MODELS OF NEUTRINO MASSES AND MIXINGS

Ferruccio Feruglio

Neutrino 2004 – Paris, June 2004

- How many light neutrinos ?
- Why $m_\nu << m_f$ (f: charged fermion) ?
- Which spectrum is favoured by theory ?
- Which is the most probable range of U_{e3} ?

HOW MANY LIGHT ν ?

- 3 active neutrinos $N = 2.984 \pm 0.009$ (invisible Z width) $(m_t, m_H) = (174.3, 115)$ GeV

- all experiments but LSND explained by 3 ν_a
  LSND $\rightarrow$ 3 ν_a + [at least] 1 ν_s

- inclusion of ν_s worsens the fits
  - solar: $\nu_e \to \sin\vartheta_s\ \nu_s + \dots$ $\sin^2\vartheta_s < 0.1 \quad (1\sigma)$ [Bahcall&Pena-Garay 2003]
  - atm: $\nu_\mu \to \nu_\tau$ favoured over $\nu_\mu \to \nu_s$
    - zenith angle dependence of high-energy ν_μ [SK,MACRO] 3σ (no matter effects for ν_s)
    - no NC interactions for ν_s [SK]
    - τ-like CC events [SK] 2σ
  - 2+2 and 3+1 fits have a poor quality [Cirelli,Marandella,Strumia,Vissani 2004]

- WMAP + LSS $\sum m_\nu < 1.4\,\mathrm{eV}\ (95\%\ \mathrm{CL}) \quad \text{for } 3\nu_a + 1\nu_s$
  [Hannestad&Raffelt 2004 Crotty, Lesgourgues, Pastor 2004]

- from now on: 3 light ν assumed

- LSND soon checked by MiniBooNE (1st event September 2002)
  no room for LSND with 3 ν_a (CPT violation disfavoured by now) [Pakvasa&Valle 0301061]

WHY $m_\nu << m_f$?

charged fermions

L is exactly conserved

- requires $\frac{y_{\nu_e}}{y_e} < 10^{-6}$ [smallest ratio is 1/100 for charged fermions in same gen.]

- theoretical prejudices
  - global symmetries are broken by quantum gravity
  - B/L violated in all attempts to unify fundamental interactions
  - B/L broken by anomalies already in the SM

- Interesting attempts in models with extra dimensions [Dienes, Dudas, Gherghetta, Arkani-Hamed,Dimopoulos, Dvali, March-Russell, Barbieri, Creminelli, Strumia]

large ED: standard Yukawa couplings to a singlet fermion ν who lives in the bulk

$$\mathcal{L}_{Yuk} = \frac{y_\nu \mathrm{v}}{\sqrt{2}} \left(\frac{M_D}{M_P} \right) \nu_a(x) \nu_s^{(0)}(x)$$

$$\nu_s(x,y) = \frac{\nu_s^{(0)}(x)}{\sqrt{V_\delta}} + \ldots$$

no experimental hints from oscillations

$\nu_s^{(n)}$ effects subdominant, if present

dimension 5, L-violating operators not sufficiently suppressed by $M_D \approx 1\,\mathrm{TeV}$

alternative models: warped compactifications, L gauged in the bulk,…
not fully realistic in their minimal realization [Grossman&Neubert'99 Gherghetta 0312392]

L is not conserved

$$\frac{(Hl)(Hl)}{\Lambda} = \frac{1}{2}\frac{v^2}{\Lambda}\nu\nu + \ldots$$ leading L-violating operator $\Delta L = 2$

$$m_\nu = y\frac{v^2}{\Lambda} \longleftrightarrow m_e = \frac{y_e}{\sqrt{2}}v$$ smallness of m_ν due to $\frac{v}{\Lambda} << 1$

[GUT scale, see-saw, leptogenesis…]

[Buchmuller, Di Bari, Plumacher 0406014
Akhmedov, Frigerio, Smirnov 0305322]

- experimental constraints
  - oscillations are insensitive to L violation
  - L violation can be tested in 0νββ decay

HM (^{76}Ge)	$T_{1/2} > 1.9\times10^{25}\ yr$	$\lvert m_{ee}\rvert < 0.35$ eV
IGEX (^{76}Ge)	$T_{1/2} > 1.6\times10^{25}\ yr$	$\lvert m_{ee}\rvert < (0.33 \div 1.35)$ eV
Cuoricino (^{130}Te)	$T_{1/2} > 5.5\times10^{23}\ yr$	$\lvert m_{ee}\rvert < (0.37 \div 1.9)$ eV

[90%CL]

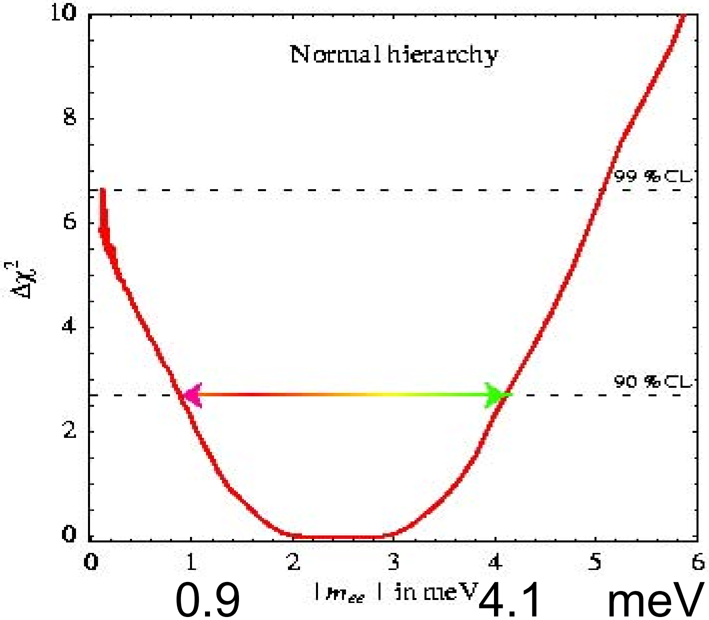
future foreseen sensitivity
on $\lvert m_{ee}\rvert$: 10 meV

uncertainty from
nuclear matrix elements

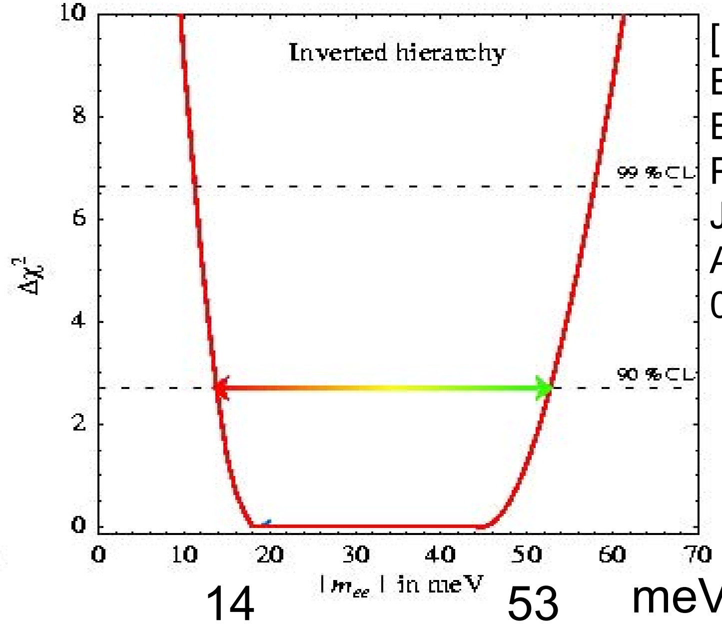
  - expected range of $\lvert m_{ee}\rvert$ can be predicted from $(\Delta m^2_{ij}, \vartheta_{ij})$

$$\lvert m_{ee}\rvert = \left\lvert \cos^2\vartheta_{13}(\cos^2\vartheta_{12}\, m_1 + \sin^2\vartheta_{12} e^{2i\alpha}\, m_2) + \sin^2\vartheta_{13} e^{2i\beta}\, m_3 \right\rvert$$

Normal Hierarchy



Inverted Hierarchy

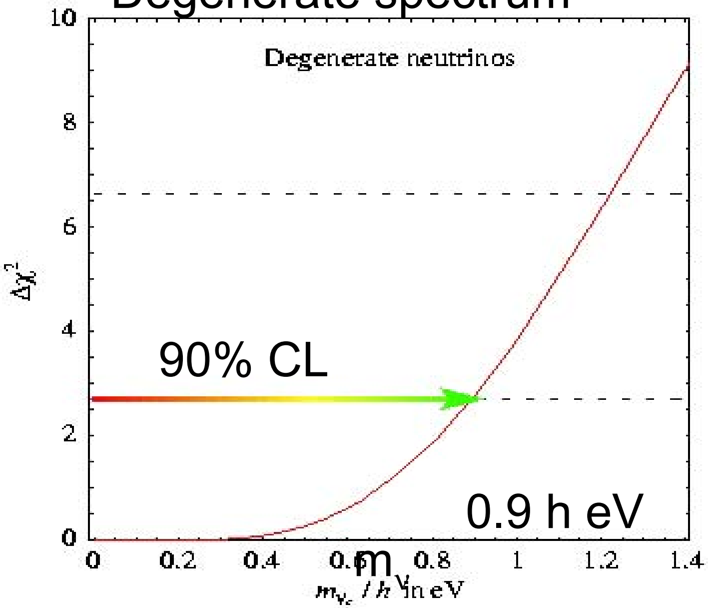


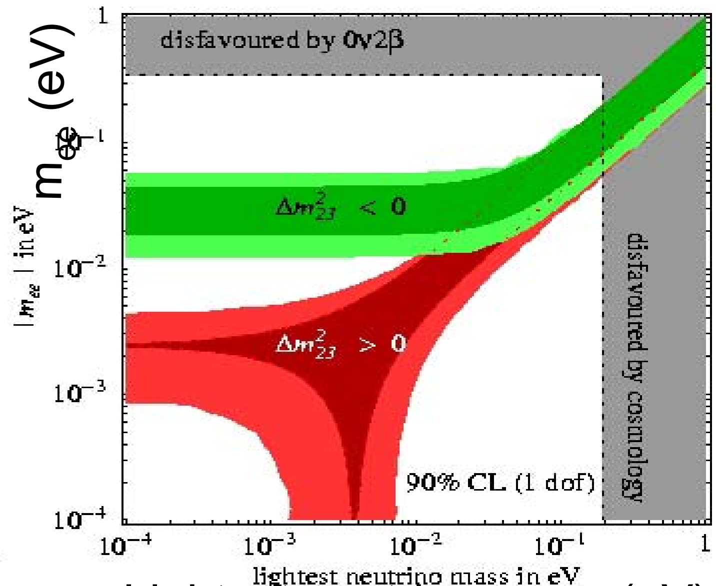
F., Strumia, Vissani

[Petcov&Pascoli 0310003
Bilenky 0403245
Bahcall, Murayama, Pena-Garay 0403167
Joaquim 0304267
Abada, Bhattacharyya 0304159]

90% CL

Degenerate spectrum





Lightest neutrino mass (eV)

h=0.6-2.8 uncertainty in nuclear matrix el.

ABSOLUTE SPECTRUM

Degenerate spectrum

- Converging evidence that this is only possible below the eV scale

$$m_{\nu_e} < 2.2 \text{ eV} \quad (95\% \text{ CL})$$ tritium β-decay

$$m_\nu < 0.9\, h \text{ eV} \quad (90\% \text{ CL})$$ 0νββ decay

$$\sum m_\nu < (0.69 \div 1.01) \text{ eV} \quad (95\% \text{ CL})$$ WMAP +LSS depending on priors

leptogenesis from out-of-equilibrium CP-violating decay of heavy ν^c prefers: $m_i < (0.12 \div 0.15)$ eV

[not an absolute bound: 1eV is still possible if ν are degenerate] [Hambye, Lin, Notari, Papucci, Strumia 0312203]

[Buchmuller, Di Bari, Plumacher 0401240 Giudice, Notari, Raidal Riotto, Strumia 2003]

- Hints for $m_i \approx 0.2 \div 0.6$ eV? 0νββ of ^{76}Ge HM by Klapdor-K. et al. 2004

- Problems with models of degenerate neutrinos

  - see-saw relation untenable: fine-tuning between M and D sectors

$$m_\nu = m_D^T M^{-1} m_D$$

connection with charged fermion masses (e.g. from GUTs) is lost

- $m_i = m$ can be understood in some symmetry limit (e.g. $SO(3)$) where angles and (mass)^2 differences are completely undetermined

$$\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}} << 1 \qquad \vartheta_{13} << \vartheta_{23} \approx \frac{\pi}{4}$$

arise from symmetry breaking terms that require a special misalignment between charged leptons and neutrinos

specific realizations already ruled out or strongly disfavoured, e.g. Flavour Democracy [Fritzsch, Xing]

$$\sin^2 2\vartheta_{23} = \frac{8}{9} \approx 0.89 \leftrightarrow \sin^2 2\vartheta_{23} > 0.9(0.84) \text{ at } 2\sigma(3\sigma)$$

no general consensus on how to realize this

- to avoid a full-fledged theory of breaking terms, we can assume anarchy in the neutrino sector

[Hall, Murayama, Weiner 2000
De Gouvea, Murayama 0301050]

$$m_\nu = m \begin{pmatrix} O(1) & O(1) & O(1) \\ O(1) & O(1) & O(1) \\ O(1) & O(1) & O(1) \end{pmatrix}$$

$\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}} << 1$ can be produced in part by the see-saw

$\vartheta_{13} << \vartheta_{23}$ accidental

$\vartheta_{23} \approx \frac{\pi}{4}$ fortuitous

Inverse Hierarchy

- the best we have, at present, is

$$m_\nu = m\begin{pmatrix} 0 & a & b \\ a & 0 & 0 \\ b & 0 & 0 \end{pmatrix} + \dots$$

corrections << |a|, |b|

- leading order determined by $L_e - L_\mu - L_\tau$ either with or without see-saw

- compatible with GUT

- leading order

$$|m_1| = |m_2| = \sqrt{|a|^2 + |b|^2} \qquad m_3 = 0$$

$$\vartheta_{13} = 0$$

good!

$$\tan\vartheta_{23} = -\frac{b}{a}$$

adjustable to maximal for |b|=|a|

$$\vartheta_{12} = 45^o$$

it's out by 6σ

$$\vartheta_{12}^{exp} = 33^o \pm 2^o$$

- turning SB terms on

$$1 - \tan^2\vartheta_{12} \approx O\left(\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}\right)$$

$$0.36 \div 0.70\ (3\sigma) \quad >> \quad 0.015 \div 0.07\ (3\sigma)$$

off by a factor > 10

difficulty common to many models

- Flavour Democracy (deg. spectrum)
- pseudoDirac structure in 12 sector

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

[exception: normal hierarchy]

- substantial contribution to ϑ_{12} from charged leptons needed

$$U_{PMNS} = U_e^+ U_\nu$$

standard parametrization

$$U_e = U_{23}^e \cdot U_{13}^e \cdot U_{12}^e$$

U_{23}^e can be absorbed in U_ν to give:

$$\vartheta_{23} \approx 45^o$$

by expanding to 1st order in $|u| \equiv \sin\vartheta_{12}^e, |v| \equiv \sin\vartheta_{13}^e << 1$

$$1 - \tan^2\vartheta_{12} = 2\sqrt{2}\,\mathrm{Re}(u+v)$$

$$|U_{e3}| = \frac{1}{\sqrt{2}}|u-v|$$

[Frampton, Petcov, Rodejohann 0401206
Altarelli, F, Masina 0402155
Romanino 0402508]

$$\tan^2\vartheta_{23} = 1 + O(u^2, v^2, uv)$$

$$\delta_{CP} = \arg(u-v)$$

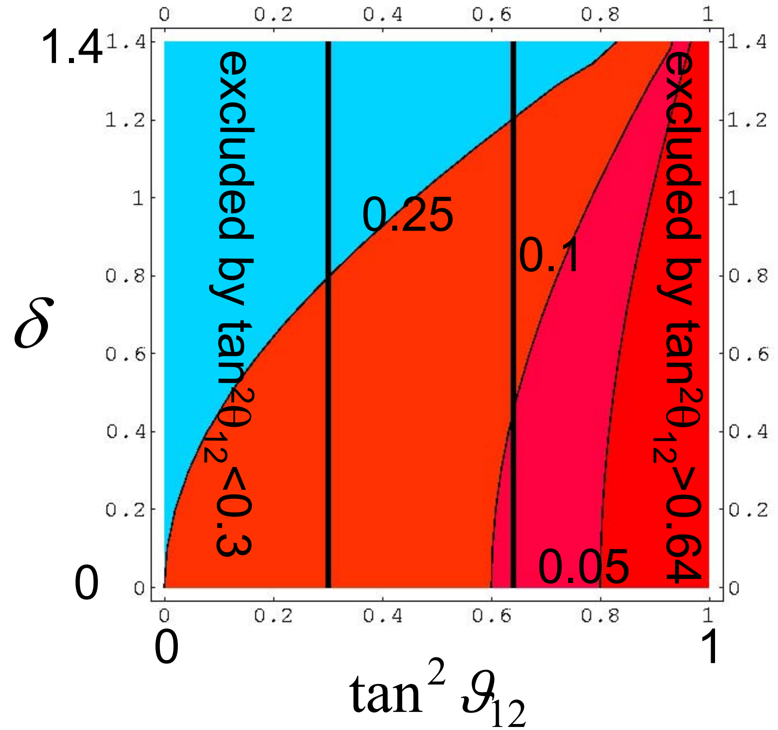
if, by analogy with the quark sector:

$$|v| << |u| \approx \vartheta_C \approx 0.22$$

- $1 - \tan^2\vartheta_{12} \approx 2\sqrt{2}\,\vartheta_C \approx 0.6$

  [right amount] [Raidal 0404046
  Minakata, Smirnov 0405088]

- $$|U_{e3}| = \frac{(1-\tan^2\vartheta_{12})}{4\cos\delta_{CP}}$$



Normal Hierarchy

- Several viable mechanisms for ϑ_{23} large

	GUT	quark mixing
ϑ_{23}^{e} and ϑ_{23}^{ν} small but $\vartheta_{23} \equiv \vartheta_{23}^{\nu} - \vartheta_{23}^{e} \approx O(1)$	O.K.	O.K.
[1] see-saw dominance of light ν^{c} equally coupled to ν_{μ} and ν_{τ} [King]	O.K.	O.K.
[2] lopsided structure of m_e or/and m_D^{ν} : $\bar{R}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & a & b \end{pmatrix} L$ [Albright, Barr Altarelli, F]	O.K.	large RH quark mixing in SU(5)

- typical texture as e.g. from U(1) flavour symmetry

[entries up to O(1) coefficients]

$$m_{\nu} = m \begin{pmatrix} \varepsilon^2 & \varepsilon & \varepsilon \\ \varepsilon & \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \\ \varepsilon & & \end{pmatrix}$$

$\vartheta_{13} \approx \varepsilon$ vanishes in the symmetry limit

ϑ_{12} large $\leftrightarrow \det[23] \approx \varepsilon$

- accidentally (semi-anarchy=SA)
- by a see-saw suppression as in [1] and [2]

U_{e3}

No reason why U_{e3} should be tiny in realistic models

- inverse hierarchy from $L_e - L_\mu - L_\tau$ [barring cancellations]

$$|U_{e3}| \geq \frac{(1-\tan^2\vartheta_{12})}{4} \approx (0.09 \div 0.18)\,[3\sigma]$$

- normal hierarchy [1st order in $\sin\vartheta_{12}^e >> \sin\vartheta_{13}^e$]

$$U_{e3} \approx \sin\vartheta_{13}^\nu - \sin\vartheta_{23} \cdot \sin\vartheta_{12}^e$$

$V_{ub} \approx \lambda^3 \quad (\lambda \approx 0.22)$

if we make a similar estimate in the quark sector

ν-dominated

$$\approx \sin\vartheta_{12}^\nu \sqrt{\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}}$$

[De Gouvea 041220]

$$\approx (0.03 \div 0.3)$$

e-dominated

$$\approx -\sin\vartheta_{23}\sqrt{\frac{m_e}{m_\mu}}$$

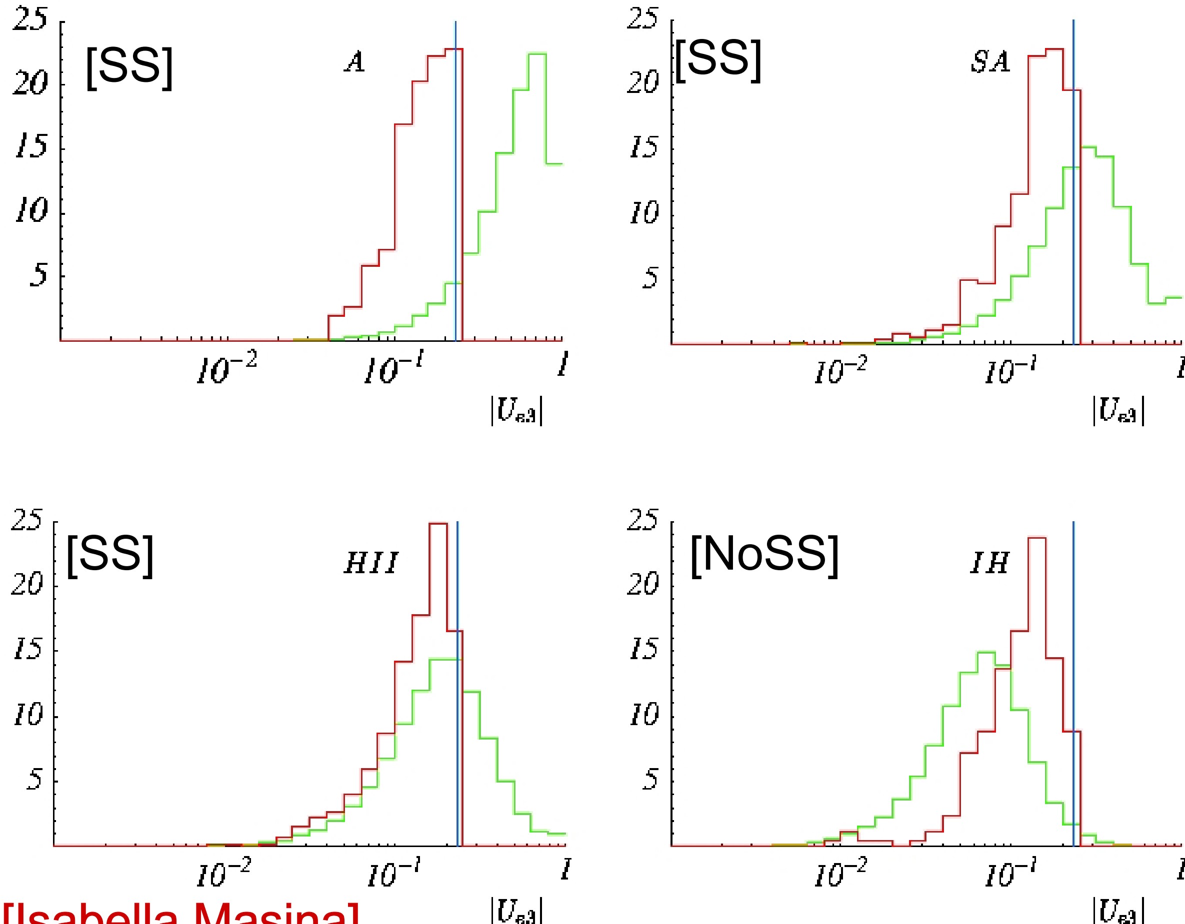
$$\approx (0.02 \div 0.1)$$

[estimates by allowing 3σ exp. and (factors ½ and 2) th. uncertainties]

- degenerate spectrum (examples)

  - Δm^2_{sol} and U_{e3} determined by RGE $\quad \approx \sqrt{\frac{\Delta m^2_{sol}}{2\Delta m^2_{atm}}} \approx (0.04 \div 0.4)$
  - Flavour Democracy $\quad \approx \sqrt{2m_e/3m_\mu} \approx (0.03 \div 0.1)$
  - Anarchy $\quad \approx O(1)$

U_{e3} in models with U(1) flavour symmetry



[Isabella Masina]

$$\begin{cases} 0.018 < r \equiv \left|\dfrac{\Delta m^2_{12}}{\Delta m^2_{23}}\right| < 0.053 \\ |U_{e3}| < 0.23 \\ 0.30 < \tan^2 \vartheta_{12} < 0.64 \\ 0.45 < \tan^2 \vartheta_{23} < 2.57 \end{cases}$$

ε optimised case by case to fit

$$m_\nu = m \begin{pmatrix} \varepsilon^2 & \varepsilon & \varepsilon \\ \varepsilon & 1 & 1 \\ \varepsilon & 1 & 1 \end{pmatrix}$$

$\varepsilon = 1$ anarchy=A

$\varepsilon < 1$ semianarchy=SA

$\varepsilon < 1$ normal hierarchy=H

$\det 23 \approx \varepsilon$

inverse hierarchy=IH

$\varepsilon < 1$

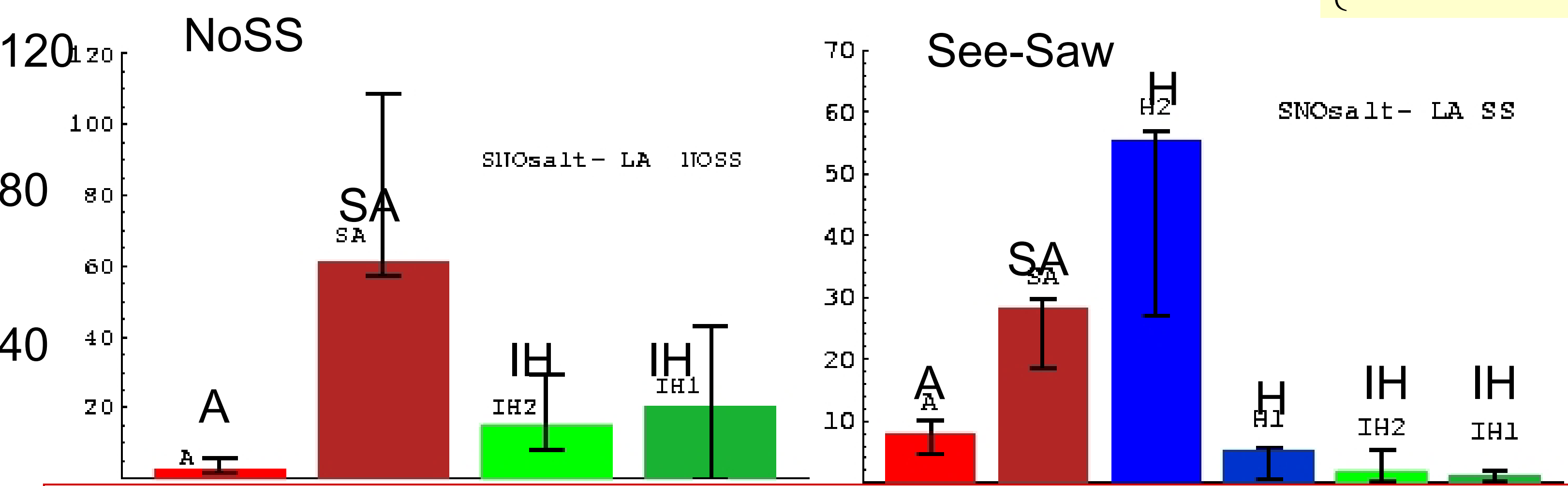
$$m_\nu = m \begin{pmatrix} \varepsilon^2 & 1 & 1 \\ 1 & \varepsilon^2 & \varepsilon^2 \\ 1 & \varepsilon^2 & \varepsilon^2 \end{pmatrix}$$

matrix elements up to unknown O(1) coeff.

Outcome of the optimisation procedure

by generating random, O(1), complex coefficients and by counting the success rate in reproducing

$$\begin{cases} 0.018 < r \equiv \left|\dfrac{\Delta m^2_{12}}{\Delta m^2_{23}}\right| < 0.053 \\ |U_{e3}| < 0.23 \\ 0.30 < \tan^2 \vartheta_{12} < 0.64 \\ 0.45 < \tan^2 \vartheta_{23} < 2.57 \end{cases}$$



ε	1	0.2	0.25	0.25	1	0.25	0.12	0.2	0.45	0.25

[Altarelli,F,Masina 2003]

- some amount of order is clearly preferred over structure-less mass matrices
- an expansion parameter close to $\lambda \approx 0.22$ is needed to account for the smallness of r and U_{e3} .

Lepton Flavour Violation

[Lavignac, Masina, Savoy]

$$BR(\mu \to e\gamma) < 1.1 \times 10^{-11}$$

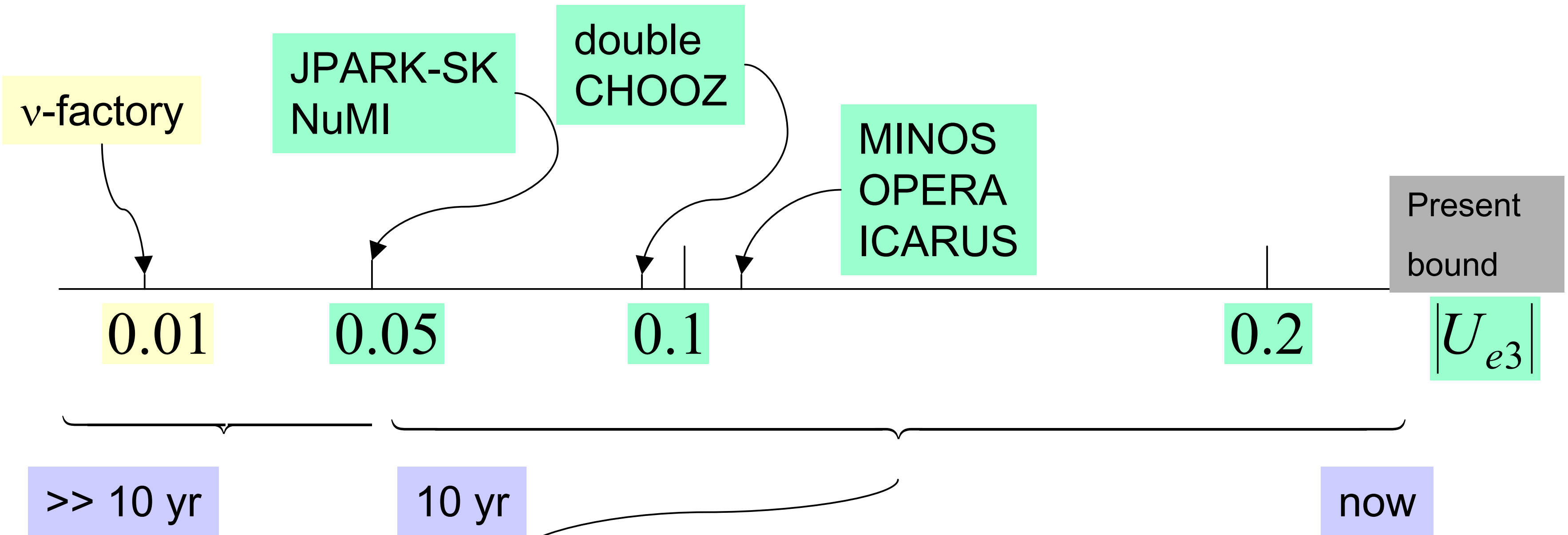
↓

$$y_{j2}^* \, y_{j1} \log\left(\frac{\Lambda}{M_j}\right) \leq O(1) \quad \longrightarrow \quad \left| y_{32}^* \, y_{31} \right| \leq O(\lambda) \approx 0.22$$

in most of the plane $(m_{\tilde{l}}, \mathrm{M}_1)$
up to $(1\,\mathrm{TeV}, 500\,\mathrm{GeV})$
at $\tan\beta = 10$
[mSUGRA, universal b.c. at Λ]

	$\overline{5}$	1	$y_{32} y_{31}$
A	(0,0,0)	(0,0,0)	O(1)
SA	(1,0,0)	(2,1,0)	O(ε)
H	(2,0,0)	(1,-1,0)	O(ε)
IH	(1,-1,-1)	(-1,1,0)	$O(\varepsilon^2)$

- Most of plausible range for Ue3 explored in 10 yr from now



|Ue3| in the range 0.05-0.23 would not favour a particular model and/or a type of spectrum

|Ue3|<0.05 would select a very narrow (not empty) subset of existing models

similar conclusion by:

Barbieri, Hambye, Romanino 0302118
Ibarra, Ross 0307051
Chen, Mahanthappa 0305088
Lebed, Martin 0312219

CONCLUSION

- small ν masses: $\not{L}$ at a scale $\Lambda \leq M_{GUT}$

- absolute spectrum
  though an open experimental question, theoretically welcome
  (but not experimentally unavoidable) properties like GUTs, see-saw and relationship with other fermion masses favour a hierarchical spectrum, with normal hierarchy less constrained than the inverse one by current knowledge of $\tan^2 \vartheta_{12}$ and U_{e3}

- Ue3 not un-measurably small in most of models.

- Even though the hierarchies among observable quantities are less pronounced than in the quark sector, an expansion parameter close to $\lambda \approx 0.22$ largely helps in reproducing the relative smallness of r and U_{e3}