MODELS OF NEUTRINO MASSES AND MIXINGS

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- How many light neutrinos?
- Why $m_v \ll m_f$ charged fermion?
- Which spectrum is favoured by theory?
- Which is the most probable range of $U_{e3}$?
HOW MANY LIGHT $\nu$ ?
3 active neutrinos \[ N = 2.984 \pm 0.009 \] (invisible Z width) 

all experiments but LSND explained by 3 \( \nu_a \) 
\[ \text{LSND} \rightarrow 3 \, \nu_a + [\text{at least}] \, 1 \, \nu_s \]

inclusion of \( \nu_s \) worsens the fits 
- solar: \( \nu_e \rightarrow \sin \vartheta \, \nu_s + \ldots \) \[ \sin^2 \vartheta < 0.1 \] (1\( \sigma \)) [Bahcall&Pena-Garay 2003]
- atm: \( \nu_\mu \rightarrow \nu_\tau \) favoured over \( \nu_\mu \rightarrow \nu_s \)
  - zenith angle dependence of high-energy \( \nu_\mu \) [SK,MACRO] 3\( \sigma \) (no matter effects for \( \nu_s \))
  - no NC interactions for \( \bar{\nu}_s \) [SK]
  - \( \tau \)-like CC events [SK] 2\( \sigma \)
- 2+2 and 3+1 fits have a poor quality [Cirelli,Marandella,Strumia,Vissani 2004]

WMAP + LSS \[ \sum m_\nu < 1.4 \text{ eV (95\% CL)} \] for 3\( \nu_a \) + 1\( \nu_s \) [Hannestad&Raffelt 2004, Crotty, Lesgourgues, Pastor 2004]

from now on: 3 light \( \nu \) assumed

LSND soon checked by MiniBooNE (1st event September 2002) 
no room for LSND with 3 \( \nu_a \) (CPT violation disfavoured by now) [Pakvasa&Valle 0301061]
WHY \( m_v << m_f \) ?

charged fermions
L is exactly conserved

- requires \( \frac{y_{\nu_e}}{y_e} < 10^{-6} \) [smallest ratio is 1/100 for charged fermions in same gen.]

- theoretical prejudices
  - global symmetries are broken by quantum gravity
  - B/L violated in all attempts to unify fundamental interactions
  - B/L broken by anomalies already in the SM

- Interesting attempts in models with extra dimensions

  large ED: standard Yukawa couplings to a singlet fermion \( \nu \) who lives in the bulk

  \[
  \mathcal{L}_{Yuk} = \frac{y_{\nu} v}{\sqrt{2}} \left( \frac{M_D}{M_P} \right) \nu_a (x) \nu_s^{(0)} (x)
  \]

  no experimental hints from oscillations

  \( \nu_s^{(n)} \) effects subdominant, if present

  dimension 5, L-violating operators not sufficiently suppressed by \( M_D \approx 1 \text{ TeV} \)

  alternative models: warped compactifications, L gauged in the bulk,…

  not fully realistic in their minimal realization

  [Dienes, Dudas, Gherghetta, Arkani-Hamed, Dimopoulos, Dvali, March-Russell, Barbieri, Creminelli, Strumia]

L is not conserved

\[ \frac{(H_d)(H_u)}{\Lambda} = \frac{1}{2} \frac{v^2}{\Lambda} \nu \nu + ... \]

leading L-violating operator \( \Delta L = 2 \)

\[ m_\nu = y \frac{v^2}{\Lambda} \quad \leftrightarrow \quad m_e = \frac{y e}{\sqrt{2}} v \]

smallness of \( m_\nu \) due to \( \frac{v}{\Lambda} \ll 1 \)

[\text{GUT scale, see-saw, leptogenesis...}]

- experimental constraints
  - oscillations are insensitive to L violation
  - L violation can be tested in 0νββ decay

| Experiment            | \( T_{1/2} > \) (yr) | \( |m_{ee}| < \) (eV) |
|-----------------------|----------------------|----------------------|
| HM (\(^{76}\text{Ge}\)) | \(1.9 \times 10^{25}\) | \(< 0.35\) |
| IGEX (\(^{76}\text{Ge}\)) | \(1.6 \times 10^{25}\) | \(< (0.33 \div 1.35)\) [90%CL] |
| Cuoricino (\(^{130}\text{Te}\)) | \(5.5 \times 10^{23}\) | \(< (0.37 \div 1.9)\) |

future foreseen sensitivity on \( |m_{ee}| : 10 \text{ meV} \)

- expected range of \( |m_{ee}| \) can be predicted from \( (\Delta m^2_{ij}, \vartheta_{ij}) \)

\[ |m_{ee}| = \cos^2 \vartheta_{13} (\cos^2 \vartheta_{12} m_1 + \sin^2 \vartheta_{12} e^{2i\alpha} m_2) + \sin^2 \vartheta_{13} e^{2i\beta} m_3 \]

[Buchmuller, Di Bari, Plumacher 0406014
Akhmedov, Frigerio, Smirnov 0305322]
Normal Hierarchy

Inverted Hierarchy

Degenerate spectrum

90% CL

h=0.6-2.8 uncertainty in nuclear matrix el.
ABSOLUTE SPECTRUM
Converging evidence that this is only possible below the eV scale:

- $m_{\nu_e} < 2.2 \text{ eV}$ (95% CL) from tritium $\beta$-decay
- $m_\nu < 0.9 \text{ eV}$ (90% CL) from $0\nu\beta\beta$ decay
- $\sum m_\nu < (0.69 \div 1.01) \text{ eV}$ (95% CL) from WMAP +LSS depending on priors

Leptogenesis from out-of-equilibrium CP-violating decay of heavy $\nu^c$ prefers:

- $m_i < (0.12 \div 0.15) \text{ eV}$ [Buchmuller, Di Bari, Plumacher 0401240, Giudice, Notari, Raidal Riotto, Strumia 2003, Hambye, Lin, Notari, Papucci, Strumia 0312203]

Hints for $m_i \approx 0.2 \div 0.6 \text{ eV}$? $0\nu\beta\beta$ of $^{76}\text{Ge}$ HM by Klapdor-K. et al. 2004

Problems with models of degenerate neutrinos:

- See-saw relation untenable: fine-tuning between M and D sectors
- Connection with charged fermion masses (e.g. from GUTs) is lost

$$m_\nu = m_D^T M^{-1} m_D$$
\[ m_i = m \] can be understood in some symmetry limit (e.g. SO(3)) where angles and \((\text{mass})^2\) differences are completely undetermined.

\[
\frac{\Delta m_{sol}^2}{\Delta m_{atm}^2} \ll 1 \quad \mathcal{G}_{13} \ll \mathcal{G}_{23} \approx \frac{\pi}{4}
\]

arise from symmetry breaking terms that require a special misalignment between charged leptons and neutrinos.

Specific realizations already ruled out or strongly disfavoured, e.g. Flavour Democracy [Fritzsch, Xing]

\[ \sin^2 2\mathcal{G}_{23} = \frac{8}{9} \approx 0.89 \leftrightarrow \sin^2 2\mathcal{G}_{23} > 0.9(0.84) \text{ at } 2\sigma(3\sigma) \]

\[ \text{to avoid a full-fledged theory of breaking terms, we can assume anarchy in the neutrino sector} \]

\[ m_\nu = m \begin{pmatrix} O(1) & O(1) & O(1) \\ O(1) & O(1) & O(1) \\ O(1) & O(1) & O(1) \end{pmatrix} \]

\[
\frac{\Delta m_{sol}^2}{\Delta m_{atm}^2} \ll 1 \\
\mathcal{G}_{13} \ll \mathcal{G}_{23} \approx \frac{\pi}{4}
\]

can be produced in part by the see-saw.

accidental

\[ \mathcal{G}_{23} \approx \frac{\pi}{4} \]

fortuitous

no general consensus on how to realize this
Inverse Hierarchy

- the best we have, at present, is
  \[ m_\nu = m \begin{pmatrix} 0 & a & b \\ a & 0 & 0 \\ b & 0 & 0 \end{pmatrix} + \ldots \]
  - corrections \( \ll |a|, |b| \)
  - leading order determined by \( L_e - L_\mu - L_\tau \)
    either with or without see-saw
  - compatible with GUT

- leading order
  \[ |m_1| = |m_2| = \sqrt{|a|^2 + |b|^2} \quad m_3 = 0 \]
  \[ \theta_{13} = 0 \]
  \[ \tan \theta_{23} = \frac{b}{a} \]
  \[ \theta_{12} = 45^\circ \]
  it’s out by \( 6\sigma \)

- turning SB terms on
  \[ 1 - \tan^2 \theta_{12} \approx O \left( \frac{\Delta m^2_{\text{sol}}}{\Delta m^2_{\text{atm}}} \right) \]
  \[ 0.36 \div 0.70 \ (3\sigma) \]
  off by a factor \( > 10 \)

  \[ \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \] [exception: normal hierarchy]

- difficulty common to many models
  - Flavour Democracy (deg. spectrum)
  - pseudoDirac structure in 12 sector
substantial contribution to $g_{12}$ from charged leptons needed

$U_{PMNS} = U^e_e U^e_\nu$

standard parametrization

$U^e_e = U^e_{23} \cdot U^e_{13} \cdot U^e_{12}$

$U^e_{23}$ can be absorbed in $U^e_\nu$ to give:

$g_{23} \approx 45^\circ$

by expanding to 1st order in 

$|u| \equiv \sin g^e_{12}, |v| \equiv \sin g^e_{13} \ll 1$

$1 - \tan^2 g_{12} = 2\sqrt{2} \text{Re}(u + v)$

$|U^e_{e3}| = \frac{1}{\sqrt{2}} |u - v|$

[Frampton, Petcov, Rodejohann 0401206
Altarelli, F, Masina 0402155
Romanino 0402508]

if, by analogy with the quark sector:

$|v| \ll |u| \approx g_C \approx 0.22$

$1 - \tan^2 g_{12} \approx 2\sqrt{2} g_C \approx 0.6$

[right amount] [Raidal 0404046
Minakata, Smirnov 0405088]

$|U^e_{e3}| = \frac{(1 - \tan^2 g_{12})}{4 \cos \delta_{CP}}$
Normal Hierarchy

- Several viable mechanisms for $\theta_{23}$ large
  - $\theta^e_{23}$ and $\theta^\nu_{23}$ small
    but $\theta_{23} \equiv \theta^\nu_{23} - \theta^e_{23} \approx O(1)$
  - see-saw dominance of light $\nu^e$ equally coupled to $\nu^\mu$ and $\nu^\tau$
    see-saw suppression as in \cite{1}
  - lopsided structure of $m_e$ or $m^\nu_D$:
    \[
    \left(\begin{array}{ccc}
    0 & 0 & 0 \\
    0 & 0 & 0 \\
    0 & a & b
    \end{array}\right)
    \]
    \text{large RH quark mixing in SU(5)}

- typical texture as e.g. from U(1) flavour symmetry
  \[
  m^\nu = m\begin{pmatrix}
  \epsilon^2 & \epsilon & \epsilon \\
  \epsilon & 1 & 1 \\
  \epsilon & 1 & 1
  \end{pmatrix}
  \]
  $\theta_{13} \approx \epsilon$, vanishes in the symmetry limit
  $\theta_{12}$ large $\leftrightarrow$ $\det[23] \approx \epsilon$
  - accidentally (semi-anarchy=SA)
  - by a see-saw suppression as in \cite{1} and \cite{2}
$U_{e3}$
No reason why $U_{e3}$ should be tiny in realistic models

- inverse hierarchy from $L_e - L_\mu - L_\tau$
  
  \[ |U_{e3}| \geq \frac{(1 - \tan^2 \theta_{12})}{4} \approx (0.09 \div 0.18) [3\sigma] \]

- normal hierarchy [1st order in \( \sin \theta_{12} \gg \sin \theta_{13} \)]

\[ U_{e3} \approx \sin \theta_{13} - \sin \theta_{23} \cdot \sin \theta_{12} \]

\[ \approx \sin \theta_{12} \sqrt{\frac{\Delta m_{sol}^2}{\Delta m_{atm}^2}} \]

\[ \approx -\sin \theta_{23} \sqrt{\frac{m_e}{m_\mu}} \]

[De Gouvea 041220]

\[ \approx (0.03 \div 0.3) \]

[estimates by allowing 3\sigma exp. and ( factors ½ and 2) th. uncertainties]

- e-dominated
  \[ \approx (0.02 \div 0.1) \]

- degenerate spectrum (examples)

  - \( \Delta m_{sol}^2 \) and $U_{e3}$ determined by RGE
  - Flavour Democracy
  - Anarchy

\[ \approx \sqrt{\frac{\Delta m_{sol}^2}{2 \Delta m_{atm}^2}} \approx (0.04 \div 0.4) \]

\[ \approx \sqrt{\frac{2m_e / 3m_\mu}{}} \approx (0.03 \div 0.1) \]

\[ \approx O(1) \]
$U_{e3}$ in models with U(1) flavour symmetry

$0.018 < r \equiv \frac{\Delta m^2_{12}}{\Delta m^2_{23}} < 0.053$

$|U_{e3}| < 0.23$

$0.30 < \tan^2 \theta_{12} < 0.64$

$0.45 < \tan^2 \theta_{23} < 2.57$

$\varepsilon$ optimised case by case to fit

$m_\nu = m \begin{pmatrix} \varepsilon^2 & \varepsilon & \varepsilon \\ \varepsilon & 1 & 1 \\ \varepsilon & 1 & 1 \end{pmatrix}$

anarchy=A

semianarchy=SA

normal hierarchy=H

inverse hierarchy=IH

matrix elements up to unknown O(1) coeff.
Outcome of the optimisation procedure

by generating random, O(1), complex coefficients and by counting the success rate in reproducing

\[
0.018 < r \equiv \frac{|\Delta m_{21}^2|}{|\Delta m_{23}^2|} < 0.053
\]

\[
|U_{e3}| < 0.23
\]

\[
0.30 < \tan^2 \vartheta_{12} < 0.64
\]

\[
0.45 < \tan^2 \vartheta_{23} < 2.57
\]

- some amount of order is clearly preferred over structure-less mass matrices

- an expansion parameter close to \( r \) and \( \lambda \sim 0.22 \) is needed to account for the smallness of \( U_{e3} \).
Lepton Flavour Violation

\[ BR(\mu \rightarrow e\gamma) < 1.1 \times 10^{-11} \]

\[ y^*_j y_j \log \left( \frac{\Lambda}{M_j} \right) \leq O(1) \]

in most of the plane \((m_\tilde{\ell}, M_1)\)
up to \((1 \text{ TeV}, 500 \text{ GeV})\)
at \(\tan \beta = 10\)

[mSUGRA, universal b.c. at \(\Lambda\)]

\[ |y^*_3 y_{31}| \leq O(\lambda) \approx 0.22 \]

\[
\begin{array}{c|c|c}
\bar{5} & 1 & y_{32} y_{31} \\
A & (0,0,0) & (0,0,0) & O(1) \\
SA & (1,0,0) & (2,1,0) & O(\epsilon) \\
H & (2,0,0) & (1,-1,0) & O(\epsilon) \\
IH & (1,-1,-1) & (-1,1,0) & O(\epsilon^2) \\
\end{array}
\]
Most of plausible range for $U_{e3}$ explored in 10 yr from now

$|U_{e3}|$ in the range 0.05-0.23 would not favour a particular model and/or a type of spectrum

$|U_{e3}|<0.05$ would select a very narrow (not empty) subset of existing models

similar conclusion by:
Barbieri, Hambye, Romanino 0302118
Ibarra, Ross 0307051
Chen, Mahanthappa 0305088
Lebed, Martin 0312219
CONCLUSION

- small $\nu$ masses: $\mathbf{L}$ at a scale $\Lambda \leq M_{GUT}$

- absolute spectrum
  though an open experimental question, theoretically welcome
  (but not experimentally unavoidable) properties like GUTs, see-saw and
  relationship with other fermion masses favour a hierarchical spectrum,
  with normal hierarchy less constrained than the inverse one
  by current knowledge of $\tan^2 \theta_{12}$ and $U_{e3}$

- $U_{e3}$ not un-measurably small in most of models.

- Even though the hierarchies among observable quantities
  are less pronounced than in the quark sector,
  an expansion parameter close to $\lambda \approx 0.22$
  largely helps in reproducing the relative smallness of $r^*$ and $U_{e3}$