

Neutrinos: Windows to New Physics

R. D. Peccei
UCLA

Neutrinos: Windows to New Physics

- Symmetries of the SM
- Disquisitions on Neutrino Masses
- Recondite Physics
 - *Extended symmetries*
 - *Patterns in the neutrino sector*
- Connecting Physical Phenomena
- Wild Speculations
 - *LSND musings*
 - *Mass varying neutrinos*
- Concluding Remarks

Symmetries of the SM

- The **SM** is invariant under **local** transformations of the $SU(3) \times SU(2) \times U(1)$ group, **spontaneously broken** to the $SU(3) \times U(1)_Q$ subgroup
- The **SM** has, additionally, **4 global symmetries** :
 $U(1)_{B+L}; U(1)_{B-L}; U(1)_{L_i-L_j}$ [$i, j = \{1, 2, 3\}$]
Last **3** are **exact**, but $U(1)_{B+L}$ has an **EW anomaly**, so is not a symmetry at quantum level
- In addition, to solve the **strong CP problem**, one often invokes the existence of another global symmetry $U(1)_{PQ}$, which has both **strong and EW anomalies**

- The existence of neutrino masses and mixings, inferred from neutrino oscillation experiments, provides evidence for physics beyond the SM
- Neutrino mixings imply that individual lepton number is not conserved. In effect, because of observed mixing lose 2 SM global symmetries:
 - mixing $\leftrightarrow U(1)_{L_i-L_j}$ no longer a symmetry
- Existence of neutrino masses also affects the symmetry structure of SM, indicating the presence of new interactions beyond those of the SM which most likely violate $U(1)_{B-L}$

Disquisitions on Neutrino Masses

- Since neutrinos have **zero charge**, besides the usual **Dirac** (**particle-antiparticle**) mass they can have also a **Majorana** (**particle-particle**) mass
- Most general mass term has the structure:

$$L = -\frac{1}{2} \begin{pmatrix} \bar{\nu}_L^c & \bar{\nu}_R \end{pmatrix} \begin{bmatrix} m_T & m_D^T \\ m_D & m_S \end{bmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}$$

where $\nu^c = C\nu^\dagger$. Clear that m_T , m_S are **Majorana** mass terms, and m_D is a **Dirac** mass term.

These **masses break SM symmetries** differently

- From a symmetry point of view
 - m_T – SU(2) Triplet with $Q_{(B-L)} = -2$
 - m_D – SU(2) Doublet with $Q_{(B-L)} = 0$
 - m_S – SU(2) Singlet with $Q_{(B-L)} = +2$
 thus their presence is connected to different physics
- What combination of m_T , m_S , and m_D determines the tiny ($m_\nu < 1 \text{ eV}$) neutrino masses observed is a function of what this underlying physics is
- Discuss each of these mass terms in turn

- Most conservative stance is to admit **existence** of ν_R , but assume **(B-L)** remains a **symmetry** of nature, so $m_\nu \equiv m_D$
- However, must understand what physics gives

$$m_D = \Gamma^\nu \langle \Phi \rangle \quad \ll \quad m_\ell = \Gamma^\ell \langle \Phi \rangle$$

- One possibility is to invoke **extra dimensions** [Arkani-Hamed Dimopoulos Dvali] Illustrate with **d=5** theory **compactified** to space of **radius L**

$$S_{\text{Einstein}} = M^{*3} / 16\pi \int d^4x \int dy \sqrt{-g_5} R_5 \quad (d=5)$$

after compactification becomes

$$S_{\text{Einstein}} = M^{*3} L / 16\pi \int d^4x \sqrt{-g_4} R_4 \quad (d=4)$$

which implies $M_{\text{Planck}}^2 = M^{*3} L$

- If **SM particles** live in **D3 brane**, but ν_R lives in **5d bulk**, then neutrino action involving ν_R reads

$$S = M^* \int d^4x dy \sqrt{-g_5} \bar{\nu}_R \gamma^\mu i \partial_\mu \nu_R + \int d^4x \sqrt{-g_4} \Gamma^\ell \bar{\nu}_R \Phi L_L$$

After **compactification**

$$M^* \int d^4x dy \sqrt{-g_5} \bar{\nu}_R \gamma^\mu i \partial_\mu \nu_R \rightarrow M^* L \int d^4x \sqrt{-g_4} \bar{\nu}_R \gamma^\mu i \partial_\mu \nu_R$$

Must rescale ν_R by factor $1/(M^* L)^{1/2} \equiv M^*/M_{\text{Planck}}$

Thus $\Gamma^\nu \equiv [M^*/M_{\text{Planck}}] \Gamma^\ell$, so get **large reduction**

- Many issues open:
 - Why is **(B-L) conserved** if ν_R exists?
 - Are **K-K tower of states** dangerous?
- Perhaps **alternate solution** is to have **no ν_R**

- Realized if one has **Triplet mass**. Two options:

$$m_T \leftrightarrow \begin{cases} \text{VEV of } T \\ \text{VEV of } T_{\text{eff}} = \Phi \tau \Phi / \Lambda \end{cases} \quad \text{break SU(2) \& (B-L)}$$

- Since $m_T \sim \langle T \rangle$, first possibility requires that $\langle T \rangle \ll \langle \Phi \rangle = v_F \sim 250 \text{ GeV}$ **difficult to justify**
- Second possibility much more natural. **Small neutrino masses** result from the **large scale Λ** entering in T_{eff} , associated with **(B-L) breaking**

$$m_T \sim \langle T_{\text{eff}} \rangle \sim v_F^2 / \Lambda \quad \text{seesaw formula}$$

- See that in this case m_ν offers **window** to **physics well beyond the EW scale** $v_F \ll \Lambda$

- Can arrive at similar **seesaw formula** from **singlet breaking** [Yanagida, Gell Mann Ramond Slansky] provided $m_S \gg m_D$
- Since m_S carries **no SM quantum numbers**, its **scale** is **unconnected** to v_F or Λ_{QCD} . It is, **related** to the **scale** of $U(1)_{B-L}$ **breaking**
- If a hierarchy exists, $m_S \gg m_D$, then the neutrino mass matrix

$$M = \begin{pmatrix} 0 & m_D^T \\ m_D & m_S \end{pmatrix}$$

has both **large** $M_N = m_S$ and **small** eigenvalues $M_\nu = -m_D^T (m_S)^{-1} m_D$ **seesaw**

- Principal lessons learned from existence of neutrino masses and mixings are two-fold:
 - i. There are additional **interactions beyond the SM**, which **break** all **global symmetries**
 - ii. Small **neutrino masses** are related to large scale where **$U(1)_{B-L}$ breaks down**
- Not really possible to tease apart whether M_ν arises from:
 - **composite operators** $M_\nu \equiv m_T \sim \langle T_{\text{eff}} \rangle \sim v_F^2/\Lambda$
 - **large v_R masses** $M_\nu = m_D^T (m_S)^{-1} m_D$
 - **combination of both**

$$M_\nu = m_T - m_D^T (m_S)^{-1} m_D$$

Recondite Physics

- Looking beyond the SM for hints to the origins of neutrino masses and mixing there are **three fruitful avenues** to follow:
 - i. Look for possible new **symmetries** and symmetry breakings [**top-down**]
 - ii. Uncover **patterns** of observed masses and mixings [**bottom-up**]
 - iii. Find **connections** between physical phenomena [**rely on experiments**]
- Will illustrate each of these avenues by means of some **appealing examples**

Extended symmetries

- Interesting insights on neutrinos emerge in Grand Unified Theories (GUTs) where quarks and leptons are in same multiplets:
- In simplest GUT $SU(5)$ ν_R transforms as a singlet under the group

$$SU(5): 10 = \{Q_L, u_R^c, l_R^c\}; 5^* = \{d_R^c, L_L\}; 1 = \{\nu_R^c\}$$

Thus no constraints exist on the mass term

$$L_{\text{mass}} = -\frac{1}{2} m_S \bar{\nu}_R \nu_R^c + h.c.$$

and there is no connection between $SU(5)$ breaking and (B-L) breaking [$m_S \neq M_{\text{GUT}}$]

- Matters different in $SO(10)$ where all quarks and leptons, including ν_R , are in the same representation:

$$SO(10): 16 = \{Q_L, u_R^c, d_R^c, L_L, \ell_R^c, \nu_R^c\}$$

- Since $U(1)_{B-L} \subset SO(10)$ now $U(1)_{B-L}$ is a local, not global, symmetry
- Mass m_S is related to GUT breaking scale, or to a possible intermediate scale
- This latter case ensues if symmetry breakdown of $SO(10)$ is through the chain:

$$SO(10) \xrightarrow{M_{GUT}} SU(3) \times SU(2) \times SU(2) \times U(1)_{B-L} \xrightarrow{m_S} SM$$

- Possible to contemplate range
 $10^{10} \text{ GeV} < m_S < 10^{16} \text{ GeV}$
- Because $16 \otimes 16 = 10 \oplus 126 \oplus 120$, one can consider Yukawa interactions containing **SO(10) Higgs** fields in these representations.
- 126_H contains a **SM singlet field** carrying **(B-L) charge**, so that naturally $m_S \sim < 126_H >$
- Simplest model, leading to quite restrictive mass spectrum, has just **2 Yukawa** coupling terms (using either $U(1)_{PQ}$ or **SUSY** to eliminate 10^*_H couplings) [**Babu Mohapatra**]
- $L_{\text{Yukawa}} = \Gamma^D 16 \cdot 16 \cdot 10_H + \Gamma^S 16 \cdot 16 \cdot 126^*_H$
with $m_D = \Gamma^D < 10_H > \ll m_S = \Gamma^S < 126^*_H >$

- A different class of models, rather than using an elementary 126_H , replace it by a composite Higgs term made up of low- dimension fields:

$$126_H \rightarrow (16_H \cdot 16_H) / M$$

so that $m_S \sim M^2_{GUT} / M$

- Irrespective of whether 126_H (and other Higgs) are composite or not, to obtain a realistic spectrum and mixings for the charged fermions and neutrinos, one imposes some flavor symmetry - typically $U(1) \times D$
- Remnants of these symmetries should be visible in mass matrices

Patterns in the neutrino sector

- Different approach (more bottom-up) is to **divine from data** the structure of underlying theory
 - In **3 neutrino** framework [ignore **LSND** for now]
 - $|\Delta m_{32}^2| = \Delta m_{\text{atmos}}^2 = (2.4 \pm 0.3) \times 10^{-3} \text{ eV}^2$
 - $|\Delta m_{21}^2| = \Delta m_{\text{solar}}^2 = (7.9 \pm 0.4) \times 10^{-5} \text{ eV}^2$
- are consistent with a **hierarchical spectrum** $m_3 \gg m_2 \gg m_1$ (or an **inverted hierarchy**), as well as **other patterns** (e.g. $m_3 \gg m_2 \approx m_1$)
- As far as mixing goes, **two** of the **angles** in the leptonic mixing matrix U_{PMNS} are **large** $s_{23} \approx 1/\sqrt{2}$; $s_{12} \approx 1/2$ (actually $s_{12} \approx 0.56$) while the third is bound at 3σ by $s_{13} < 0.22$ and could be **small**

- Thus, approximately, letting $s_{13} = \varepsilon$, one has

$$U_{\text{PMNS}} \approx \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & \varepsilon e^{-i\delta} \\ -1 & \sqrt{3} & 1 \\ \frac{2\sqrt{2}}{2\sqrt{2}} & \frac{2\sqrt{2}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & -\sqrt{3} & 1 \\ \frac{2\sqrt{2}}{2\sqrt{2}} & \frac{2\sqrt{2}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

- Often assumed that pattern seen reflects some (perhaps approximate) **family symmetry** in the neutrino mass matrix $M_\nu = U_{\text{PMNS}}^* m_\nu^{\text{diag}} U_{\text{PMNS}}^\dagger$ where

$$m_\nu^{\text{diag}} = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 e^{i\alpha_2} & 0 \\ 0 & 0 & m_3 e^{i\alpha_3} \end{bmatrix}$$

- Because uncertainty in masses m_i is large and Majorana phases α_i unknown, it is difficult to reconstruct the matrix M_ν . Better to focus on mixing angles
- In this respect, an interesting starting point is provided by matrices M_ν which produce $s_{13}=0$ and give maximal mixing $s_{23} = 1/\sqrt{2}$. It is easy to see that matrices M_ν that are 2-3 symmetric [C.S. Lam]

$$M_\nu = \begin{bmatrix} X & A & A \\ A & B & C \\ A & C & B \end{bmatrix}$$

precisely accomplish this

- This **permutation symmetry** can be part of a larger discrete or continuous symmetry [many models exist: A_4 , S_3 , Z_4 , D_4]
- Since this **symmetry is approximate** in nature ($m_\mu \neq m_\tau$), the way that the symmetry is broken determines the correlation among these angles:

$$s_{23}^2 - 1/2 = F(s_{13}) \quad F(0)=0$$

with $F(s_{13})$ a model dependent function

- Another approach [**Smirnov**] uses the observation that:

$$\theta_{12} + \theta_{12}^{\text{had}} \approx \theta_{23} + \theta_{23}^{\text{had}} \approx \pi/4$$

to build models that display this **quark-lepton complementarity (QLC)**. **Is above coincidence?**

- Models are a bit forced. Illustrate with an example [Minakata Smirnov]. Recall that:

$$U_{\text{PMNS}} = U_l^\dagger U_\nu \quad ; \quad U_{\text{CKM}} = U_u^\dagger U_d$$

Imagine that mixing in the doublet Higgs sector comes only from the “down” side (thus $U_u = 1$) and GUT forces:

$$U_l = U_d = U_{\text{CKM}}$$

Then get interesting QLC relation if seesaw forces

$$U_\nu = U_{\text{bm}} = \frac{1}{2} \begin{bmatrix} \sqrt{2} & \sqrt{2} & 0 \\ -1 & 1 & \sqrt{2} \\ 1 & -1 & \sqrt{2} \end{bmatrix}$$

then $U_{\text{PMNS}} = U_{\text{CKM}}^\dagger U_{\text{bm}}$

Connecting Physical Phenomena

- Third approach for exploring the neutrino window is by looking for **physical inter-relations**. Best example is provided by **Leptogenesis**
- The ratio of baryon to photon density in the Universe now $\eta = n_B / n_\gamma$ is a measure of the primordial **matter-antimatter asymmetry** in the Universe

$$\eta = [(n_{\text{Matter}} - n_{\text{Antimatter}})_{\text{Primordial}}] / n_\gamma$$

and is well determined by **WMAP** and BBN:

$$\eta = (6.097 \pm 0.206) \times 10^{-10}$$

- The heavy ν_R needed for the **seesaw** provide a nice mechanism for generating $\eta \neq 0$. In this **Leptogenesis scenario** [Fukugita Yanagida] a primordial **lepton-antilepton asymmetry** is generated from out of equilibrium decays of **heavy Majorana neutrinos** [$N \rightarrow lH$; $N \rightarrow l^\dagger H^\dagger$]

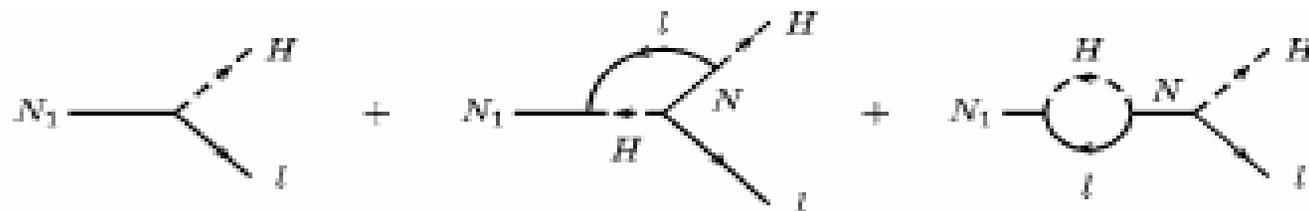


Figure 1 Tree level and one-loop diagrams contributing to heavy neutrino decays whose interference leads to Leptogenesis.

- Because the $(B+L)$ current is anomalous, **sphaleron processes** [Kuzmin Rubakov Shaposhnikov] transmute this **lepton number asymmetry** into a **baryon number asymmetry**.

- Since $\eta \approx 0.35 \eta_L$ an **asymmetry** $\eta_L \sim 1.7 \times 10^{-9}$ will produce the desired baryon asymmetry. Now,

$$\eta_L \approx 7 \varepsilon \kappa / g^*$$

where ε is a measure of the **CP-asymmetry** in the decay of the lightest heavy neutrino N_1 , κ takes into account of possible **washout** of the asymmetry, and $g^* \sim 100$ counts the effective degrees of freedom at $T \sim M_1$

- ε is related to the neutrino spectrum:

$$\varepsilon = -[3/16\pi^2] M_1 / v_F^2 \text{Im} (\Gamma^* M_\nu \Gamma^\dagger)_{11} / (\Gamma \Gamma^\dagger)_{11}$$

- For $T \sim M_1 \sim 10^{10}$ GeV, one needs **very light** neutrino masses ($m_\nu < \text{eV}$) to obtain the typical parameters needed for the CP asymmetry ($\varepsilon \sim 10^{-6}$) and the washout ($\kappa \sim 10^{-2}$) to get $\eta_L \sim 1.7 \times 10^{-9}$

Result for κ is
independent of
 initial abundances
 $10^{-3} \text{eV} < m_\nu < \text{eV}$

Buchmuller Di Bari
 Plumacher

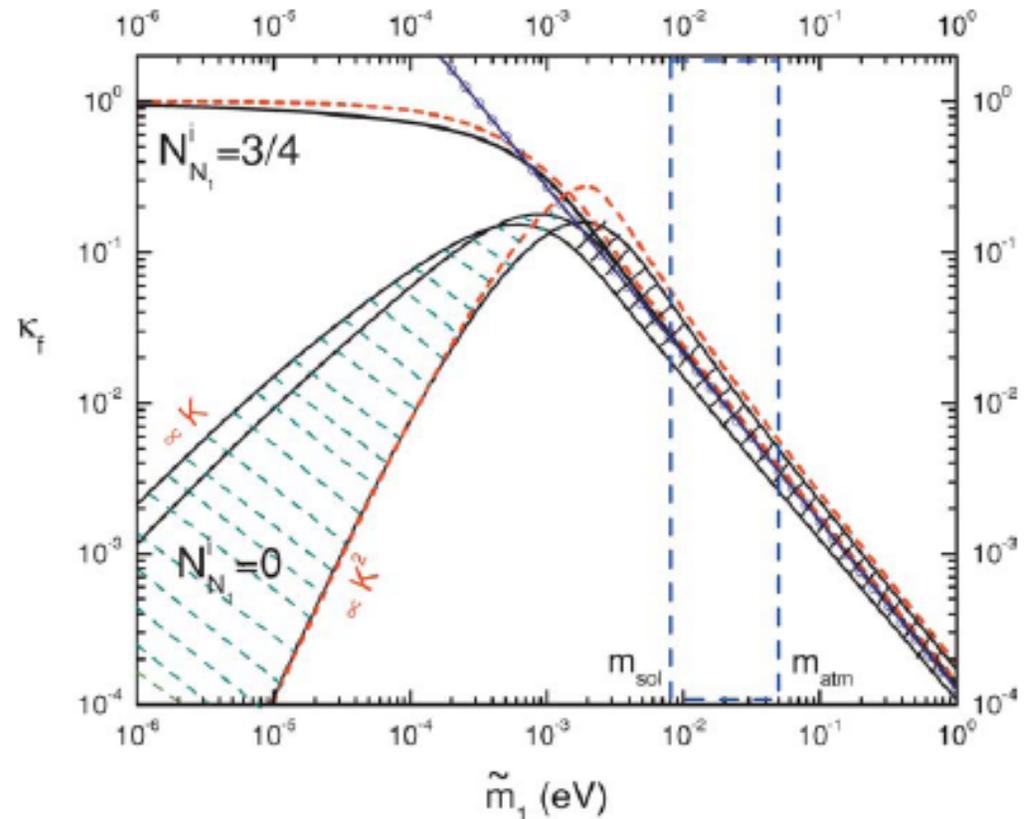


Figure 3 Final efficiency factor when the washout term ΔW is neglected. From (69).

- Since ε cannot be too small for **Leptogenesis** to work, this provides a **lower bound** on M_1 [**Davidson Ibarra**]. For $\tilde{m}_1 > 10^{-3} \text{ eV}$ one finds

$$M_1 > 2 \times 10^9 \text{ GeV} \quad \text{BDP}$$

- Furthermore, **Leptogenesis** also provides an **upper bound** on the **light neutrinos** masses, since washout rate $W \sim \sum_i m_i^2$. One finds

$$m_i < 0.1 \text{ eV} \quad \text{BDP}$$

- Inference from **Leptogenesis** of **heavy neutrinos** with masses in the 10^{10} GeV range associated with a **sub eV light neutrino spectrum** is very encouraging!

- **Leptogenesis** is a triumph for neutrinos and, if η is generated this way, we owe our existence to the **CP phases** in the **neutrino sector**!
- However, the fact that **Leptogenesis** occurred at temperatures $T \sim M_1 > 2 \times 10^9 \text{ GeV}$ has significant import for SUSY theories
- If the **reheating temperature** after inflation T_R is too high in SUGRA one **overproduces gravitinos**, with catastrophic consequences since gravitino decay products destroy the light elements produced in BBN. To avoid trouble:

$$T_R < 10^7 \text{ GeV} \quad \text{Kawasaki et al}$$
- But **Leptogenesis** argues $T_R > M_1 > 2 \times 10^9 \text{ GeV}$

- There are solutions to **gravitino problem**, but these in general alter the **“normal” SUSY** expectations coming from **SUGRA**:

$$m_{3/2} > 100 \text{ TeV}$$

Gherghetta et al

$$\tilde{g} \text{ is LSP } \tilde{\tau} \text{ is NLSP}$$

Fujii et al

- **LFV** provides another example of **tension** between **SUSY** and **Leptogenesis**. Prediction for $\mu \rightarrow e\gamma$ are **model dependent** but sensitive to mass of heavy neutrinos [$\text{BR} \sim (M_3 \ln M_X/M_3)^2$] and to satisfy present bounds $M_3 < 10^{13} \text{ GeV}$
- Seeking compatibility between **SUSY** and **Leptogenesis** leads to **testable** experimental predictions and **insights into neutrino physics**

Wild Speculations

LSND musings

- There well may be more surprises in the neutrino sector. A prime example is provided by the **LSND** result
- **LSND** result for $\bar{\nu}_\mu - \bar{\nu}_e$ oscillations

$$\Delta m^2 \sim 1 \text{ eV}^2 ; s^2 \sim 10^{-3}$$

cannot be reconciled in a **3 ν** framework.

- We await **Mini BooNe** results, but if phenomena is true it requires **different physics**
 - sterile neutrinos
 - CPT violation

- Easy to find candidates for **sterile neutrinos** in string/GUT theories
 E_6 27 \supset 9 SU(3) singlets \supset 2 SU(2)xU(1) singlets
- What is challenging is to get $m_{st} \sim eV$. Normal trick is to use **discrete symmetry** to set $m_{st}=0$ and get a **small mass** from **breaking** of symmetry (e.g **2-3 symmetry** Mohapatra et al)
- Models are quite complex and **phenomenology** of 3+1 models **shaky** (3+2 better)
- **CPT violation** is a much bolder suggestion [Barenboim et al] since allows **differences** between $\bar{\nu}_\mu - \bar{\nu}_e$ and $\nu_\mu - \nu_e$ oscillations

- Unfortunately, phenomenology does not work better here!

All data \leftrightarrow CPT OK

LSND \leftrightarrow CPT viol.

- Barenboim and Mavromatos claim, however, that matters are improved by including quantum decoherence effects

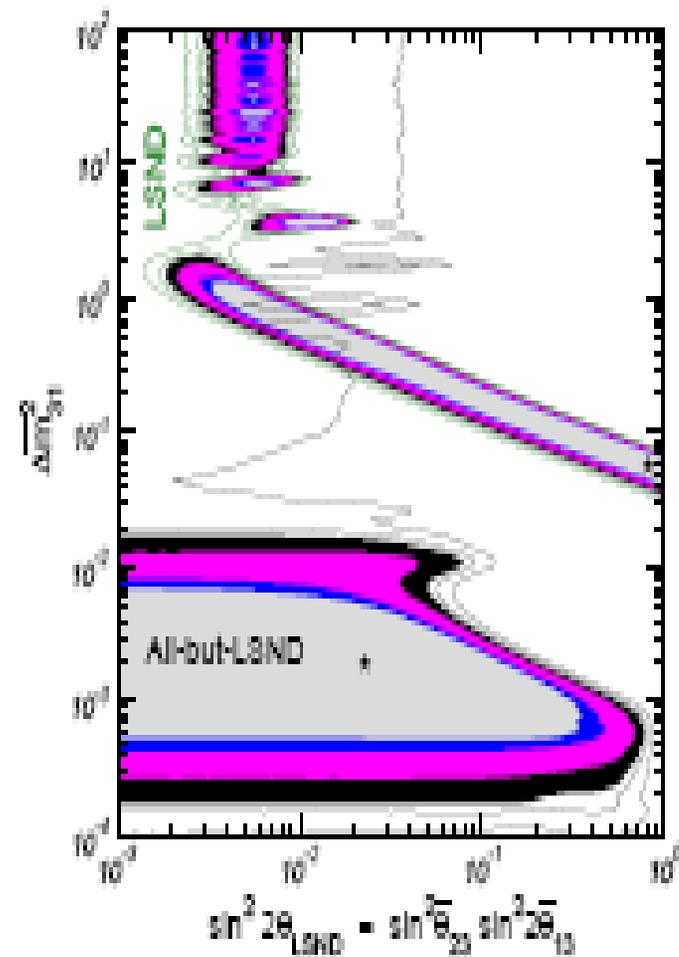


Figure 6: 90%, 95%, 99%, and 3σ CL allowed regions (filled) in the $(\Delta m_{\text{LSND}}^2, \sin^2 2\theta_{\text{LSND}})$ plane required to explain the LSND signal together with the corresponding allowed regions from our global analysis of all-but-LSND data. The contour lines correspond to $\Delta\chi^2 = 13$ and 18 (2.2σ and 3.0σ , respectively).

Gonzales-Garcia
Maltoni Schwetz

Mass varying neutrinos

- Neutrinos with **environment-dependent masses** have been suggested recently by **Fardon Nelson Wiener (FNW)** to account for the dark energy the Universe
- Data from **WMAP** provides accurate values for the **cosmological parameters** $\Omega = 8\pi G_N \rho / 3 H^2$ and Ω_i with $\Omega = \sum_i \Omega_i$:

$$\Omega - 1 = -0.010 \pm 0.016 \mp 0.009 \quad \textit{flat Universe}$$

$$\Omega_m = 0.234 \pm 0.035 \quad \textit{25\% matter}$$

$$\Omega_{de} = 0.72 \pm 0.04 \quad \textit{75\% dark energy}$$

Furthermore, **matter** is dominantly dark matter and the *dark energy* has **negative pressure**

- In more detail, in a **flat Universe**, combining **WMAP** data with that from the **Supernova Legacy Survey** gives for the dark energy equation of state:

$$\omega = p_{\text{de}} / \rho_{\text{de}} = -0.97 \pm 0.07 \pm 0.09$$

consistent with the dark energy being a **cosmological constant** ($\omega_{\Lambda} = -1$)

- Even though **neutrinos** are a **subdominant component** ($\Omega_{\nu} < 0.014$) of the Universe's density now, what **FNW** suggested is that ρ_{de} **tracks** ρ_{ν} . More precisely, in the **FNW** picture **neutrinos** and dark energy are assumed to be **coupled**

- In the **NR regime** examined by **FNW**

$$\rho_{\text{dark}} = m_{\nu} n_{\nu} + \rho_{\text{de}}(m_{\nu})$$

- Because the two components are coupled, **neutrino masses** are determined **dynamically** by **minimizing** the above

$$n_{\nu} + \partial \rho_{\text{de}}(m_{\nu}) / \partial m_{\nu} = 0$$

neutrino masses depend on **density**: $m_{\nu} = m_{\nu}(n_{\nu})$

- The **equation of state** for the dark sector follows from the energy conservation equation

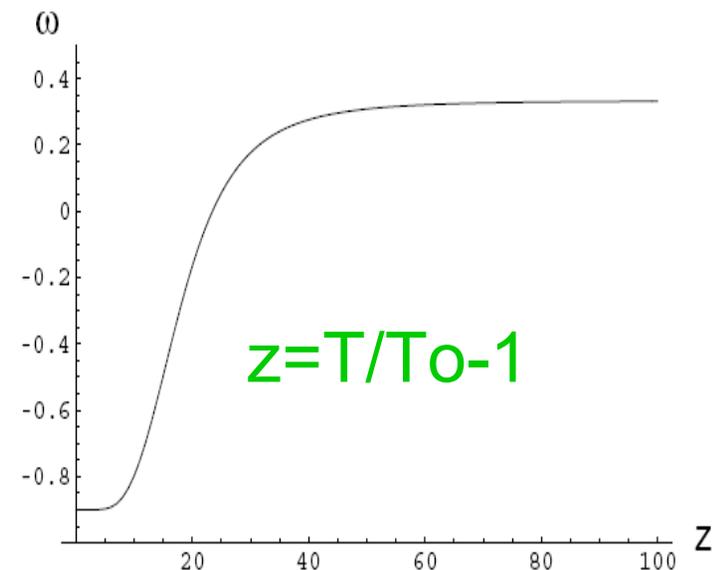
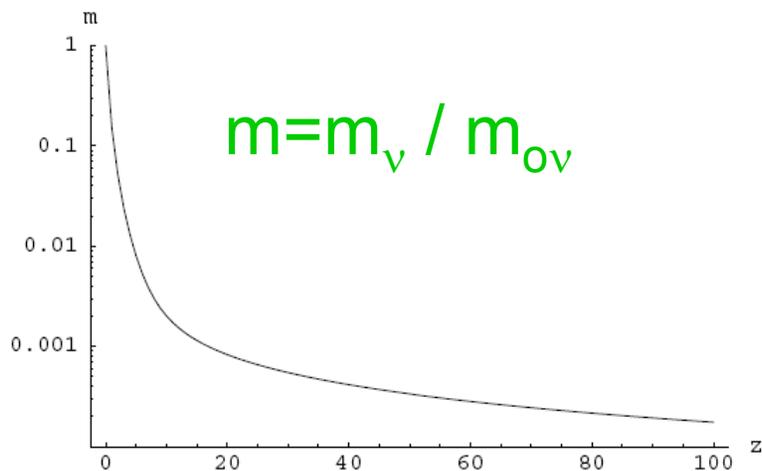
$$\partial \rho_{\text{dark}} / \partial t = -3H(\rho_{\text{dark}} + p_{\text{dark}}) = -3H\rho_{\text{dark}}(\omega + 1)$$

and in **NR limit** one finds

$$\omega + 1 = m_{\nu} n_{\nu} / \rho_{\text{dark}} = m_{\nu} n_{\nu} / [m_{\nu} n_{\nu} + \rho_{\text{de}}]$$

- If $\omega \approx -1$ the neutrino contribution to ρ_{dark} is a **small fraction** of ρ_{de} . Furthermore, one can show [Peccei] that model reduces to having a **cosmological constant**, which **runs** with m_ν
- Thus $\rho_{\text{de}} = -p_{\text{de}} \equiv V(m_\nu)$ and $m_\nu = m_\nu(n_\nu)$ is determined by the **minimization condition**

$$n_\nu + \partial V(m_\nu) / \partial m_\nu = 0$$
- Typical example $V(m_\nu) \sim m_\nu^{-\alpha}$ leads to



- There are many issues one can raise concerning **neutrino models** of dark energy:
 - What physics fixes $V(m_\nu)$?
 - What **dynamical principle** demands that $\partial \rho_{\text{dark}} / \partial m_\nu = 0$?
 - Many models lead to **dynamical instability**, since $c_s^2 = \partial p / \partial \rho < 0$
- Nevertheless, idea intriguing since it associates the dark energy sector, through the **seesaw mechanism**, to the **$SU(3) \times SU(2) \times U(1)$ singlet sector** connected with **heavy neutrinos**
- Because this sector is difficult to probe, easy to imagine that the **physics** which determines the **Universe's late dynamics** lurks there

Concluding Remarks

- Neutrino Physics has opened windows into phenomena **beyond the SM**, associated in the first instance with **(B-L)-breaking**
- It is also providing hints of possible **flavor symmetries** and of **unification**, although no unequivocal theoretical direction has surfaced
- Deeper puzzles and mysteries may well surface in the future, involving **neutrinos** and their **role** in the **Universe**