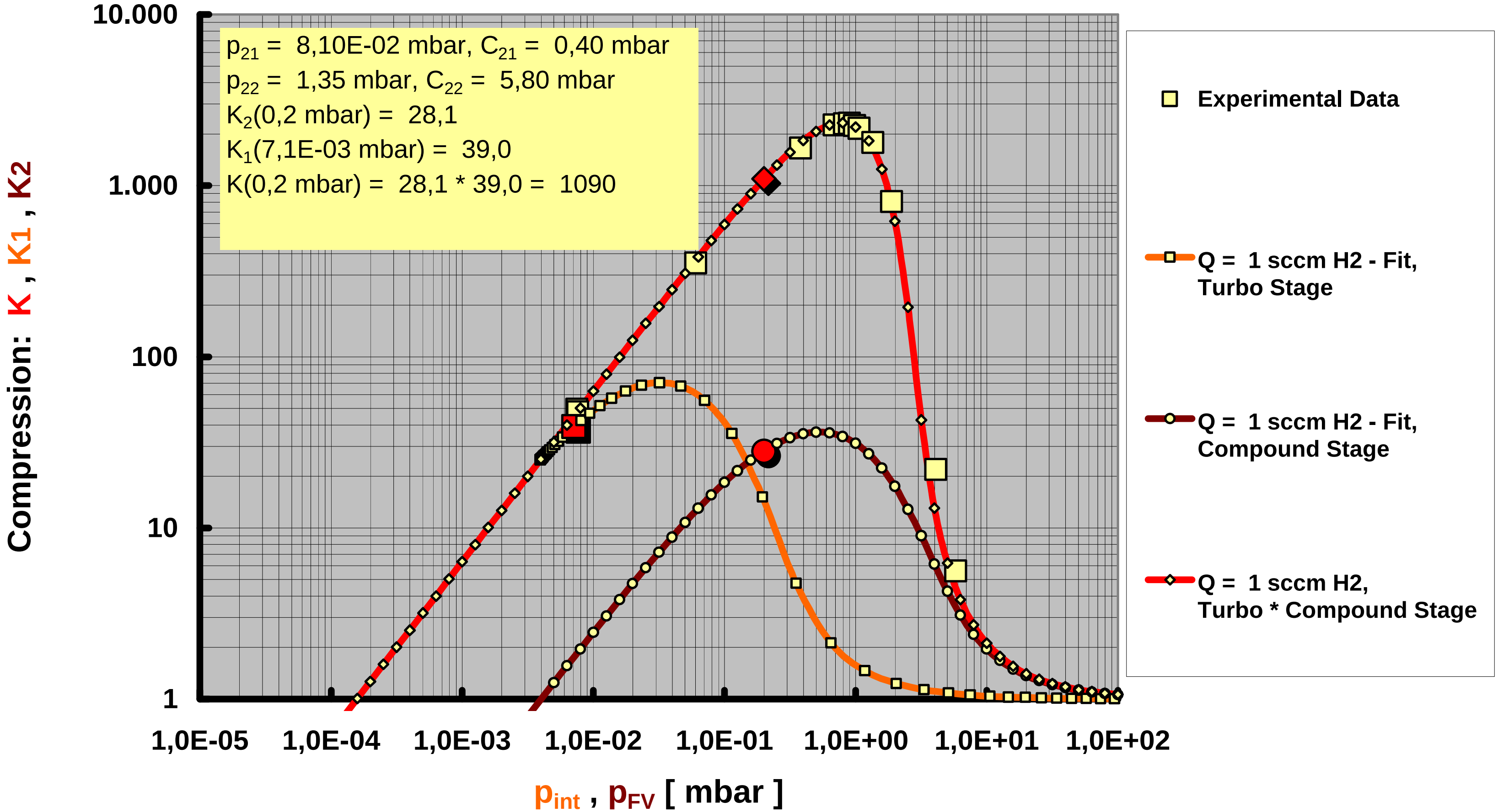
TMP with Compound Stage (100 ISO-K; Rotational Speed: 1000 Hz)

Compression - Hydrogen

The separation of turbo and compound stage works.



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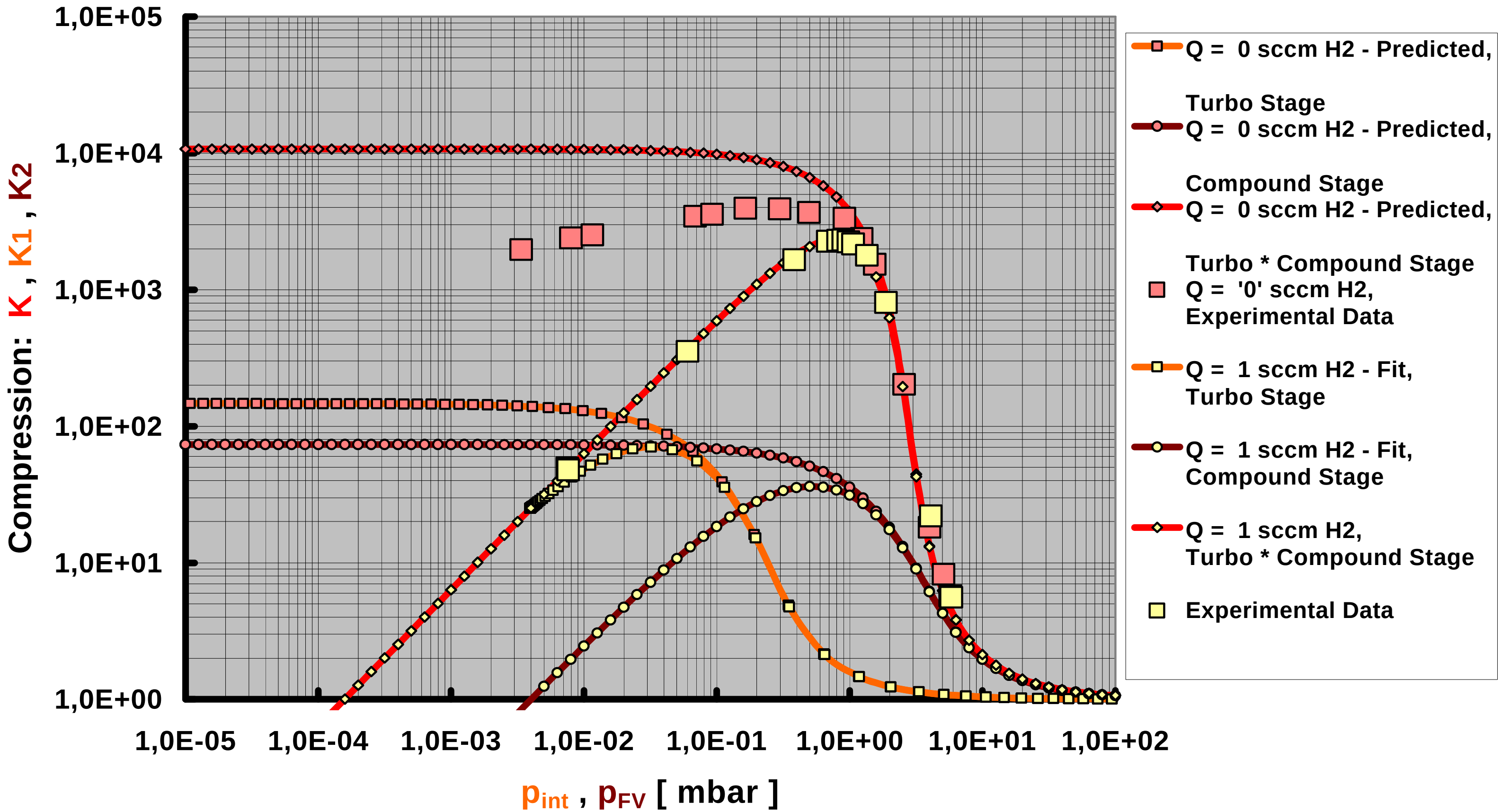
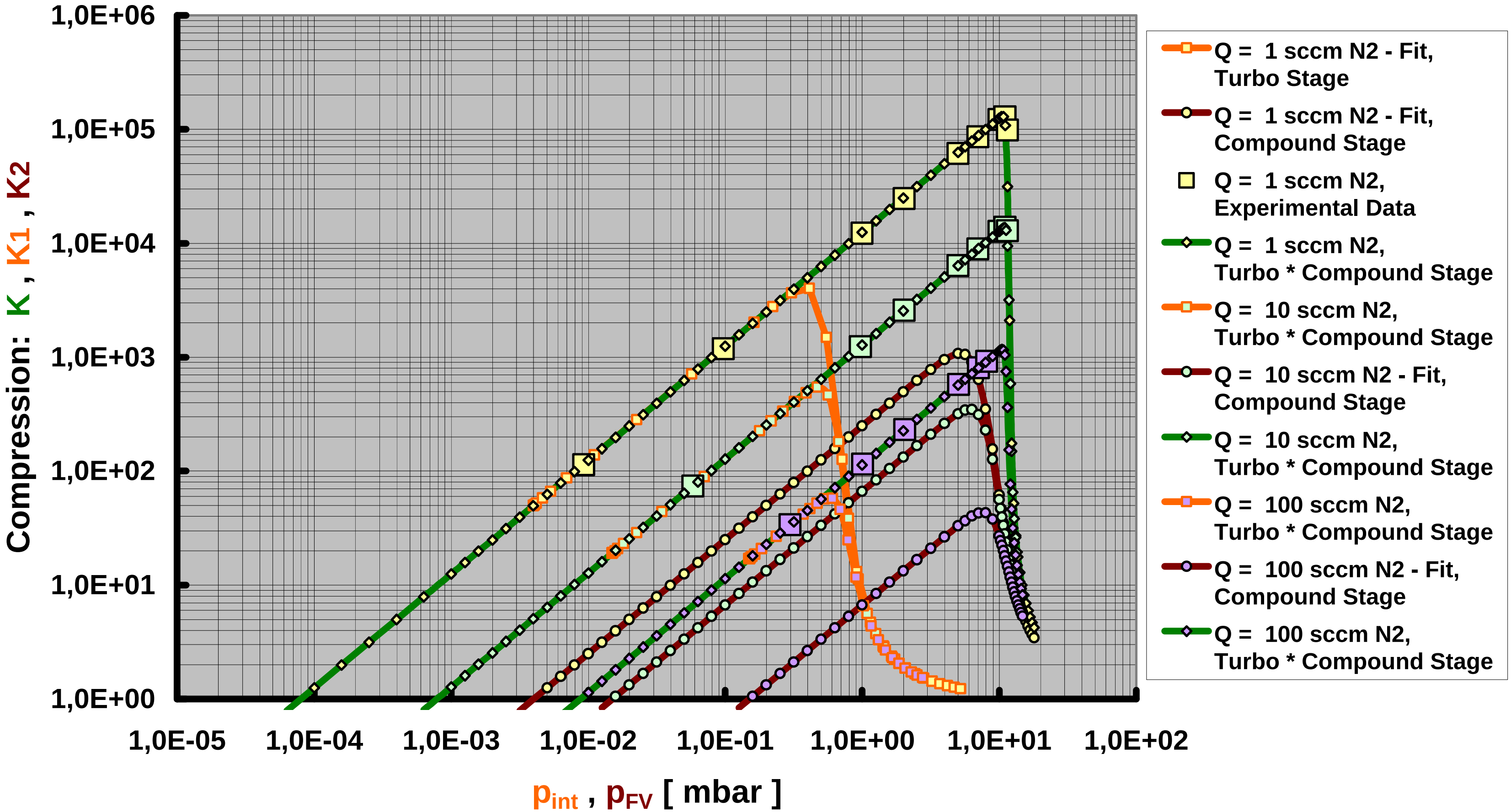
TMP with Compound Stage (100 ISO-K; Rotational Speed: 1000 Hz)
Compression - Hydrogen
The true zero-throughput compression curves can be calculated.



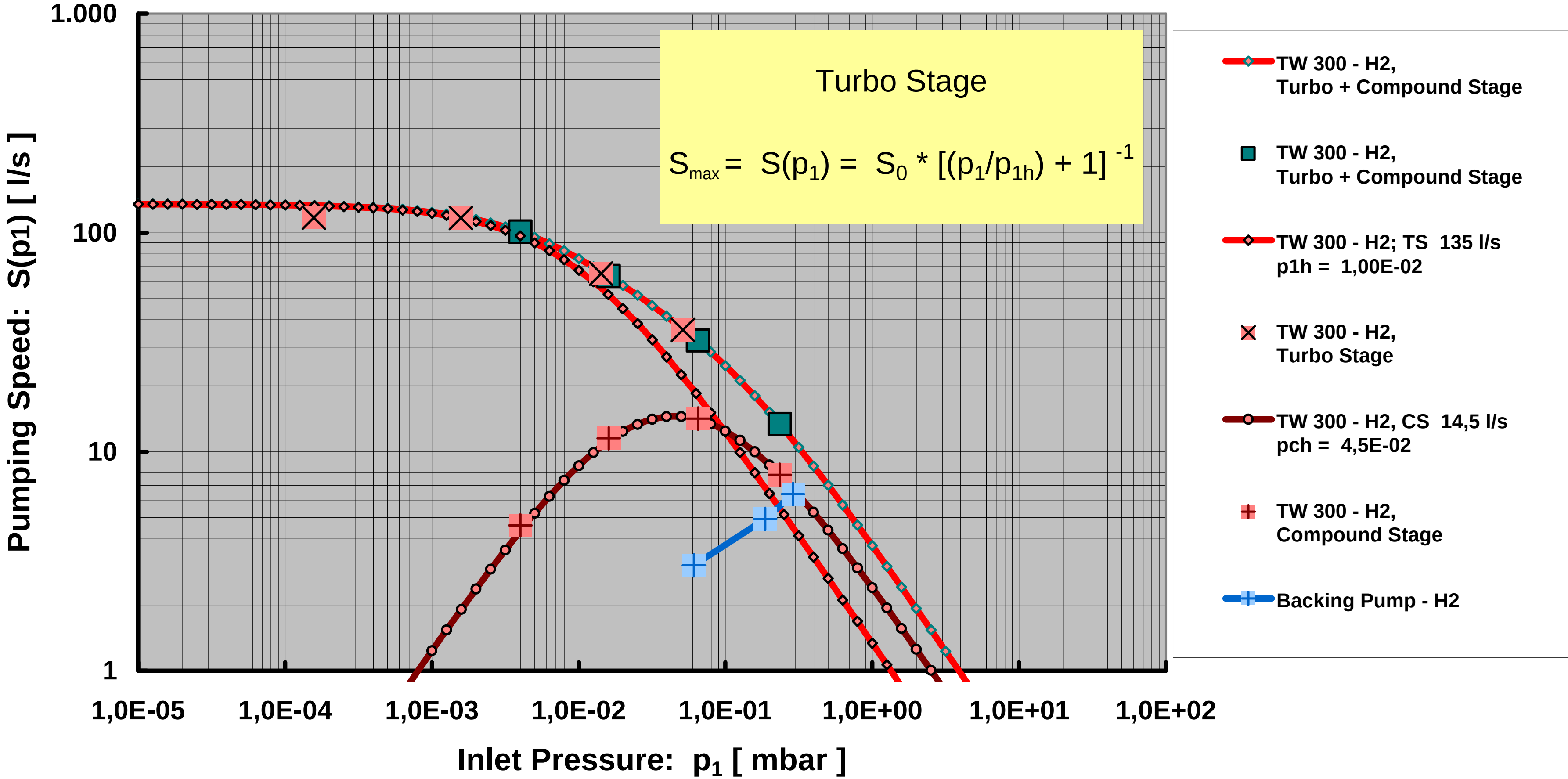
TMP with Compound Stage (100 ISO-K; Rotational Speed: 1000 Hz)
Compression - Nitrogen



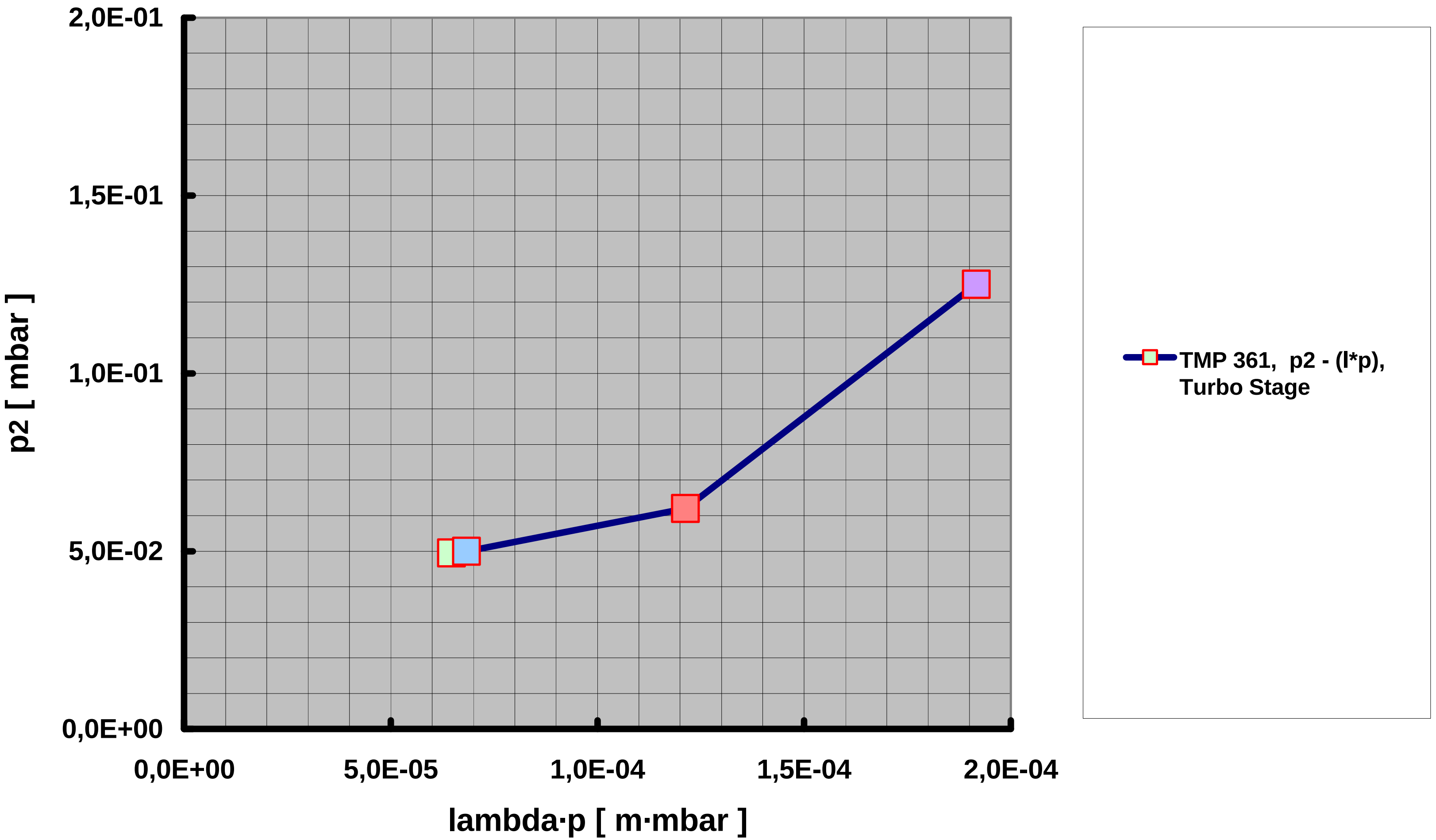
TMP with Compound Stage (100 ISO-K; Rotational Speed: 1000 Hz)

Pumping Speed for Hydrogen

Turbo Stage and Compound Stage



p_2 is a nonlinear function of lambda·p !



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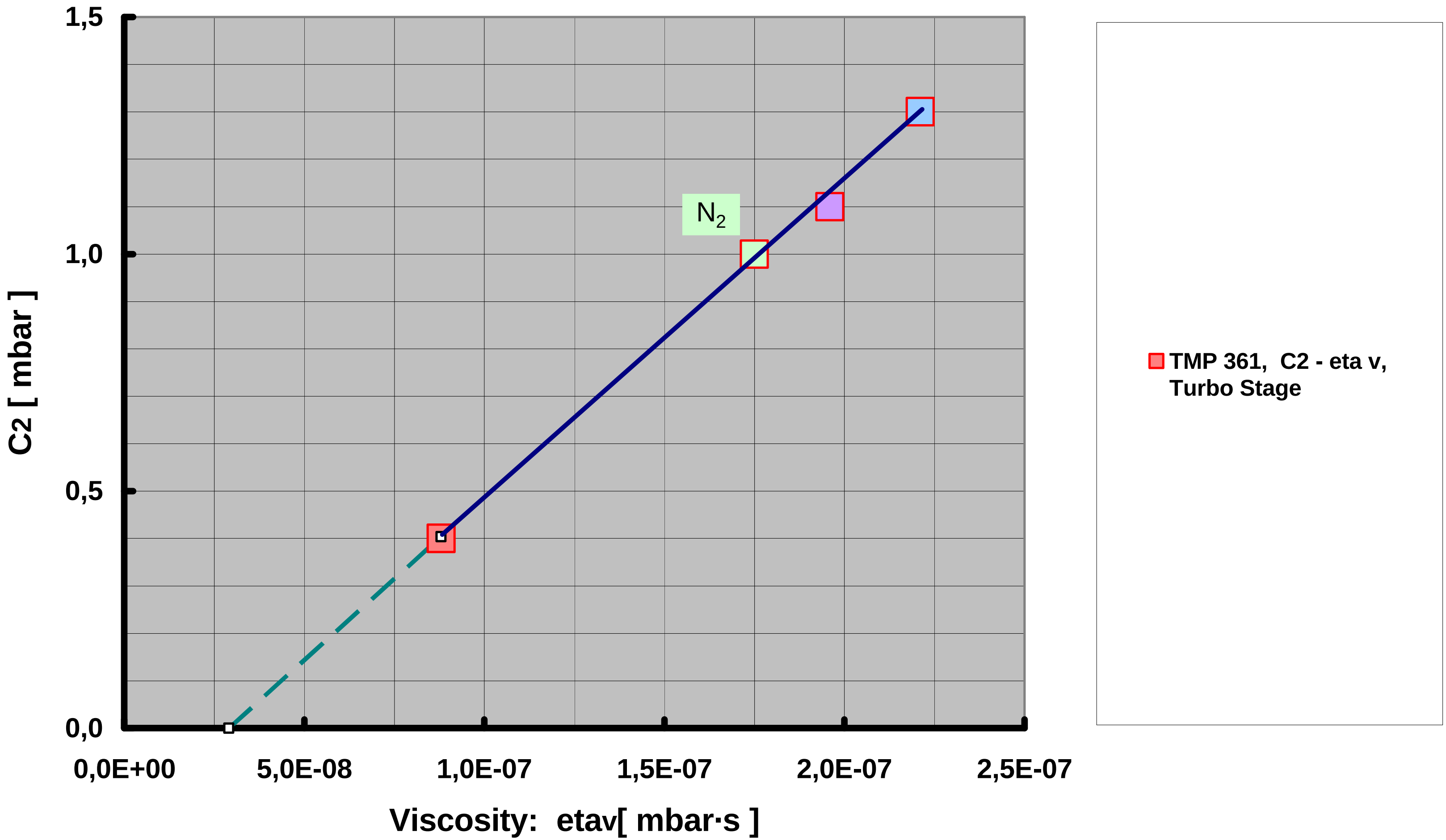
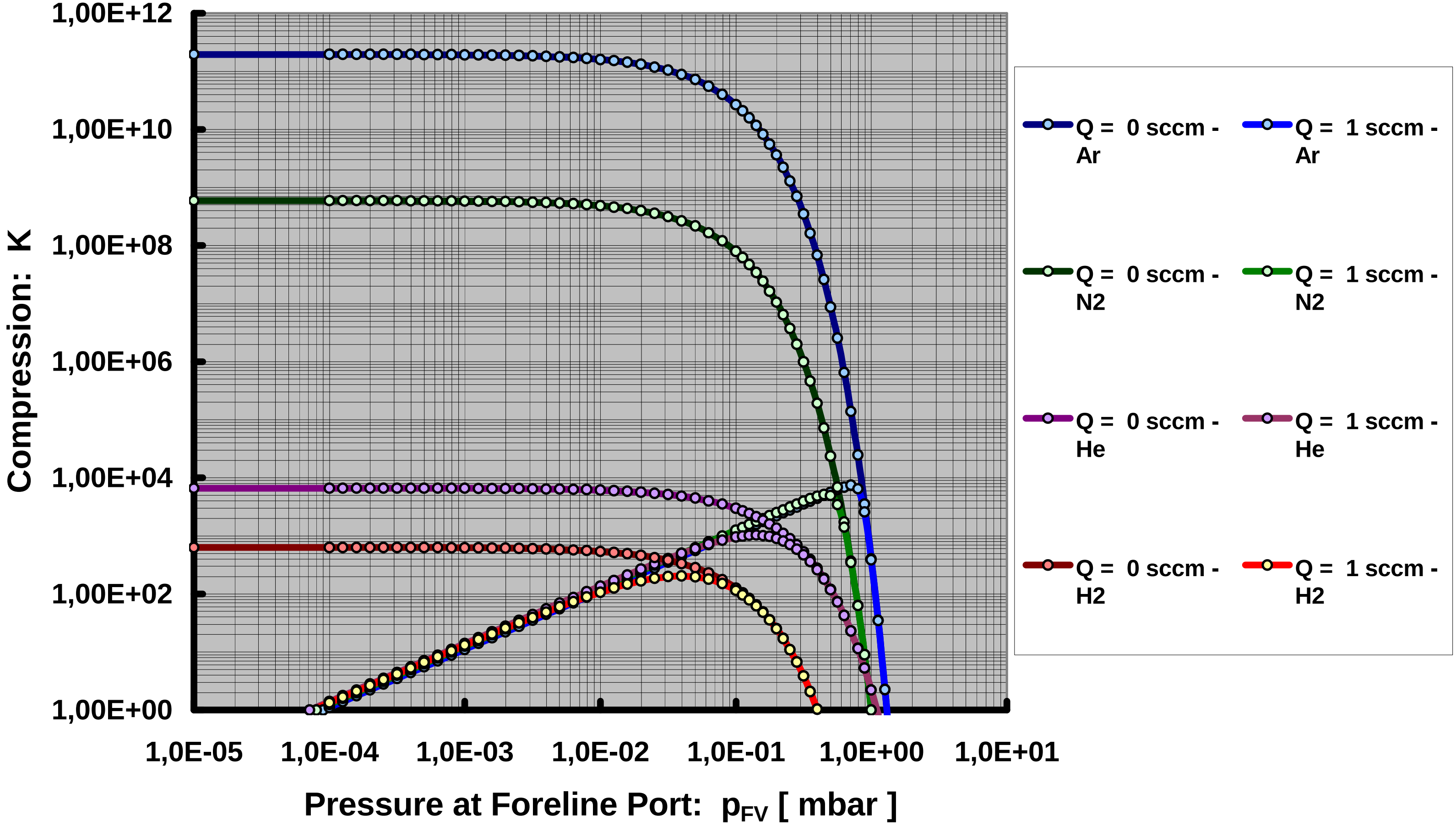
There is a strong correlation between C_2 and viscosity !



TMP w/o Compound Stage (100 ISO-K; Rotational Speed: 750 Hz)
Compression -
Hydrogen, Helium, Nitrogen, Argon



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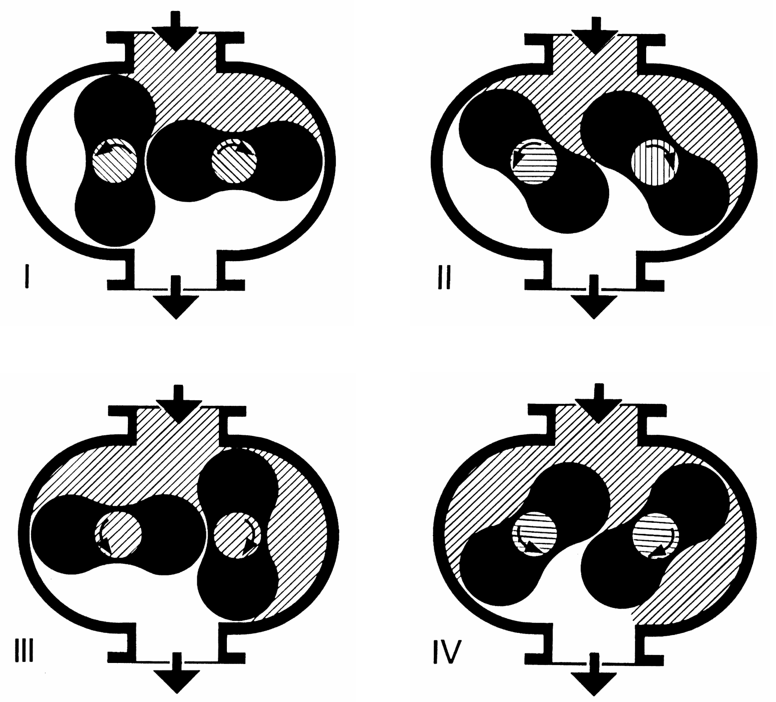
Principle of Roots Operation



- Two impellers rotate past each other in close proximity.
- In impeller positions **I** and **II**, the volume in the intake flange is increased.
- In position **III**, part of the volume is sealed off from the intake side.
- In position **IV**, this volume is opened to the discharge side.
- As the impellers rotate further, the compressed gas is ejected via the discharge flange.

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How to calculate the compression curve of a Roots blower ?

Use the same mathematical formalism as shown above in case of turbomolecular pumps !

Analytical model:

Roots blower (RB) in one dimension:

$x = 0$: High-vacuum flange (= inlet flange)

$x = L$: Foreline port; L is taken as the effective length of the Roots blower

Basic assumptions

Ansatz:

$$Q = Q_{max} - Q_{bs} - Q_{bl}$$

Net gas throughput	=	Maximum gas throughput in forward direction	–	Backstreaming	–	Backleakage

$$p_1 \cdot S_{max} = p_{HV} \cdot S_{max} - r \cdot (1/h_v) \cdot (p + p_2) \cdot (dp/dx) - p_{FV} \cdot S_{bl}$$

h_v : Dynamic viscosity in the range of viscous flow

p_2 : Empirical parameter (depends on the geometry of the Roots blower)
{ $p_2 = p_2(Q)$ }

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Backleakage
in Roots blowers

$\mathbf{Q_{bl} = p_{FV} \cdot S_{bl}}$ = Backleakage

$\mathbf{S_{bl}}$: Volume flow rate of gas transported by the rotors from the fore-vacuum side to the high-vacuum side, adsorption of gas on the fore-vacuum side – desorption of gas on the high-vacuum side, depends on foreline pressure

$\mathbf{p_{FV}}$: Foreline pressure

$\mathbf{p_{HV}}$: High-vacuum pressure

Differential equation

Differential equation: $(r/h_v)\cdot(p + p_2)\cdot(dp/dx) = (p_{HV} - p_1)\cdot S_{max} - p_{FV}\cdot S_{bl}$

Boundary condition: $p(x = 0) = p_{HV}$ (= high-vacuum pressure)

x : Position inside the RB $\{ 0 \leq x \leq L \}$

p : Pressure inside the RB $\{ p(x = 0) = p_{HV} \leq p = p(x) \leq p_{FV} = p(x = L) \}$

p_1 : Lowest high-vacuum pressure which can be attained with the RB at the prescribed gas throughput Q $\{ p_1 = p_1(Q) \}$

h_v : Dynamic viscosity in the range of viscous flow

S_{max} : Maximum pumping speed at the inlet flange at the prescribed gas throughput Q $\{ S_{max} = Q/p_1 = S(p_1) \}$

r : Backstreaming coefficient $\{ r = r(Q) \}$

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Solution of the differential equation

First step: Neglect backleakage ($S_{bl} = 0$)

Then the solution of the differential equation is:

$$p(x) = +[\,(p_{HV} + p_2)^2 + 2\cdot(p_{HV} - p_1)\cdot C_2\cdot(x/L)\,]^{1/2} - p_2$$

$$C_2 = h_v\cdot S_{max}\cdot L/r$$

p_2 and C_2 are the crucial parameters of the model.

High-vacuum pressure vs. foreline pressure

$$p(x = L) = p_{FV} = +[(p_{HV} + p_2)^2 + 2 \cdot (p_{HV} - p_1) \cdot C_2]^{1/2} - p_2$$

Þ $$(p_{HV} + p_2 + C_2)^2 = (p_{FV} + p_2 + C_2)^2 - 2 \cdot (p_{FV} - p_1) \cdot C_2$$

Þ $$(p_{HV} + p_2 + C_2)^2 - (p_{FV} + p_2)^2 = 2 \cdot (p_1 + p_2) \cdot C_2 + C_2^2$$

High-vacuum pressure and foreline pressure form a hyperbola !

Asymptotic behaviour:

1. $p_{FV} = p_1$ Þ $\mathbf{p_{HV} = p_{FV}}$
2. $p_{FV} >> C_2$ Þ $\mathbf{p_{HV} = p_{FV} - C_2}$

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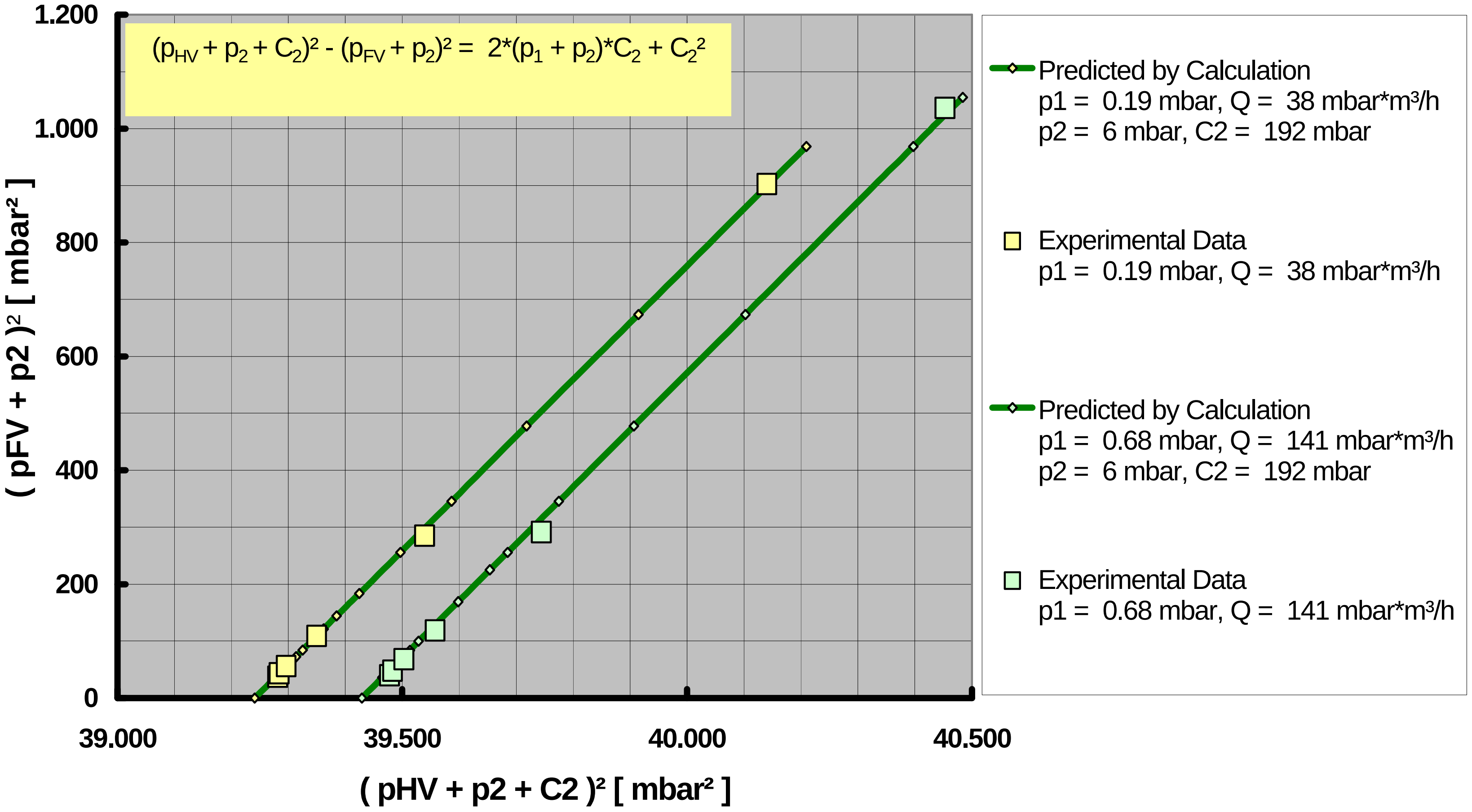
RuVac WA 251 + ScrewLine SP 630
p_{HV} and p_{FV} form a hyperbola



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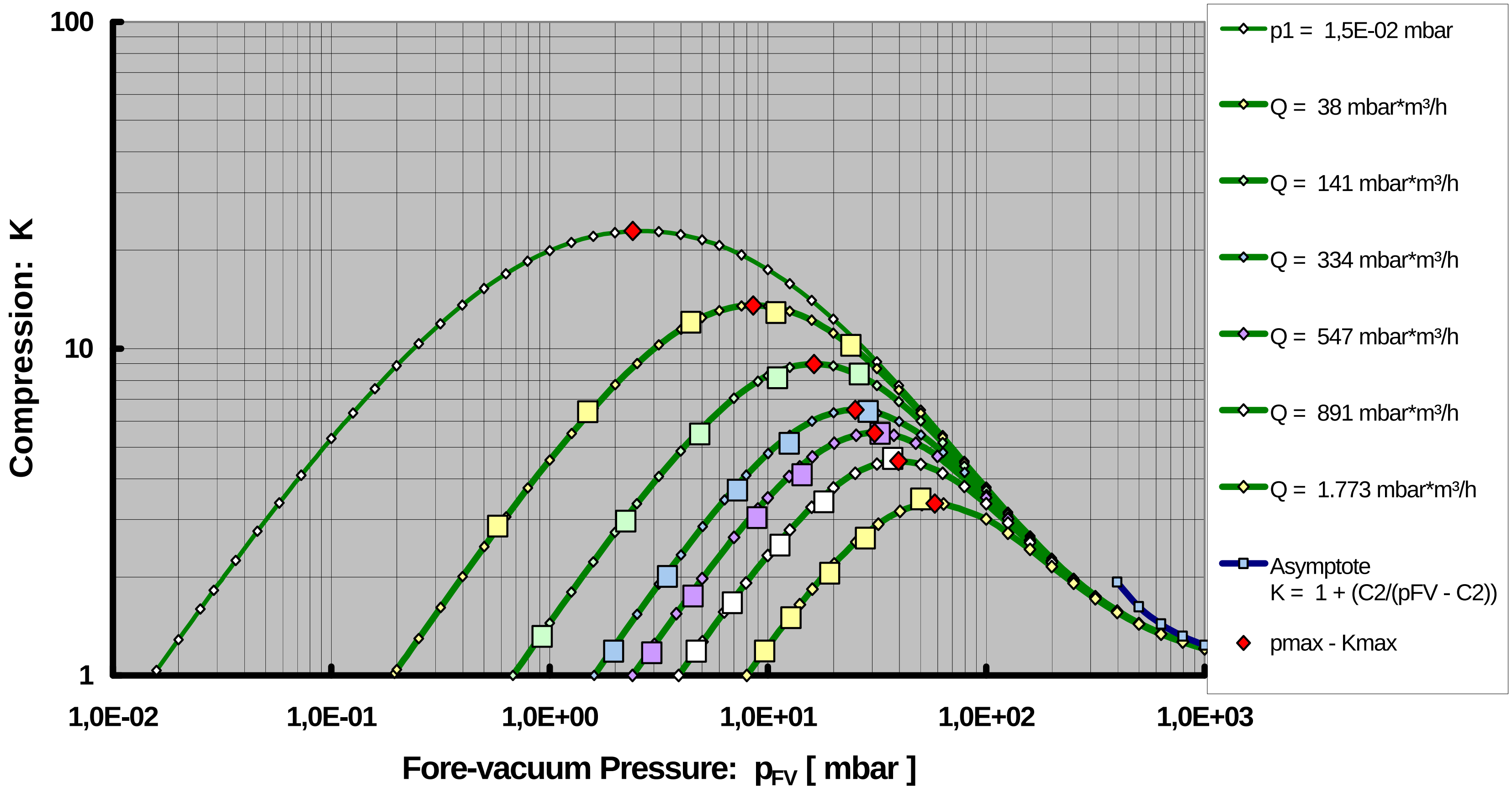
RuVac WA 251 + ScrewLine SP 630 (50 Hz)

Compression for Air

$p_2 =$ 6 mbar, $C_2 =$ 192 mbar, $S_{bl,0}/S_{max} =$ 0.02, $p_{bl} =$ 0.165 mbar



Intuition

The analysis of the compression curves leads to the following intuition:

Þ **One pair of parameters (p_2, C_2) can be used for all throughputs.**

Þ **C_2 = $h_v \cdot S_{max} \cdot L/r$ does not depend on throughput.**

Þ **Consequently, S_{max}/r does not depend on throughput.**

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Compression vs. foreline pressure

Compression = K = $K(p_{FV})$ = p_{FV}/p_{HV}

Asymptotic behaviour:

1. $p_{FV} = p_1$ þ **K = 1**

2. $p_{FV} >> C_2$ þ **K = 1 + [$C_2/(p_{FV} - C_2)$]**

At prescribed gas throughput $Q = p_1 \cdot S_{max}(p_1)$ the maximum of the compression occurs at the following foreline pressure:

$$p_{FV,\,Kmax} = p_{max} = [\, 2 \cdot p_1 \cdot p_2 + (p_2 + C_2) \cdot (2 \cdot p_1 \cdot y)^{1/2} \,]/(2 \cdot p_2 + C_2)$$

$$@ \; [\, 2 \cdot p_1 \cdot (2 \cdot p_2 + C_2) \,]^{1/2}$$

$$y = 2 \cdot (p_1 + p_2) + C_2$$

Zero throughput compression

$Q = 0$

þ $(p_{HV} + p_2 + C_2)^2 = (p_{FV} + p_2 + C_2)^2 - 2 \cdot p_{FV} \cdot C_2$

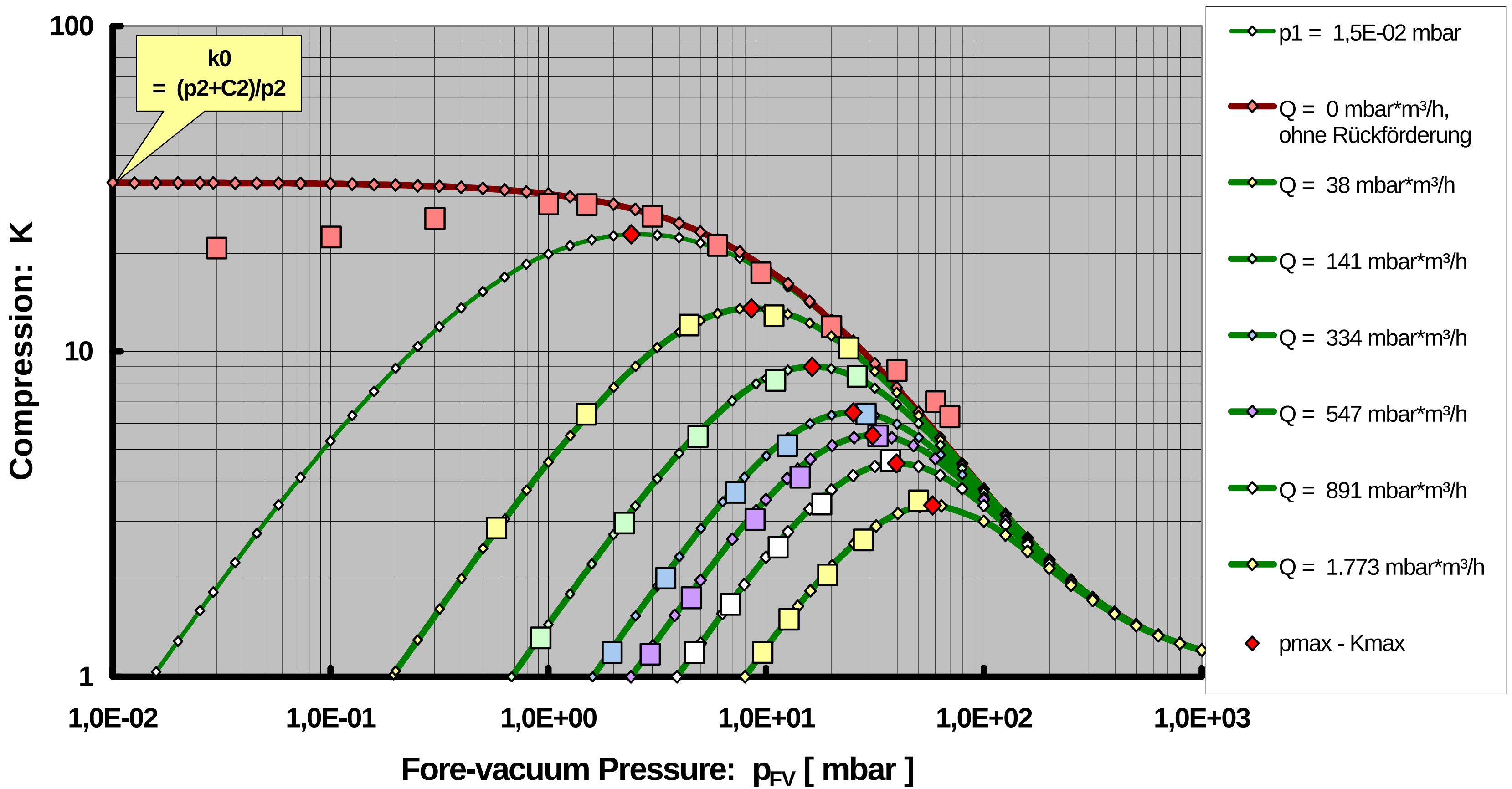
In the limit $p_{FV} << p_2$ we obtain for the zero throughput compression:

þ K_0 ® $k_0 = (p_2 + C_2)/p_2$

RuVac WA 251 + ScrewLine SP 630 (50 Hz)

Compression for Air

p_2 = 6 mbar, C_2 = 192 mbar, $S_{bl,\,0}/S_{max}$ = 0.02, p_{bl} = 0.165 mbar



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Zero throughput compression taking backleakage into account

Substitute p_1 by $p_1 + [\ (p_{FV} * S_{bl}(p_{FV}))/S_{max}\ ]$!

The Ansatz

$$S_{bl}(p_{FV}) = S_{bl,\,0} \cdot [\ p_{bl}/(p_{bl} + p_{FV})\ ]$$

leads to excellent agreement with the experiment.

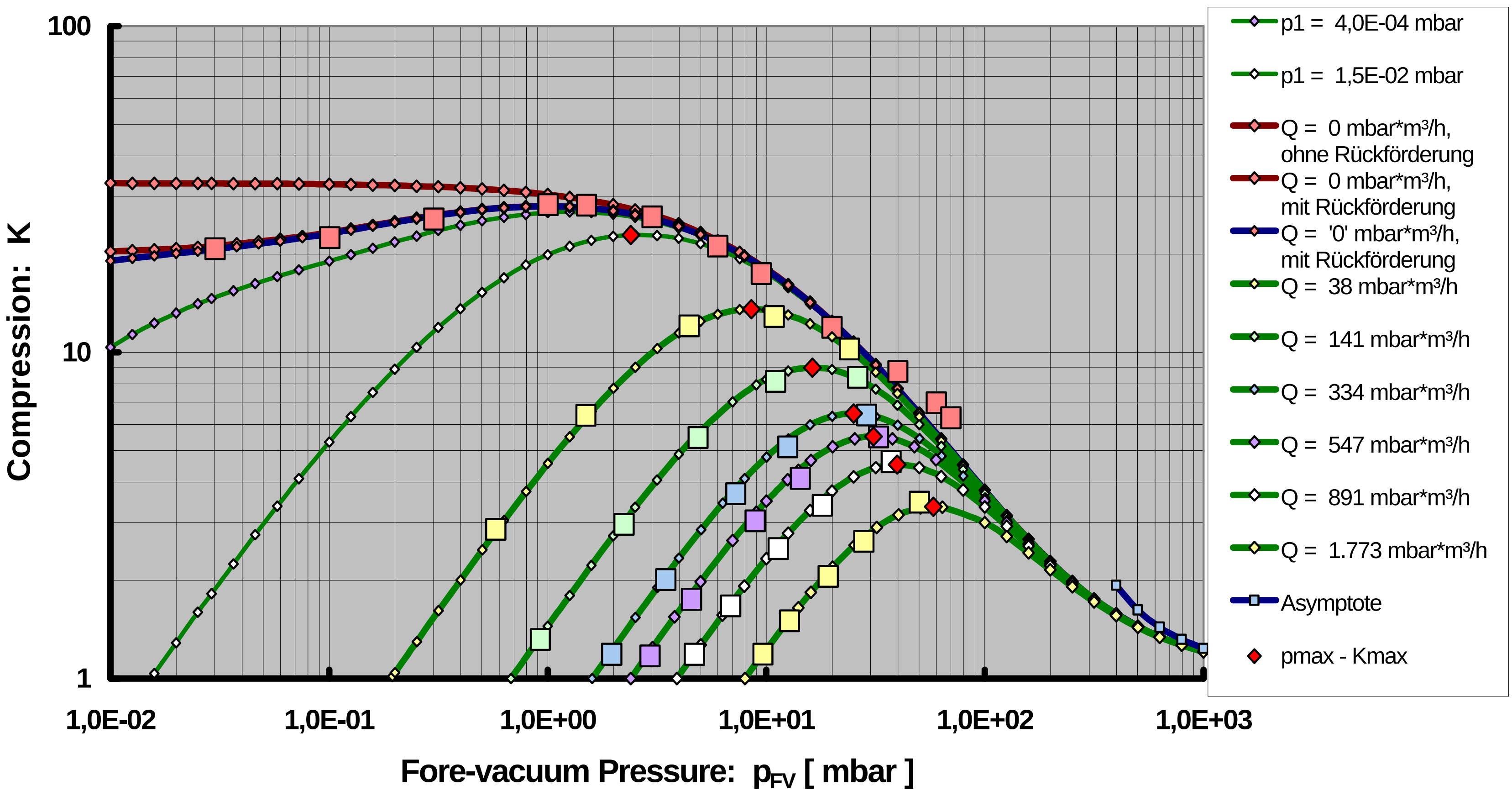
$$\begin{array}{llll} p_{FV} \cdot S_{bl}(p_{FV}) & \circledR & p_{FV} \cdot S_{bl,\,0} & \text{for } p_{FV} << p_{bl} \\ & \circledR & p_{bl} \cdot S_{bl,\,0} & \text{for } p_{FV} >> p_{bl} \end{array}$$

$$K_0 \ \circledR \ k_{0,\,bl} = (p_2 + C_2)/[\ p_2 + C_2 \cdot (S_{bl,\,0}/S_{max})\ ]$$

RuVac WA 251 + ScrewLine SP 630 (50 Hz)

Compression for Air

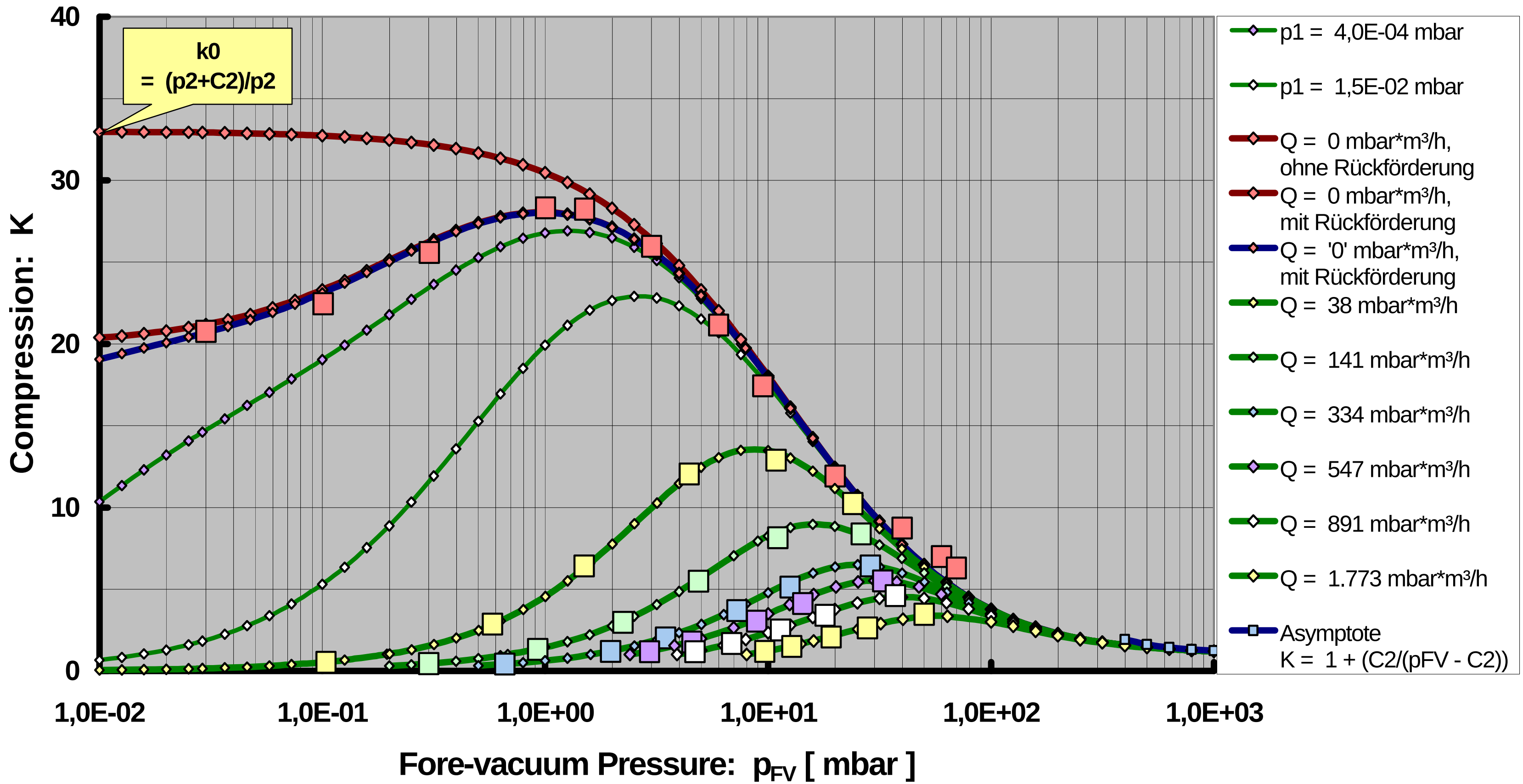
p_2 = 6 mbar, C_2 = 192 mbar, $S_{bl,0}/S_{max}$ = 0.02, p_{bl} = 0.165 mbar



RuVac WA 251 + ScrewLine SP 630 (50 Hz)

Compression for Air

p_2 = 6 mbar, C_2 = 192 mbar, $S_{bl,\,0}/S_{max}$ = 0.02, p_{bl} = 0.165 mbar



The conductance of a Roots blower

Using the analytical results of the model the following expression for the conductance can be derived:

$$C = [\, S_{max}/(2 \cdot C_2)\,] \cdot (p_{HV} + p_{FV} + 2 \cdot p_2)$$

Note that

$$C_{molecular} = (p_2/C_2) \cdot S_{max} = S_{max}/(k_0 - 1)$$

can be taken as the conductance of the Roots blower in the range of molecular flow.

Þ $$C = C_{molecular} \cdot [\, 1 + (p_{HV} + p_{FV})/(2 \cdot p_2)\,]$$

On the compression in turbomolecular pumps and Roots blowers

Summary

- An analytical model has been presented for calculating and systematically analysing the compression curves for turbomolecular pumps, Wide Range turbomolecular pumps and Roots blowers.

- Analytical expressions for the compression at finite ($Q > 0$) and at zero throughput ($Q = 0$) are provided by the model.

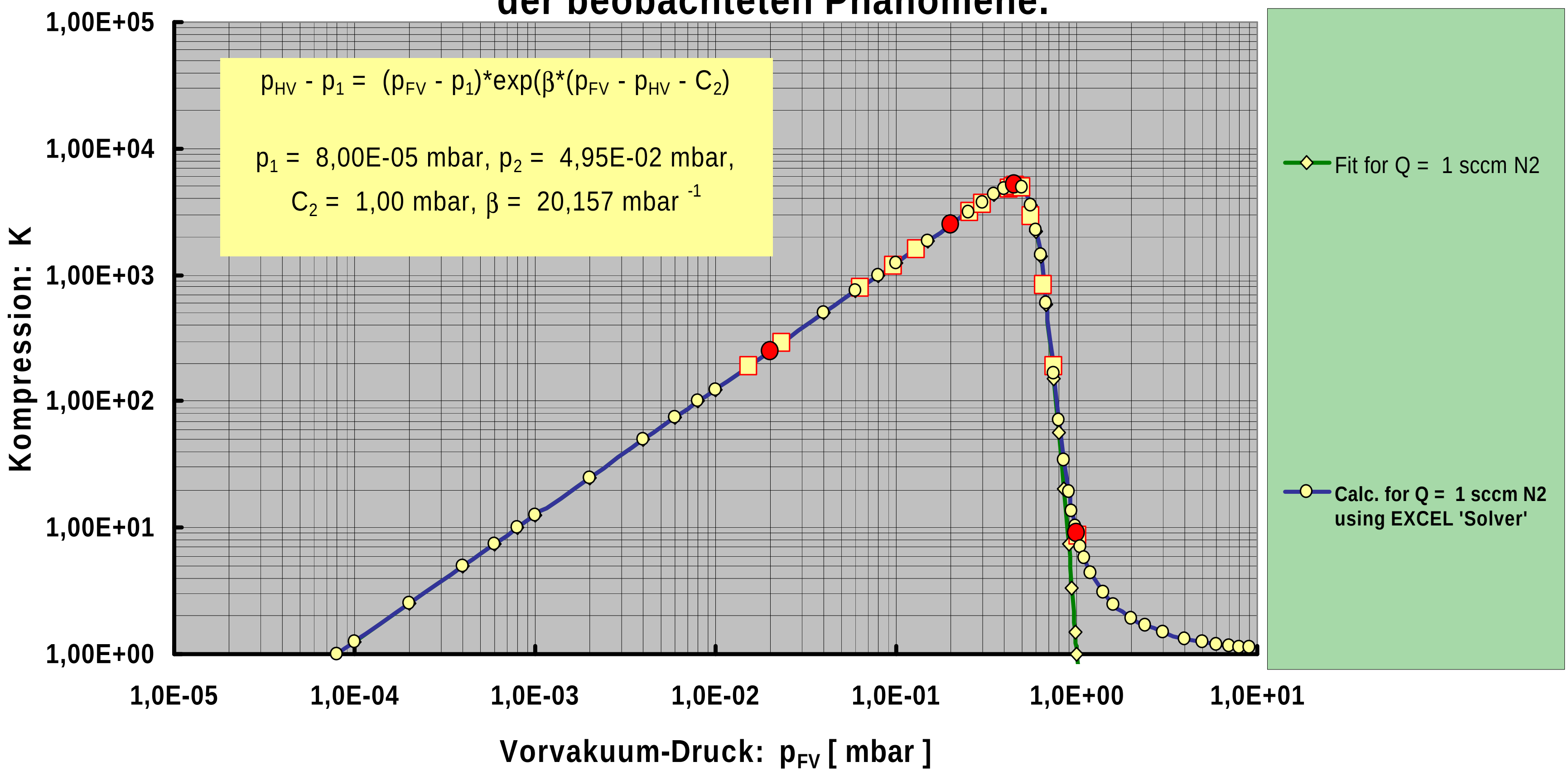
- Only three parameters (p_1, p_2, C_2) are required to describe the performance of a classical turbo stage, a compound stage or a Roots blower stage.
  There is strong evidence, that that the compression curve for zero gas throughput can be derived from a compression curve for a finite gas throughput.

- Comparison with experimental data shows that the analytical model provides an excellent qualitative and quantitative description of the observed phenomena.

Klassische TMP (100 ISO-K; 16 KF; Drehzahl: 750 Hz)

Kompression für Stickstoff

Das Modell liefert eine exzellente quantitative Beschreibung der beobachteten Phänomene.



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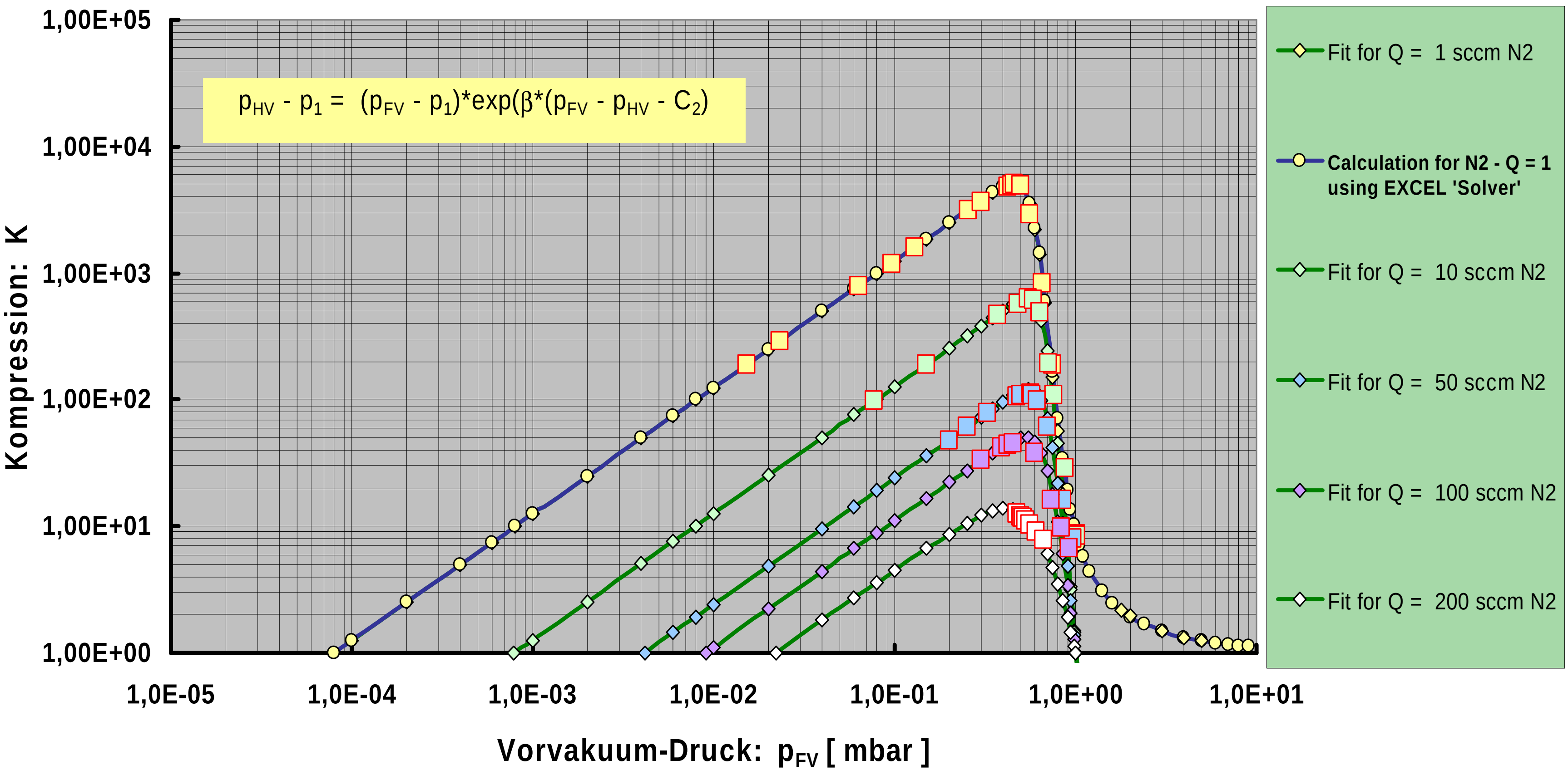
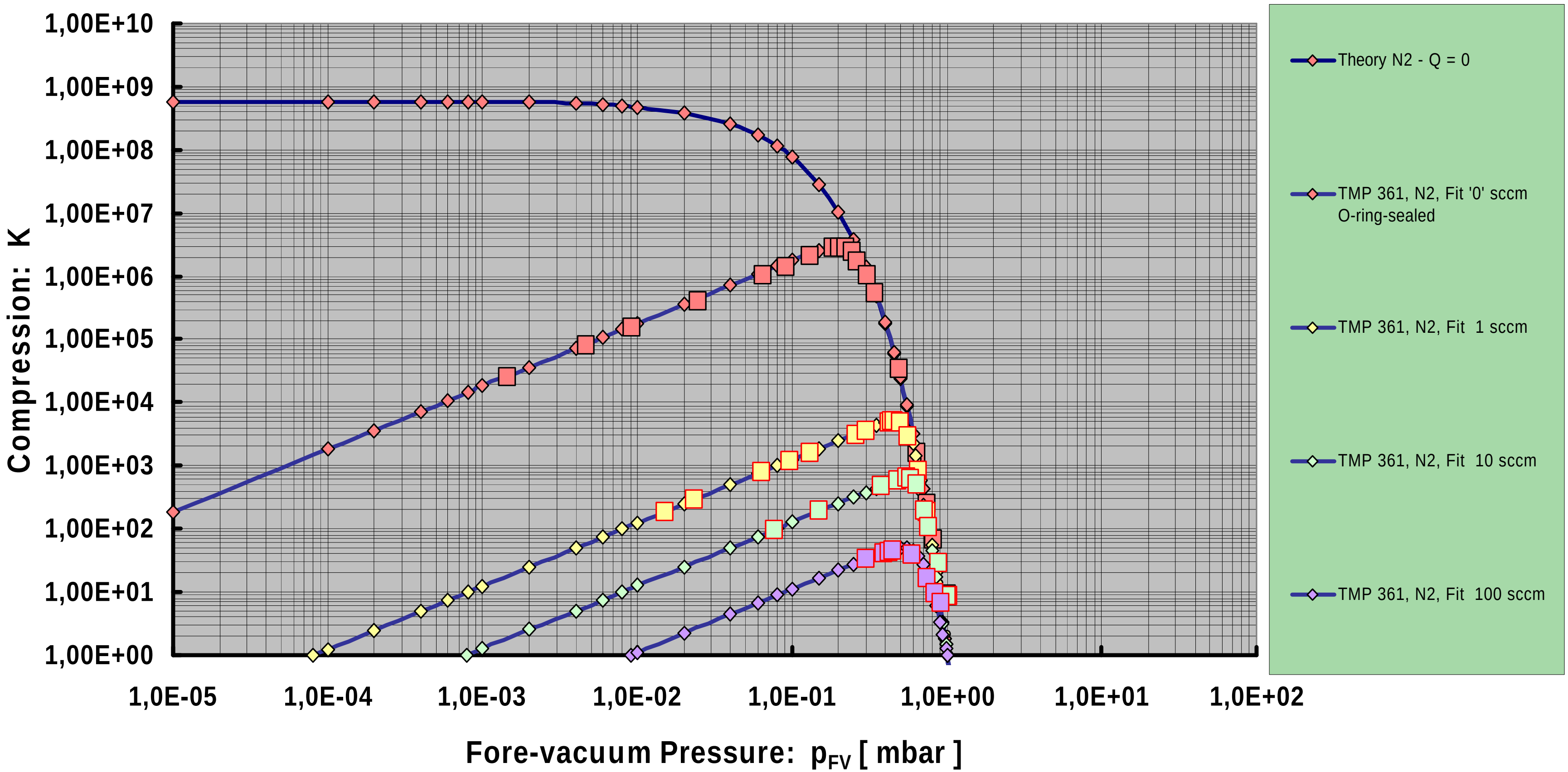
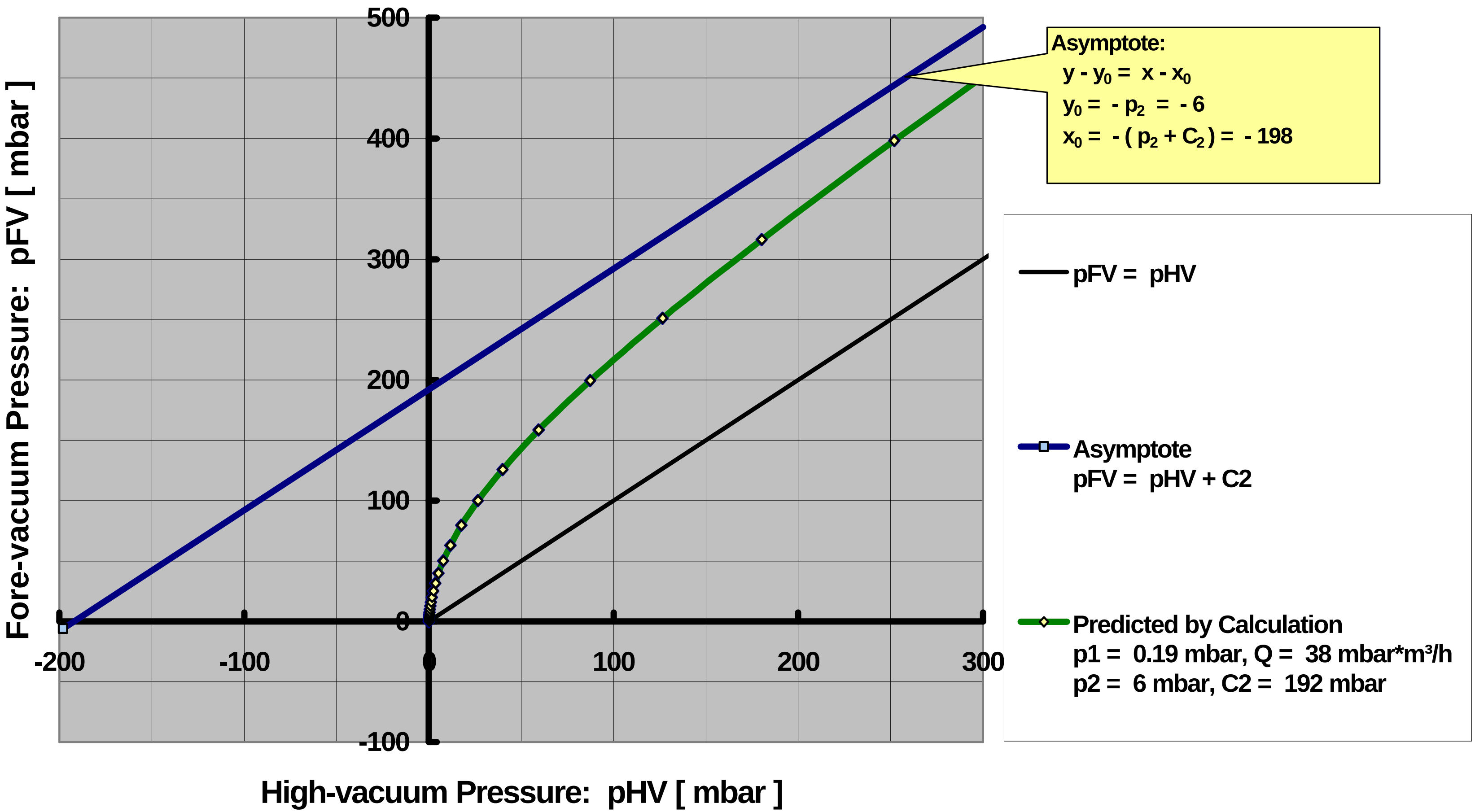
Klassische TMP (100 ISO-K; 16 KF; Drehzahl: 750 Hz)
Kompression für Stickstoff



Classic TMP (100 ISO-K; 16 KF)
Compression for Nitrogen



RuVac WA 251 + ScrewLine SP 630
p_{HV} and p_{FV} form a hyperbola



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TMP with Compound Stage (100 ISO-K; Rotational Speed: 1000 Hz)
Pumping Speed for Nitrogen, Helium and Hydrogen
Turbo Stage and Compound Stage