

Nuclear modifications of Structure Functions

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MOTIVATIONS AND OUTLINE

- ❖ *Main motivation of our studies was the development of a **quantitative model** providing predictions and corresponding uncertainties to be used in the analysis of present and future data from nuclear targets **in a wide kinematic range of x and Q^2***

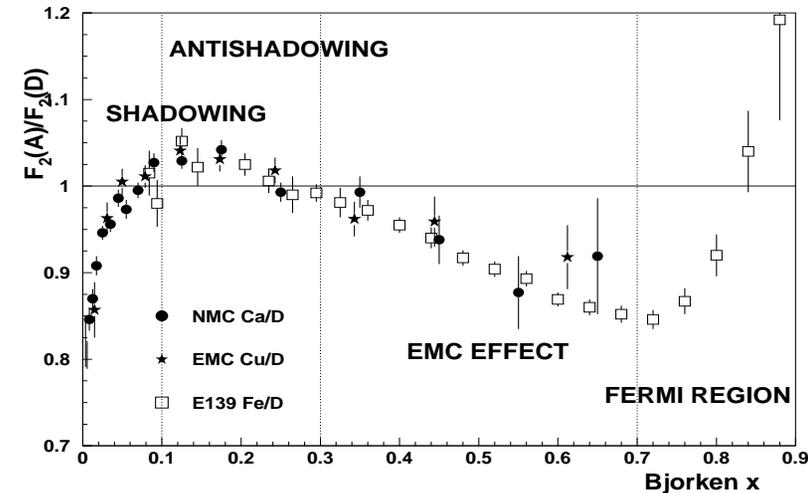
- ❖ ***Analysis of nuclear structure functions (SF)***
 - *Data and basic mechanisms of nuclear DIS*
 - *Description of the approach*
 - *Results and interpretation*

- ❖ ***Applications of the model***
 - *Nuclear parton distribution functions (nPDF)*
 - *Drell-Yan pA cross-sections*
 - *(Anti)neutrino-nucleus inelastic cross-sections*

NUCLEAR STRUCTURE FUNCTIONS

❖ **GLOBAL APPROACH** aiming to obtain *quantitative calculations* covering the complete range of x and Q^2 available (S. Kulagin and R.P., NPA 765 (2006) 126-187):

- Scale controlling nuclear processes $L_I = (Mx)^{-1}$
Distance between nucleons $d = (3/4\pi\rho)^{1/3} \sim 1.2\text{Fm}$
- $L_I < d$
For $x > 0.2$ nuclear DIS \sim *incoherent sum* of contributions from bound nucleons
- $L_I \gg d$
For $x \ll 0.2$ *coherent effects* of interactions with few nucleons are important



❖ **DIFFERENT EFFECTS** on structure functions (SF) are taken into account:

$$F_i^A = F_i^{p/A} + F_i^{n/A} + F_i^{\pi/A} + \delta F_i^{\text{coh}}$$

- $F_i^{p(n)/A}$ bound proton(neutron) SF with *Fermi Motion, Binding (FMB) and Off-Shell effect (OS)*
- $F_i^{\pi/A}$ *nuclear Pion excess correction (PI)*
- δF_i^{coh} contribution from coherent nuclear interactions: *Nuclear Shadowing (NS)*

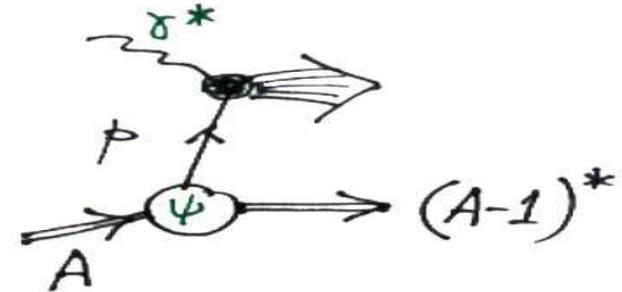
INCOHERENT NUCLEAR SCATTERING

- ❖ **FERMI MOTION AND BINDING** in nuclear structure functions can be calculated from the *convolution of nuclear spectral function and (bound) nucleon SFs*:

$$F_2^A(x, Q^2) = F_2^{p/A} + F_2^{n/A}$$

$$F_2^{p/A} = \int d\varepsilon d^3\mathbf{k} \mathcal{P}_p(\varepsilon, \mathbf{k}) \left(1 + \frac{k_z}{M}\right) F_2^p(x', Q^2, k^2)$$

where $x' = Q^2 / (2k \cdot q)$ and $k = (M + \varepsilon, \mathbf{k})$.



- ❖ Since bound nucleons are **OFF-MASS-SHELL** there appears dependence on the *nucleon virtuality* $k^2 = (M + \varepsilon)^2 - \mathbf{k}^2$:

$$F_2(x, Q^2, k^2) = F_2(x, Q^2) \left(1 + \delta f_2(x)(k^2 - M^2)/M^2\right).$$

where we have introduced an off-shell structure function **$\delta f_2(x)$**

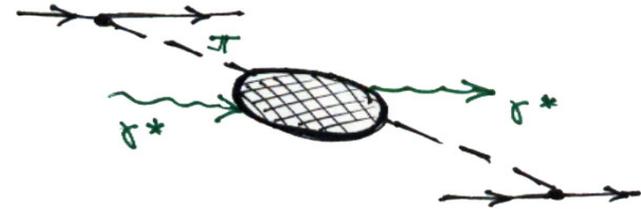
- ❖ *Hadronic/nuclear input:*

- Proton/neutron SFs computed in NNLO pQCD + TMC + HT from fits to DIS data
- Two-component nuclear spectral function (mean-field + correlated part) based on Ciofi & Simula

NUCLEAR PION CORRECTION

- ❖ Leptons can *scatter off mesons* which mediate interactions among bound nucleons:

$$F_i^{\pi/A}(x, Q^2) = \int_x dy f_{\pi/A}(y) F_i^\pi(x/y, Q^2)$$



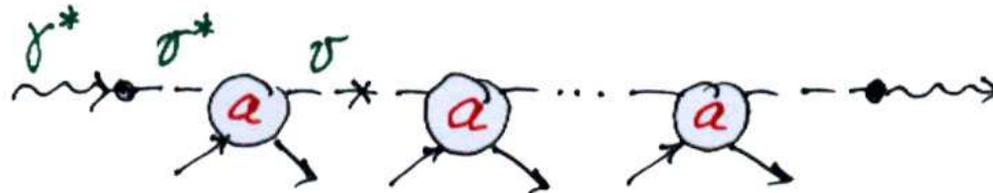
- ❖ Contribution from nuclear pions (mesons) to *balance nuclear light cone momentum* $\langle y \rangle_\pi + \langle y \rangle_N = M_A / (A M)$. The pion distribution function is localized at $y \leq p_F / M \sim 0.3$ so that the *pion contribution is at $x < 0.3$* . The correction is driven by the average number of “pion excess” $n_\pi = \int dy f_\pi(y)$ and $n_\pi / A \sim 0.1$ for *heavy nuclei*. It modifies the nuclear sea quark distributions, but not the valence quarks.

- ❖ *Hadronic/nuclear input:*

- Pion Parton Density Functions from fits to Drell-Yan data by Gluck, Reya & Schienbein
- $f_{\pi/A}(y)$ calculated using constraints of light-cone momentum conservation and equations of motion for pion-nucleon system

COHERENT NUCLEAR EFFECTS

- ❖ **SHADOWING** correction comes from *multiple interactions of the hadronic component of virtual photon* during the propagation through matter. This is described following the Glauber-Gribov approach:



$$\delta\mathcal{R} = \frac{\delta F_2^{\text{coh}}}{F_2^N} \approx \frac{\delta\sigma^{\text{coh}}}{\sigma}; \quad \delta\sigma^{\text{coh}} = 2 \text{Im} \left(ia^2 C_2^A(a) \right)$$

$$C_2^A(a) = \int_{z_1 < z_2} d^2\mathbf{b} dz_1 dz_2 \rho_A(\mathbf{b}, z_1) \rho_A(\mathbf{b}, z_2) \exp \left[i \int_{z_1}^{z_2} dz' (a \rho_A(\mathbf{b}, z') - k_L) \right]$$

$a = \sigma(i + \alpha)/2$ is the *(effective) scattering amplitude* ($\alpha = \text{Re } a / \text{Im } a$) in forward direction, $k_L = Mx(1 + m_v^2/Q^2)$ is longitudinal momentum transfer in the process $v^* \rightarrow v$ (accounts for finite life time of virtual hadronic configuration).

- ❖ *Hadronic/nuclear input:*

- Nuclear number densities $\rho_A(r)$ from parameterizations based on elastic electron scattering data

PARAMETERS OF THE MODEL

- ❖ **EFFECTIVE CROSS-SECTION** and amplitude which are used to describe the weighted sum of *hadronic components of virtual photon*:

$$\bar{a}_T = \bar{\sigma}_T(i + \alpha)/2$$

$$\bar{\sigma}_T = \sigma_1 + \frac{\sigma_0 - \sigma_1}{1 + Q^2/Q_0^2}$$

where $\sigma_1 = 0$ fixed and $\sigma_0 \equiv 27$ mb, $\alpha \equiv -0.2$ in order to match VMD model at low Q^2 . The parameter Q_0^2 controlling the *transition to partonic regime* is free.

- ❖ **OFF-SHELL FUNCTION**:

$$\delta f_2(x) = C_N(x - x_1)(x - x_0)(1 + x_0 - x)$$

where $x_1 \equiv 0.05$ from conservation of nuclear valence number, C_N and x_0 free par's.

- ❖ C_N , x_0 and Q_0^2 describe the modification of bound (off-shell) nucleon and are extracted phenomenologically from nuclear DIS data on ratios $\mathcal{R}_2(A, B) = F_2^A/F_2^B$:
- Electron and muon scattering from BCDMS, EMC, E139, E140, E665 and NMC
 - Wide range of targets He, Li, Be, C, Al, Ca, Fe, Cu, Ag, Sn, Au, Pb

RESULTS FROM NUCLEAR DIS

- ◆ Excellent agreement with existing data for the entire kinematics:

$$\chi^2/d.o.f. = 459/556$$

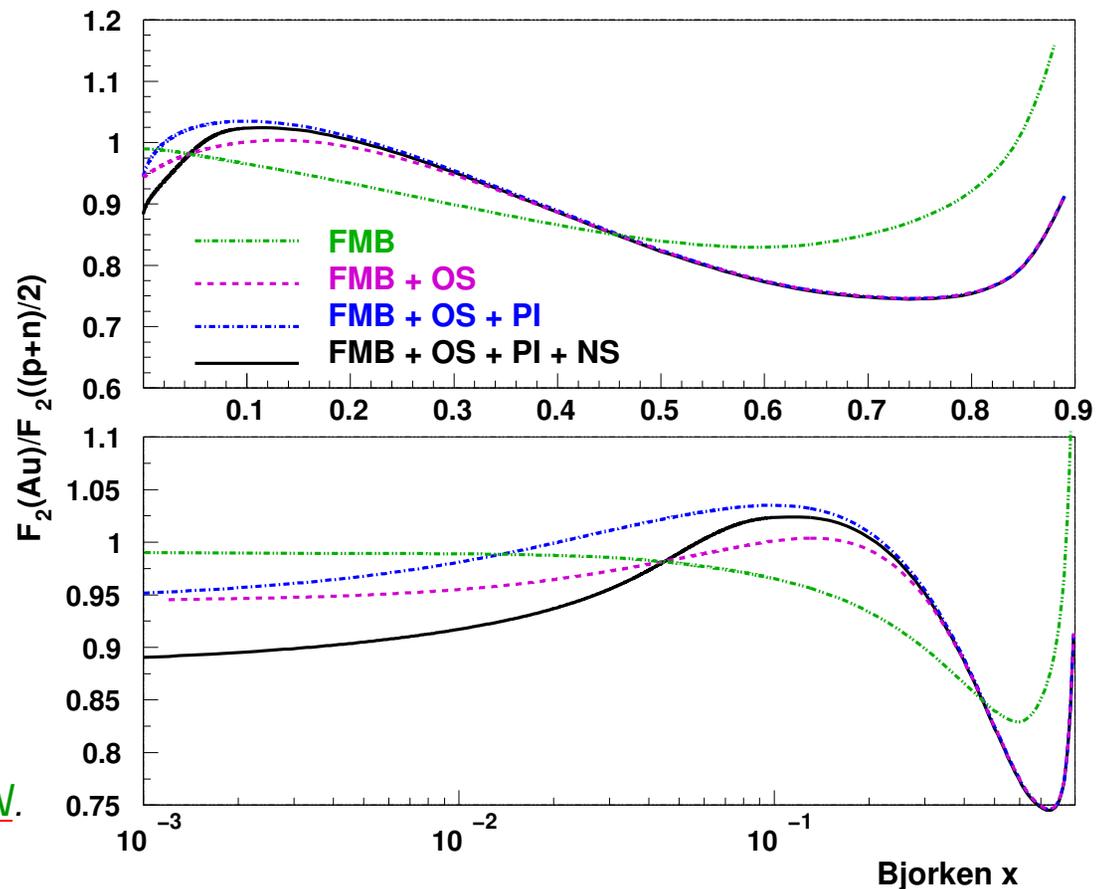
- ◆ Independent fits to different subsets of nuclei provide parameters consistent with the global fit:

$$C_N = 8.10 \pm 0.30$$

$$x_0 = 0.448 \pm 0.005$$

$$Q_0^2 = 1.43 \pm 0.06 \text{ GeV}^2$$

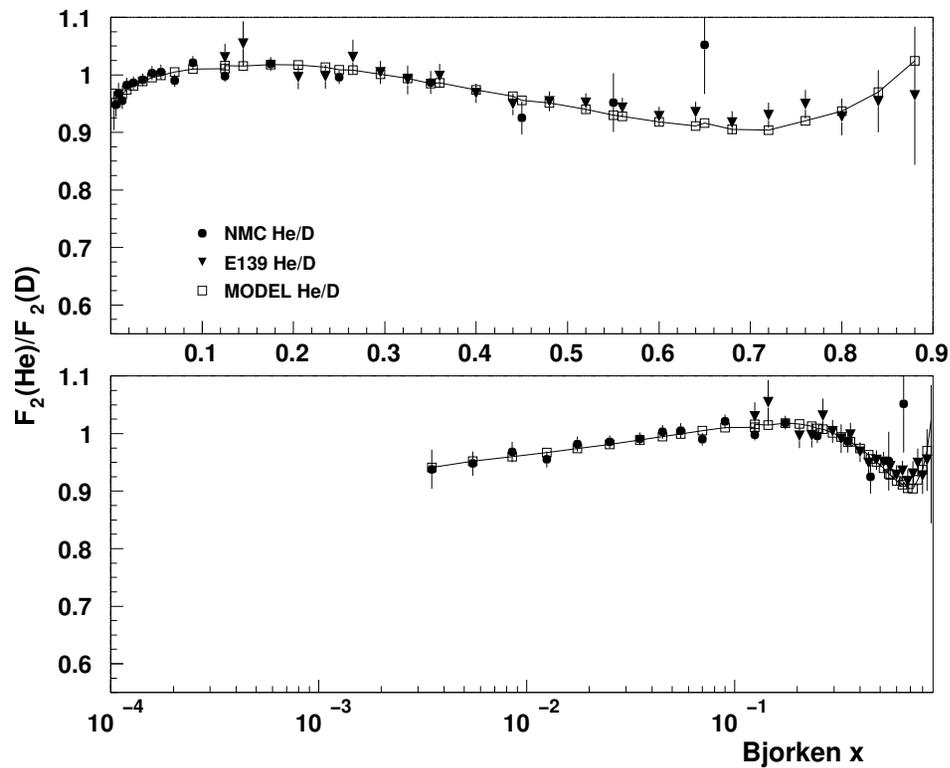
⇒ Common par's related to structure of BOUND NUCLEON.



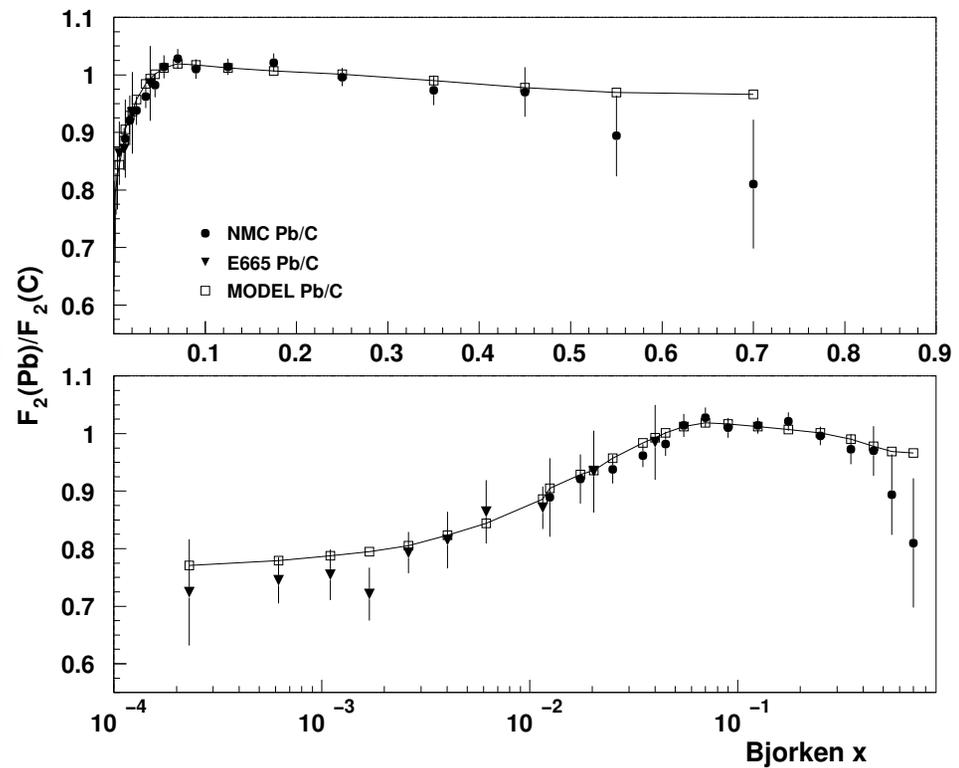
- ◆ Detailed study of systematic uncertainties: $\delta C_N=0.5$, $\delta x_0=0.007$, $\delta Q_0^2=0.2 \text{ GeV}^2$

COMPARISON WITH NUCLEAR DIS DATA

◆ $^4\text{He}/\text{D}$

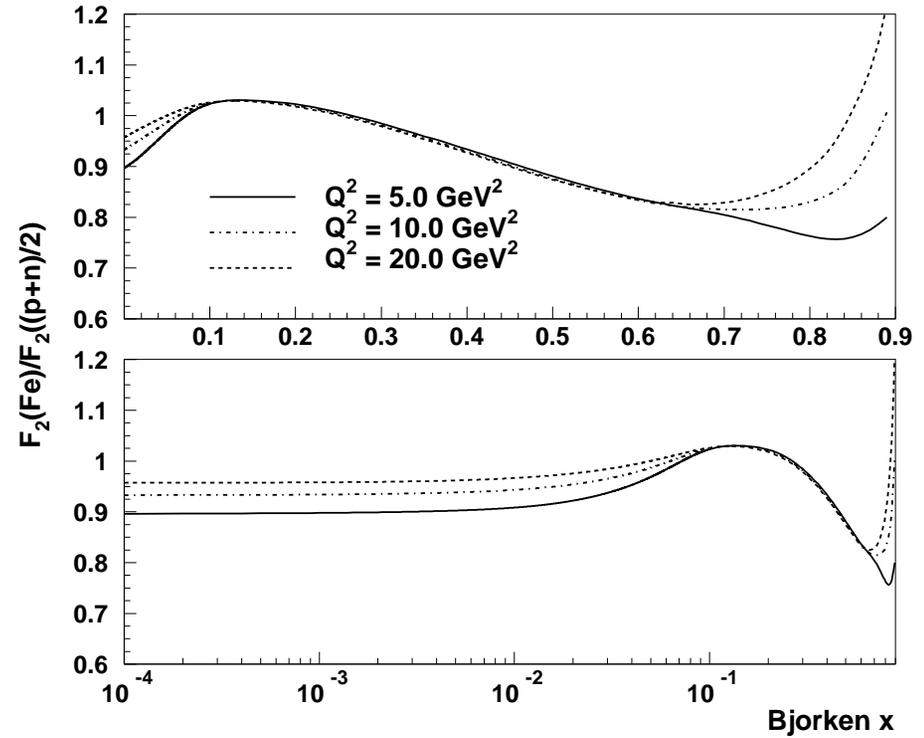
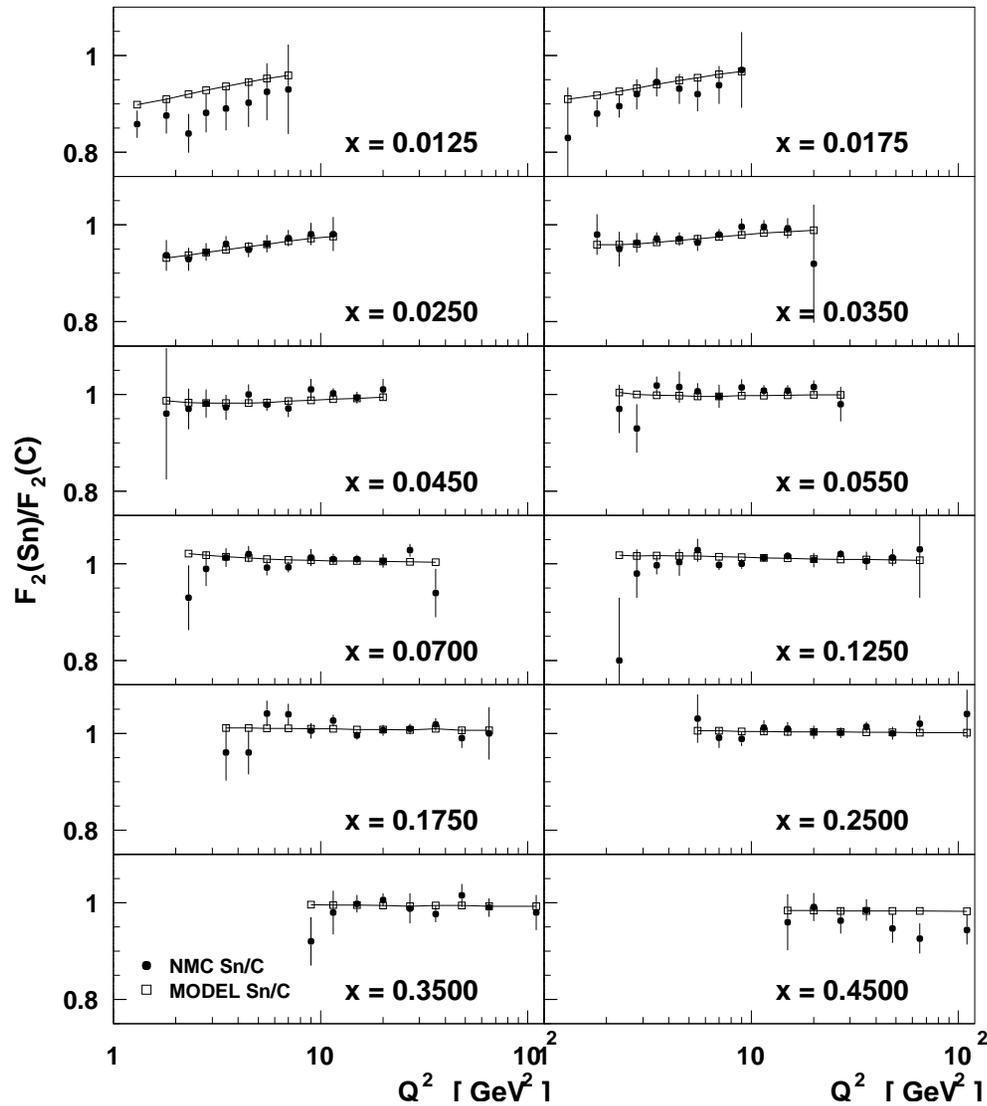


◆ $^{207}\text{Pb}/^{12}\text{C}$



*Good agreement for all measured nuclei from ^4He to ^{207}Pb
(for details see hep-ph/0412425)*

ANALYSIS OF Q^2 DEPENDENCE OF \mathcal{R}_2



Significant Q^2 dependence only for $x < 0.1$ (shadowing) and $x > 0.65$ (TMC expansion)

OFF-SHELL FUNCTION

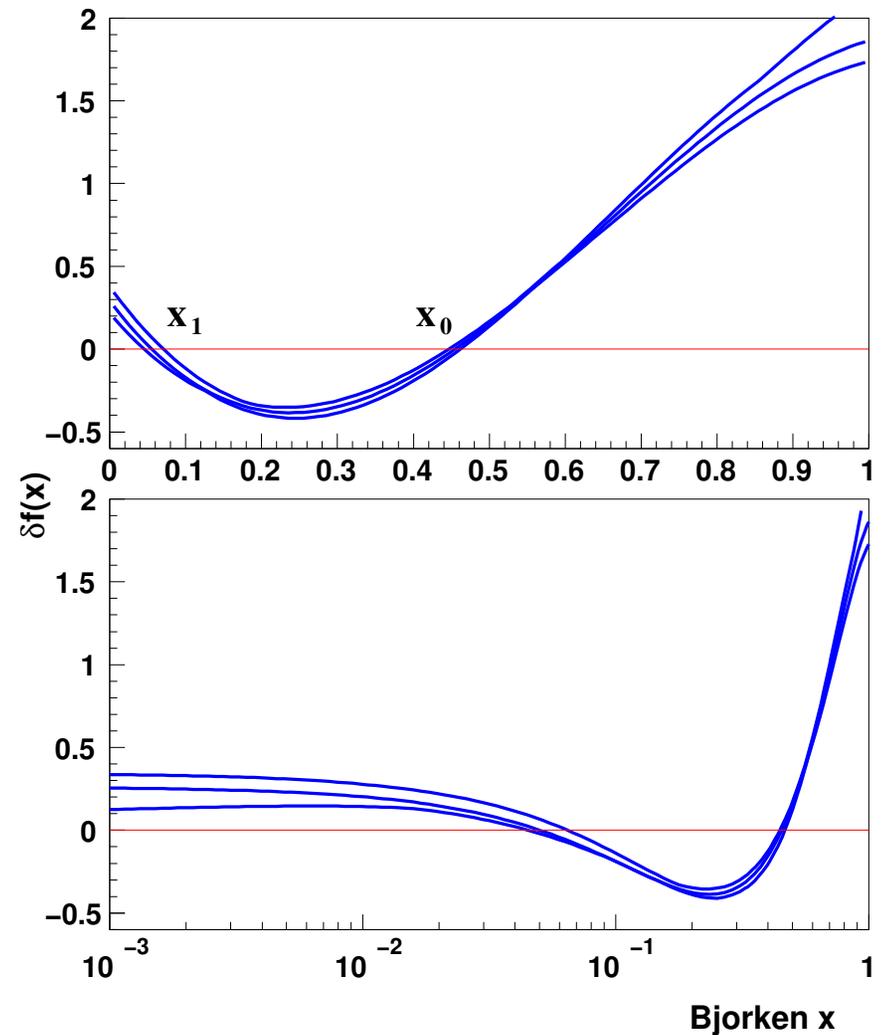
❖ The Off-Shell structure function provides an estimate of the *modification of the bound nucleon radius in nuclear medium*

⇒ x_0 and the slope $\delta f'(x_0)$ suggest an increase of the radius of nucleon valence region by $\sim 10\%$ in Fe

❖ Inclusive nuclear DIS can be used to probe the off-shell function $\delta f(x)$

⇒ *Additional information could be provided by semi-inclusive DIS data*

❖ Interesting to *check the universality of δf for all partons, which was suggested by normalization of nuclear valence number*



EFFECTIVE CROSS-SECTION

❖ The monopole form $\bar{\sigma} = \sigma_0 / (1 + Q^2/Q_0^2)$ describes well the existing data on nuclear shadowing for $Q^2 < 20 \text{ GeV}^2$

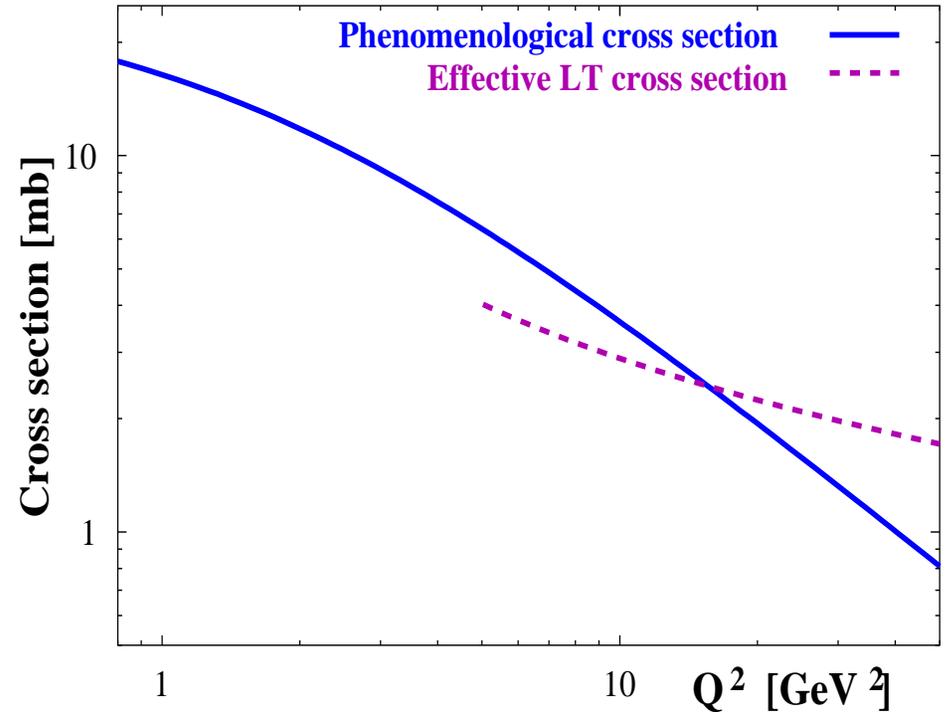
⇒ No constraint from current data on $\bar{\sigma}$ at $Q^2 > 20 \text{ GeV}^2$

❖ An effective cross-section at high Q^2 can be calculated by imposing the conservation of the nuclear valence quark number (baryon number):

$$\delta N_{\text{val}}^{\text{off-shell}} + \delta N_{\text{val}}^{\text{shadowing}} = 0$$

⇒ Nuclear data indicate substantial High Twist contributions at $Q^2 < 10 \text{ GeV}^2$

⇒ Effective Leading Twist cross-section allows the calculation of nuclear PDFs

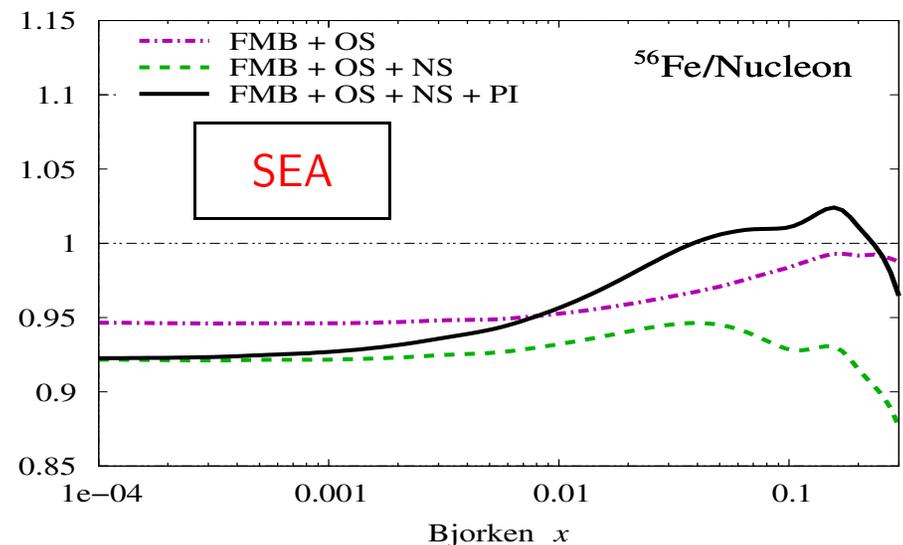
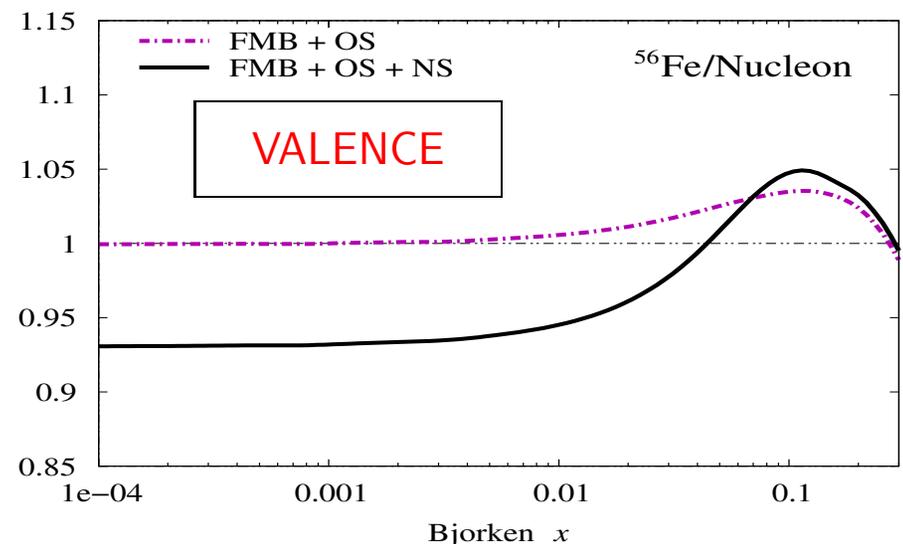


CORRECTION TO VALENCE AND SEA QUARKS

- ❖ *Nuclear corrections to parton distributions (nuclear PDFs) can be derived naturally from our analysis of SFs*
- ❖ *Universality of off-shell function implies an universal nuclear convolution for all partons*
- ❖ *For isoscalar targets the nuclear pion (meson) correction to valence dist. cancels out*
- ❖ *Nuclear shadowing depends on C-parity $q^\pm = q \pm \bar{q}$. Multiple scattering correction is enhanced for valence, however total relative effect is similar for valence and sea.*

$$\frac{\delta \bar{q}_A(x)}{\bar{q}_N(x)} = \delta \mathcal{R}^+ + \frac{q_{\text{val}/N}(x)}{2\bar{q}_N(x)} (\delta \mathcal{R}^+ - \delta \mathcal{R}^-)$$

⇒ *Remarkable cancellation between pion and shadowing effects for \bar{q}*



ISOSPIN DEPENDENCE OF NUCLEAR PDFs

- ◆ Different *Fermi motion and binding corrections* for isoscalar $q_0 = u + d$ and isovector $q_1 = u - d$ quark distributions:

$$\mathcal{P}^{p+n} = A\mathcal{P}_0; \quad \mathcal{P}^{p-n} = (Z - N)\mathcal{P}_1$$

isoscalar and isovector spectral functions \mathcal{P}_0 and \mathcal{P}_1 are different distributions

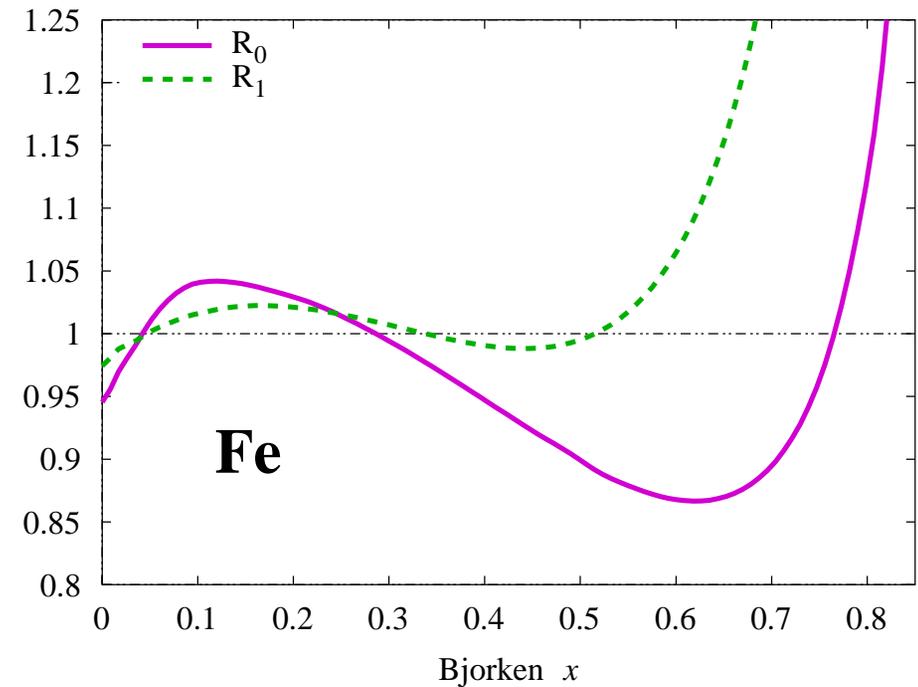
⇒ *Isospin effect in the FMB correction*

⇒ *Flavour-dependent nuclear correction*

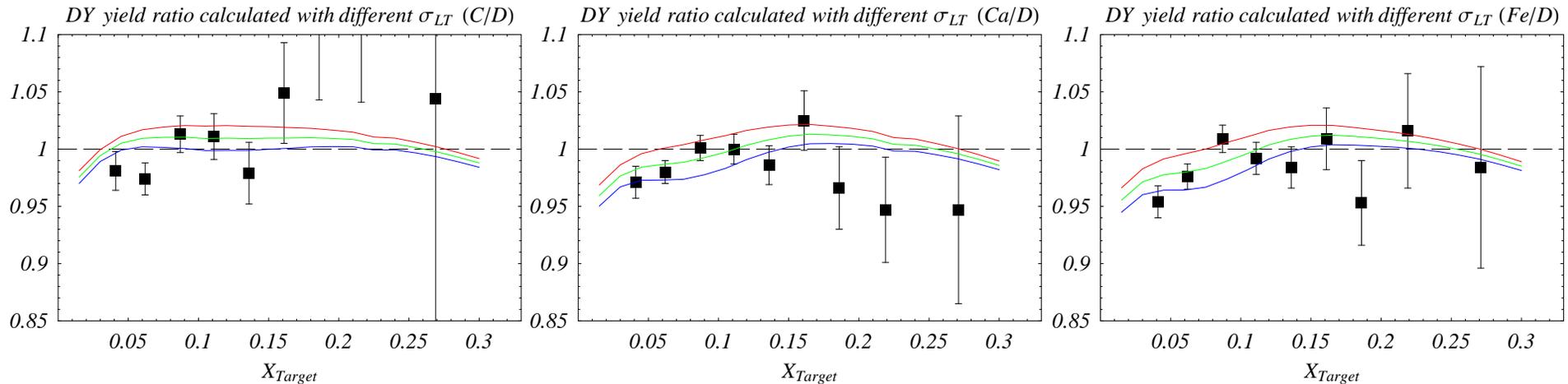
- ◆ *Convolution* of q_0 and q_1 quark distributions with isoscalar f_0 and isovector f_1 nucleon distributions in nucleus

$$q_{0/A} = Af_0 \otimes q_{0/p}; \quad q_{1/A} = (Z - N)f_1 \otimes q_{1/p}$$

- ◆ *Implementation of isospin (quark flavour) effects for nuclear shadowing is in progress*



APPLICATION TO DRELL-YAN pA CROSS-SECTIONS



❖ Extraction of Leading Twist and calculation of nuclear parton distributions can be checked against *Drell-Yan data from E772 experiment*:

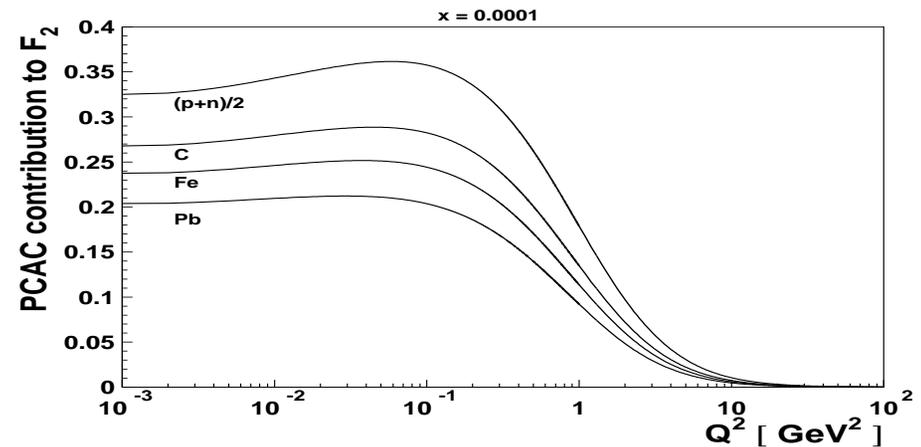
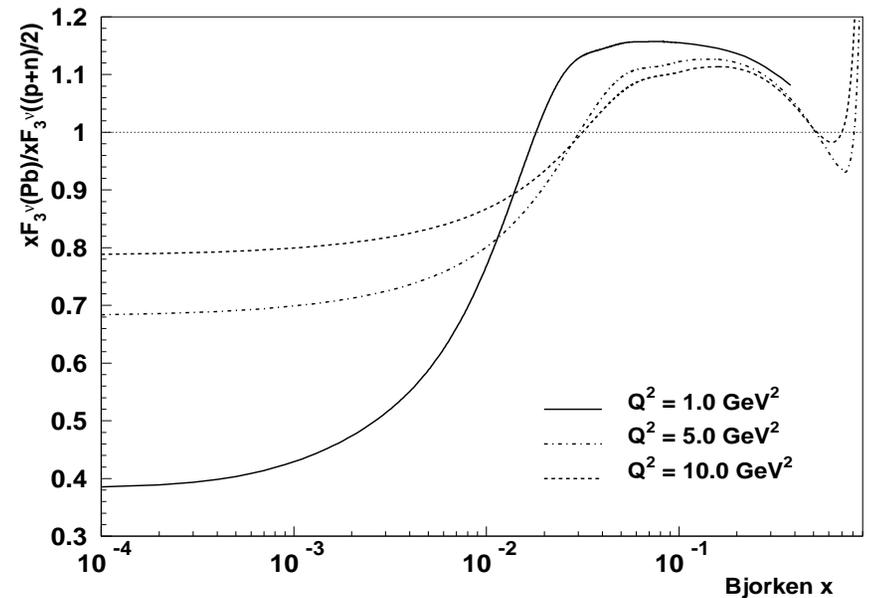
- Different process and kinematics with respect to DIS data ($Q^2 \geq 16 \text{ GeV}^2$);
- As a first test, *calculate Drell-Yan cross-sections in the LO approximation* (with NNLO PDFs);
- Exact kinematics (mass) of points not available from paper.

⇒ *Predictions are in reasonable agreement with C/D, Ca/D and Fe/D ratios.*

❖ Cancellation from shadowing *reconciles nuclear pion excess with Drell-Yan data*

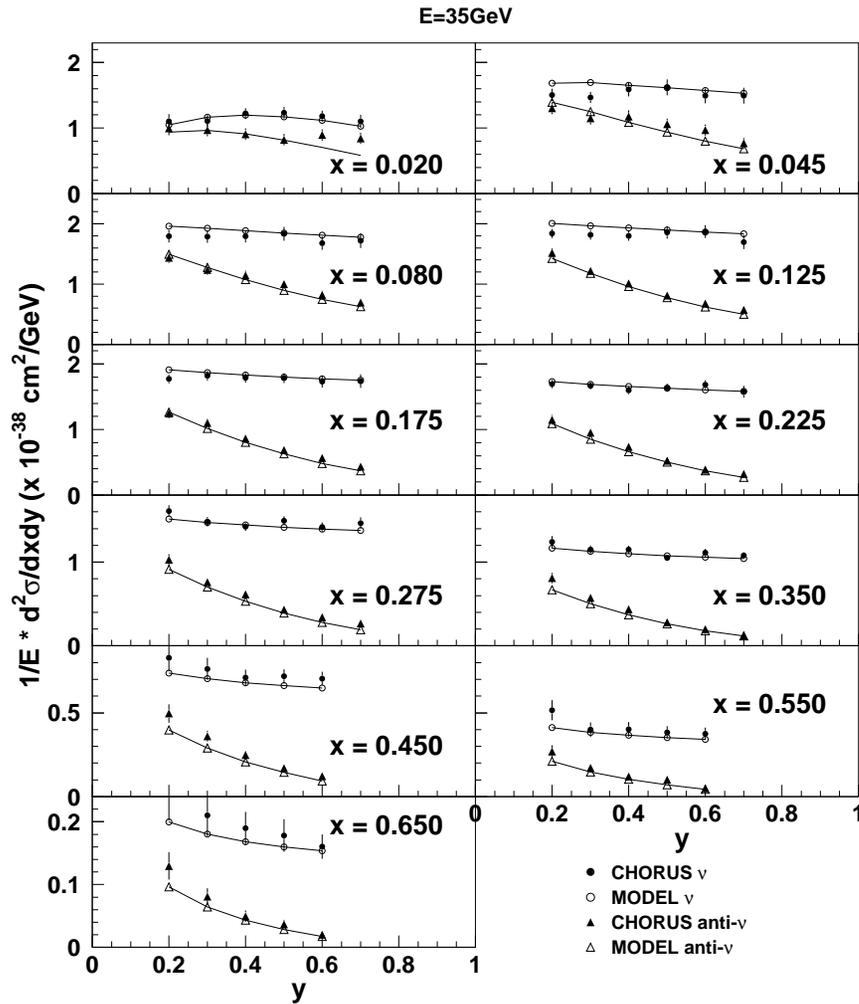
APPLICATION TO $\nu(\bar{\nu})A$ CROSS-SECTIONS

- ❖ Similarities between neutrino and charged lepton scattering for **Leading Twist** (high Q^2) are based on description in terms of **universal process-independent PDFs**. Nuclear corrections to PDFs (nuclear PDFs) are also process-independent.
- ❖ **High Twist** contributions are generally **process-dependent** and differ for neutrino and charged lepton scattering (low Q^2):
 - The axial current dominates neutrino cross-sections at $Q^2 \ll 1\text{GeV}^2$. **The PCAC relation** links structure functions to virtual pion cross section (Adler). Coherent nuclear corrections are driven by **multiple scattering effects in the virtual pion cross section**.
 - **Non PCAC contributions** (e.g. axial-vector states $a_1, \rho\pi$) are relevant in the **transition region** of $Q^2 \sim 1\text{GeV}^2$ and $x \sim (1 - 5)10^{-2}$.

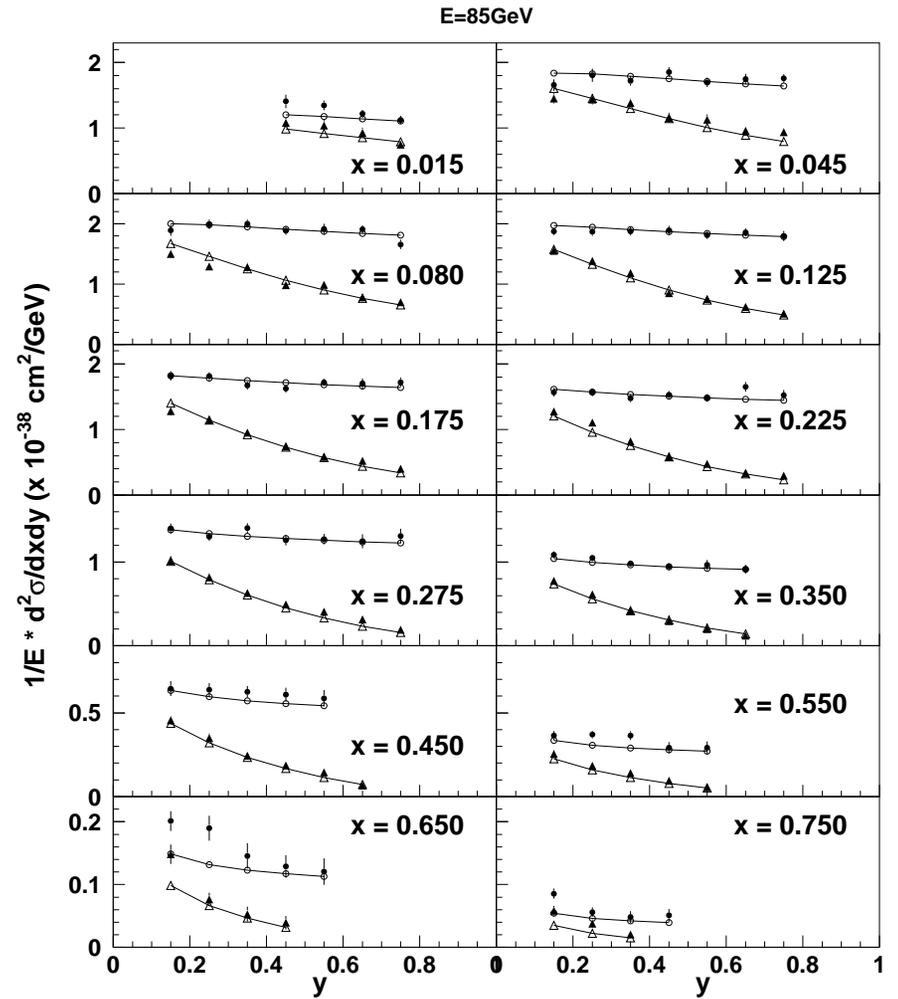


COMPARISON WITH $\nu(\bar{\nu})A$ DATA

CHORUS $\nu(\bar{\nu})$ Pb



NuTeV $\nu(\bar{\nu})$ Fe



Predictions of our model for differential cross sections

SUMMARY

- ❖ *A detailed and quantitative study of existing data from charged lepton-nucleus scattering has been performed in a wide kinematic region of x and Q^2*
- ❖ *A model was developed which includes the QCD treatment of nucleon structure functions as well as major nuclear effects like shadowing, Fermi motion and nuclear binding, nuclear pion and off-shell corrections to bound nucleon structure functions*
- ❖ *Nuclear parton distribution functions (nPDFs) can be derived naturally from our treatment of nuclear structure functions, by extracting the Leading Twist contribution*
- ❖ *Applications to Drell-Yan pA cross-sections and to (anti)neutrino-nucleus cross-sections are in agreement with existing experimental data*